Big Bang Nucleosynthesis and the statistical foundations behind it

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Abstract

The standard Big Bang Nucleosynthesis, based on the Maxwell–Boltzmann energy distribution and well-measured nuclear cross-sections, successfully explains the observed abundances of deuterium and helium but shows persistent tension for lithium. This report summarizes the physical conditions and reaction mechanisms of BBN, as well as the impact of adopting Tsallis' statistics. The q-generalized distribution alters the population of particles at different energies, affecting the nuclear reaction rates and element yields. Previous studies suggest that minor deviations from thermal equilibrium can lower the predicted lithium abundance while preserving the overall agreement of standard BBN with observations.

1. Introduction

Big Bang Nucleosynthesis (BBN) is one of the most important processes in cosmology, marking the period during which the first atomic nuclei were formed in the early Universe. It occurred roughly between one second and three minutes after the Big Bang, when the temperature of the Universe fell from about 10^{10} K to10^{8} K [1], where protons and neutrons combined through a network of nuclear reactions to form the lightest elements, most notably hydrogen, deuterium, heluim, and a trace amount of lithium. The relative abundances of these nuclei provide a direct link between nuclear physics and cosmology, offering a powerful test to our understanding of the early Universe.

The standard theory of BBN is based on well-established physics, assuming that the Universe was homogeneous and isotropic, expanding according to the Friedmann equations, and that the reacting particles followed the Maxwell–Boltzmann (M–B) velocity distribution. The predicted abundances of deuterium and helium show remarkable agreement with observations [2], making BBN one of the cornerstones of the Big Bang model.

Despite its overall performance, some unexplained differences still remain between theoretical predictions and astronomical measurements, notably the abundance of lithium, where the observed amount of ^{7}Li is several times lower than predicted by standard BBN calculations [2]. Although many possible explanations have been explored, from stellar depletion to nuclear data uncertainties, one interesting approach is to examine whether the Maxwell–Boltzmann distribution itself is strictly valid under early-universe conditions.

In a rapidly expanding systems which exhibit meta-equilibrium states, heavy correlations between states, presence of long-range interactions, and other non-equilibrium cases, classical Boltzmann–Gibbs statistics may not fully describe the system, therefore Tsallis proposed a generalized, non-extensive statistical mechanics, where the entropy is defined as

$$S_q = k\frac{1 - \sum_i p_i^q}{q - 1}$$

introducing a parameter q that quantifies the degree of non-extensivity [3]. When $q \to 1$, Tsallis statistics reduces to the standard Boltzmann–Gibbs statistics. Applying this to cosmological conditions can lead to modified energy distributions and reaction rates.

In this report, I aim to summarize the key ideas of Big Bang Nucleosynthesis, outline the role of reaction cross-sections and statistical assumptions, and discuss how adopting the Tsallis non-extensive formalism could influence the predicted light-element abundances.

2. Big Bang Nucleosynthesis

About 1 s after the Big Bang, the temperature dropped to around 10^{10} K (approximately 1 MeV), allowing neutrinos (and antineutrinos) to decouple from other particles. At this stage, the Universe was primarily composed of neutrinos, electrons, photons, and a small excess of baryons (specifically neutrons and protons) [1].

A single cosmological parameter, the baryon-to-photon ratio $\eta = n_b/n_\gamma$, determines the overall outcome of nucleosynthesis. Its value, inferred from the cosmic microwave background, is $\eta \approx 6 \times 10^{-10}$ [2]. Once η is known, the entire sequence of nuclear reactions that produced the light elements can be calculated.

The relative numbers of protons and neutrons are determined by the following charged-current weak interactions:

$$\begin{aligned} n + e^+ &\longleftrightarrow p + \bar{\nu}_e \\ p + e^- &\longleftrightarrow n + \nu_e \\ n &\longleftrightarrow p + e^- + \bar{\nu}_e \end{aligned}$$

