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Q1

1. Domain & Range

(a) $f(x) = \sqrt{1-x^2}$

$1 - x^2 \geq 0$ Domain : $[-1, 1]$

$|x| \leq 1$

Range $[0, 1]$

(b) $\frac{x}{x+1}$

$x + 1 \neq 0$ Domain : $R - \{-1\}$

$x \neq -1$

Range : $R - \{1\}$

$y = \frac{x}{x+1}$

$y(x+1) = x$

$yx + y = x$

$y = x - yx$

$y = x(1-y)$

$x = \frac{y}{1-y}$

$1 - y \neq 0$

$y \neq 1$

(c) $f(x) = \lfloor x \rfloor$ G.I.F.

Domain $\mathbb{R}$

Range $\mathbb{Z}$

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2.

one to one (injective)
↳ different inputs give different outputs.

onto (surjective)
↳ every possible output in codomain is covered.

bijective
↳ both injective and surjective.

(a) $F(x) = x^3 + 1$ → Injective ✓
Onto ✓
Bijective ✓

(b) $F(x) = x^2$ → Injective ✗
Onto ✗
Bijective ✗

(c) $F(x) = 2x - 1$ → Injective ✓
onto ✓
bijective ✓

3. even and odd functions.

$F(-x) = F(x)$ → even
$F(-x) = -F(x)$ → odd

(a) $F(x) = x^4 - 3x^2$
$F(-x) = x^4 - 3x^2$ ∴ even

(b) $F(x) = x^3 - x$
$F(-x) = -x^3 + x$ ∴ odd.
$= -(x^3 - x)$
$= -F(x)$

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(c) $F(x) = e^{x}$
$F(-x) = e^{-x}$ $\therefore$ neither even nor odd.

4. (a) $F(x) = |x|$

Domain $\mathbb{R}$
Range $[0, \infty)$
Nature decreasing on $(-\infty, 0]$
increasing on $[0, \infty)$
continuous everywhere.

(b) $F(x) = [x]$

Domain $\mathbb{R}$
Range $\mathbb{Z}$
Nature Step function
discontinuous at integers.
constant on intervals
$[n, n+1)$
$n \in \mathbb{Z}$

(c) $F(x) = \log_2 x$

Domain $x > 0$
Range $\mathbb{R}$
Nature. Strictly increasing,
continuous on $(0, \infty)$
Vertical asymptote at $x = 0$.

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5. $F(x) = x^2$

Domain All real no.
Range $[0, \infty)$
Many to one $F(1) = F(-1) = 1$

6. $f(x, y) = \sqrt{y - x^2}$

$y - x^2 \geq 0$
$y \geq x^2 \rightarrow$ parabola $y = x^2$

Domain $\{(x, y) \in \mathbb{R}^2 . y \geq x^2\}$

7. $F(x, y) = x^2 + y^2$

$c = 1, 4, 9$. $x^2 + y^2 = c$

$c = 1 \rightarrow$ radius 1
$c = 4 \rightarrow$ radius 2
$c = 9 \rightarrow$ radius 3

radius $\sqrt{c}$

8. $F(x, y) = x^4 - x^2 y^2 + y^4$

$F_x = 4x^3 - 2xy^2$
$F_y = -2x^2 y + 4y^3$

$F_{xx} = 12x^2 - 2y^2$
$F_{yy} = -2x^2 + 12y^2$

$F_{xy} = -4xy$

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$F_x(-1,1) = -2$
$F_y(-1,1) = 2$
$F_{xx}(-1,1) = 10$
$F_{yy}(-1,1) = 10$
$F_{xy}(-1,1) = 4$

9. $F(x,y) = \ln(x^2+y^2)$:

$$F_x = \frac{2x}{x^2+y^2} \qquad F_y = \frac{2y}{x^2+y^2}$$

$$F_{xy} = 2x \cdot \frac{\partial}{\partial y}\left(\frac{1}{x^2+y^2}\right)$$

$$= 2x\left(-\frac{2y}{(x^2+y^2)^2}\right)$$

$$= -\frac{4xy}{(x^2+y^2)^2}$$

$$F_{yx} = \frac{\partial}{\partial x} F_y = 2y\left(\frac{-2x}{(x^2+y^2)^2}\right)$$

$$= -\frac{4xy}{(x^2+y^2)^2}$$

$\therefore F_{xy} = F_{yx}$

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10. $F: R^2 \to R$.

not diff. at origin.
continuous at origin

$F(x) = \sqrt{|x|} + \sqrt{|y|}$