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Dynamics of toroidal bodies in a fluid

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Mathematical model

Consider the motion of heavy homogeneous toroidal bodies of circular cross section in a fluid. To describe the motion we introduce two coordinate systems: a fixed one, $Oxyz$, and the moving one, $O_1e_1e_2e_3$,(see Fig. 1).

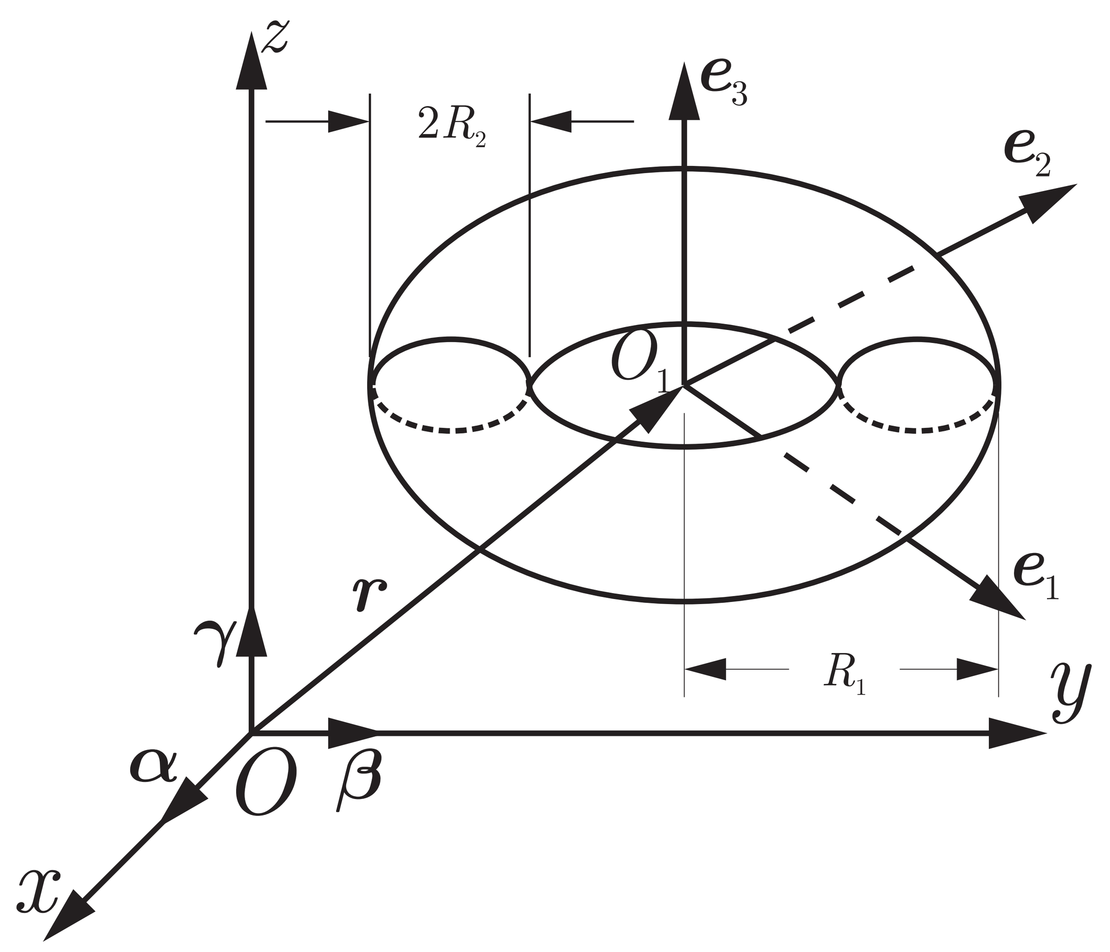

Figure 1: Coordinate systems

Equations of motion

The motion is governed by the following equations:

$$\begin{gathered}\mathbf{C}\dot{\boldsymbol{v}} = \mathbf{C}\boldsymbol{v}\times\boldsymbol{\omega} - \mu\boldsymbol{\gamma} + \boldsymbol{F},\\ \mathbf{I}\dot{\boldsymbol{\omega}} = \mathbf{I}\boldsymbol{\omega}\times\boldsymbol{\omega} + \mathbf{C}\boldsymbol{v}\times\boldsymbol{v} + \boldsymbol{G},\\ \dot{\boldsymbol{r}} = \mathbf{Q}^T\boldsymbol{v},\quad \dot{\boldsymbol{\alpha}} = \boldsymbol{\alpha}\times\boldsymbol{\omega},\quad \dot{\boldsymbol{\beta}} = \boldsymbol{\beta}\times\boldsymbol{\omega},\quad \dot{\boldsymbol{\gamma}} = \boldsymbol{\gamma}\times\boldsymbol{\omega},\\ \mathbf{Q} = \begin{pmatrix}\alpha_1 & \beta_1 & \gamma_1\\ \alpha_2 & \beta_2 & \gamma_2\\ \alpha_3 & \beta_3 & \gamma_3\end{pmatrix},\end{gathered} \tag{1}$$