At temperatures $T > 1\,\mathrm{MeV}$, neutrons and protons are in thermal equilibrium. As the Universe expands, the temperature drops, resulting in more protons than neutrons. Thus, if the weak interaction rates were efficient, then the neutron/proton ratio would continue to drop as the Universe expands and cools. However, the reaction rate per neutron depends on the number density of electron neutrinos, which falls as a result of cosmic expansion. Once this rate becomes smaller than the cosmic expansion rate, the neutron-to-proton ratio "freezes out." As the expansion rate was dominated by radiation, the Friedmann equation gives the expansion rate as

$$\frac{\dot{a}}{a} \equiv H = \sqrt{\frac{8\pi G\rho}{3}} \propto T^2$$

where H is the Hubble parameter and ρ the total energy density. Since the weak interaction rates scale as T^4, they eventually drop below the expansion rate($\propto T^2$) as the Universe cools. The weak interactions thus cease to be effective around $k_\mathrm{B}T \simeq 0.8\,\mathrm{MeV}$ and at this stage, the neutron-to-proton ratio can be calculated to be approximately $n/p \simeq 1/6$, which serves as the initial condition for BBN.

The first bound nucleus to form is deuterium (D), through the reaction$p + n \leftrightarrow \mathrm{D} + \gamma$. At early times, the extremely high photon density destroyed deuterium almost immediately after its formation. Since baryons were vastly outnumbered by photons, even the high-energy tail contained enough photons with energies exceeding the deuteron binding energy to dissociate any newly formed deuterons. As a result, deuterium could not survive until the photon temperature dropped well below the binding energy, delaying further nucleosynthesis. This is known as the 'deuteron bottleneck'. The creation and destruction rates of Equation are equal when the Universe is at a temperature of $k_B T \approx 0.06\mathrm{MeV}$, and at this point, the production of deuterons begins in earnest. Now, a rapid series of fusion reactions followed:

$$\begin{aligned} d + p &\longrightarrow {}^3\mathrm{He} + \gamma & \qquad d + d &\longrightarrow {}^3\mathrm{He} + n \\ d + d &\longrightarrow {}^3\mathrm{H} + p & {}^3\mathrm{He} + n &\longrightarrow {}^3\mathrm{H} + p \\ d + d &\longrightarrow {}^4\mathrm{He} + \gamma & d +^3 He &\longrightarrow {}^4\mathrm{He} + p \\ d +^3 H &\longrightarrow {}^4\mathrm{He} + n \end{aligned}$$

once a considerable amount of ^{4}He is produced, several mass-7 nuclides aren also formed from the following reactions:

$$\begin{aligned} {}^3\mathrm{H} + {}^4\mathrm{He} &\longrightarrow {}^7\mathrm{Li} + \gamma & \qquad {}^3\mathrm{H} + {}^4\mathrm{He} &\longrightarrow {}^7\mathrm{Be} + \gamma \\ {}^7\mathrm{Be} + n &\longrightarrow {}^7\mathrm{Li} + p & {}^7\mathrm{Li} + p &\longrightarrow 2\ {}^4\mathrm{He} \end{aligned}$$

Nearly all the available neutrons was incorporated into ^{4}He, yielding a mass fraction of about $Y_p \approx 0.25$. Smaller fractions of deuterium and ^{3}He survived, while a small amount of ^{7}Liwas produced primarily through the decay of ^{7}Be after the end of BBN.

When the predicted abundances of these elements are compared with observational data from metal-poor stars and intergalactic gas clouds the agreement for deuterium and helium is excellent, providing strong evidence for standard BBN [2]. However, the observed primordial lithium abundance is significantly lower than predicted, this is a long-standing discrepancy known as the "lithium problem." While several explanations have been proposed, including astrophysical depletion and nuclear uncertainties, there is also a question of whether the assumed equilibrium Maxwell–Boltzmann distribution really describes particle velocities in the extreme conditions of the early Universe. This issue connects directly to the calculation of nuclear reaction rates, which depend on the product of the cross-section and the energy distribution of the reacting particles. The following section, therefore, reviews the concept of nuclear cross-sections and explains how they are used to determine reaction rates in the standard BBN model.

3. Nuclear Reaction Cross-Sections and Rates

Nucleosynthesis, as these rates determine how quickly different species are converted into one another. For a reaction between two nuclei i and j, the quantity of interest is the reaction rate per particle pair, given by

$$\langle \sigma v \rangle = \int_0^\infty \sigma(E)\, v(E)\, f(E)\, dE,$$

where $\sigma(E)$ is the energy-dependent cross-section, $v(E)$ is the relativevelocity, and $f(E)$ is the normalized distribution of particle energies[2]. In the standard approach, $f(E)$ follows theMaxwell–Boltzmann distribution,

$$f(E)\, dE = 2\sqrt{\frac{E}{\pi (k_\mathrm{B} T)^3}} \exp\left(-\frac{E}{k_\mathrm{B} T}\right) dE,$$

which assumes complete thermal equilibrium in a homogeneous plasma.

