EE 4301 - Communications Systems 1

Lecture 5

Dr. C.K.W.Seneviratne

Department of Electrical and Information Engineering,

University of Ruhuna

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Frequency Domain Description of AM

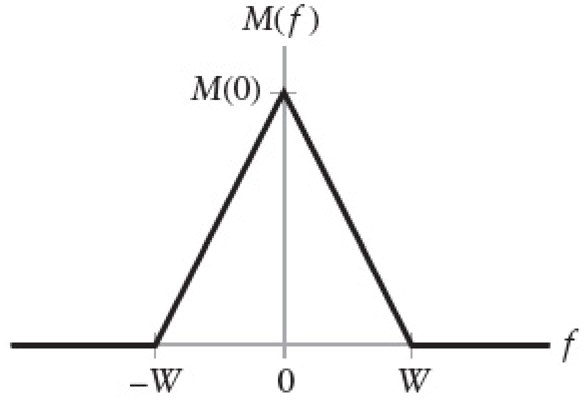
- Consider the time domain signal

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

  - Find the Fourier transform of AM wave *s(t)*.
  - *Answer:*
  - $S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$

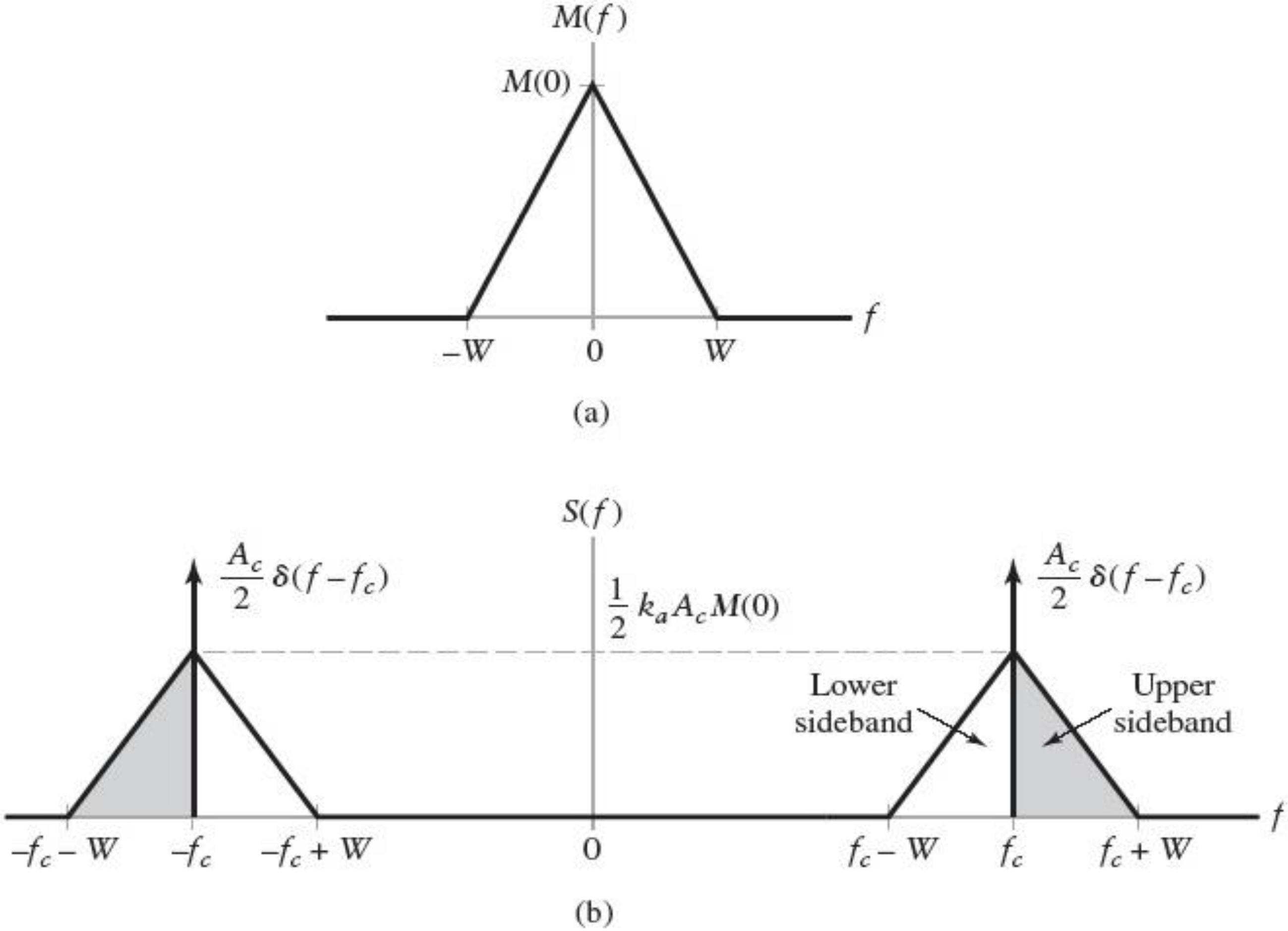
Representation of Spectrum of AM

- Assume that m(t) is a band limited signal in the interval $-W < f_m < W$ and has the following shape
