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Unit – 2 \rightsquigarrow Recurrence Relation and Propositional Logic

Unit – 2.1 \rightsquigarrow Recurrence Relation

Method – 2 \rightsquigarrow Solution of Linear Recurrence Relation using Undetermined Coefficient Method

Examples of Method-2: Solution of Linear Recurrence Relation using Undetermined Coefficient Method

A	1	<p>Solve the recurrence relations $a_n = 6a_{n-1}$ using the method of undetermined coefficients.</p> <p>Answer: $C_1(6)^n$</p>
A	2	<p>Solve the recurrence relations $a_n = a_{n-1} + 2a_{n-2}$ using the method of undetermined coefficients.</p> <p>Answer: $C_1(2)^n + C_2(-1)^n$</p>
A	3	<p>Solve the recurrence relations $a_n = 9a_{n-1} - 27a_{n-2} + 27a_{n-3}$ using the method of undetermined coefficients.</p> <p>Answer: $(C_1 + C_2n + C_3n^2)(3)^n$</p>
B	4	<p>Solve the following recurrence relations using the method of undetermined coefficients:</p> <p>(1) $a_n - 7a_{n-1} + 10a_{n-2} = 0$; $a_0 = 0, a_1 = 3$ (2) $a_n + 2a_{n-1} - 15a_{n-2} = 0$; $a_0 = 0, a_1 = 1$ (3) $a_n = 2a_{n-1} - a_{n-2}$; $a_1 = 1.5, a_2 = 3$ (4) $a_n - 4a_{n-1} + 4a_{n-2} = 0$; $a_0 = 1, a_1 = 6$</p> <p>Answer : (1) $a_n = (5)^n - (2)^n$ (2) $a_n = \frac{1}{8}(3)^n - \frac{1}{8}(-5)^n$ (3) $a_n = (1.5n)(1)^n$ (4) $a_n = (1 + 2n)(2)^n$</p>

Unit -2 Recurrence Relation and Propositional Logic

C	5	Find the unique solution to recurrence relation with the given initial conditions: $a_n = 10a_{n-1} - 32a_{n-2} + 32a_{n-3} ; n \geq 3, a_0 = 5, a_1 = 18, a_2 = 76$ Answer: $a_n = (2 + n)(4)^n + 3(2)^n$
B	6	Find solution to recurrence relation with the given initial conditions: $a_n = 5a_{n-1} + 3 ; n \geq 1, a_1 = 2$ Answer: $a_n = \frac{11}{20}(5)^n - \frac{3}{4}$
B	7	Solve the given recurrence relation using the method of undetermined coefficients $a_n - a_{n-1} - 2a_{n-2} = n^2 ; n \geq 2$. Answer: $a_n = C_1(-1)^n + C_2(2)^n - 4 - \frac{5}{2}n - \frac{1}{2}n^2$
B	8	Solve the given recurrence relation using the method of undetermined coefficients $a_n = 2a_{n-1} + 3a_{n-2} + 5^n ; n \geq 2, a_0 = -2, a_1 = 1$. Answer: $a_n = -\frac{17}{24}(-1)^n - \frac{27}{8}(3)^n + \frac{25}{12}(5)^n$
C	9	Solve the given recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = n + 3^n$ using undetermined coefficients method. Answer : $a_n = (C_1 + C_2n)(2)^n + 4 + n + (3)^{n+2}$
C	10	Solve the given recurrence relation using the method of undetermined coefficients $a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3n ; n \geq 2, a_0 = 2, a_1 = -2$. Answer: $a_n = -5(2)^n + \frac{7}{4}(3)^n - n2^{n+1} + \frac{21}{4} + \frac{3}{2}n$

