MGE-MSR-01 – Sensors and State Estimation

Introduction to Exercise 2
WS 20/21

Tomislav Medic

Institut für Geodäsie und Geoinformation
Universität Bonn

Timetable

eCampus PPT submission deadline: Tuesday 12.01.2020., 23:59

Discussion Time: Wednesday 13.01.2021., 08:30.

Topic: Exercises deal with three different topics

- Exercise 1: Inertial navigation
- **Exercise 2: Kalman filtering**
- Exercise 3: Laser scanning

Date	Preliminary discussion	Exercise examination
	Begin of lectures	
	Exercise 1	–
16.12.2020	**Exercise 2**	**Exercise 1**
13.01.2021	**Exercise 3**	**Exercise 2**
	–	Exercise 3
	End of lectures	

Learning objectives

- Understanding theory of Kalman Filtering

(functional and stochastic model, steps, system and measurement model, tuning)

- Implementing simplified 2D Extended Kalman Filter

(+ visualization, interpretation and comparison of the results)

Main literature:

- MGE-MSR-01 – Sensors & State Estimation

(Lectures: Kalman Filter I.,II., III.)

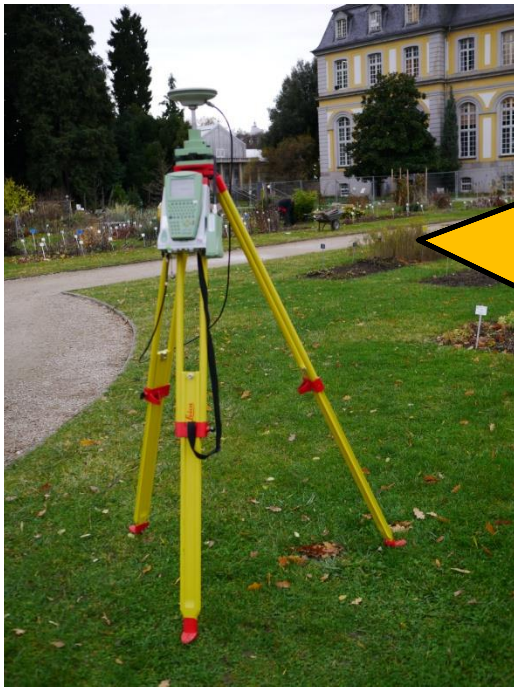
- Groves, P. D.: Navigation Using Inertial Sensors,

University College London, UK, IEEE A&E Systems magazine, February 2015, Part II of II.

Components of the MSS

Kinematic MSS for laser scanning in motion

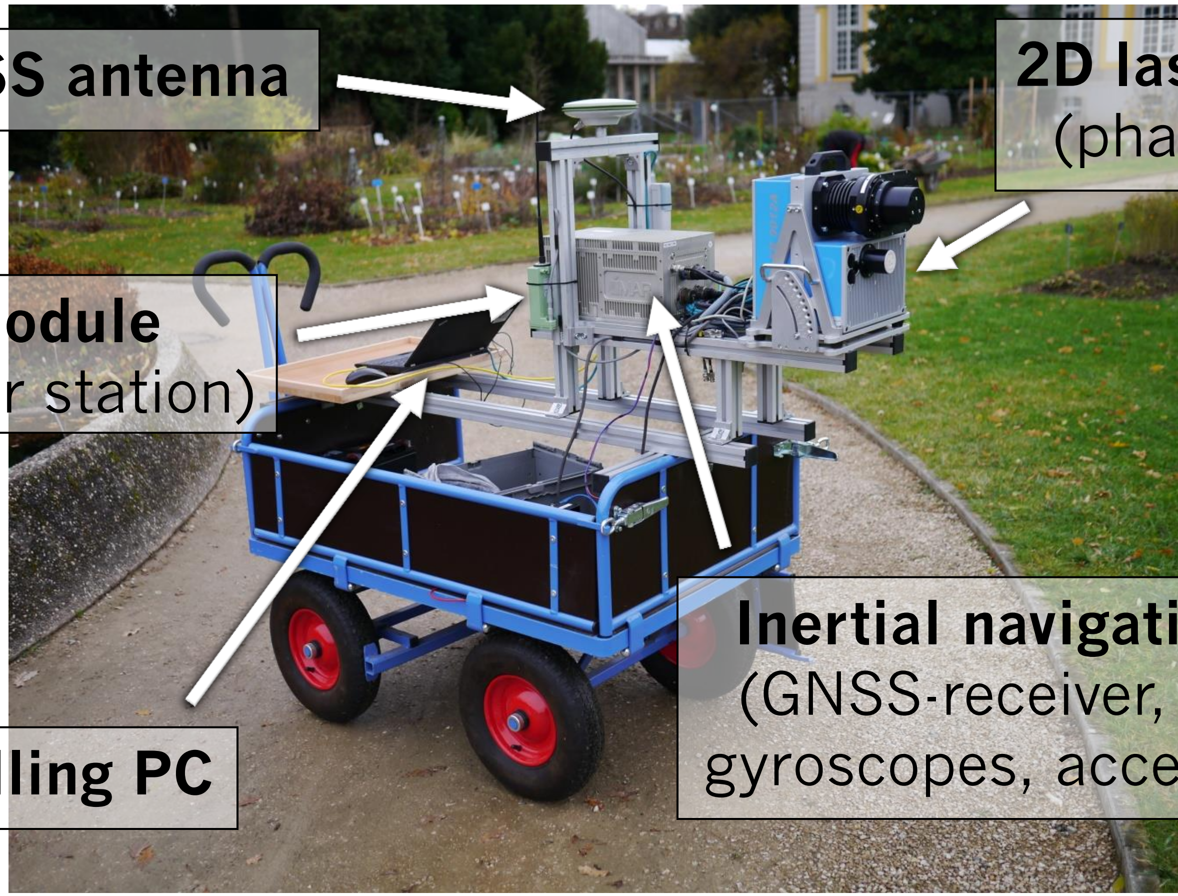
GNSS master station (sending observations to MSS → RTK-GNSS)



