

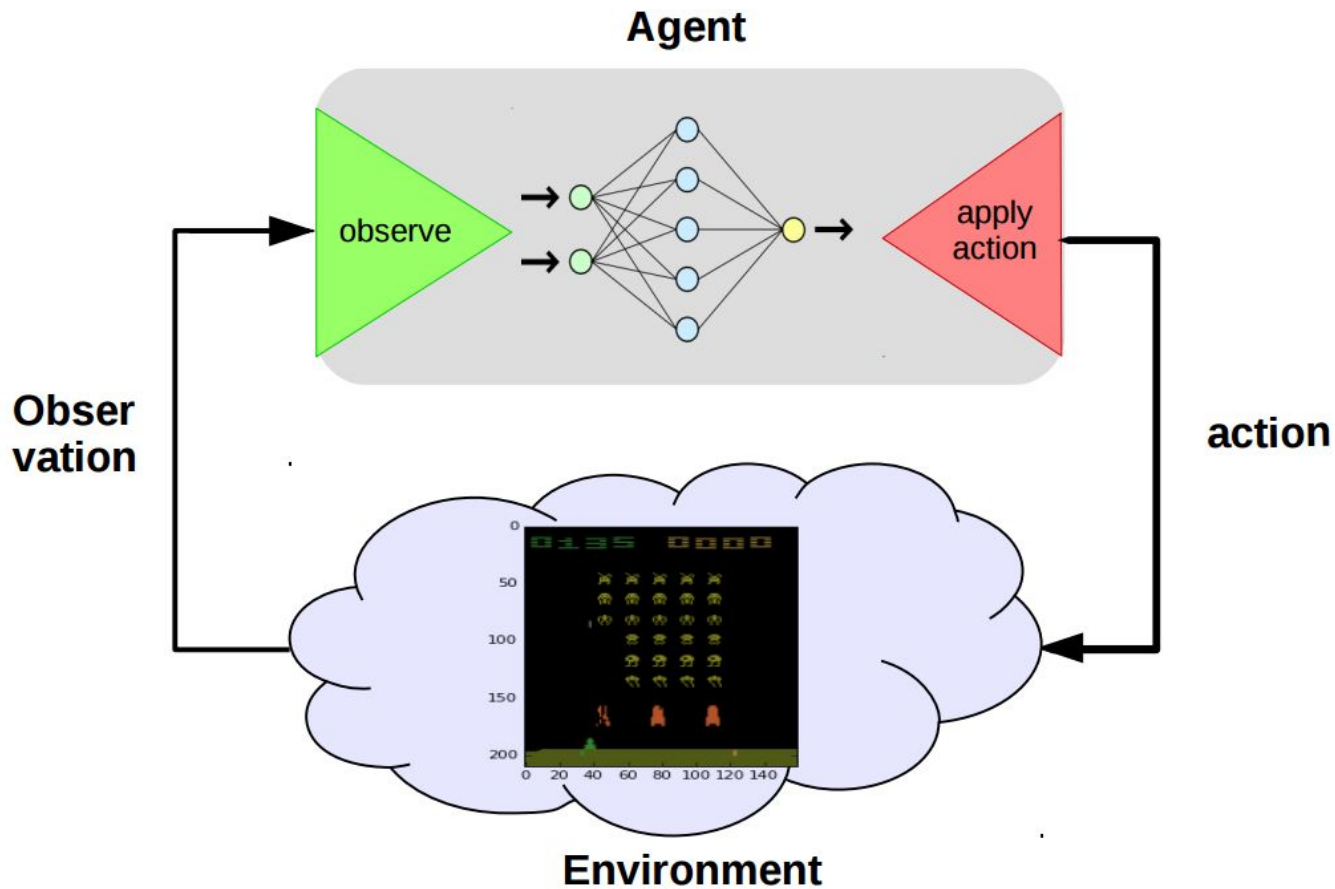
# Lecture 04: Approximate Q-learning, DQN

**Radoslav Neychev**

# References

These slides are almost the exact copy of Practical RL course week 4 slides.  
Special thanks to YSDA team for making them publicly available.

Original slides link: [week04\\_approx\\_rl](#)



$$G_t = \sum_{t'=t}^T \gamma^{(t'-t)} r_{t'}$$

$$Q^\pi(s, a) = E_\pi[G_t | s_t = s, a_t = a]$$

$$V^\pi(s) = E_\pi[G_t | s_t = s]$$

## Recurrent relations

$$Q^\pi(s, a) = E_{s_{t+1}}[r_t + \gamma V^\pi(s_{t+1})]$$

$$Q^\pi(s, a) = E_{s_{t+1}, a_{t+1} \sim \pi}[r_t + \gamma Q^\pi(s_{t+1}, a_{t+1})]$$

For all  $\pi, s, a$ :  $Q^{\pi^*}(s, a) \geq Q^{\pi}(s, a)$

$$\pi^*(s) = \operatorname{argmax}_a Q^{\pi^*}(s, a)$$

## **Bellman optimality equation**

$$Q^*(s_t, a) = E_{s_{t+1}}[r_t + \max_{a'} Q^*(s_{t+1}, a')]$$

## Training step

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha(r_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t))$$

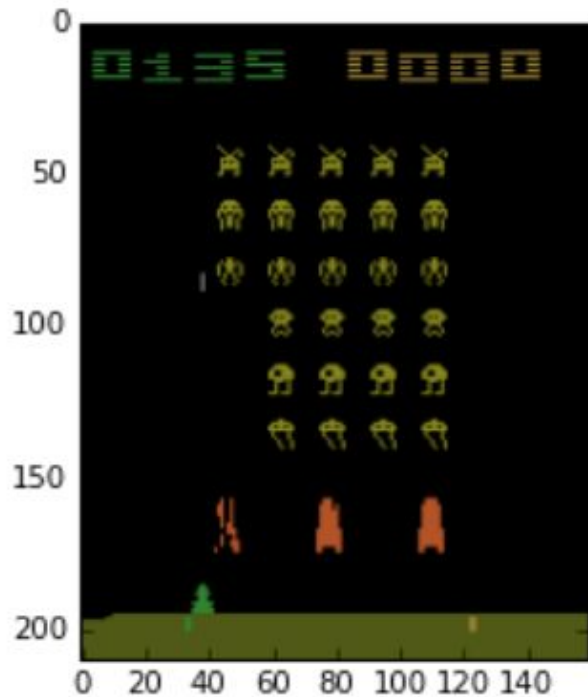
## Q-learning as MSE minimization

$$L = (r_t + \gamma \max_{a'} \boxed{Q(s_{t+1}, a')} - Q(s_t, a_t))^2$$

Const

$$\nabla L = 2 \cdot (r_t + \gamma \max_{a'} \boxed{Q(s_{t+1}, a')} - Q(s_t, a_t))$$

**What's wrong here?**



How many states are there?  
approximately

$$|S| = 2^{210 \cdot 160 \cdot 8 \cdot 3}$$

# Q-learning: make it continuous

For now states and actions are discrete. What if states are **not**?