Unit – 2.2 \rightsquigarrow Propositional Logic

Method 1 \rightsquigarrow Statements

Examples of Method-1: Statements

A	1	<p>Which of the following are proposition?</p> <p>p : Do not go.</p> <p>q : $4 > 3$</p> <p>r : 4 is an integer.</p> <p>Answer: q and r are proposition</p>
A	2	<p>Which of the following are not proposition?</p> <p>p : $x \in \{8, 9, 10, 11\}$</p> <p>q : $2 > 3$</p> <p>r : x is perfect square.</p> <p>Answer: p and r are not proposition</p>
A	3	<p>Which of the following are proposition?</p> <p>p : grass is green.</p> <p>q : $2 + 5 = 5$</p> <p>s : Javaharlal Nehru is the first prime minister of india.</p> <p>Answer: p, q and s are proposition</p>
A	4	<p>Which of the following are proposition?</p> <p>p : Look Out!</p> <p>q : How far is it to the next town?</p> <p>r : $x + 2 = 2x$</p> <p>s : $x + 2 = 2x$ when $x = -2$</p> <p>Answer: s is a proposition</p>

A	5	<p>Which of the following are proposition?</p> <p>p : 5 is a Prime Number.</p> <p>q : 8 is an odd number.</p> <p>r : Did you lock the door?</p> <p>s : Happy Birthday!</p> <p>Answer: p and q are proposition</p>
A	6	<p>Assign the truth value to the following sentences if it is proposition.</p> <p>p : 4 is an integer.</p> <p>q : $4 > 7$</p> <p>r : India Won cricket world cup of 1983.</p> <p>s : You are innocent.</p> <p>Answer: p : T, q : F, r : T,</p> <p>s : No truth value as it is not a proposition</p>
A	7	<p>Assign the truth value to the following sentences if it is proposition.</p> <p>p : Water boils at 100 degrees celsius at sea level.</p> <p>q : All birds can fly.</p> <p>r : Sum of angles of triangle is 360°.</p> <p>s : The sky is blue.</p> <p>Answer: p : T, q : F, r : F,</p> <p>s : No truth value as it is not a proposition</p>
A	8	<p>Assign the truth value to the following sentences if it is proposition.</p> <p>p : Dublin is a capital of Ireland.</p> <p>q : $5 - 11 = 7$</p> <p>r : 2 is an even number.</p> <p>s : Kuala Lumpur is the capital of Malasia.</p> <p>t : You and me.</p> <p>Answer: p : T, q : F, r : T, s : T,</p> <p>t : No truth value as it is not a proposition</p>

A	9	<p>Assign the truth value to the following sentences if it is proposition.</p> <p>p : How are you?</p> <p>q : Bangkok is the capital of Thailand.</p> <p>r : September has 30 days.</p> <p>s : A square has five sides.</p> <p>t : A cow has 12 legs.</p> <p>Answer: p : No truth value as it is not a proposition, q : T, r : F, s : F, t : F</p>
A	10	<p>Assign the truth value to the following sentences if it is proposition.</p> <p>p : A triangle has 3 sides.</p> <p>q : 57 is a prime number.</p> <p>r : $12 + (-4) = 8$</p> <p>s : London is the capital of England.</p> <p>Answer: p : T, q : F, r : T, s : T</p>

Method 2 \Rightarrow Logical Connectives

Examples of Method-2.1: Conjunction

A	1	<p>Find the conjunction of propositions p and q.</p> <p>Where, p : Shyam is happy.</p> <p>q : Shyam is rich.</p> <p>Answer: Shyam is happy and he is rich.</p>
A	2	<p>Find the conjunction of propositions p and q.</p> <p>Where, p : Today is Monday.</p> <p>q : It is raining today.</p> <p>Answer: Today is Monday and it is raining.</p>
A	3	<p>Define truth value for the following conjunction:</p> <p>(1) $4 + 2 = 6$ and Virat Kohli won the gold medal in Olympic.</p> <p>(2) Sardar Patel was born in Guajrat and he was a lawyer.</p> <p>Answer: (1) F, (2) T</p>
A	4	<p>Define truth value for following conjunction:</p> <p>(1) All rational numbers are complex numbers and $2 + 8 = 10$.</p> <p>(2) $x + 2 = 6$ if $x = 4$ and $x + 2 = 6$ if $x = 6$.</p> <p>Answer: (1) F, (2) F</p>
A	5	<p>Determine truth value for following conjunction:</p> <p>(1) Paris is in France and London is in England.</p> <p>(2) $x + 3 = 6$ if $x = 8$ and $x + 5 = 6$ if $x = 10$</p> <p>Answer: (1) T, (2) F</p>