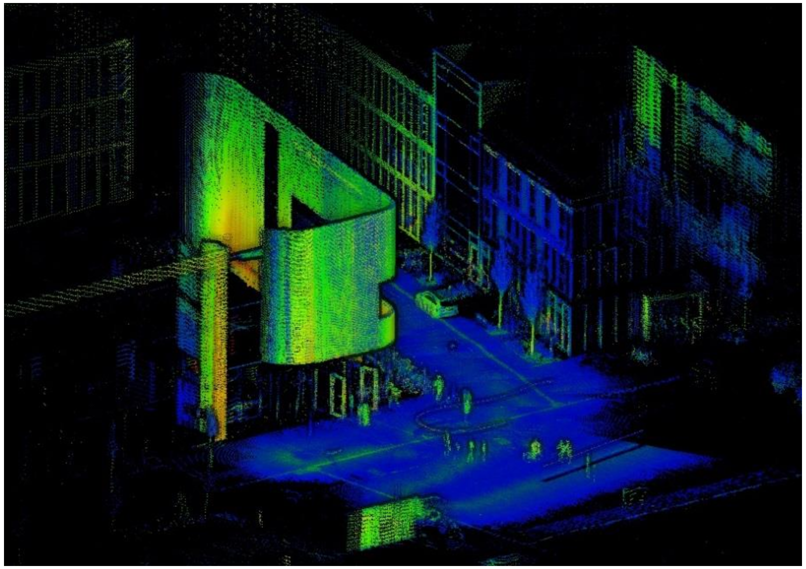
Same as in Exercise 1

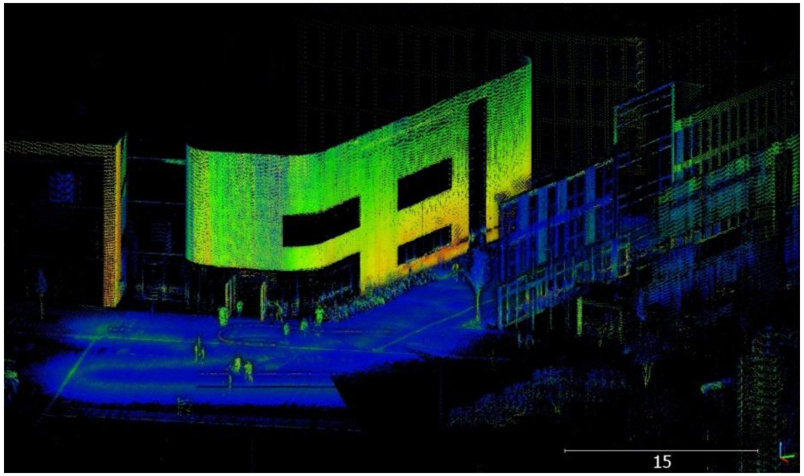
Platform

Components of the MSS



Measurements





Theoretical part (1)

- **Describe Kalman Filter:**
  - Algorithm diagram / flow-chart
  - Two main steps of the KF
  - All formulas
  - All variables in formulas
  - Input data
  - Output data
  - Necessary initial values
  - Difference between KF and EKF

Prediction

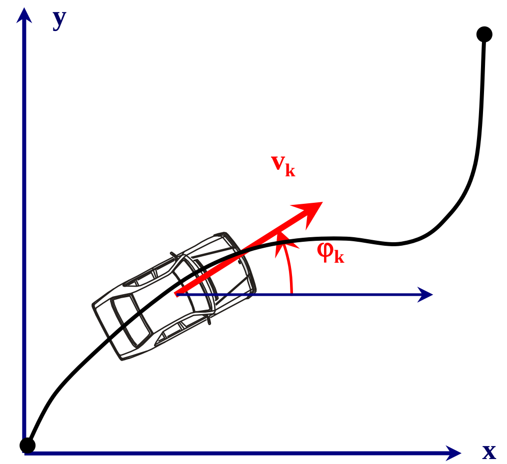
$$
\begin{aligned}
\mathbf{x}_k^- &= \mathbf{\Phi}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_k \\
\mathbf{P}_k^- &= \mathbf{\Phi}\mathbf{P}_{k-1}\mathbf{\Phi}^T + \mathbf{G}\mathbf{Q}\mathbf{G}^T
\end{aligned}
$$

$$
\begin{aligned}
K_k &= P_k^- H^T (H P_k^- H^T + R)^{-1} \\
\mathbf{x}_k &= \mathbf{x}_k^- + K_k (z_k - H\mathbf{x}_k^-) \\
P_k &= (I - K_k H) P_k^-
\end{aligned}
$$

