Problem 1

Question:

Suppose that f is a function from a finite set A to a finite set B where $|A| = a$ and $|B| = b$, and $a > b$. Use the Pigeon-hole Principle and the definition of 1-1 to prove that f cannot be 1-1

Proof:

A function f from $S_1 \to S_2$ is 1-1, if for any 2 elements $a, b \in S_1$: if $f(a) = f(b)$ then $a = b$. In the above case, we have 2 finite sets: A, B, where $|A| > |B|$. We can think of each element a in A as the pigeons, and each element b in B as the pigeon-holes. The act of placing the pigeon in a hole, is what the function f is doing. This means that for f to be 1-1, each hole can only have 1 pigeon. But we know that the number pigeons is greater than the number of holes: $a > b$. This means that there must be at least 2 pigeons in the same hole. Mathematically, this means that there exists $a_1, a_2 \in A$ where $a_1 \neq a_2$, BUT $f(a_1) = f(a_2)$. This proves that the function f that maps A to B CANNOT be 1-1.