[EXP]

(a) Find the expected value of the sum of the sum and product of two independent die rolls.

(b) You roll a die, and if the result is prime you roll two more dice, and if it isn't prime you roll *three* more dice. Find the expected number of pips showing on the top faces of all of the dice rolled (so, either three dice or four dice).

A_1 : First die

A_2 : Second die.

(a) $E(A_1 + A_2 + A_1A_2) = E(A_1) + E(A_2) + E(A_1A_2)$

$= E(A_1) + E(A_2) + E(A_1)E(A_2)$ ← Independent

$= 3.5 + 3.5 + 3.5^2 = 19.25$

(b) Prime : $\{2, 3, 5\}$. $P(\text{prime}) = \frac{1}{2}$

$E(\text{pips}) = E(\text{pips} \mid \text{prime}) + E(\text{pips} \mid \text{prime}^c)$

$= [E(\text{prime}) + E(2 \text{ dies})] \cdot P(\text{prime})$

$+ [E(\text{prime}^c) + E(3 \text{dies})] \cdot P(\text{prime}^c)$

$= \left(\frac{10}{3} + 7\right) \cdot \frac{1}{2} + \left(\frac{11}{3} + 10.5\right) \cdot \frac{1}{2}$

$= \frac{1}{2} + \frac{7}{2} + \frac{21}{4} = \frac{49}{4}$