The nuclear cross-section $\sigma(E)$ represents the probability that a reaction will occur at a given energy. For charged-particle reactions, the cross-section decreases sharply at low energies due to the Coulomb barrier. To express this behaviour moreconveniently, it is written in terms of the astrophysical S-factor:

$$\sigma(E) = \frac{S(E)}{E} \exp[-2\pi\eta(E)]\,,$$

where $\eta(E) = Z_i Z_j e^2/(\hbar v)$ is the Sommerfeld parameter[4]. The exponential term accounts for the quantum-mechanical tunneling probability through the Coulomb barrier, while $S(E)$ varies slowly with energy and encapsulates the underlying nuclear physics.

The integrand of the rate equation contains two competing effects: the exponential decay of the Maxwell–Boltzmann distribution at high energy, and the tunneling probability that rises with energy. Their product produces a sharply peaked function known as the *Gamow peak*, which identifies the most effective energy range for reactions in a thermal plasma [1]. For typical BBN temperatures of$0.1 - 1\,\mathrm{MeV}$, the relevant energies lie within a few tens of keV—the so-called *Gamow window*. Experimental measurements of cross-sections in this energy range are therefore crucial for accurate BBN predictions.

In the standard formalism, the assumption of the Maxwell–Boltzmann energy distribution allows the rate integral to be evaluated analytically or numerically for any given reaction. The resulting rates are then input into large reaction-network codes that track the abundances of light elements as the Universe expands and cools[2]. Because the energy distribution enters directly into the integrand, any deviation from the Maxwell–Boltzmann form—such as that predicted by non-extensive Tsallis statistics—can lead to modified reaction rates and potentially different element abundances.

4. Motivation for Using Tsallis Statistics

The standard Big Bang Nucleosynthesis assumes the primordial plasma to be in complete thermal equilibrium and that the particle velocities follow the Maxwell–Boltzmann distribution. This assumption is valid when collisions occur frequently enough to maintain equilibrium and when the system is extensive—that is, the total entropy is additive over subsystems. However, the early Universe was a rapidly expanding high energy plasma with long-range electromagnetic interactions, density fluctuations, and finite relaxation times, which could produce small but significant departures from ideal equilibrium [4]. Under these conditions, the classical Boltzmann–Gibbs statistics may no longer fully capture the microscopic dynamics of the particles.

Tsallis introduced a non-extensive entropy function to generalize the classical statistical mechanics defined as

$$S_q = k\,\frac{1 - \sum_i p_i^q}{q - 1},$$

where p_i denotes the probability of a microstate and q is a real parameter that characterizes the degree of non-extensivity [3]. For $q = 1$, the standard Boltzmann–Gibbs entropy is recovered. Values of $q > 1$ correspond to systems with sub-additive entropy, often associated with stronger correlations or long-range forces, while $q < 1$ corresponds to super-additive behaviour. The resulting energy distribution takes the form

$$f_q(E)\,dE \propto E^{1/2}\left[1 - (1 - q)\frac{E}{k_{\mathrm{B}}T}\right]^{\frac{1}{q-1}} dE,$$

which smoothly approaches the Maxwell–Boltzmann distribution when $q \to 1$.

5. Discussion

A simple comparison between the Maxwell–Boltzmann and Tsallis distributions, shown in Figure 1, illustrates this effect. When $q > 1$, the distribution falls off more rapidly at high energies, while for $q < 1$, the high-energy tail is enhanced. Even minor deviations from $q = 1$ on the order of 10^{-2} can produce noticeable changes in predicted abundances.