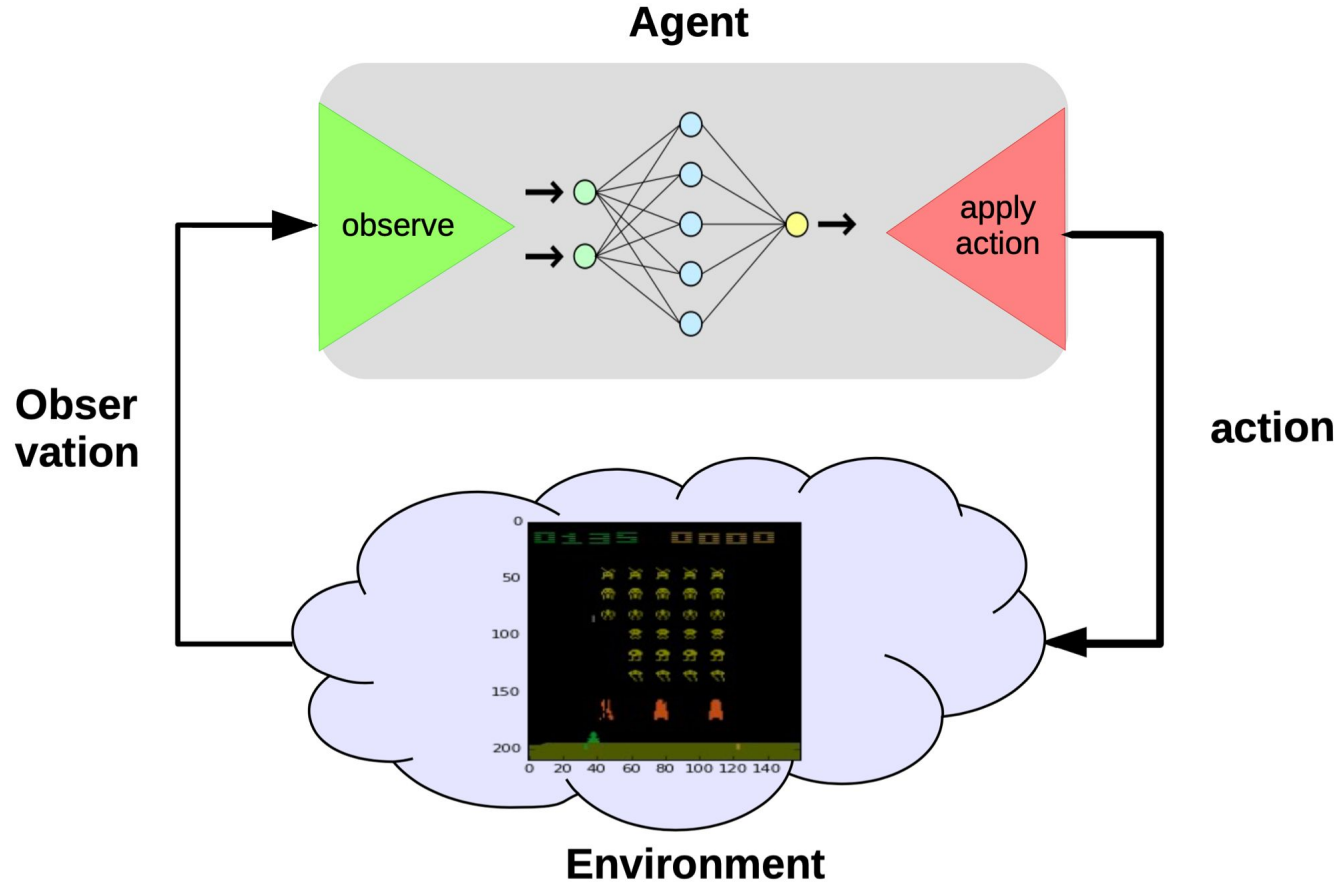
Minimize loss given  $\langle \mathbf{s}, \mathbf{a}, \mathbf{r}, \mathbf{s}' \rangle$

$$L = [Q(s_t, a_t) - Q^{true}(s_t, a_t)]^2$$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]^2$$

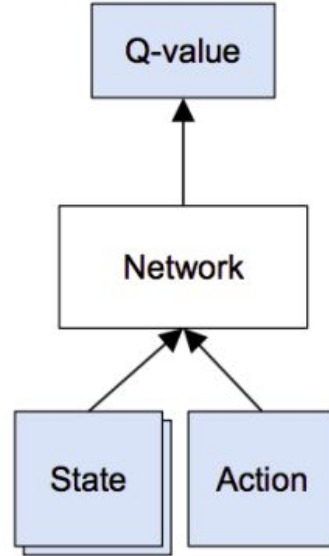


# Approximate Q-learning



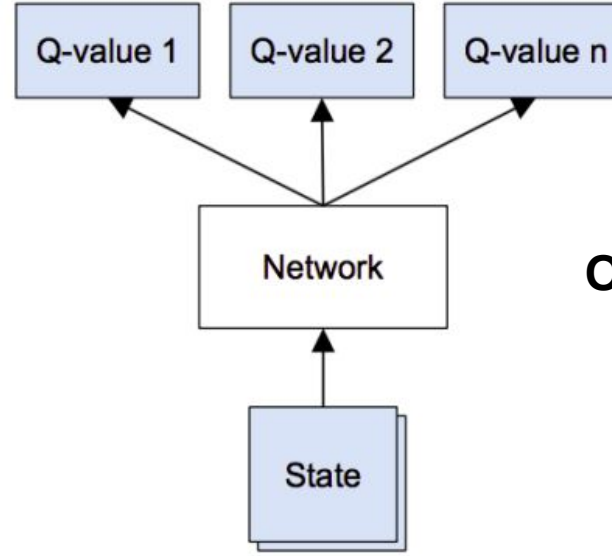
# Possible architectures

**Continuous  
control or large  
number of  
actions**



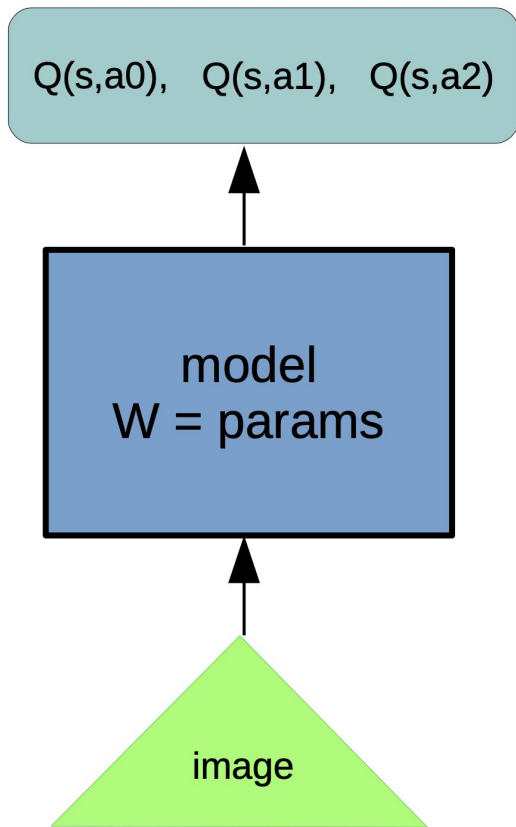
Given **(s,a)**  
Predict  $Q(s,a)$

**One pass for all  
actions**



Given **s** predict all q-values  
 $Q(s,a_0)$ ,  $Q(s,a_1)$ ,  $Q(s,a_2)$

# Approximate Q-learning



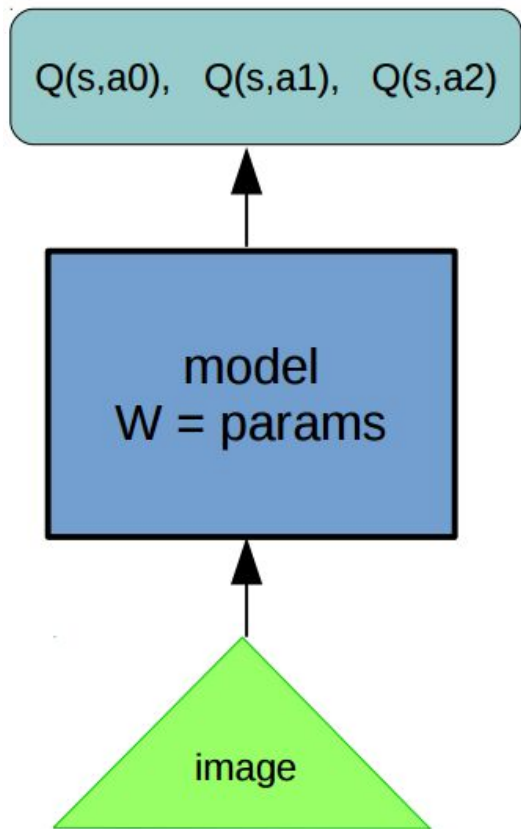
$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} \hat{Q}(s_{t+1}, a')$$

$$L = \left( Q(s_t, a_t) - \left[ r + \gamma \cdot \max_{a'} Q(s_{t+1}, a') \right] \right)^2$$

Consider const

$$w_{t+1} = w_t - \alpha \cdot \frac{\delta L}{\delta w}$$

# Approximate Q-learning



**Objective:**

$$L = \left( Q(s_t, a_t) - \underbrace{\hat{Q}(s_t, a_t)}_{\text{consider const}} \right)^2$$

**Q-learning:**

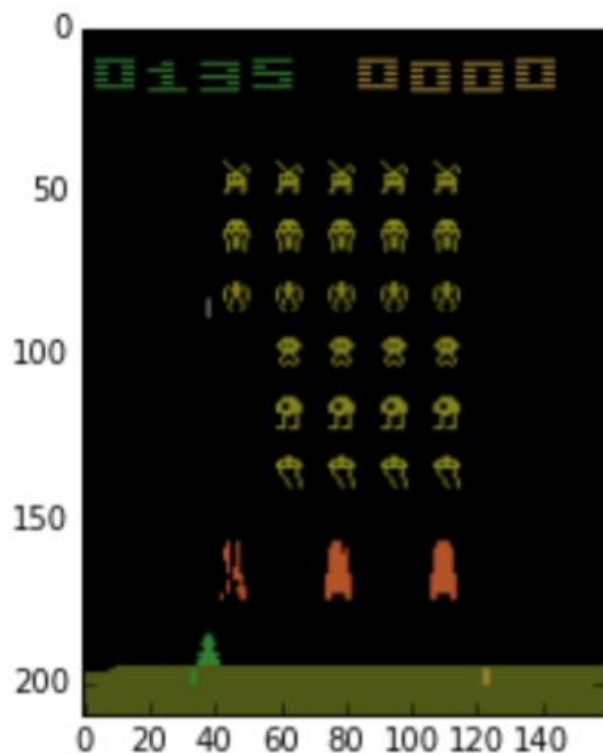
$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

