(a) Suppose that the conditional probability of an email (chosen uniformly at random from a large collection of emails) containing the phrase "additional income", given that the email is spam, is 14%. Suppose that the conditional probability of an email being spam, given that it contains the phrase "additional income", is 88%. Find the ratio of the probability that an email is spam to the probability that an email contains the phrase "additional income".

(b) We flip a weighted coin that has probability $\frac{3}{4}$ of turning up heads. If we get heads, we roll a six-sided die, and otherwise we roll an eight-sided die. Given that the die turns up 4, what is the conditional probability that the coin turned up heads?

(a) A: contains the phrase "additional Income"

B: email is spam

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 14\%$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = 88\%$$

$$\frac{P(B)}{P(A)} = \frac{P(B|A)}{P(A|B)} = \frac{44}{7}$$

(b) A: coin turn up head. E: die turns up 4

$$P(A|E) = \frac{P(E|A) \cdot P(A)}{P(E)} = \frac{P(E|A) \cdot P(A)}{P(E|A) P(A) + P(E|A^c) P(A^c)}$$

$$P(E|A) = \frac{1}{6} \quad P(E|A^c) = \frac{1}{8} \quad P(A) = \frac{3}{4} \quad P(A^c) = \frac{1}{4}$$

$$P(A|E) = \frac{1}{6} \cdot \frac{3}{4} \bigg/ \left(\frac{1}{6} \cdot \frac{3}{4} + \frac{1}{8} \cdot \frac{1}{4} \right) = 0.8$$