B

6

Construct the truth table for $(q \wedge r)$.**Answer:**

q	r	$(q \wedge r)$
T	T	T
T	F	F
F	T	F
F	F	F

Examples of Method-2.2: Disjunction

A	1	<p>Find the disjunction of propositions p and q.</p> <p>Where, p : Mohan is scientist.</p> <p>q : Shyam is scientist.</p> <p>Answer: Mohan is scientist or Shyam is scientist.</p>
A	2	<p>Find the disjunction of propositions p and q.</p> <p>Where, p : Today is Monday.</p> <p>q : It is raining today.</p> <p>Answer: Today is Monday or it is raining .</p>
A	3	<p>Define truth value for following disjunction:</p> <p>(1) Tata group was title sponsor of IPL 2023 or Amazon was title sponsor of IPL 2023.</p> <p>(2) Penguin can fly or Ostrich can fly.</p> <p>Answer: (1) T, (2) F</p>
A	4	<p>Define truth value for following disjunction:</p> <p>(1) Square root of negative number is natural number or square root of negative number is real number.</p> <p>(2) An acute angle measures less than 90° or right-angle measures less than 90°.</p> <p>Answer: (1) F, (2) T</p>

Examples of Method-2.3: Negation

A	1	<p>Write negation for each of the following propositions:</p> <p>p : Robin is present.</p> <p>q : He has money to purchase food.</p> <p>r : She will accept my proposal.</p> <p>s : Everybody will talk during the program.</p> <p>Answer: $\neg p$: Robin is not present.</p> <p>$\neg q$: He has no money to purchase food.</p> <p>$\neg r$: She will never accept my proposal.</p> <p>$\neg s$: Nobody will talk during the program.</p>
A	2	<p>Write negation for each of the following propositions:</p> <p>p : James and Jack mentioned this place in their tutoring place.</p> <p>q : She can make cookies and pasta for all of us.</p> <p>r : I am going to school.</p> <p>Answer: $\neg p$: Neither James nor Jack mentioned this place in their tutoring place.</p> <p>$\neg q$: She can neither make cookies nor pasta for all of us.</p> <p>$\neg r$: I am not going to school.</p>

B	3	<p>Translate the following symbolic form into sentences:</p> <p>(1) $\neg r \wedge q$</p> <p>(2) $\neg q \vee p$</p> <p>(3) $\neg p \wedge \neg q$</p> <p>(4) $p \vee \neg r$</p> <p>Where, p : Today I am not happy. q : The river is clean. r : $3 > 2$</p> <p>Answer: (1) $3 \not> 2$ and the river is clean.</p> <p>(2) The river is not clean or today i am not happy.</p> <p>(3) Today i am happy and the river is not clean.</p> <p>(4) Today I am not happy or $3 \not> 2$.</p>															
B	4	<p>Construct the truth table for compound propositions $p \vee \neg q$.</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$p \vee \neg q$</th></tr> </thead> <tbody> <tr> <td>T</td><td>T</td><td>T</td></tr> <tr> <td>T</td><td>F</td><td>T</td></tr> <tr> <td>F</td><td>T</td><td>F</td></tr> <tr> <td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	p	q	$p \vee \neg q$	T	T	T	T	F	T	F	T	F	F	F	T
p	q	$p \vee \neg q$															
T	T	T															
T	F	T															
F	T	F															
F	F	T															
B	5	<p>Construct the truth table for compound propositions $\neg(\neg p \vee \neg q)$.</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$\neg(\neg p \vee \neg q)$</th></tr> </thead> <tbody> <tr> <td>T</td><td>T</td><td>T</td></tr> <tr> <td>T</td><td>F</td><td>F</td></tr> <tr> <td>F</td><td>T</td><td>F</td></tr> <tr> <td>F</td><td>F</td><td>F</td></tr> </tbody> </table>	p	q	$\neg(\neg p \vee \neg q)$	T	T	T	T	F	F	F	T	F	F	F	F
p	q	$\neg(\neg p \vee \neg q)$															
T	T	T															
T	F	F															
F	T	F															
F	F	F															