Correction

Theoretical part (2)

- Describe selected **system model** for this exercise (Lecture: Kalman_Filter_III, Example 2)



Presumptions:

- constant angular rate
- constant acceleration

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{w_k})$$

$$\begin{bmatrix} x_k \\ y_k \\ \varphi_k \\ \dot{\varphi}_k \\ v_k \\ a_k \end{bmatrix} = f\left(\begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \varphi_{k-1} \\ \dot{\varphi}_{k-1} \\ v_{k-1} \\ a_{k-1} \end{bmatrix}\right) = \begin{bmatrix} x_{k-1} + \cos(\varphi_{k-1}) v_{k-1} \Delta t \\ y_{k-1} + \sin(\varphi_{k-1}) v_{k-1} \Delta t \\ \varphi_{k-1} + \dot{\varphi}_{k-1} \Delta t \\ \dot{\varphi}_{k-1} + w_{\dot{\varphi}} \Delta t \\ v_{k-1} + a_{k-1} \Delta t \\ a_{k-1} + w_a \Delta t \end{bmatrix}$$

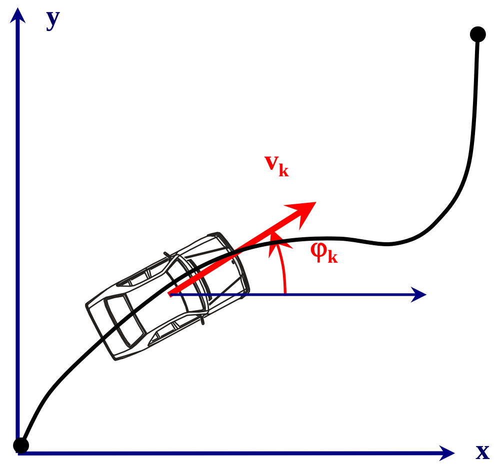
Theoretical part (2)

- Describe selected **measurement model** (Lecture: Kalman_Filter_III, Example 2)

Measurements:

- acceleration,
- angular rate,
- GNSS

if loop !



$$\mathbf{z}_k = \begin{bmatrix} x_{gps,k} \\ y_{gps,k} \\ a_k \\ \omega_k \end{bmatrix} = \mathbf{H}\mathbf{x}_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x}_k$$

Measurement model

- **Measurements have different measurement rate!**

if there_is_GNSS_measurements

Check this part of the code!

$$\mathbf{z}_k = \begin{bmatrix} x_{gps,k} \\ y_{gps,k} \\ a_k \\ \omega_k \end{bmatrix} = \mathbf{H}\mathbf{x}_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x}_k$$

else

$$\mathbf{z}_k = \begin{bmatrix} a_k \\ \omega_k \end{bmatrix} = \mathbf{H}\mathbf{x}_k = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x}_k$$

Input Values (1)

MSS measurements (IMAR.mat data files)

1. GNSS observations

- Transform ellipsoidal coordinates to UTM coordinates (N, E)

(Necessary adaptation for 2D EKF, see “General Task” in Exercise 2 sheet)

$x = \text{imar\_data.gpsUTM}[:,0]$ # East UTM [m]

$y = \text{imar\_data.gpsUTM}[:,1]$ # North UTM [m]

Input Values (2)

MSS measurements (IMAR.mat data files)

2. Acceleration (in the movement direction x-axis of the body frame – simplification)

a = -imar_data.acceleration[:,0] # acceleration x [m/s^2]

(with removed gravity influence!)

3. Angular rate (around z axis of the body frame, only yaw/heading angle)

omega = imar_data.angularvelocity[:,2] # angular velocity z [rad/s]

Initial Values

Functional model

$$\begin{bmatrix} x_k \\ y_k \\ \varphi_k \\ \dot{\varphi}_k \\ v_k \\ a_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + \cos(\varphi_{k-1}) v_{k-1} \Delta t \\ y_{k-1} + \sin(\varphi_{k-1}) v_{k-1} \Delta t \\ \varphi_{k-1} + \dot{\varphi}_{k-1} \Delta t \\ \dot{\varphi}_{k-1} + \boxed{w_{\dot{\varphi}}} \Delta t \\ v_{k-1} + a_{k-1} \Delta t \\ a_{k-1} + \boxed{w_a} \Delta t \end{bmatrix}$$

- **System noise**
- Angular acceleration

  0.1 [rad/s]
- Linear jerk

  1.5 [m/s^3]

Measurement noise

- GNSS - 0.05m
- Acceleration - empirically - std(a(1:1000))
- Angular Rate – empirically - std(omega(1:1000))

Initial Values

Functional model

$$\begin{bmatrix} x_k \\ y_k \\ \varphi_k \\ \dot{\varphi}_k \\ v_k \\ a_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + \cos(\varphi_{k-1}) v_{k-1} \Delta \\ y_{k-1} + \ldots (\varphi_{k-1}) v_{k-1} \Delta \\ \varphi_{k-} \ldots \Delta t \\ \dot{\varphi}_{k-} \\ \ldots \\ \ldots \end{bmatrix}$$

- **System noise**
- A... ar acceleration
  - 0.1 [rad/s]
  - ... 5 [m/s^3]

Filter tuning

- change system and measurement noise parameters
- Analyse the differences in trajectory

Me...