- The bandwidth of the message is W.



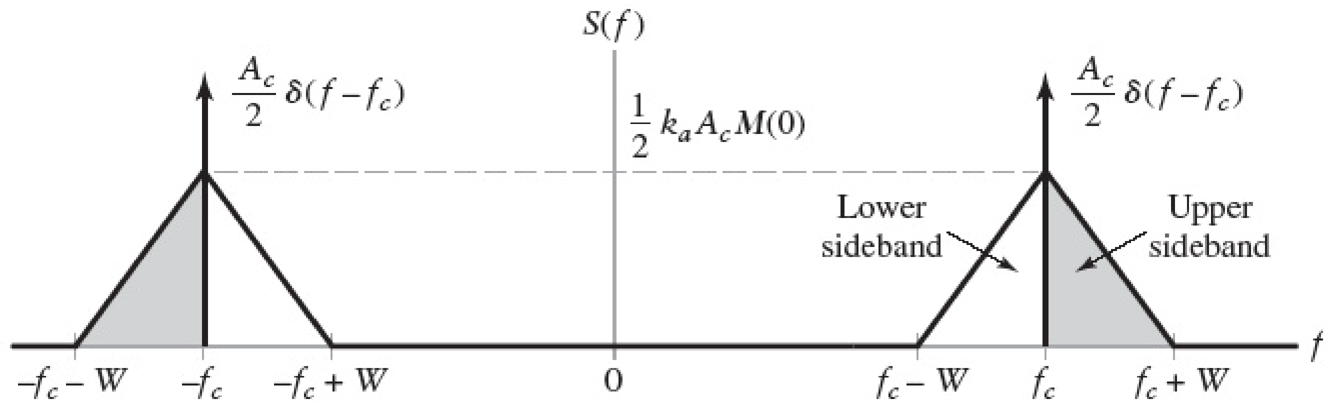
(a)

Representation of Spectrum of AM



(*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of AM wave $s(t)$.

Representation of Spectrum of AM



(b)

- Portion of the spectrum of AM above f_c called as the Upper sideband and the symmetric portion below the f_c called as the Lower sideband.
- Highest frequency of the AM wave is $f_c + W$ and lowest frequency of the AM wave is $f_c - W$.
- The difference among them is called as transmission bandwidth B_T which is equal to the twice of the message bandwidth W.

