Index

Unit – 3 ⇝ Relation

Method – 1 ⇝ Properties of Relation

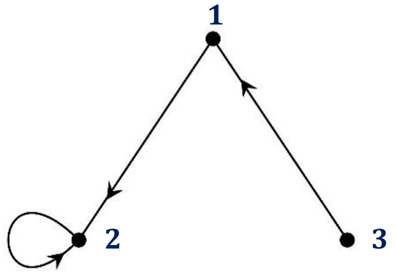
Examples of Method-1: Properties of Relation

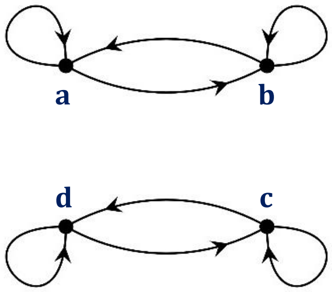
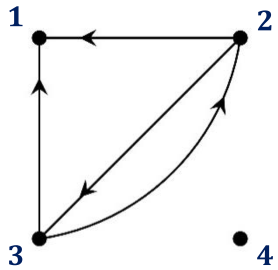
A	1	For each of these relations on the set $\{1, 2, 3, 4\}$, determine whether it is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. (**1**) $R_1 = \{(1, 1), (2, 2), (3, 3)\}$ (**2**) $R_2 = \{(1, 1), (1, 3), (1, 2), (3, 1), (3, 2), (3, 3), (4, 4)\}$ (**3**) $R_3 = \{(1, 2), (1, 3), (1, 4), (2, 3), (3, 1), (4, 1), (3, 4)\}$ (**4**) $R_4 = A \times A$ **Answer: R_1 ⇝ Symmetric, Antisymmetric, Transitive** **R_2 ⇝ Transitive** **R_3 ⇝ Irreflexive** **R_4 ⇝ Reflexive, Symmetric, Transitive**
A	2	Let $A = \{1, 2, 3, 4, 5, 6\}$ and define a relation R on A as $R = \{(a, b) : b - a = 2\}$ Check whether R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. **Answer: R is irreflexive, asymmetric and antisymmetric**
B	3	Let R be the relation on the set of integers such that aRb if and only if $a = b$ or $a = -b$. Check whether R is reflexive, symmetric, transitive or not. **Answer: R is reflexive, symmetric and transitive**

C	4	Determine whether the relation R on the set of all set of all people is reflexive, symmetric, antisymmetric or transitive, where aRb if and only if **(1)** a is taller than b. **(2)** a is 3 inches shorter than b. **(3)** a and b were born on the same day. **(4)** a has the same first name as b. **(5)** a is grandparent of b. **(6)** a is brother of b. **Answer:** (1) **R is transitive and antisymmetric** (2) **R is antisymmetric** (3) **R is reflexive, symmetric and transitive** (4) **R is reflexive, symmetric and transitive** (5) **R is antisymmetric** (6) **R is transitive**
C	5	Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric or transitive, where $(x, y) \in R$ if and only if (**1**) $x \neq y$ (**2**) $xy \geq 1$ (**3**) $x = y^2$ (**4**) $xy = 0$ **Answer:** (1) **R is symmetric** (2) **R is symmetric and transitive** (3) **R is antisymmetric** (4) **R is symmetric**
C	6	Let relation R defined on $\mathbb{R}$ as $R = \{ (a, b) : a \leq b^3;\ a, b \in \mathbb{R} \}$. Check whether R is reflexive, symmetric, transitive or not. **Answer: R is neither reflexive nor symmetric nor transitive**
B	7	Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{ (L_1, L_2) : L_1 \text{is perpendicular to } L_2\}$. Check whether R reflexive, symmetric or transitive. **Answer: R is symmetric**

Method – 2 ⇝ Matrix and Graph Representation of a Relation

Examples of Method-2: Matrix and Graph Representation of a Relation

A	1	Represent given relation R on { 1, 2, 3, 4 } with a matrix. $R = \{ (1,2), (1,3), (1,4), (2,3), (2,4), (3,4) \}$ **Answer: Matrix Representation of a Relation:** $M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$
A	2	Represent given relation R on { 1, 2, 3 } with a matrix. $R = \{ (1,1), (1,2), (1,3), (2,2), (2,3), (3,3) \}$ **Answer: Matrix Representation of a Relation:** $M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$
A	3	Draw the directed graph representing relation on { 1, 2, 3 }: $R = \{ (1,2), (2,2), (3,1) \}$ **Answer: Digraph of a Relation:** 

B	4	Draw the directed graph representing the following relation on a set $\{a, b, c, d\}$: $R = \{ (a,a), (a,b), (b,a), (b,b), (c,c), (c,d), (d,c), (d,d) \}$ **Answer: Digraph of a Relation:** 
B	5	Represent given relation R on $\{ 1, 2, 3, 4 \}$ with a matrix also draw a directed graph of it. $R = \{ (2,1), (2,3), (3,1), (3,2) \}$ **Answer: Matrix Representation of a Relation:** $$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$ **Digraph of a Relation:** 