**SARSA:**

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot Q(s_{t+1}, a_{t+1})$$

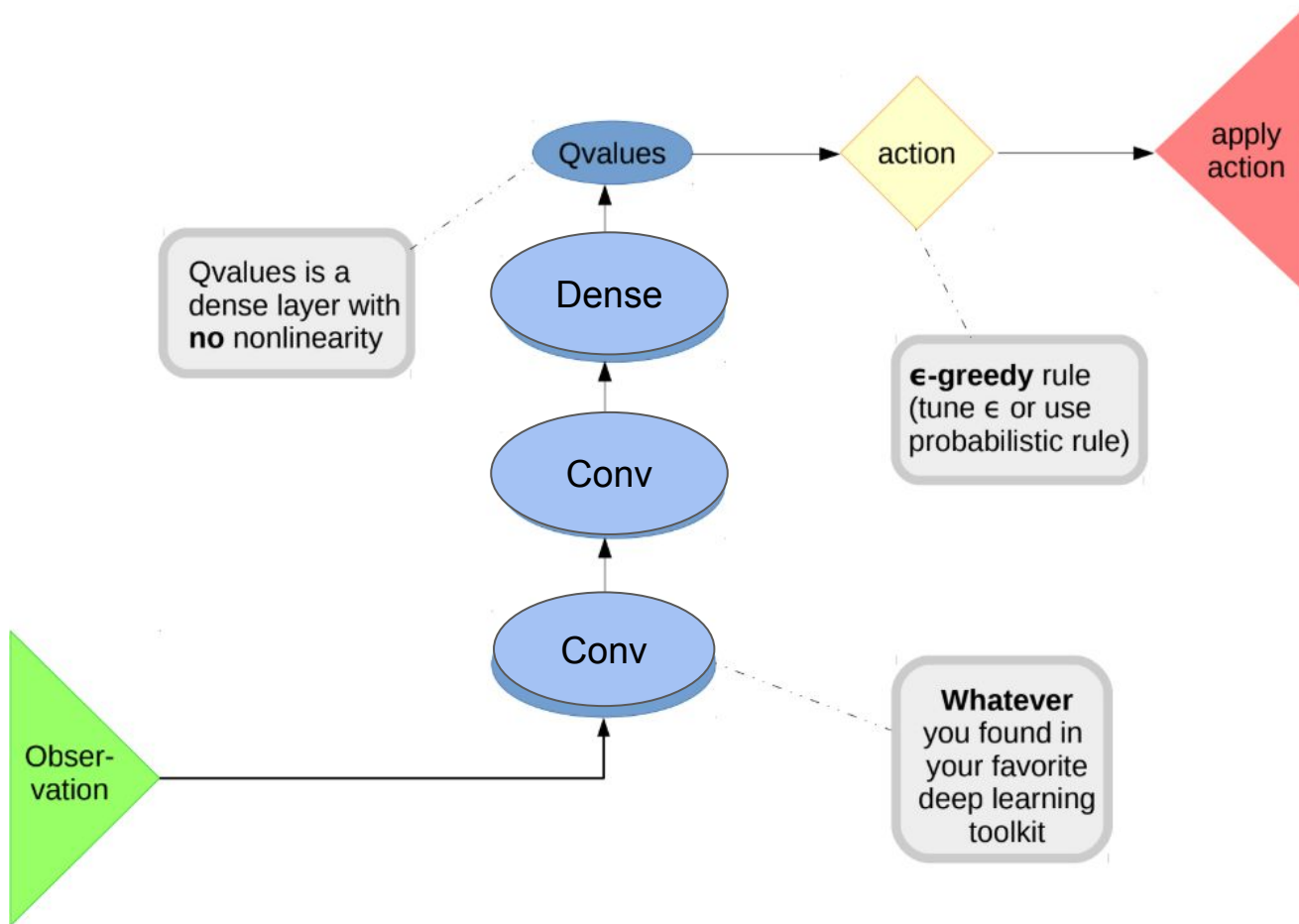
**Expected Value SARSA:**

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot E_{a' \sim \pi(a|s)} Q(s_{t+1}, a')$$

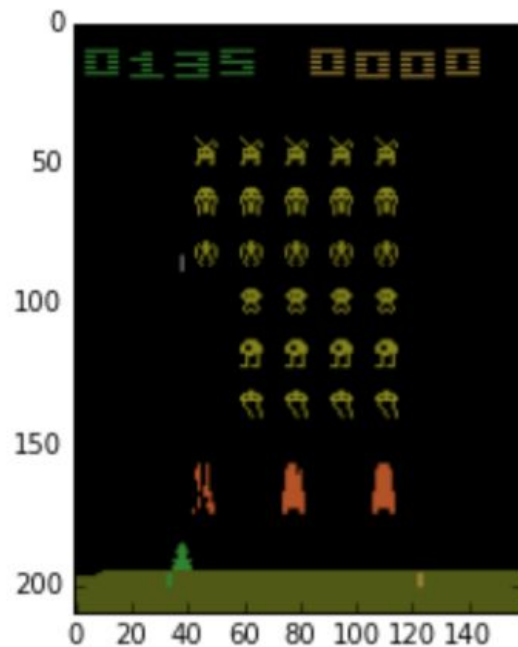


What kind of network digests images well?

# Basic deep Q-learning



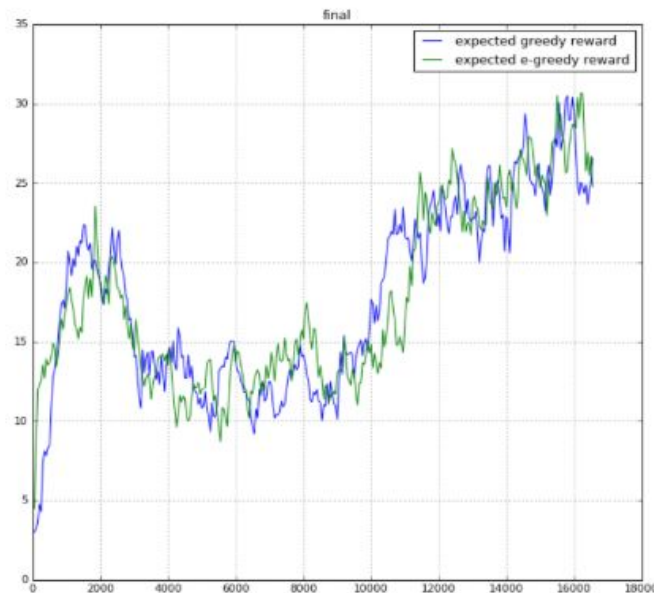




How bad it is if agent spends  
next 1000 ticks under the left rock?  
(while training)



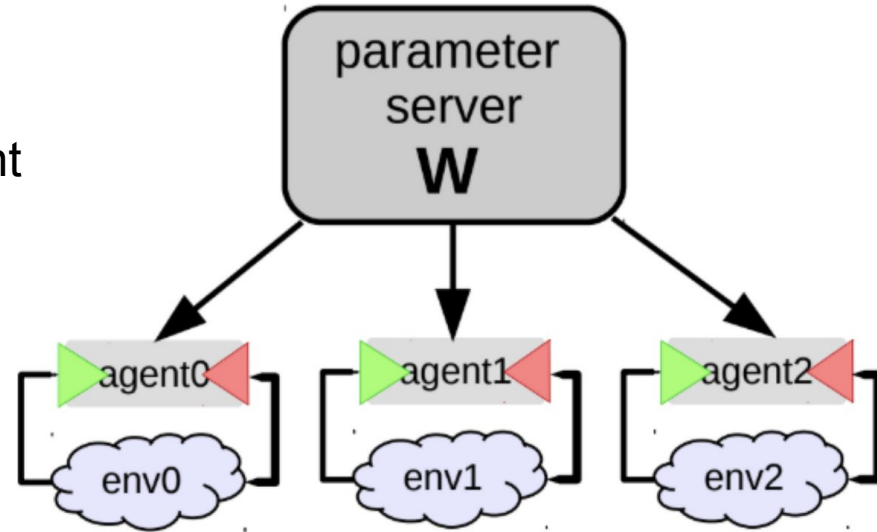
- Training samples are **not** “i.i.d”,
- Model forgets parts of environment it hasn't visited for some time
- Drops on learning curve
- **Any ideas?**



# Multiple agents trick

**Idea:** Throw in several agents with shared  $\mathbf{W}$ .

- Chances are, they will be exploring different parts of the environment
- More stable training
- Requires a lot of interaction