Examples of Method-2.4: Conditional

B	1	<p>Express the following statements in symbolic form:</p> <p>For, p : It is cold.</p> <p>q : It is raining.</p> <p>(1) If it is cold, then it is raining.</p> <p>(2) If it is raining, then it is cold.</p> <p>(3) If it is not raining, then it is not cold.</p> <p>(4) If it is not cold, then it is raining.</p> <p>Answer: (1) $p \rightarrow q$, (2) $q \rightarrow p$, (3) $\neg q \rightarrow \neg p$, (4) $\neg p \rightarrow q$</p>
B	2	<p>Express the following statements in symbolic form:</p> <p>For, p : The material is interesting.</p> <p>q : The exercise are challenging.</p> <p>r : The course is enjoyable.</p> <p>(1) If the material is interesting, then the exercise are challenging and conversely.</p> <p>(2) If material is not interesting and exercise are not challenging, then the course is not enjoyable.</p> <p>Answer: (1) $(p \rightarrow q) \wedge (q \rightarrow p)$, (2) $(\neg p \wedge \neg q) \rightarrow \neg r$</p>
B	3	<p>Express the following compound propositions as an English sentence by using p, q :</p> <p>Where, p : It is below freezing.</p> <p>q : It is snowing.</p> <p>(1) $q \rightarrow p$</p> <p>(2) $\neg q \rightarrow \neg p$</p> <p>(3) $\neg p \rightarrow \neg q$</p> <p>Answer: (1) If It is snowing, then it is below freezing.</p> <p>(2) If It is not snowing, then it is not below freezing.</p> <p>(3) If It is not below freezing, then it is not snowing.</p>

B	4	<p>Determine whether the following conditional statements are true or false:</p> <p>(1) If $1 + 1 = 3$, then $2 + 3 = 5$.</p> <p>(2) If $1 + 1 = 3$, then dogs can fly.</p> <p>(3) If $1 + 1 = 2$, then cats can fly.</p> <p>(4) If $2 + 2 = 4$, then $1 + 2 = 3$.</p> <p>Answer: (1) T, (2) T, (3) F, (4) T</p>															
B	5	<p>Construct the truth table for $(p \vee \neg q) \rightarrow q$.</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$(p \vee \neg q) \rightarrow q$</th></tr> </thead> <tbody> <tr> <td>T</td><td>T</td><td>T</td></tr> <tr> <td>T</td><td>F</td><td>F</td></tr> <tr> <td>F</td><td>T</td><td>T</td></tr> <tr> <td>F</td><td>F</td><td>F</td></tr> </tbody> </table>	p	q	$(p \vee \neg q) \rightarrow q$	T	T	T	T	F	F	F	T	T	F	F	F
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T	T	T															
T	F	F															
F	T	T															
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B	6	<p>Construct the truth table for $(p \rightarrow q) \vee (\neg p \rightarrow q)$.</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$(p \rightarrow q) \vee (\neg p \rightarrow q)$</th></tr> </thead> <tbody> <tr> <td>T</td><td>T</td><td>T</td></tr> <tr> <td>T</td><td>F</td><td>T</td></tr> <tr> <td>F</td><td>T</td><td>T</td></tr> <tr> <td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	p	q	$(p \rightarrow q) \vee (\neg p \rightarrow q)$	T	T	T	T	F	T	F	T	T	F	F	T
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T	T	T															
T	F	T															
F	T	T															
F	F	T															
B	7	<p>Express the inverse, converse and contrapositive statements of conditional statement given below:</p> <p>"If a man is a gentleman, then he is considerate of others".</p> <p>Answer: (1) Converse: "If man is considerate of others, then he is a gentleman".</p> <p>(2) Inverse: "If man is not a gentleman, then he is not considerate of others".</p> <p>(3) Contrapositive: "If man is not considerate of others, then he is not a gentleman".</p>															

