Find the error and redo:

How many subsets of a 100-element set have more than one element?

Solution: There are $\binom{100}{2}$ subsets of a 100-element set with two elements, since this counts unordered pairs of distinct elements. Similarly, there are $\binom{100}{3}$ subsets with three elements. Continuing in this way and using the sum rule (these cases are mutually exclusive), there are

$$\sum_{k=2}^{99} \binom{100}{k}$$

subsets of a 100-element set that have more than one element.

Fixing the error:

A subset B of a set A is valid only if all elements of B are also elements of A. Using this definition, note that a subset of a 100-element set can be the set itself. The above solution omitted the 100-element subset. To fix this, change the summation to

$$\sum_{k=2}^{100} \binom{100}{k},$$

which now includes the full 100-element subset. **Totally different approach:**

Think of the 100-element set S as a bit string of length 100. Any subset of S corresponds to a bit string where a 1 means the element is included and a 0 means it is not. Each position has two possibilities, so by the product rule there are 2^{100} subsets in total.

Now remove the subsets with exactly one element (there are 100 of them) and the empty subset (one of them). This leaves

$$2^{100} - 100 - 1 = 2^{100} - 101$$

subsets of a 100-element set that have more than one element.

- **Who Contributed:** Dhvan Shah was the main contributor, doing the formal write-up of the solution. Pranav Bonthu and Michael Ku compared their solutions to Dhvan's and provided feedback and edits.
- **Resources**: No resources were used.
- **Main points:** Based on group discussions, we felt that this problem