*Question: your agent is a real robot car.  
Will there be any problems?*



## Idea:

Store several past interactions

$\langle s, a, r, s' \rangle$

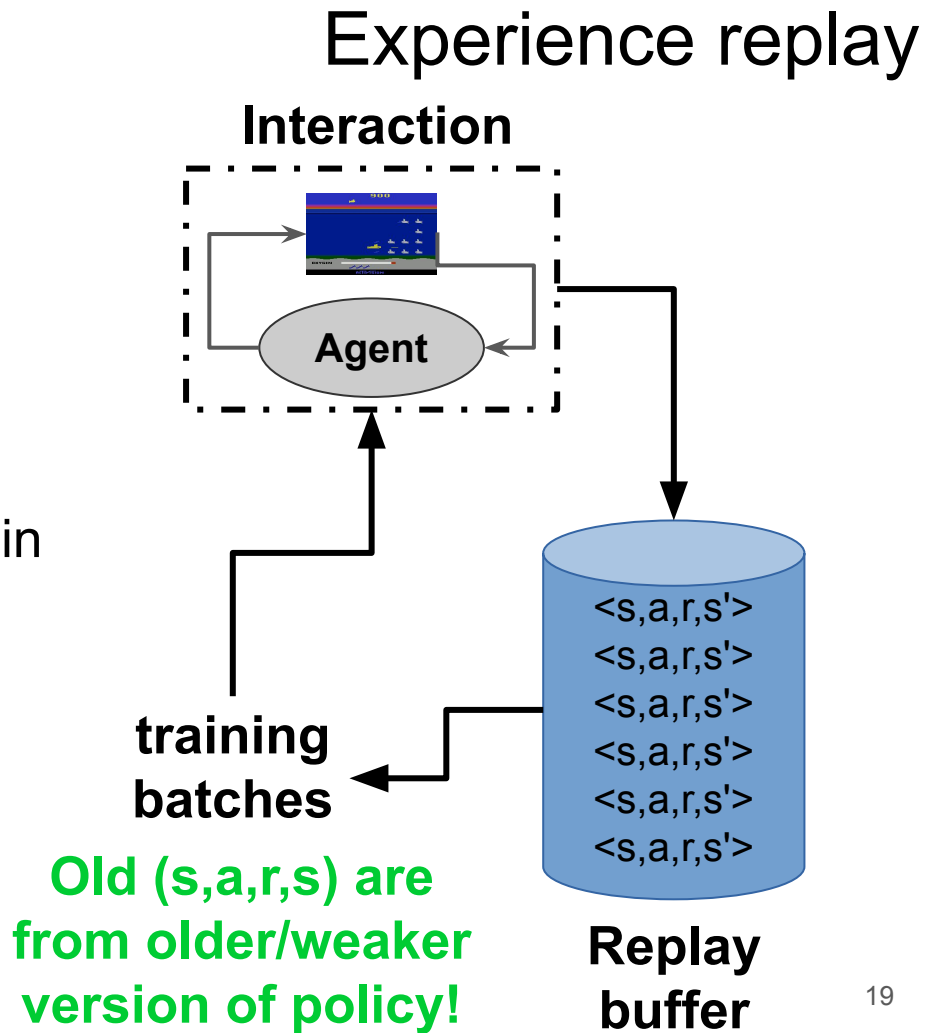
Train on random subsamples

## Training curriculum:

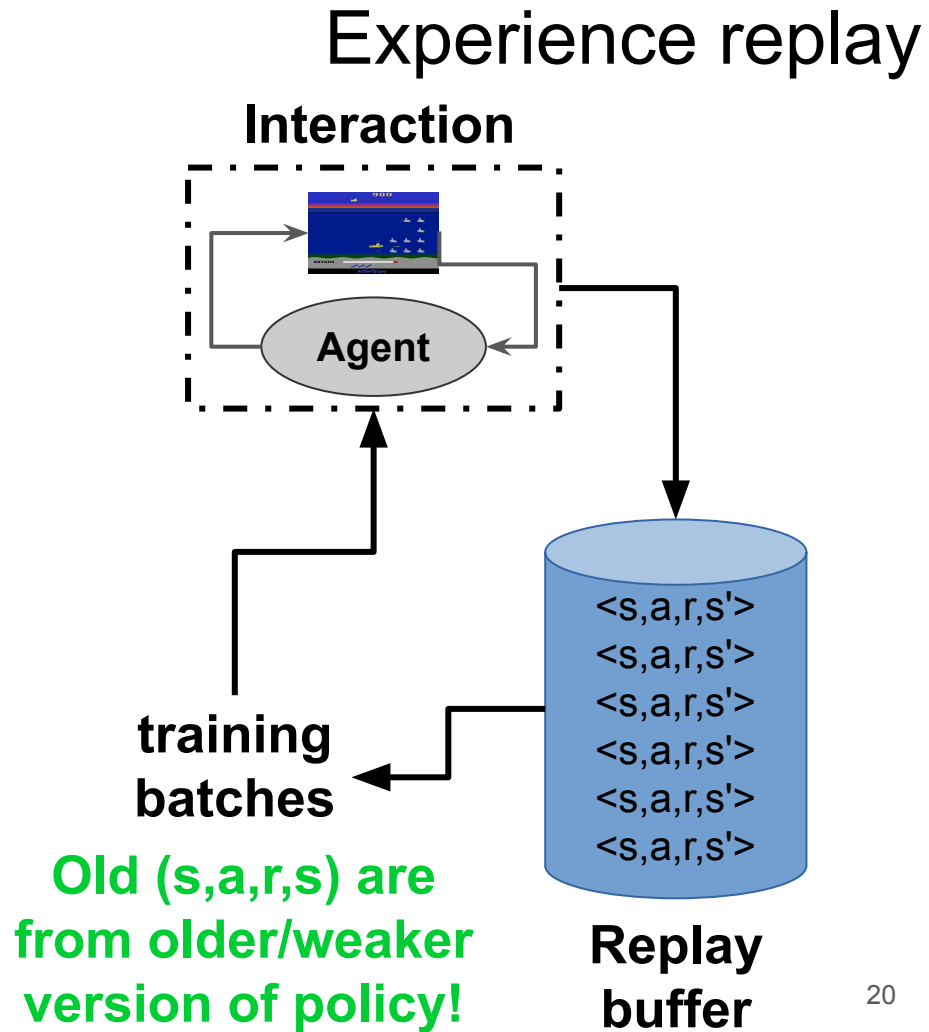
- Play 1 step and record it
- Pick N random transitions to train

## Profit:

you don't need to revisit same  $(s, a)$  many times to learn it.



- Atari DQN  $> 10^5$  interactions
- Closer to i.i.d.  
pool contains several sessions
- Older interactions were  
obtained under weaker policy



# Experience replay

- You approximate  $Q(s, a)$  with a neural network
- You use **experience replay** when training

**Question:** which of those algorithms will fail?

- Q-learning
- SARSA
- CEM
- Expected Value SARSA

# Experience replay

- You approximate  $Q(s, a)$  with a neural network
- You use **experience replay** when training

Agent trains off-policy on an older version of himself

**Question:** which of those algorithms will fail?

Off-policy methods work, On-policy methods are super slow (fail)

- Q-learning
- **SARSA**
- **CEM**
- Expected Value SARSA

When training with on-policy methods,

- use no (or small) experience replay
- compensate with parallel game sessions



**Left or right?**

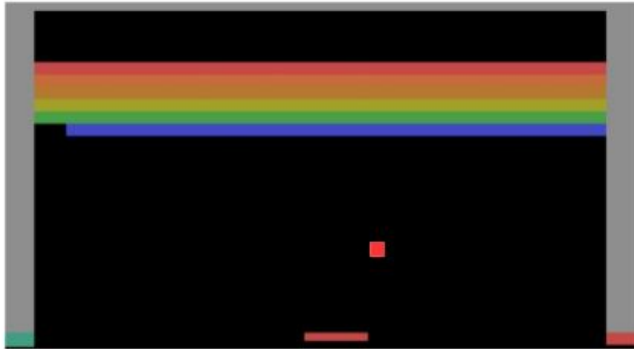


Idea:

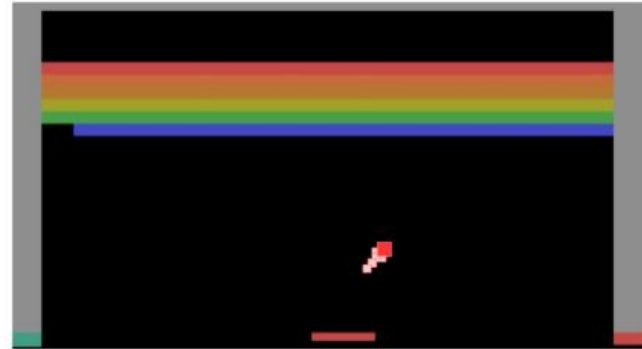
$$s_t \neq o(s_t)$$

$$s_t \approx (o(s_{t-n}), a_{t-n}, \dots, o(s_{t-1}), a_{t-1}, o(s_t))$$

e.g. ball movement in breakout



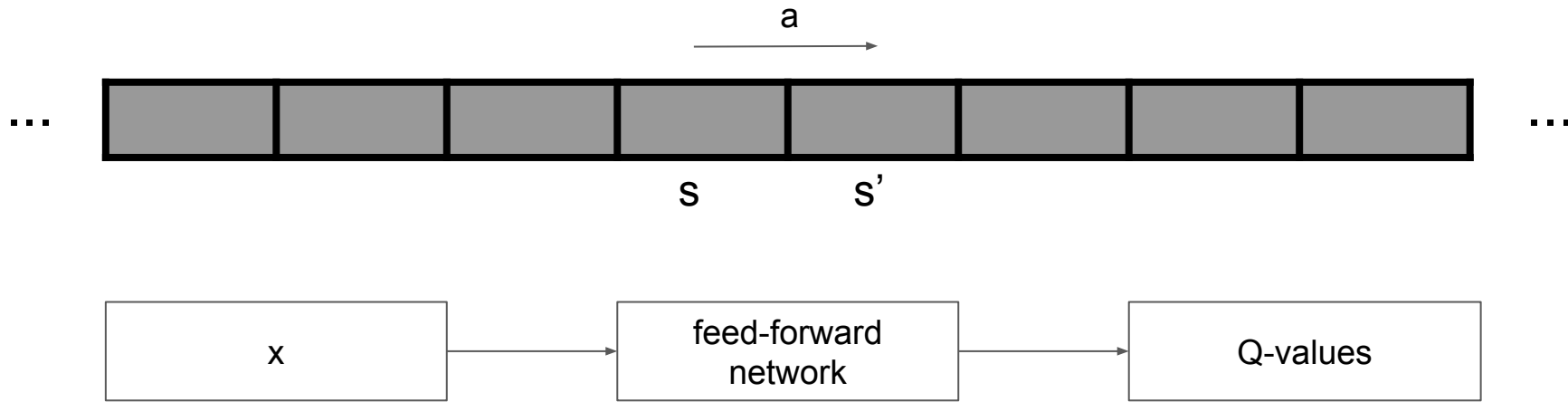
· One frame



· Several frames

- Nth-order markov assumption
- Works for velocity/timers
- Fails for anything longer than N frames
- Impractical for large N

# Autocorrelation



Target is based on prediction

$$Q(s, a) \text{ correlates with } Q(s', a)$$

# Target network

**Idea:** use network with frozen weights to compute the target

$$L(\Theta) = E_{s \sim S, a \sim A} [(Q(s, a, \Theta) - (r + \gamma \max_{a'} Q(s', a', \Theta^-)))^2]$$

where  $\Theta^-$  is the frozen weights

↑  
**Const**

**Hard target network:**

Update  $\Theta^-$  every **n** steps and set its weights as  $\Theta$

# Target network

**Idea:** use network with frozen weights to compute the target

$$L(\Theta) = E_{s \sim S, a \sim A} [(Q(s, a, \Theta) - (r + \gamma \max_{a'} Q(s', a', \Theta^-)))^2]$$

where  $\Theta^-$  is the frozen weights

↑  
**Const**

**Hard target network:**

Update  $\Theta^-$  every **n** steps and set its weights as  $\Theta$

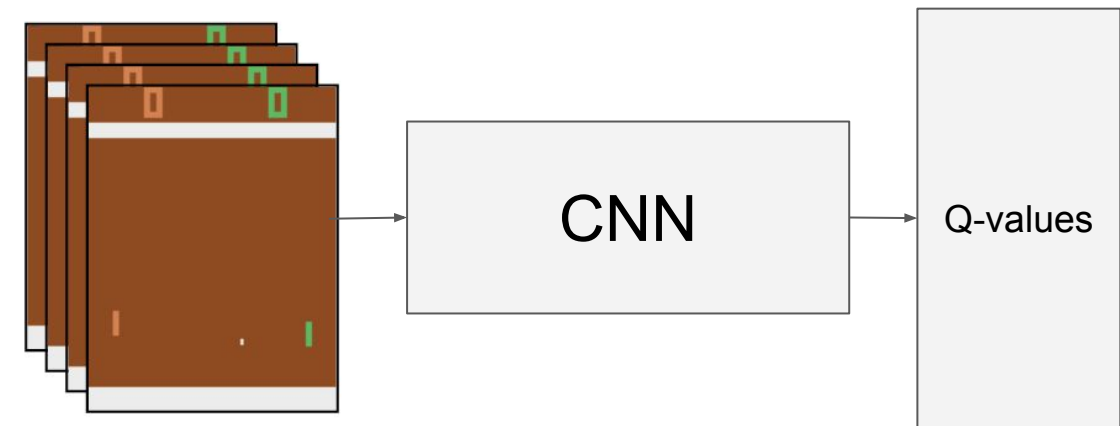
**Soft target network:**

Update  $\Theta^-$  every step:

$$\Theta^- = (1 - \alpha)\Theta^- + \alpha\Theta$$

# Playing Atari with Deep Reinforcement Learning

(2013, Deepmind)



4 last frames as input

Update weights using:

$$L(\Theta) = E_{s \sim S, a \sim A} [(Q(s, a, \Theta) - (r + \gamma \max_{a'} Q(s', a', \Theta^-)))^2]$$

Update  $\Theta^-$  every 5000 train steps

Experience replay



$10^6$  last transitions

- Q-learning can be applied to solve the environments with continuous states as well
- But we haven't said anything about continuous **actions**!
- Remember what  $Q(s, a)$  and  $V(s)$  functions do, we will need them even outside the Q-learning
- Remember about the i.i.d. property!
  - RL algorithms usually affect the environment, hence the distribution of the data they are trained on

# Backlog



# Problem of overestimation

We use “max” operator to compute the target

$$L(s, a) = (Q(s, a) - (r + \gamma \max_{a'} Q(s', a')))^2$$

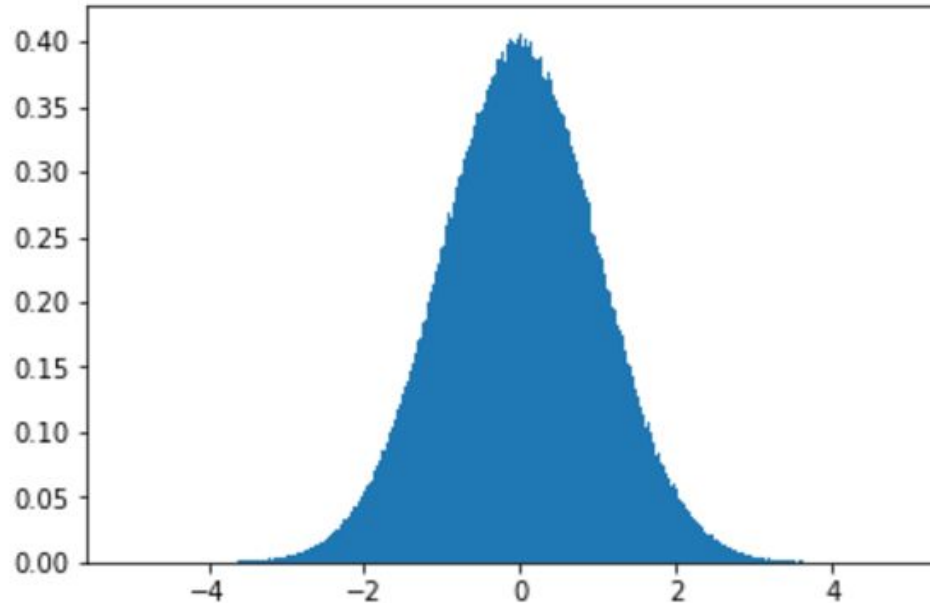
**We have a problem**

(although we want  $E_{s \sim S, a \sim A}[L(s, a)]$  to be equal zero)