B	8	<p>Express the inverse and contrapositive statements of converse statement given below:</p> <p>"If a steel rod stretches, then it has been heated".</p> <p>Answer: (1) Inverse: "If a steel rod does not stretch, then it has not been heated".</p> <p>(2) Contrapositive: "If it has not been heated, then steel rod does not stretch".</p>
B	9	<p>Express the inverse and contrapositive statements of conditional statement given below:</p> <p>"If x is rational, then x is real".</p> <p>Answer: (1) Inverse: "If x is not rational, then x is not real"</p> <p>(2) Contrapositive: "If x is not real, then x is not rational"</p>

Examples of Method-2.5: Biconditional

A	1	<p>Determine whether each of the following biconditional statements is true or false:</p> <p>(1) $1 + 1 = 3$ if and only if $2 + 3 = 5$.</p> <p>(2) $1 + 1 = 3$ if and only if dogs can fly.</p> <p>(3) $1 + 1 = 2$ if and only if cats can fly.</p> <p>(4) $2 + 2 = 4$ if and only if $1 + 2 = 3$.</p> <p>Answer: (1)F , (2) T, (3) F, (4) T</p>															
B	2	<p>Construct the truth table for $(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$.</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$</th></tr> </thead> <tbody> <tr> <td>T</td><td>T</td><td>T</td></tr> <tr> <td>T</td><td>F</td><td>T</td></tr> <tr> <td>F</td><td>T</td><td>T</td></tr> <tr> <td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	p	q	$(p \rightarrow q) \Leftrightarrow (\neg p \vee q)$	T	T	T	T	F	T	F	T	T	F	F	T
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T	T	T															
T	F	T															
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C	3	<p>Construct the truth table for $(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$.</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$</th></tr> </thead> <tbody> <tr> <td>T</td><td>T</td><td>T</td></tr> <tr> <td>T</td><td>F</td><td>T</td></tr> <tr> <td>F</td><td>T</td><td>T</td></tr> <tr> <td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	p	q	$(\neg p \Leftrightarrow \neg q) \Leftrightarrow (p \Leftrightarrow q)$	T	T	T	T	F	T	F	T	T	F	F	T
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C	4	<p>Construct the truth table for $(p \Leftrightarrow q) \Leftrightarrow ((p \wedge q) \vee (\neg p \wedge q))$.</p> <p>Answer:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$(p \Leftrightarrow q) \Leftrightarrow ((p \wedge q) \vee (\neg p \wedge q))$</th></tr> </thead> <tbody> <tr> <td>T</td><td>T</td><td>T</td></tr> <tr> <td>T</td><td>F</td><td>T</td></tr> <tr> <td>F</td><td>T</td><td>F</td></tr> <tr> <td>F</td><td>F</td><td>F</td></tr> </tbody> </table>	p	q	$(p \Leftrightarrow q) \Leftrightarrow ((p \wedge q) \vee (\neg p \wedge q))$	T	T	T	T	F	T	F	T	F	F	F	F
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T	T	T															
T	F	T															
F	T	F															
F	F	F															

C

5

Construct the truth table for $\neg(p \vee (q \wedge r)) \Leftrightarrow ((p \vee q) \wedge (p \vee r))$.**Answer:**

p	q	r	$\neg(p \vee (q \wedge r)) \Leftrightarrow ((p \vee q) \wedge (p \vee r))$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Method 3 \rightsquigarrow Precedence Rule