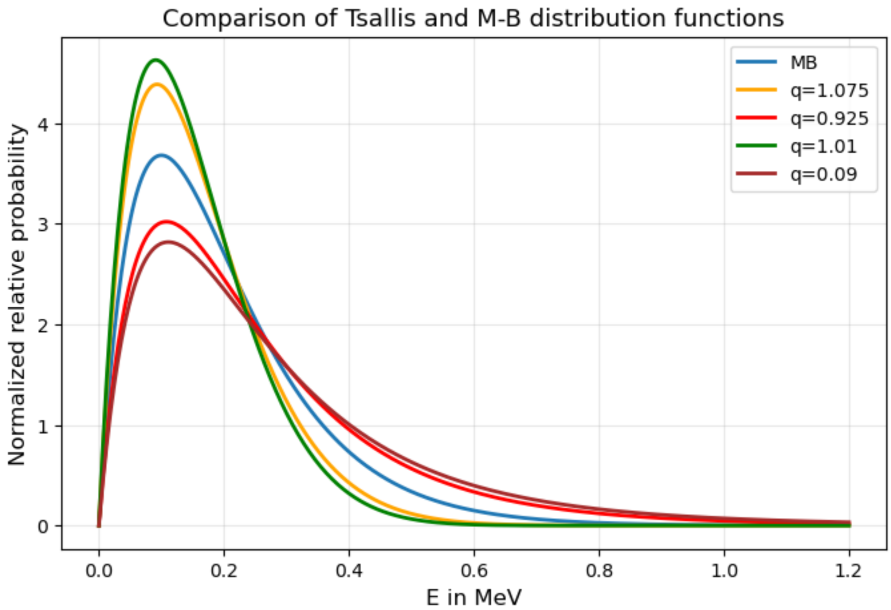

Figure 1: Comparison between Maxwell–Boltzmann ($q = 1$) and Tsallis distributions for slightly different values of q. The high-energy tail becomes suppressed for $q > 1$ and enhanced for $q < 1$.

Introducing Tsallis statistics modifies the way reaction rates are calculated. The non-extensive parameter q adjusts the relative weight of low- and high-energy particles, there by altering the number of collisions occurring within the Gamow window [4]. Even minor deviations ($|q - 1| \lesssim 0.01$) can significantly shift this window and modify the effective reaction rates. For $q > 1$, the distribution falls off faster than the Maxwell–Boltzmann case, reducing the number of energetic collisions and lowering reaction rates, while $q < 1$ enhances the high-energy tail, increasing reaction rates.

Numerical studies using the non-extensive framework show that for q values close to unity, most light-element abundances remain consistent with observational constraints [4]. However, small shifts in q can slightly reduce the predicted ^{7}Li abundance toward observed levels, with only minor effects on deuterium and helium. This sensitivity highlights how even small deviations from perfect equilibrium may have observable cosmological consequences.

While Tsallis statistics provide a physically motivated way to explore non-equilibrium effects in the early Universe, several limitations remain. The parameter q is phenomenological and not derived from first principles, and the early Universe was likely too well-coupled for large departures from equilibrium. Therefore, the Tsallis framework should be regarded as a tool for testing the robustness of standard BBN predictions rather than a replacement for them [4]. Overall, incorporating non-extensive statistics offers valuable insight into how minor statistical deviations can influence nucleosynthesis outcomes and demonstrates the sensitivity of cosmological predictions to underlying microscopic physics.

6. Conclusion

This report outlined the essential ideas behind BBN and explored how the use of non-extensive Tsallis statistics can modify the statistical assumptions that underlie reaction rate calculations.

Replacing the Maxwell–Boltzmann distribution with its q-generalized counterparts slightly alters the population of high-energy particles and, in turn, the nuclear reaction rates. Results from previous studies [4] suggest that small departures from $q = 1$ can marginally reduce the predicted lithium abundance without significantly affecting deuterium or helium. Though not a complete solution, this approach highlights the sensitivity of BBN to non-equilibrium effects and offers a useful framework to test the limits of the standard model.

Future work may involve connecting the Tsallis parameter q to physical mechanisms. More detailed numerical simulations and improved nuclear data could help clarify whether non-extensive effects played a measurable role in primordial nucleosynthesis. Regardless of the outcome, examining BBN through the lens of generalized statistics deepens our understanding of the interplay between nuclear physics, thermodynamics, and cosmology.

References

[1] R. Cooke, *Big Bang Nucleosynthesis*, Durham University, 2024.

[2] B. D. Fields,"The Primordial Lithium Problem," *Annual Review of Nuclear and Particle Science*, vol. 61, pp. 47–68, 2011,doi:10.1146/annurev-nucl-102010-130445.

[3] C. Tsallis, *Possible Generalization of Boltzmann–Gibbs Statistics*, Journal of Statistical Physics, 52(1–2), 479–487 (1988).

[4] M. Agrawal, *Nuclear Reaction Rate Theory with Tsallis' Statistics and its Application in BBN*, B.Tech. Thesis, IIT Roorkee (2019).