Conventional AM - Single Tone Modulation

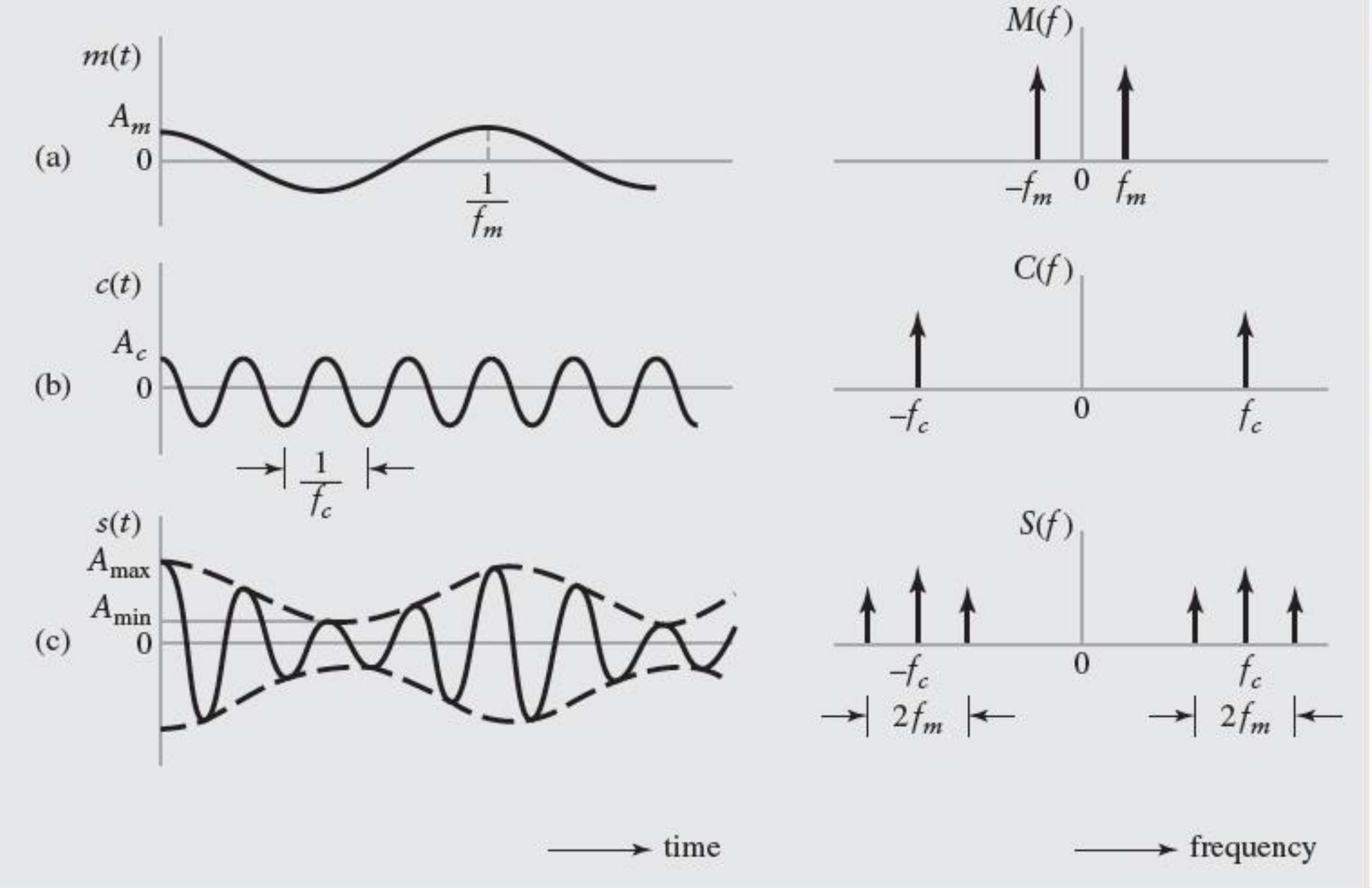
- Consider a modulating wave $m(t) = A_m \cos(2\pi f_m t)$ and carrier wave $c(t) = A_c \cos(2\pi f_c t)$.
  - Find the time domain expression for AM wave.
  - Find the Fourier transform of AM wave.
  - Illustrate the time domain and frequency domain representation of AM wave.
  - Answer

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c A_m k_a}{2} \cos(2\pi(f_c + f_m)t) + \frac{A_c A_m k_a}{2} \cos(2\pi(f_c - f_m)t)$$

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c A_m k_a}{4}[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] + \frac{A_c A_m k_a}{4}[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

Conventional AM - Single Tone Modulation

- Time domain and frequency domain characteristics of amplitude modulation produced by a sinusoidal signal.