Examples of Method-3: Precedence Rule

A	1	<p>By using preference rule put parentheses at appropriate place for following compound propositions:</p> <p>(1) $p \rightarrow q \rightleftharpoons p \wedge \neg p \vee \neg q$</p> <p>(2) $\neg p \wedge q \rightarrow \neg p \vee \neg q$</p> <p>(3) $(p \rightarrow q) \wedge q \rightarrow r \rightarrow (p \rightarrow r)$</p> <p>Answer: (1) $(p \rightarrow q) \rightleftharpoons (p \wedge ((\neg p) \vee (\neg q)))$</p> <p>(2) $((\neg p) \wedge q) \rightarrow ((\neg p) \vee (\neg q))$</p> <p>(3) $((p \rightarrow q) \wedge q) \rightarrow r \rightarrow (p \rightarrow r)$</p>
A	2	<p>By using preference rule put parentheses at appropriate place for following compound propositions:</p> <p>(1) $p \vee q \rightarrow \neg p \wedge \neg q$</p> <p>(2) $p \vee q \vee r \rightarrow \neg p \vee r$</p> <p>(3) $p \wedge \neg p \vee \neg q \rightarrow q$</p> <p>Answer: (1) $(p \vee q) \rightarrow ((\neg p) \wedge (\neg q))$</p> <p>(2) $((p \vee q) \vee r) \rightarrow ((\neg p) \vee r)$</p> <p>(3) $((p \wedge (\neg p)) \vee (\neg q)) \rightarrow q$</p>

Method 4 \rightsquigarrow Well Formed Formula

Examples of Method-4: Well Formed Formula

A	1	<p>Which of the following are not well-formed formula?</p> <p>(1) $(p \rightarrow (p \wedge q))$</p> <p>(2) $\neg p \wedge \neg q$</p> <p>(3) $(p \wedge q) \rightarrow q$</p> <p>(4) $p \wedge q \rightarrow \neg(p)$</p> <p>(5) $(\neg p \vee (q \wedge r))$</p> <p>Answer: (2), (3), (4)</p>
A	2	<p>Which of the following are well-formed formula?</p> <p>(1) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$</p> <p>(2) $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$</p> <p>(3) $(p \wedge (\neg p \vee \neg q)) \rightarrow q$</p> <p>(4) $((p \rightarrow r) \vee (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$</p> <p>Answer: None of them are well formed formula</p>
A	3	<p>Which of the following are well-formed formula?</p> <p>(1) $((p \rightarrow r) \vee q)$</p> <p>(2) $((p \rightarrow r) \vee r) \vee p$</p> <p>(3) $(p \rightarrow (r \wedge (\neg p))) \rightarrow q$</p> <p>Answer: (1), (2)</p>

Method 5 \rightsquigarrow Tautologies

Examples of method-5: Tautologies

B	1	Show that the following propositions are tautology: (1) $p \vee \neg(p \wedge q)$ (2) $\neg(p \wedge q) \Leftrightarrow ((\neg p) \vee (\neg q))$
B	2	Show that the following propositions are tautology: (1) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ (2) $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
B	3	Show that the following propositions are contradictions: (1) $(\neg q \wedge p) \wedge q$ (2) $(p \wedge q) \wedge (\neg p \vee \neg q)$
B	4	Show that the following propositions are contradictions: (1) $p \wedge (\neg p \wedge q)$ (2) $(p \wedge q) \wedge \neg(p \vee q)$
B	5	Show that the following propositions are contingency: (1) $(p \wedge q) \Leftrightarrow p$ (2) $(p \wedge (p \rightarrow \neg q)) \rightarrow q$
B	6	Show that the following propositions are contingency: (1) $(p \wedge q) \rightarrow (r \wedge (\neg p \wedge \neg q))$ (2) $(p \rightarrow \neg q) \Leftrightarrow (p \vee q)$