# Problem of overestimation

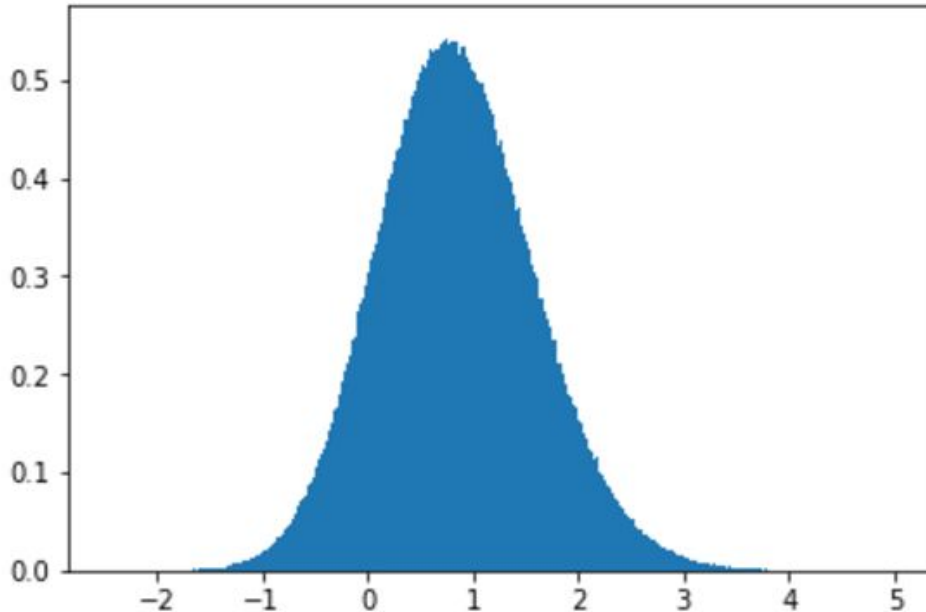
Normal distribution  
 $3 \cdot 10^6$  samples

mean:  $\sim 0.0004$



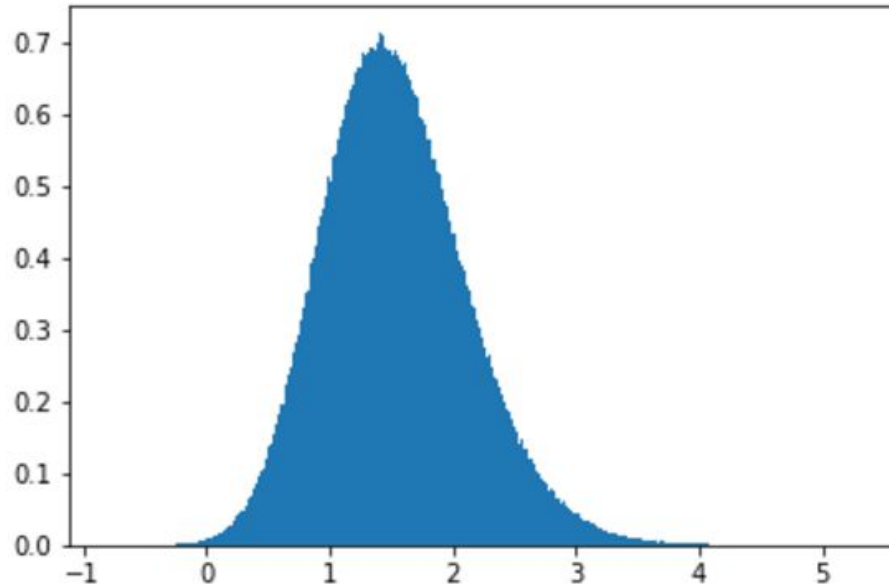
# Problem of overestimation

Normal distribution  
 $3 \cdot 10^6 \times 3$  samples  
Then take maximum of every tuple  
mean:  $\sim 0.8467$

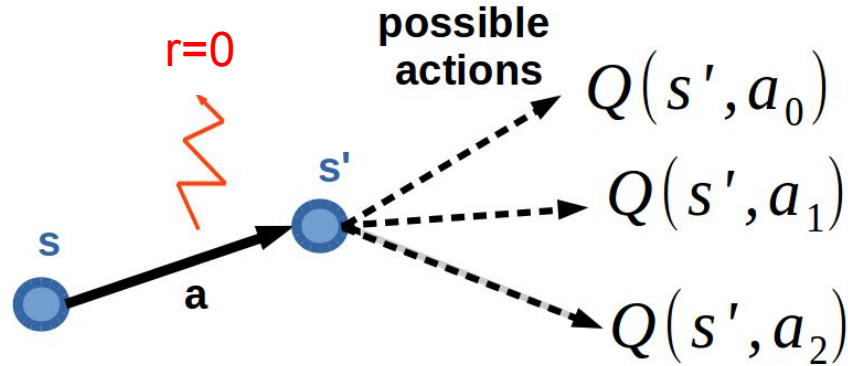


# Problem of overestimation

Normal distribution  
 $3 \times 10^6 \times 10$  samples  
Then take maximum of every tuple  
mean:  $\sim 1.538$



# Problem of overestimation

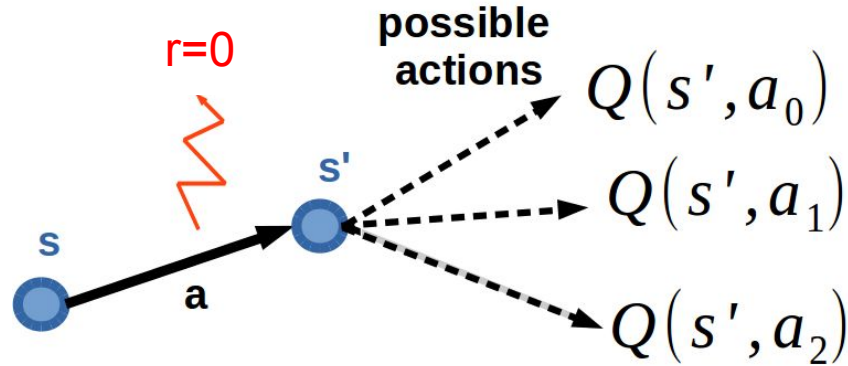


Suppose true  $Q(s', a')$  are equal to **0** for all  $a'$

But we have an approximation (or other) error  $\sim N(0, \sigma^2)$

So  $Q(s, a)$  should be equal to **0**

# Problem of overestimation



But if we update  $Q(s, a)$  towards  $r + \gamma \max_{a'} Q(s', a')$   
we will have overestimated  $Q(s, a) > 0$  because

$$E[\max_{a'} Q(s', a')] \geq \max_{a'} E[Q(s', a')]$$

# Double Q-learning

(NIPS 2010)

$$y = r + \gamma \max_{a'} Q(s', a')$$

- Q-learning target

$$y = r + \gamma Q(s', \operatorname{argmax}_{a'} Q(s', a'))$$

- Rewritten Q-learning target

**Idea:** use two estimators of q-values:  $Q^A, Q^B$

They should compensate mistakes of each other because they will be independent  
Let's get argmax from another estimator!

$$y = r + \gamma Q^A(s', \operatorname{argmax}_a Q^B(s', a'))$$

- Double Q-learning target

---

**Algorithm 1** Double Q-learning

---

```
1: Initialize  $Q^A, Q^B, s$ 
2: repeat
3:   Choose  $a$ , based on  $Q^A(s, \cdot)$  and  $Q^B(s, \cdot)$ , observe  $r, s'$ 
4:   Choose (e.g. random) either UPDATE(A) or UPDATE(B)
5:   if UPDATE(A) then
6:     Define  $a^* = \arg \max_a Q^A(s', a)$ 
7:      $Q^A(s, a) \leftarrow Q^A(s, a) + \alpha(s, a) (r + \gamma Q^B(s', a^*) - Q^A(s, a))$ 
8:   else if UPDATE(B) then
9:     Define  $b^* = \arg \max_a Q^B(s', a)$ 
10:     $Q^B(s, a) \leftarrow Q^B(s, a) + \alpha(s, a) (r + \gamma Q^A(s', b^*) - Q^B(s, a))$ 
11:   end if
12:    $s \leftarrow s'$ 
13: until end
```

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Can we combine this algorithm with DQN?



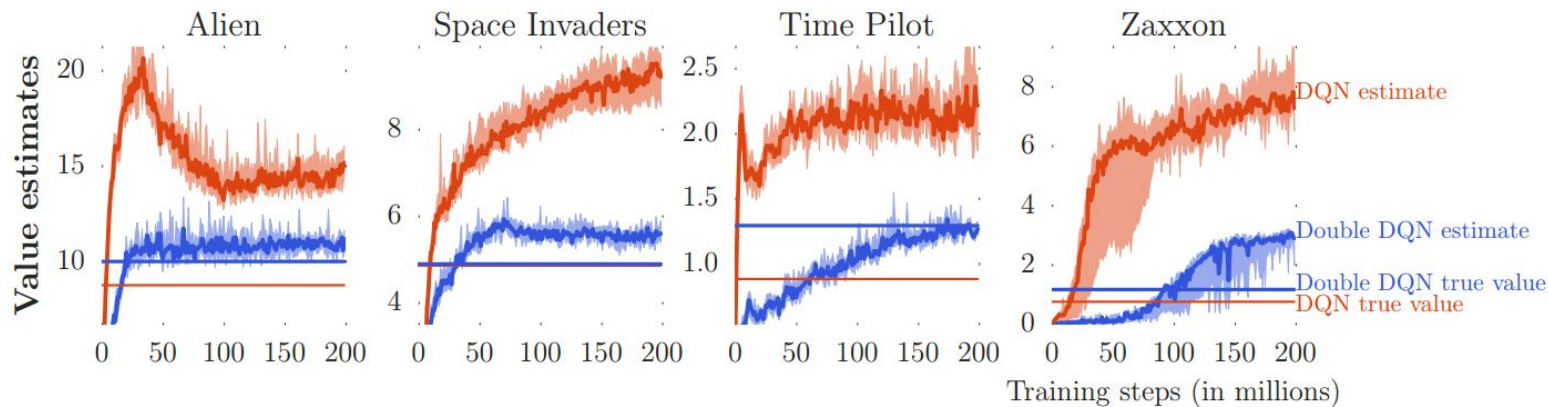
# Deep RL with Double Q-learning

(Deepmind, 2015)