where $\boldsymbol{v}$ is the linear velocity of the body, $\boldsymbol{\omega}$ is the angular velocity of the body, $\boldsymbol{F}$, $\boldsymbol{G}$ are resistance force and torque, $\mathbf{C}$ is the matrix taking into account the mass of the body and the added masses, $\mathbf{I}$ is a matrix taking into account the tensor of inertia of the body and the tensor of added moments of inertia, $\boldsymbol{a}$ and $\boldsymbol{b}$ are the constant vector taking into account circular motion of a fluid through a hole of the body, $\boldsymbol{r}$ is the radius-vector of the center of mass of the body, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ are the unit vectors directed along axes of the fixed coordinate system. Usually forces is represented as follows:

$$\begin{gathered}F_i = -f_i v_i|v_i| + (\boldsymbol{a}\times\boldsymbol{\omega})_i,\\ G_i = -g_i\omega_i|\omega_i| + (\boldsymbol{b}\times\boldsymbol{\omega})_i + (\boldsymbol{b}\times\boldsymbol{v})_i,\end{gathered}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the vectors (usually are constant) taking into account circular motion of a fluid through a hole of the body, and f_i, g_i are the drag coefficients. But in real physical processes these parameters may depend on orientation of body and its linear and angular velocities.

Experiment Processing

We have performed number of the Motion Capture experiments with natural model of torus. Photo collage of the experiment is shown in Figure 2.
The results of experiments were smoothed and differentiated with Savitzky-Golay method. Were computed trajectories of the center of mass, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ vectors, linear and angular velocities and there derivatives. Using this data the forces and torques have been computed, also we create database for learning artificial neural network. We have tried approximate forces and torques with dependence on linear velocity $\boldsymbol{v}$, angular velocity $\boldsymbol{\omega}$ and torus orientation $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$. The 80% of data we using for learning artificial neural network, and the 20% of data using for testing. A quality diagram shown in Figure 3. The forces and torques from experiment and from artificial neural network computation shown in Figure 4.