B	7	<p>Check whether the given formulas are tautologies, contradiction or contingency.</p> <p>(1) $(p \rightarrow (p \vee q))$</p> <p>(2) $(\neg p \rightarrow (p \rightarrow q))$</p> <p>(3) $\neg(p \rightarrow q) \rightarrow p$</p> <p>(4) $(p \wedge (p \rightarrow q)) \rightarrow q$</p> <p>(5) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$</p> <p>(6) $((p \wedge q) \rightleftharpoons p)$</p> <p>Answer: (1) Tautology, (2) Tautology, (3) Tautology (4) Tautology, (5) Tautology, (6) contingency</p>
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Method 6 \rightsquigarrow Equivalence of Formulas

Examples of Method-6: Logical Equivalence

B	1	<p>Show that, $\neg(p \rightleftharpoons q) \equiv (p \rightleftharpoons \neg q)$.</p> <p>Hint:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$p \rightleftharpoons \neg q$</th></tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>F</td></tr> <tr><td>T</td><td>F</td><td>T</td></tr> <tr><td>F</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>F</td><td>F</td></tr> </tbody> </table>	p	q	$p \rightleftharpoons \neg q$	T	T	F	T	F	T	F	T	T	F	F	F
p	q	$p \rightleftharpoons \neg q$															
T	T	F															
T	F	T															
F	T	T															
F	F	F															
B	2	<p>Show that $(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$.</p> <p>Hint:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$p \rightarrow q$</th></tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>F</td><td>F</td></tr> <tr><td>F</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	p	q	$p \rightarrow q$	T	T	T	T	F	F	F	T	T	F	F	T
p	q	$p \rightarrow q$															
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F	T	T															
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B	3	<p>Show that, $\neg(p \vee q) \equiv (\neg p \wedge \neg q)$.</p> <p>Hint:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$\neg(p \vee q)$</th></tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>F</td></tr> <tr><td>T</td><td>F</td><td>F</td></tr> <tr><td>F</td><td>T</td><td>F</td></tr> <tr><td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	p	q	$\neg(p \vee q)$	T	T	F	T	F	F	F	T	F	F	F	T
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B	4	<p>Show that, $\neg(p \wedge q) \equiv (\neg p \vee \neg q)$.</p> <p>Hint:</p> <table border="1"> <thead> <tr> <th>p</th><th>q</th><th>$\neg(p \wedge q)$</th></tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>F</td></tr> <tr><td>T</td><td>F</td><td>T</td></tr> <tr><td>F</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	p	q	$\neg(p \wedge q)$	T	T	F	T	F	T	F	T	T	F	F	T
p	q	$\neg(p \wedge q)$															
T	T	F															
T	F	T															
F	T	T															
F	F	T															

B	5	<p>Show that, $(p \rightleftharpoons q) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.</p> <p>Hint:</p> <table><tr><td>p</td><td>q</td><td>$(p \rightleftharpoons q)$</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td></tr></table>	p	q	$(p \rightleftharpoons q)$	T	T	T	T	F	F	F	T	F	F	F	T																					
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B	6	<p>Show that, $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$.</p> <p>Hint:</p> <table><tr><td>p</td><td>q</td><td>r</td><td>$p \rightarrow (q \wedge r)$</td></tr><tr><td>T</td><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>T</td><td>F</td><td>F</td></tr><tr><td>T</td><td>F</td><td>T</td><td>F</td></tr><tr><td>T</td><td>F</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>T</td><td>T</td></tr><tr><td>F</td><td>T</td><td>F</td><td>T</td></tr><tr><td>F</td><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>F</td><td>T</td></tr></table>	p	q	r	$p \rightarrow (q \wedge r)$	T	T	T	T	T	T	F	F	T	F	T	F	T	F	F	F	F	T	T	T	F	T	F	T	F	F	T	T	F	F	F	T
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B	7	<p>Show that, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.</p> <p>Hint:</p> <table><tr><td>p</td><td>q</td><td>r</td><td>$p \wedge (q \vee r)$</td></tr><tr><td>T</td><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>T</td><td>F</td><td>T</td></tr><tr><td>T</td><td>F</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>T</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>F</td><td>F</td></tr></table>	p	q	r	$p \wedge (q \vee r)$	T	T	T	T	T	T	F	T	T	F	T	T	T	F	F	F	F	T	T	F	F	T	F	F	F	F	T	F	F	F	F	F
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F	F	T	F																																			
F	F	F	F																																			