Conventional AM - Single Tone Modulation

- Consider the AM wave

$$s(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

where $\mu = k_a A_m$

- Here, μ is called as modulation factor or percentage modulation when it is expressed as a percentage.
- This μ value must be kept below unity to avoid distortion in the envelop.
- When μ is less then unity and the maximum and minimum value of the envelop are A_{max} and A_{min}, we can write

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$$

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Conventional AM - Single Tone Modulation

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2} \cos(2\pi (f_c + f_m) t) + \frac{A_c \mu}{2} \cos(2\pi (f_c - f_m) t)$$

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c \mu}{4}[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] + \frac{A_c \mu}{4}[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

- Consider the sinusoidal modulation based on the following parameters.
  - Carrier amplitude A_c =1, Carrier frequency $f_c = 0.4Hz$ and Modulation frequency $f_m = 0.05Hz$. Discuss the effect of μ when
    - μ = 0.5
    - μ = 1.0
    - μ=2.0

Sideband and Carrier Power of AM

- The average power delivered to a 1 ohm resistor by s(t) comprised of three components
  - Carrier power = $\frac{1}{2}A^2{}_c$
  - Upper side frequency power = $\frac{1}{8}\mu^2 A^2{}_c$
  - Lower side frequency power =$\frac{1}{8}\mu^2 A^2{}_c$
  - Power efficiency = η = $\frac{Total\ Sideband\ Power}{Total\ Power}$ = $\frac{\mu^2}{2+\mu^2}$ x100%

Sideband and Carrier Power of AM

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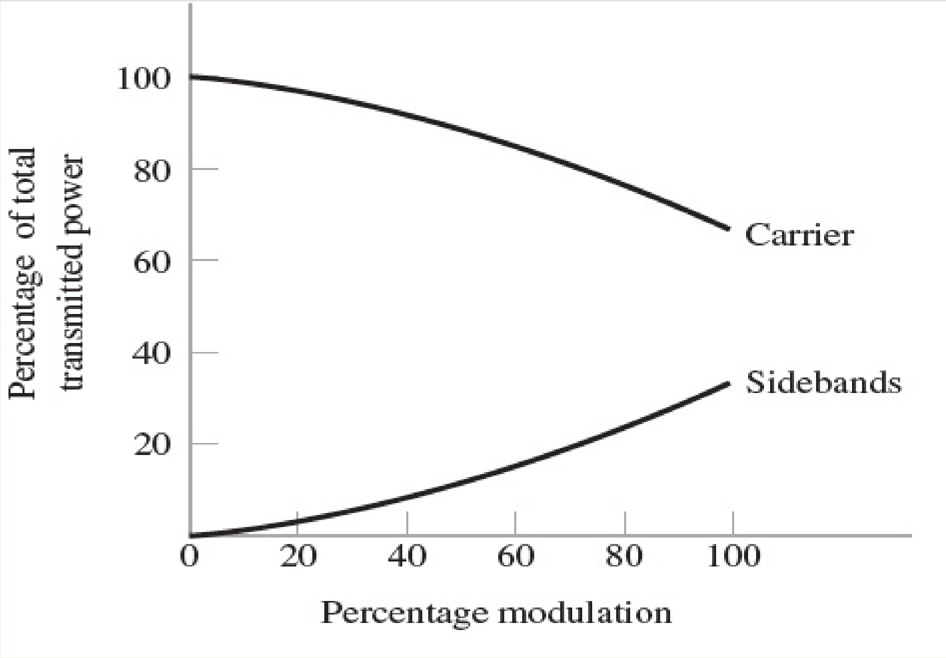

Sideband and Carrier Power of AM

- Example : Determine the power efficiency of AM wave for the tone modulation when

  $a)\ \mu$ =0.5

  $b)\ \mu$ =0.3

Answers :

a) 11.11%

b) 4.3%