- GN...
- Acc... (1:10...)
- Angular Ra... mp... ally - std(omega(1:1000))

Output Values

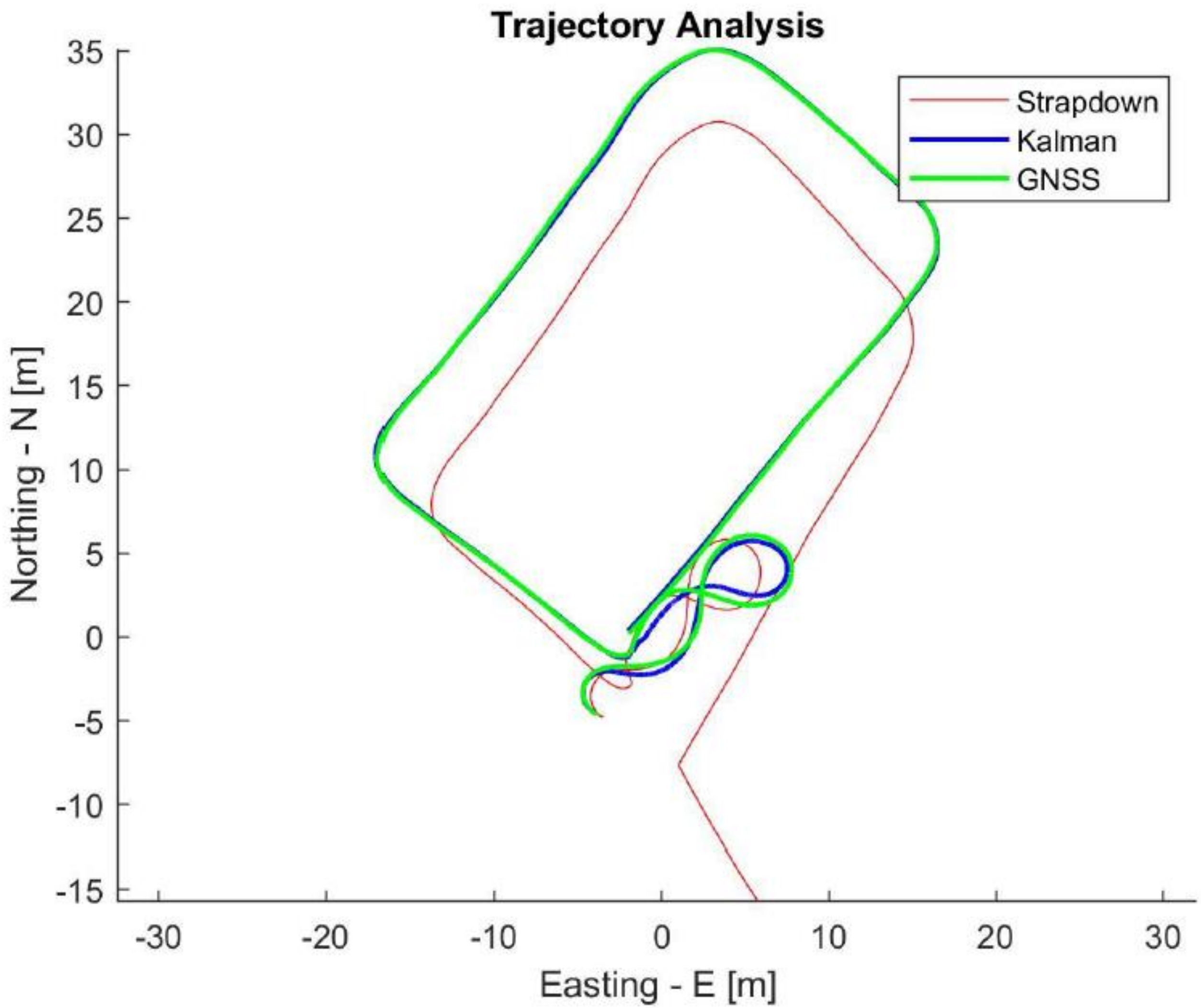
Output: State vector solution for each time stamp

[t]