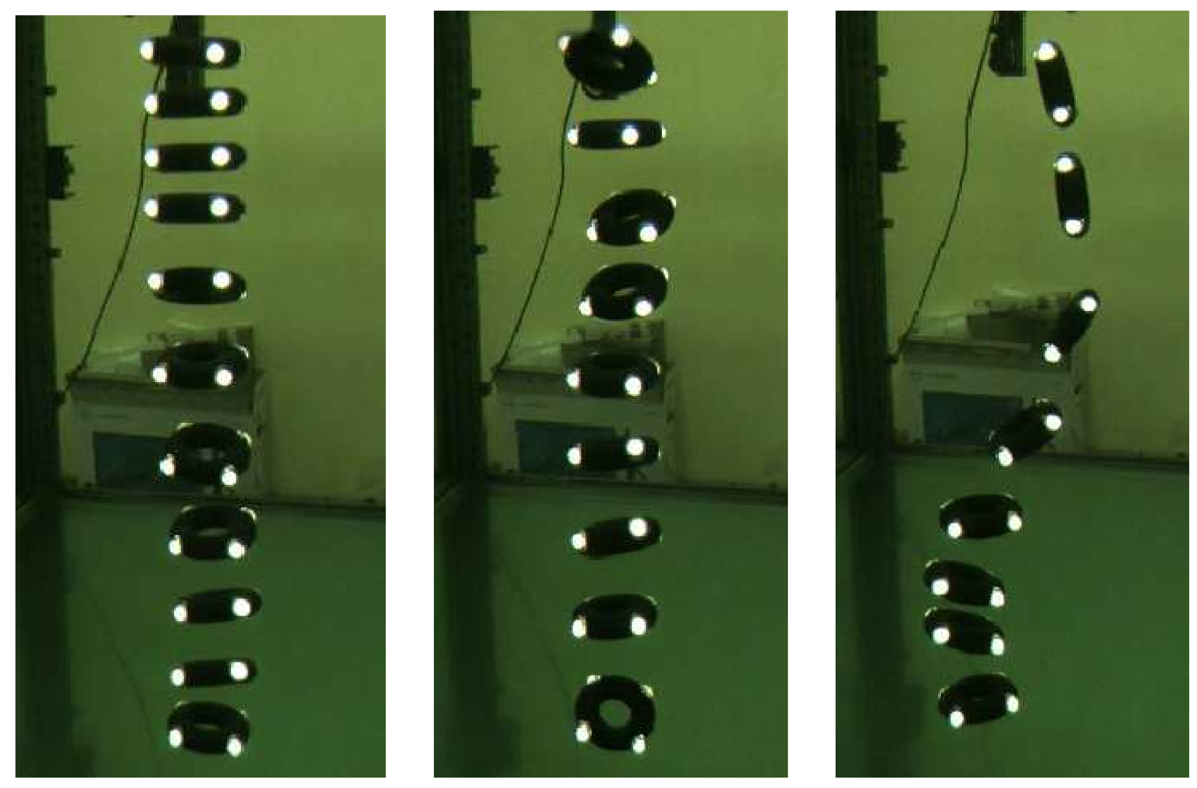

Figure 2: Photo collages of the falling torus

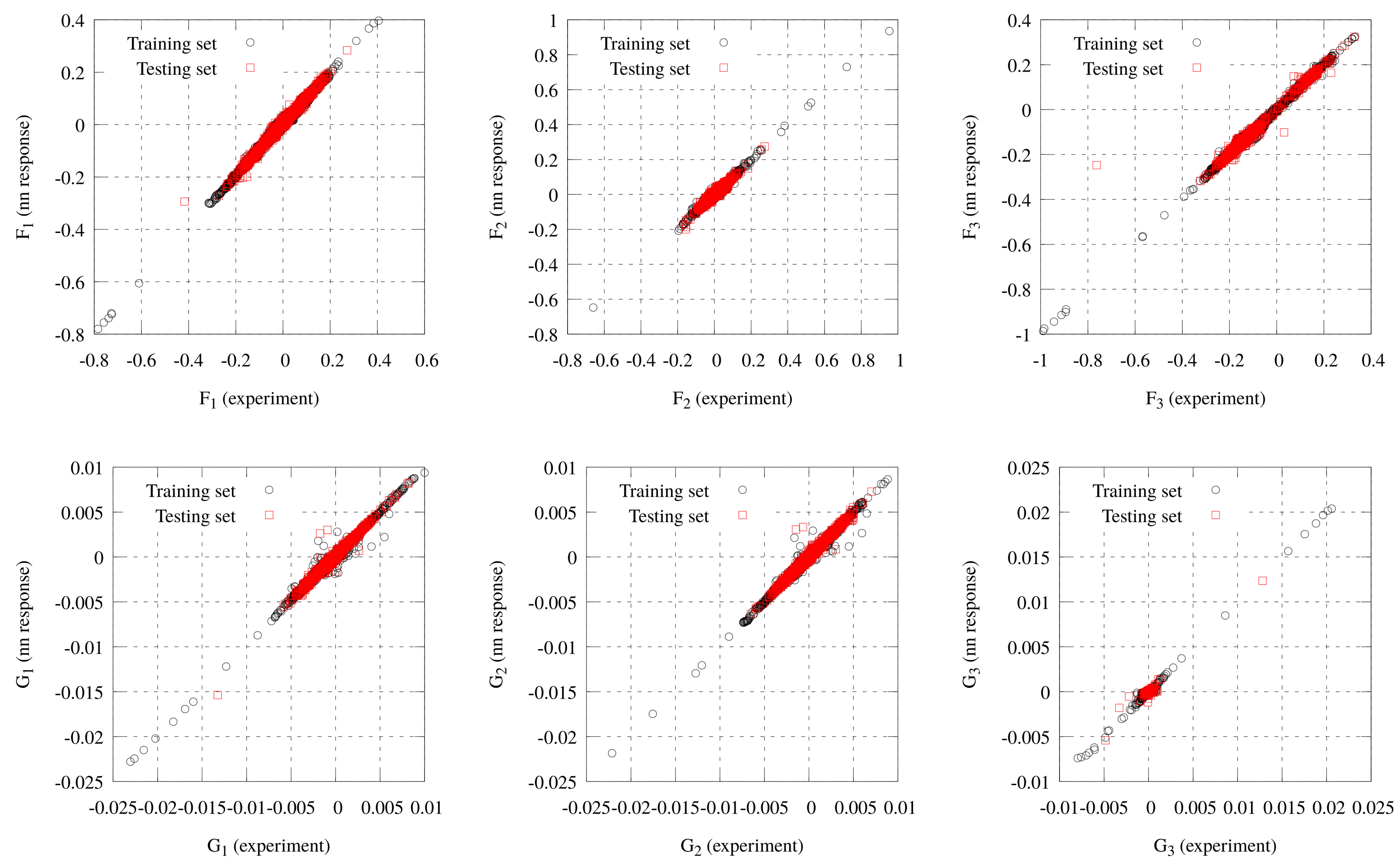

Figure 3: The results of the neural network learning

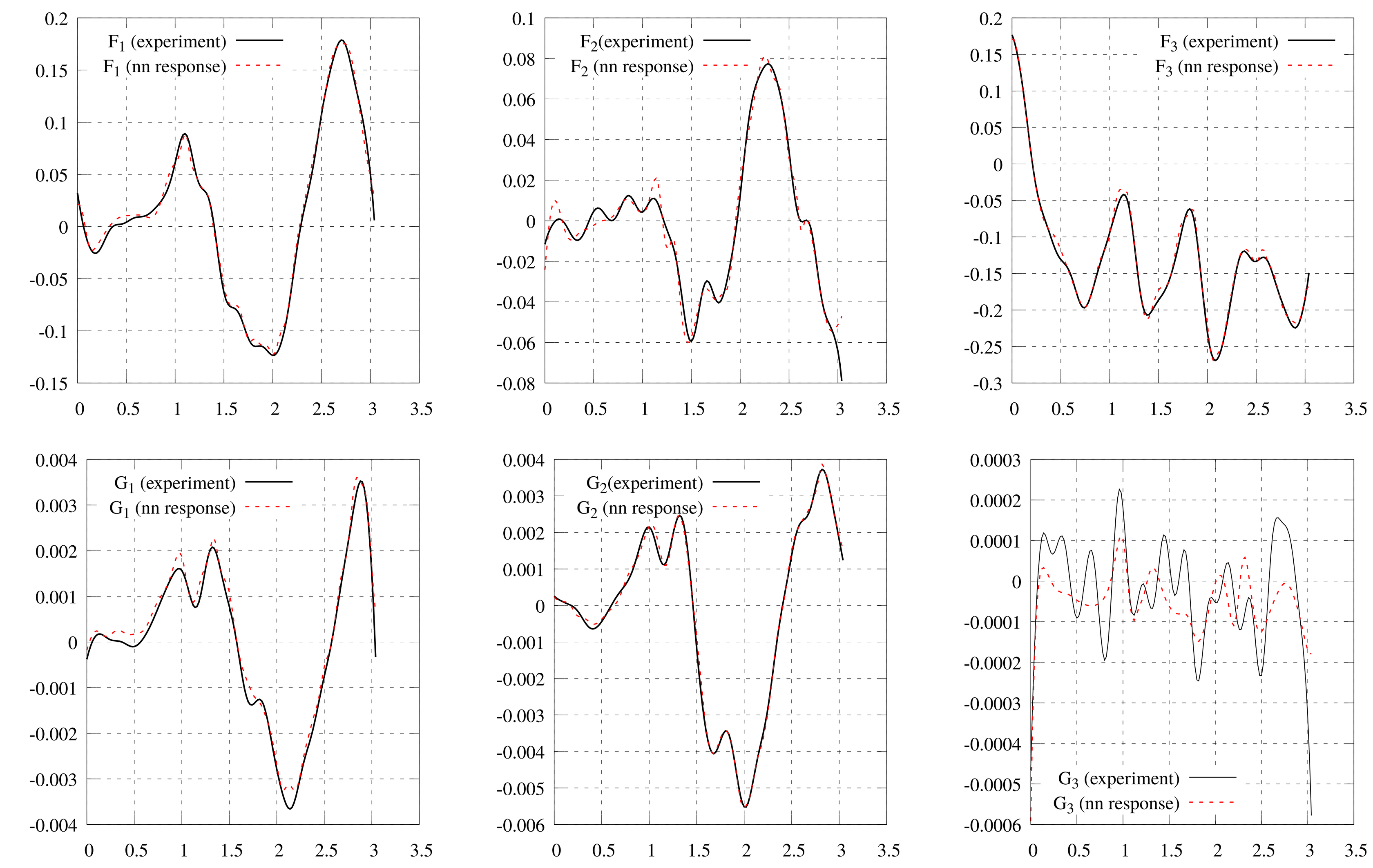

Figure 4: Comparison of forces and torques of the experiment with the results of calculations

The approximation of forces and torques using artificial neural network could be use in numerical solution of equations of motion. However correct result may be obtained on short time interval. We suppose that artificial neural network could be use in studying of dependencies of forces and torques on linear and angular velocities and body orientation.

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