**Idea:** use main network to choose action!

$$y_{dqn} = r + \gamma \max_{a'} Q(s', a', \Theta^-)$$

$$y_{ddqn} = r + \gamma Q(s', \operatorname{argmax}_{a'} Q(s', a', \Theta), \Theta^-)$$



	DQN	Double DQN	Double DQN (tuned)
Median	47.5%	88.4%	116.7%
Mean	122.0%	273.1%	475.2%

# Double Q-learning visualization



# Experience Replay

State	Action	Reward	Next state
s <sub>0</sub>	a <sub>0</sub>	0	s <sub>1</sub>
s <sub>1</sub>	a <sub>1</sub>	0	s <sub>2</sub>
...	...	...	...
s <sub>(n-1)</sub>	a <sub>(n-1)</sub>	0	s <sub>n</sub>
<b>s<sub>n</sub></b>	<b>a<sub>n</sub></b>	<b>100</b>	<b>s<sub>(n+1)</sub></b>
s <sub>(n+1)</sub>	a <sub>(n+1)</sub>	0	s <sub>(n+2)</sub>
...	...	...	...

# Dueling Network Architectures for Deep RL

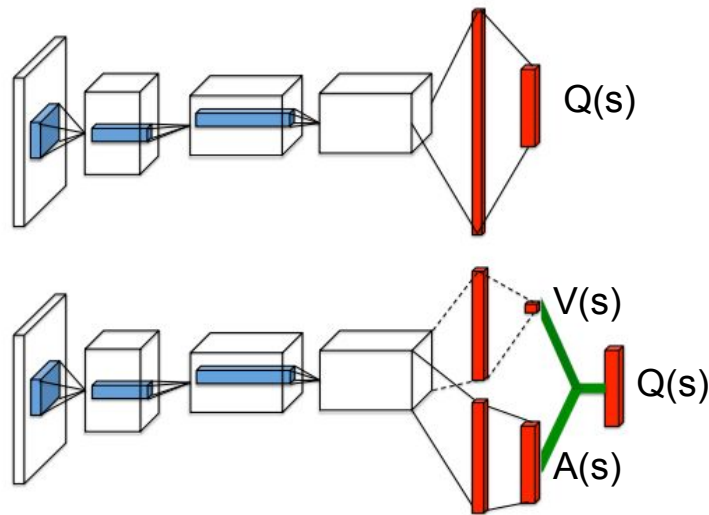
(2016, Deepmind)

**Idea:** change the network's architecture.

Recall:

Advantage Function  $A(s,a) = Q(s,a) - V(s)$

So,  $Q(s,a) = A(s,a) + V(s)$



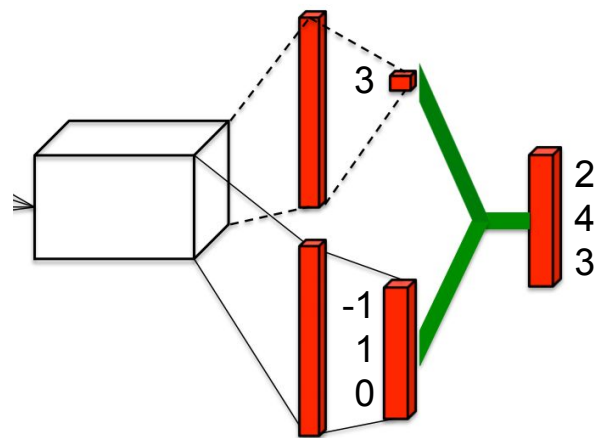
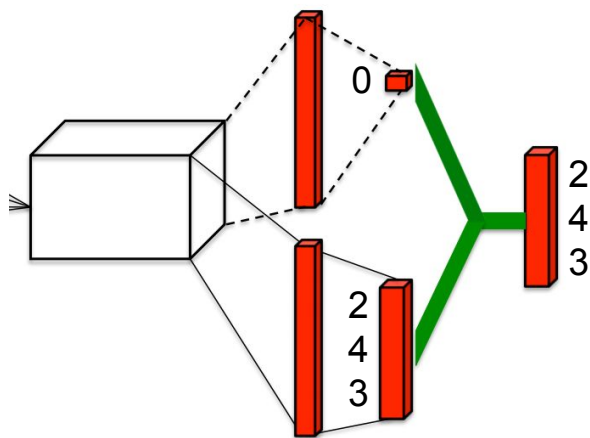
Do you see the problem?

# Dueling Network Architectures for Deep RL

(2016, Deepmind)

Here is one extra freedom degree!

Example:



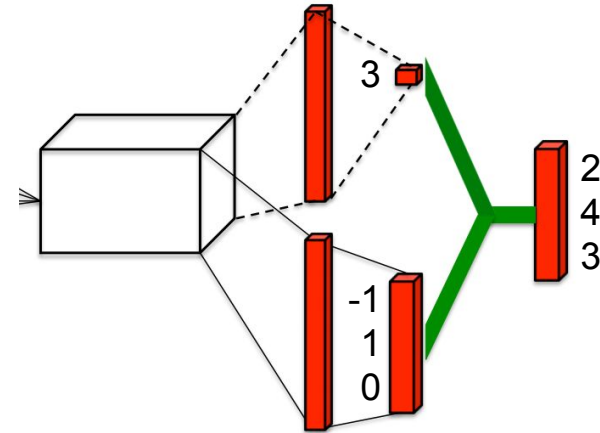
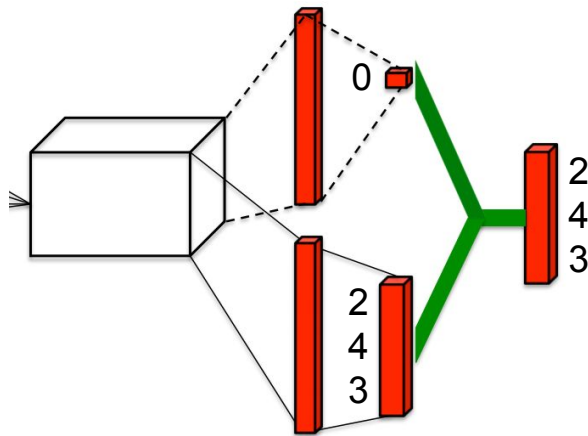
Which one is good?

# Dueling Network Architectures for Deep RL

(2016, Deepmind)

Here is one extra freedom degree!

Example:



No one

# Dueling Network Architectures for Deep RL

(2016, Deepmind)

**Solution:** require  $\max_{a' \in |\mathcal{A}|} A(s, a'; \theta, \alpha)$  to be equal to zero!

So the **Q-function** is computed as:

$$Q(s, a; \theta, \alpha, \beta) = V(s; \theta, \beta) + \left( A(s, a; \theta, \alpha) - \max_{a' \in |\mathcal{A}|} A(s, a'; \theta, \alpha) \right)$$

# Dueling Network Architectures for Deep RL

(2016, Deepmind)

**Solution:** require  $\max_{a' \in |\mathcal{A}|} A(s, a'; \theta, \alpha)$  to be equal to zero!

So the **Q-function** is computed as:

$$Q(s, a; \theta, \alpha, \beta) = V(s; \theta, \beta) + \left( A(s, a; \theta, \alpha) - \max_{a' \in |\mathcal{A}|} A(s, a'; \theta, \alpha) \right)$$

Authors of this papers also introduced this way to compute Q-values:

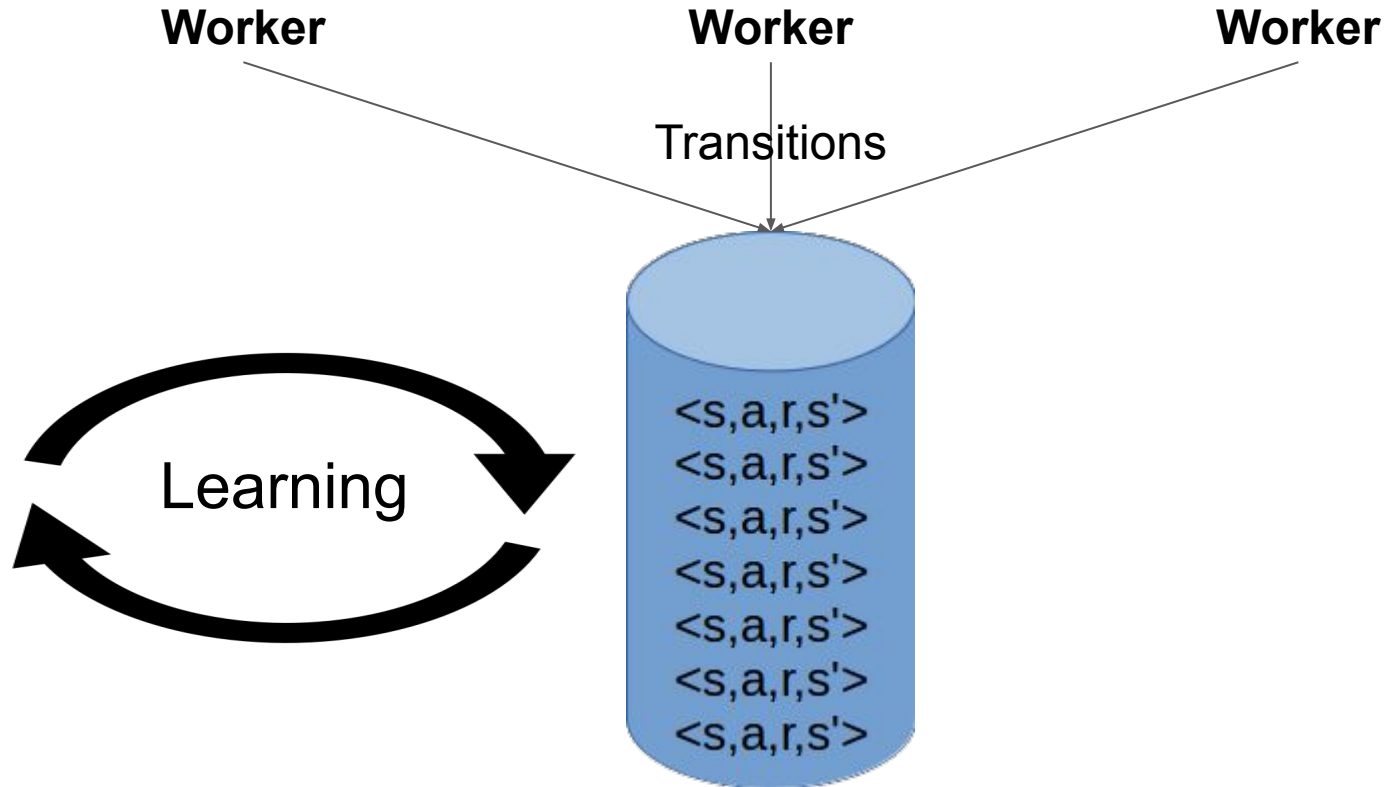
$$Q(s, a; \theta, \alpha, \beta) = V(s; \theta, \beta) + \left( A(s, a; \theta, \alpha) - \frac{1}{|\mathcal{A}|} \sum_{a'} A(s, a'; \theta, \alpha) \right)$$

They wrote that this variant increases stability of the optimization  
(The fact that this loses the original semantics of Q doesn't matter)



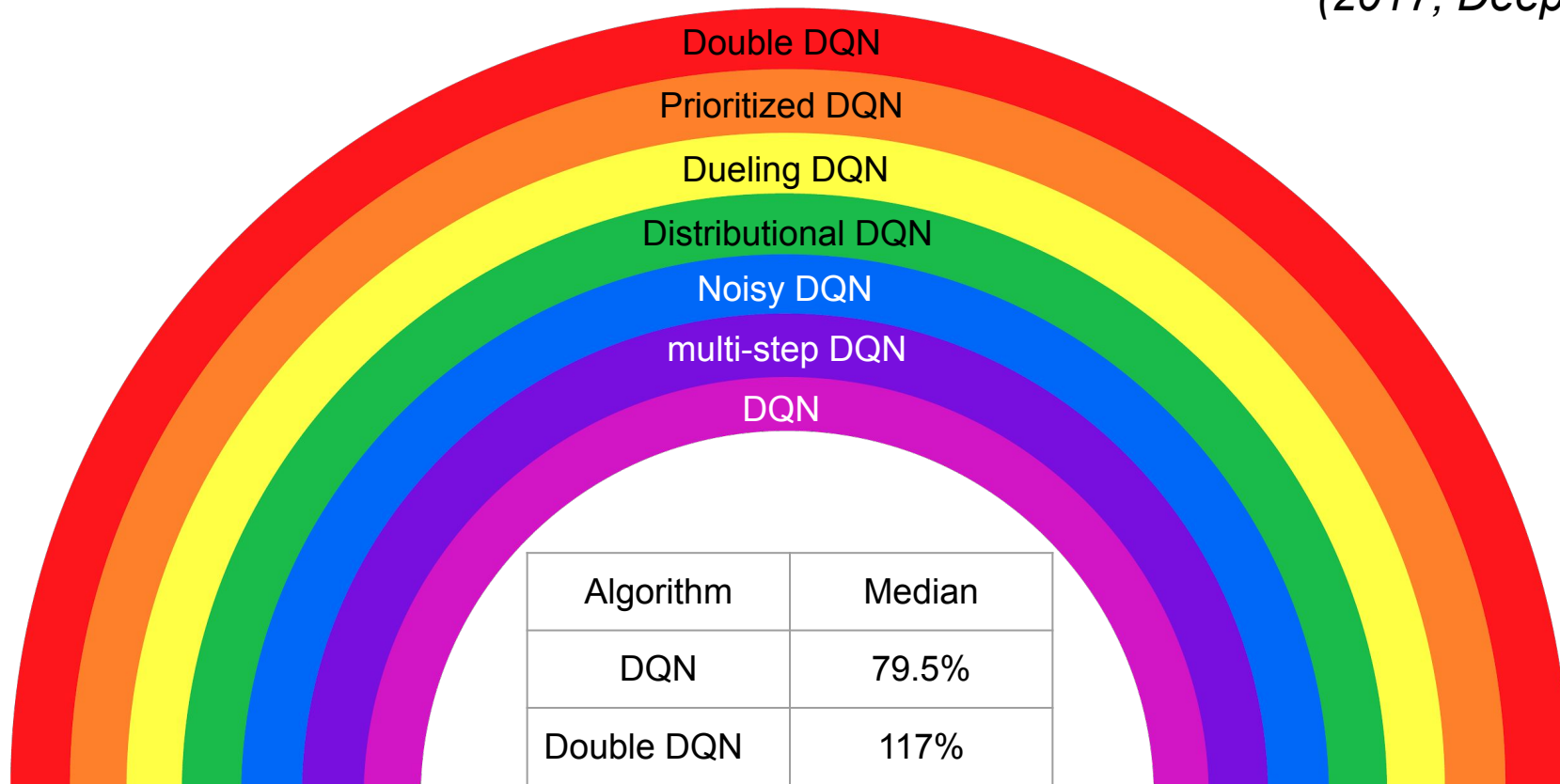
# Asynchronous Methods for Deep RL

(2016, Deepmind)



# Rainbow

(2017, Deepmind)



Algorithm	Median
DQN	79.5%
Double DQN	117%
Rainbow	223%

# R2D2

(2018, Deepmind)

LSTM

Reward re-scaling

Distributed Prioritized  
Experience Replay

Double DQN

n-step DQN

Dueling DQN



Median performance: **1920%** of human performance!

# Prioritized Experience Replay

(2016, Deepmind)

**Idea:** sample transitions from xp-replay cleverly

We want to set probability for every transition. Let's use the absolute value of TD-error of transition as a probability!

$$\text{TD-error } \delta = Q(s, a) - (r + \gamma Q(s', \arg\max_{a'} Q(s', a', \Theta), \Theta^-))$$

$$p = |\delta|$$

$$P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha} \text{ where } \alpha \text{ is the priority parameter (when } \alpha \text{ is 0 it's the uniform case)}$$

**Do you see the problem?**

**Transitions become non i.i.d. and therefore we introduce the bias.**

# Prioritized Experience Replay

(2016, Deepmind)

**Solution:** we can correct the bias by using importance-sampling weights

$$w_i = \left( \frac{1}{N} \cdot \frac{1}{P(i)} \right)^\beta \quad \text{where } \beta \text{ is the parameter}$$

So we sample using  $P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha}$  and multiply error by  $w_i$

# Prioritized Experience Replay

(2016, Deepmind)

## Additional details

We also normalize weights by  $1 / \max_i w_i$  (here is no mathematical reason)

When we put transition into experience replay, we set maximal priority  $p_t = \max_{i < t} p_i$