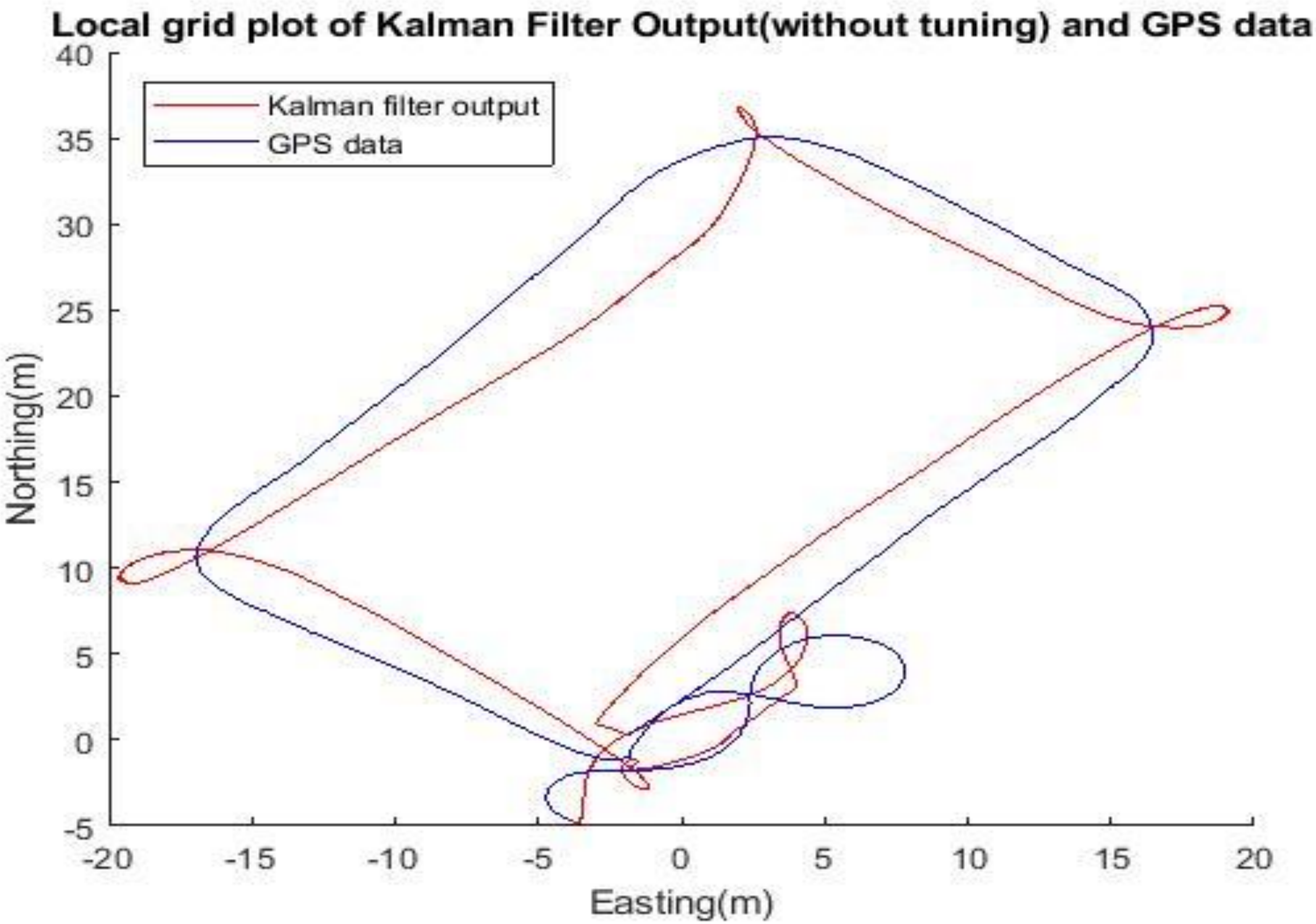
$$\begin{bmatrix} x_k \\ y_k \\ \varphi_k \\ \dot{\varphi}_k \\ v_k \\ a_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + \cos(\varphi_{k-1}) v_{k-1} \Delta t \\ y_{k-1} + \sin(\varphi_{k-1}) v_{k-1} \Delta t \\ \varphi_{k-1} + \dot{\varphi}_{k-1} \Delta t \\ \dot{\varphi}_{k-1} + w_{\dot{\varphi}} \Delta t \\ v_{k-1} + a_{k-1} \Delta t \\ a_{k-1} + w_a \Delta t \end{bmatrix}$$

