Vectors

1. What is a *vector*?

   - How do we define a vector?
   - Why do we need vectors?
   - What is a *vector space*? (can be skipped for later)

2. What is an *inner product*?

   - Why do we need an inner product?
   - What is a geometric meaning of an inner product?

3. What is a *projection*?

   - When (why) do we need a projection?

Matrices

1. What is a matrix?

   - What kinds of operations available for matrices?
   - Why do we need matrices?
   - Is a matrix a vector?

2. What is a determinant?

   - How do we compute determinants for 2×2 or 3×3 matrices? (How about for $n \times n$ matrices?)

3. What is a *cross product*?

   - What does cross product computes geometrically?
   - How do we compute the volume of a parallelopiped?
   - What does it imply when the determinant of a 3×3 matrix is zero?

Homework

- Reading assignment

  - Chapter §1.1 ~§1.3

- Writing assignment (due **Sep. 9th, 9am**)

  - Answer the following question in complete sentences. Note that there is no correct answer to all questions. Freely write your thoughts and ideas with logical explanations.

    1. Is the set of all $n \times n$ matrices a vector space?
    2. For n-dimensional vectors

    $$\mathbf{x} = (x_1, \cdots, x_n), \quad \mathbf{y} = (y_1, \cdots, y_n),$$

    the inner product $\mathbf{x} \cdot \mathbf{y}$ can be defined as
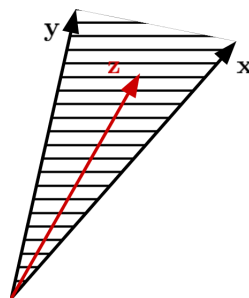
    $$\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + \cdots + x_n y_n.$$

    If the formula

    $$\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\| \cdot \|\mathbf{y}\| \cdot \cos \theta$$

    holds, how can we define the angle θ between $\mathbf{x}$ and $\mathbf{y}$?

    3. Let $\mathbf{x}$ and $\mathbf{y}$ be 2-dimensional vectors. Let the *cone* (shown below) of $\mathbf{x}$, $\mathbf{y}$ be the region between two vectors from $\mathbf{x}$ to $\mathbf{y}$ counter clockwise. Given a third 2-dimensional vector $\mathbf{z}$, how can we check whether $\mathbf{z}$ lies in the cone of $\mathbf{x}$ and $\mathbf{y}$?

  - Type or write neatly, convert to pdf, then upload to LMS.