B 8 Show that, $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

Hint:

p	q	r	$p \vee (q \wedge r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

B 9 Show that, $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$.

Hint:

p	q	r	$(p \vee q) \rightarrow r$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

B 10 Show that, $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.

Hint:

p	q	r	$\neg p \rightarrow (q \rightarrow r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

Method 7 \rightsquigarrow Normal Forms

Examples of Method-7.1: Disjunctive Normal Form (DNF)

A	1	Find disjunctive normal form of $\neg (p \rightarrow (q \wedge r))$. Answer: $(p \wedge \neg q) \vee (p \wedge \neg r)$
B	2	Find disjunctive normal form of $(p \rightarrow q) \wedge (q \rightarrow r)$. Answer: $(\neg p \wedge \neg q) \vee (q \wedge \neg q) \vee (\neg p \wedge r) \vee (q \wedge r)$
C	3	Find disjunctive normal form of $(p \rightarrow q) \wedge (\neg p \wedge q)$. Answer: $(\neg p \wedge q) \vee (q \wedge \neg p)$
B	4	Find disjunctive normal form of $(p \wedge (p \rightarrow q)) \rightarrow q$. Answer: $\neg p \vee (p \wedge \neg q) \vee q$

Examples of Method-7.2: Conjunctive Normal Form

C	1	Find conjunctive normal form of $\neg(p \wedge q) \Leftrightarrow (p \wedge q)$. Answer: $(p \vee q) \wedge (\neg p \vee \neg q)$
B	2	Find conjunctive normal form of $p \wedge (p \rightarrow q)$. Answer: $p \wedge (\neg p \vee q)$
C	3	Find conjunctive normal form of $(p \wedge q) \vee (\neg p \wedge q \wedge r)$. Answer: $(p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p) \wedge (q) \wedge (q \vee r)$
B	4	Find conjunctive normal form of $(\neg p \rightarrow r) \wedge (p \rightarrow q)$ Answer: $(p \vee q) \wedge (q \vee \neg p)$
B	5	Find conjunctive normal form of $\neg((p \vee \neg q) \wedge \neg r)$. Answer: $(\neg p \vee r) \wedge (q \vee r)$

Examples of Method-7.3: Principal Disjunctive Normal Form

A	1	Find principal disjunctive normal form of $\neg(p \rightarrow (q \wedge r))$. Answer: $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r)$
C	2	Find principal disjunctive normal form of $\neg(p \vee q) \Leftrightarrow (p \wedge q)$. Answer: $(p \wedge \neg q) \vee (\neg p \wedge q)$
A	3	Find principal disjunctive normal form of $\neg((p \vee \neg q) \wedge \neg r)$. Answer: $(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$
C	4	Find principal disjunctive normal form of $(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$. Answer: $(p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$

Examples of Method-7.4: Principal Conjunctive Normal Form

A	1	Find principal conjunctive normal form of $\neg(p \rightarrow (q \wedge r))$. Answer: $(p \vee q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$
B	2	Find principal conjunctive normal form of $\neg(p \rightleftharpoons q)$. Answer: $(p \vee q) \wedge (\neg q \vee \neg p)$
B	3	Find principal conjunctive normal form of $\neg((p \vee \neg q) \wedge \neg r)$. Answer: $(\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$

***** End of the Unit *****