Solutions From previous years

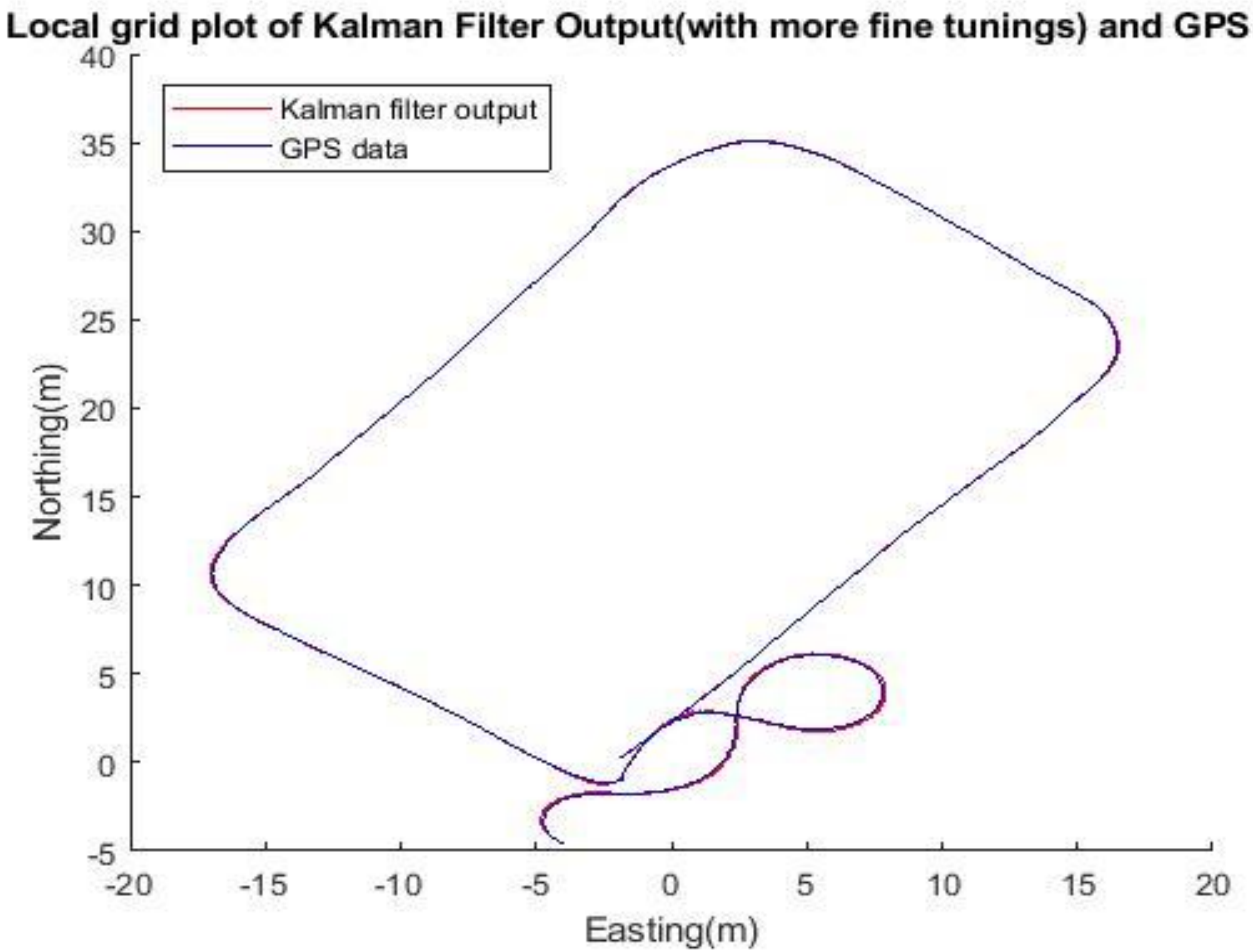
Previous years



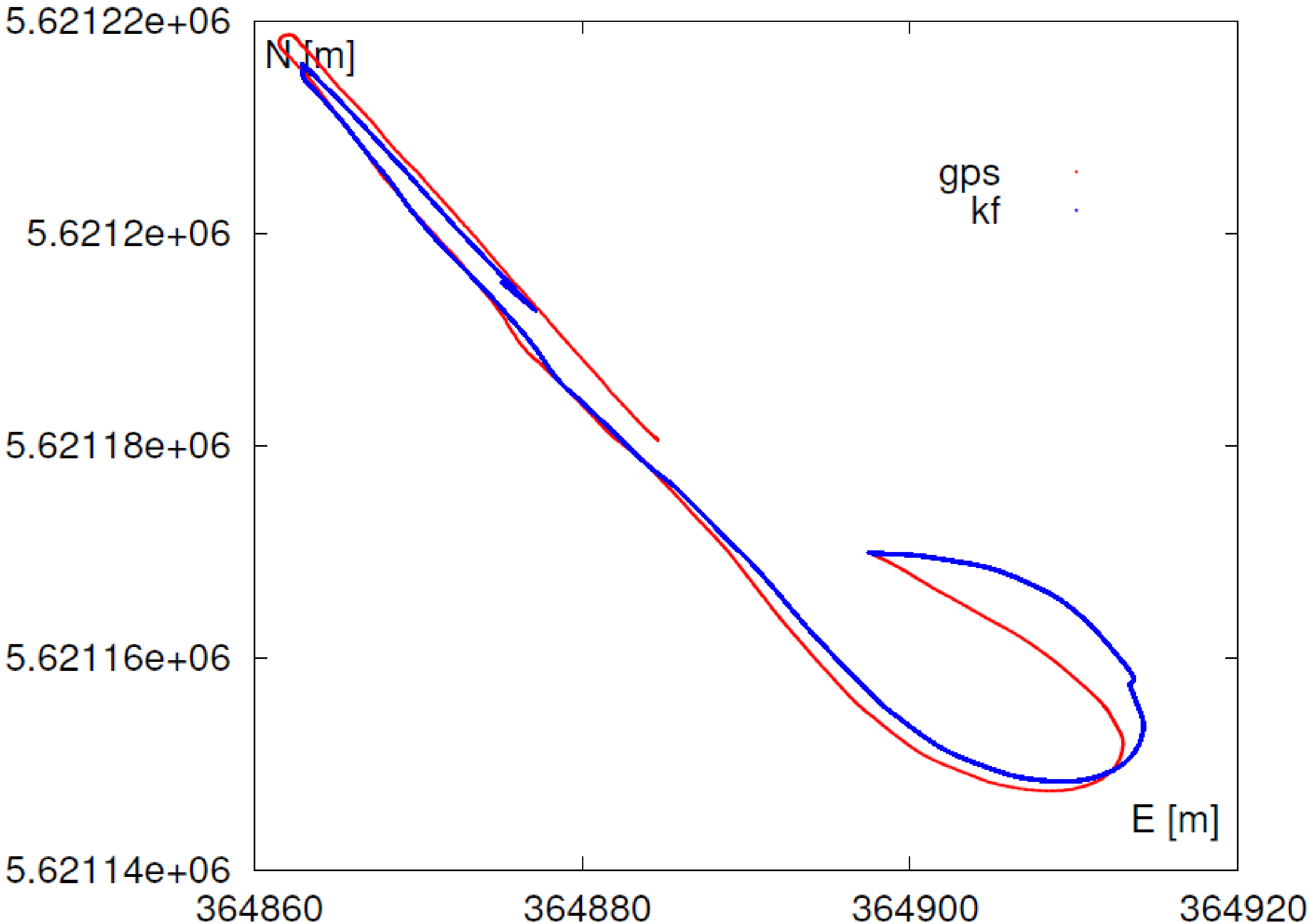
Previous years



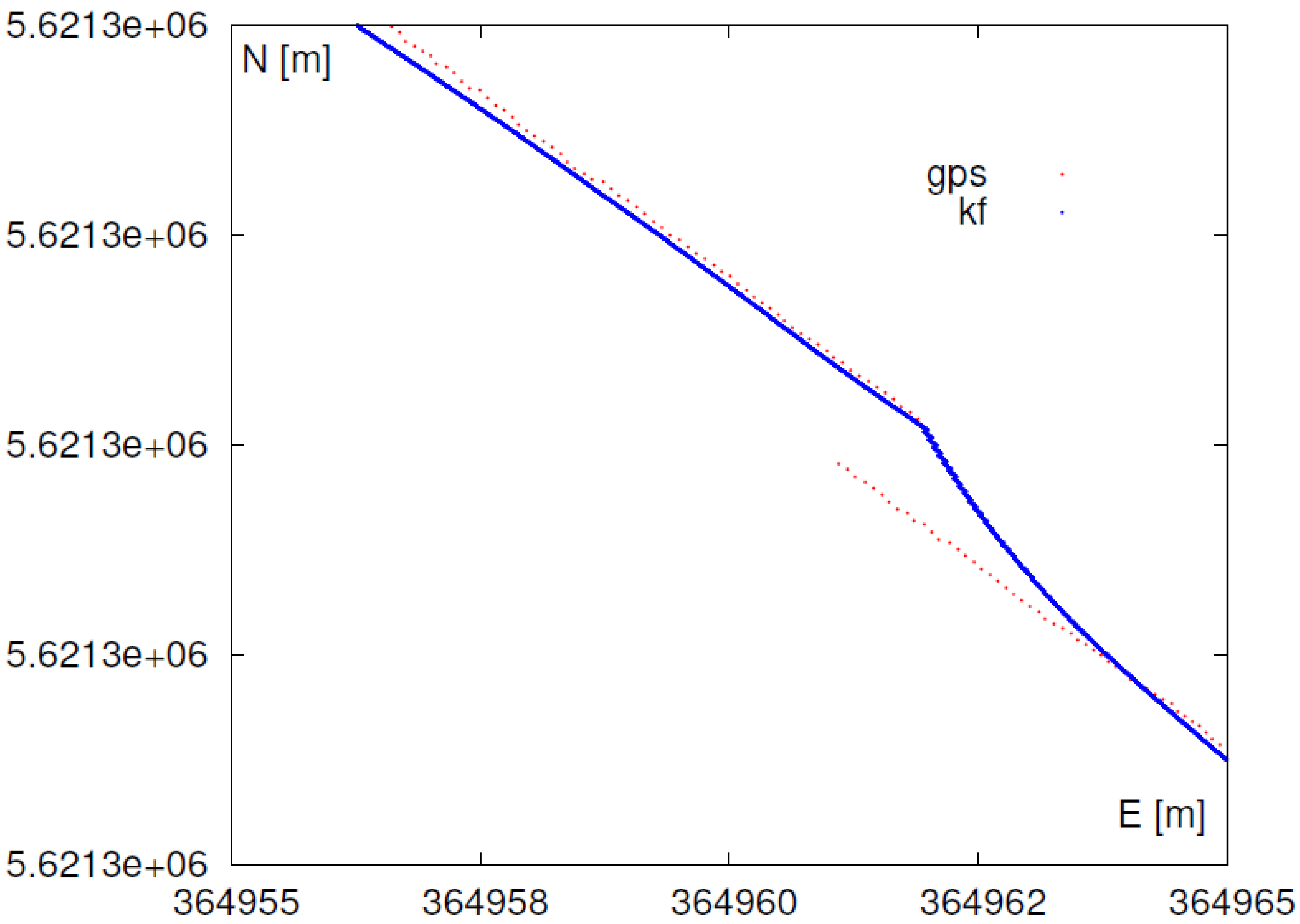
Previous years



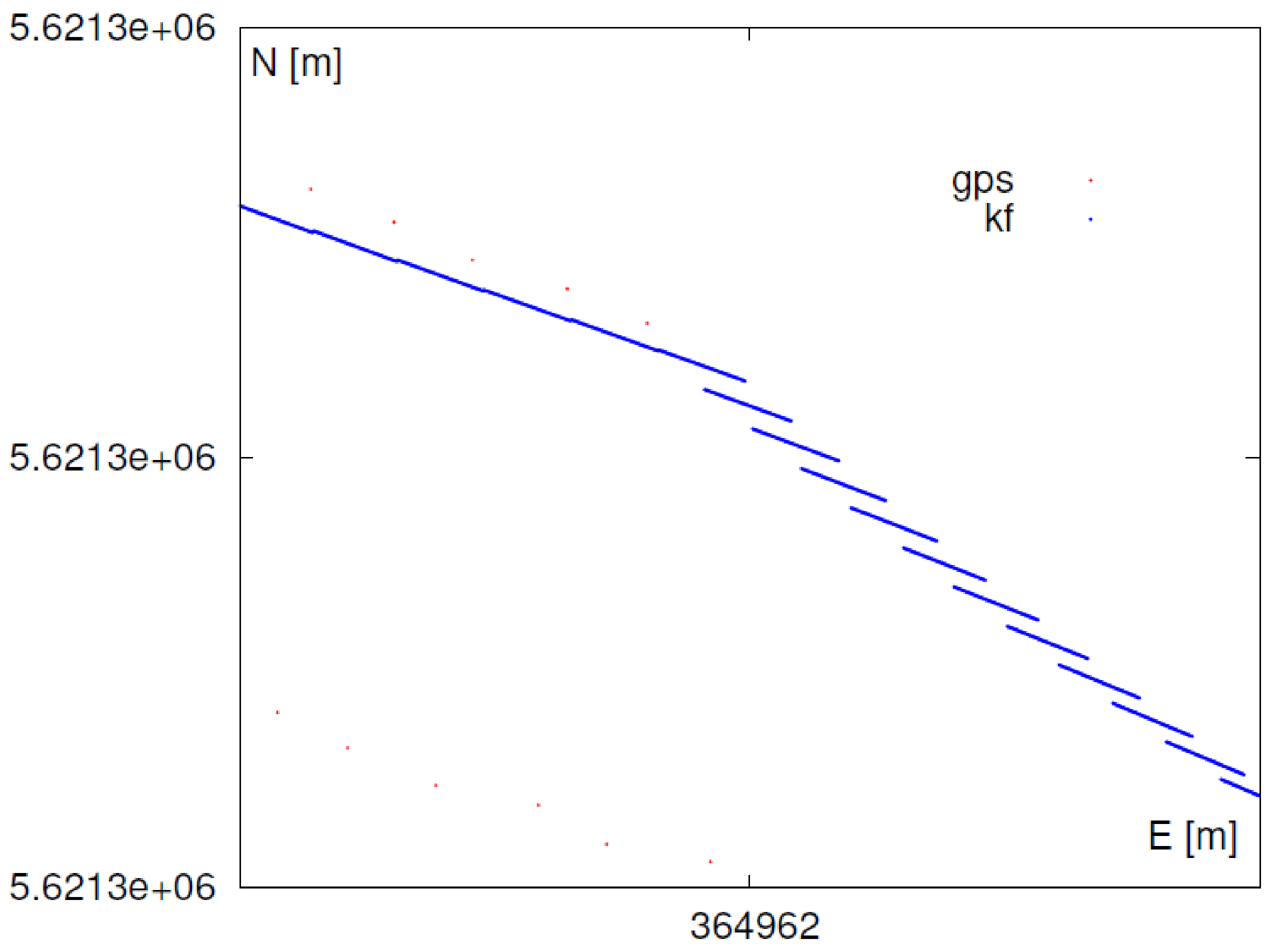
Previous years



Previous years



Previous years



Results & Comparison

Remarks for plots:

- Title
- Equally scaled axis
- Colour visible on the screen (not white & yellow)
- Axis titles (with readable measurement units)
- UTM (N,E) [m] -> reduce UTM to local coordinates for better readability(reduction to t0)
- Multiple plot views (e.g. overview & detailed view)
- Use in-built functions from previous exercise!

Thank you for your attention
Questions or comments?