<table>
<tr><td>C</td><td>6</td><td>

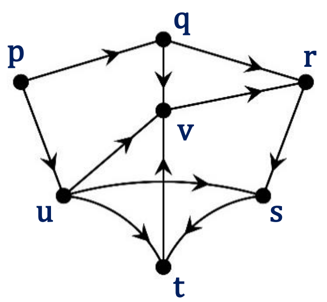
Represent given relation R on $\{ p, q, r, s, t, u, v \}$ with a matrix also draw a directed graph of it.

$$R = \left\{ \begin{matrix} (p, q), (p, u), (q, r), (q, v), (r, s), (s, t), \\ (t, v), (u, s), (u, t), (u, v), (v, r) \end{matrix} \right\}$$

Answer: Matrix Representation of a Relation:

$$\mathbf{M_R} = \begin{array}{c} \\ p \\ q \\ r \\ s \\ t \\ u \\ v \end{array} \begin{array}{c} \begin{matrix} p & q & r & s & t & u & v \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Digraph of a Relation:



</td></tr>
<tr><td>C</td><td>7</td><td>

Give the relation R defined on $S = \{ 1, 2, 3, 5 \}$ from following matrix.

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Answer: $\mathbf{R = \{ (1,1), (1,2), (2,2), (2,3), (3,3), (3,5), (5,1) \}}$

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</table>

Method – 3 ⇝ Partition and Covering of a Set

Examples of Method-3: Partition and Covering of a Set

A	1	Let $S = \{2, 4, 6, 8, 12, 14, 15, 17\}$ be a set and $A_1 = \{4, 6, 12\}$, $A_2 = \{2\}$, $A_3 = \{12, 14\}$, $A_4 = \{15, 17\}$, $A_5 = \{8, 14\}$ be subsets of S. Determine which of the collection is covering or partition: $P_1 = \{A_1, A_2, A_4\}$, $P_2 = \{A_1, A_2, A_3, A_4, A_5\}$ $P_3 = \{A_1, A_2, A_4, A_5\}$, $P_4 = \{A_1, A_3, A_4, A_5\}$ **Answer: P_1 is neither covering nor partition,** **P_2 is covering but not partition,** **P_3 is covering and partition,** **P_4 is neither covering nor partition.**
B	2	Let $P = \{p, q, r, x, y, z, a, b, c\}$ be a set. Determine any two such collection which are partition of set P. **Hint: Make a collection of nonempty and disjoint subsets of S.**
B	3	Let $P = \{p, q, r, x, y, z, a, b, c\}$ be a set. Determine any two such collection which are covering but not partition of set P. **Hint: Make a collection of nonempty and not disjoint subsets of S.**
B	4	Let $P = \{p, q, r, x, y, z, a, b, c\}$ be a set. Determine any two such collection which are neither covering nor partition of set P. **Hint: Some elements of the set must not be in the collection.**

Method – 4 ⇝ Equivalence Relation

Examples of Method-4: Equivalence Relation

A	1	Let R be the relation on the set of real numbers such that aRb if and only if $a - b$ is an integer. Is R an equivalence relation? **Answer: R is an equivalence relation.**
A	2	Show that the "divides" relation on the set of positive integers in not an equivalence relation. **Hint: Check that R is reflexive, symmetry and transitive or not.**
B	3	Let X be a nonempty set and $\mathcal{P}(X)$ the power set of X. Define the "subset" relation S on $\mathcal{P}(X)$ as follows: For all $A,\ B \in \mathcal{P}(X),\ (A,\ B) \in S \Leftrightarrow A \subseteq B$. Check whether relation S is equivalence or not. **Answer: R is not equivalance relation**
B	4	Let m be an integer with $m > 1$. Show that the relation $R = \{(a, b) \mid a \equiv b \pmod{m}\}$ is an equivalence relation on the set of integers. **Hint: Check that R is reflexive, symmetry and transitive or not.**
B	5	Show that the relation R in the set P of points in a plane given by $R = \left\{(X,\ Y) : \begin{array}{c}\text{Distance of the point X from the origin is same} \\ \text{as the distance of the point Y from the origin}\end{array}\right\}$, is an equivalence relation. **Answer: Check that R is reflexive, symmetry and transitive or not.**
B	6	Let R be the relation on $\mathbb{R}$ such that xRy if and only if $\lvert x - y \rvert < 1$. Show that R is not an equivalence relation. **Hint: Check that R is reflexive, symmetry and transitive or not.**

B	7	Let A be a set of nonzero integers and let $\approx$ be the relation on $A \times A$ defined by $(a, b) \approx (c, d)$ whenever $ad = bc$. Prove that $\approx$ is an equivalence relation. **Hint: Check that R is reflexive, symmetry and transitive or not.**
C	8	Let X be the set of all nonempty subsets of $\{1, 2, 3\}$. Define a relation R on X as follows: $\forall A, B \in X, ARB \Leftrightarrow$ the least element of A equals the least element of B. Prove that R is an equivalence relation on X. **Hint: Check that R is reflexive, symmetry and transitive or not.**
C	9	Which of these relations on the set of all people are equivalence relations? **(1)** $\{(a, b) \mid$ a and b are the same age $\}$ **(2)** $\{(a, b) \mid$ a and b have the same parents $\}$ **(3)** $\{(a, b) \mid$ a and b have met $\}$ **(4)** $\{(a, b) \mid$ a and b speak a common language $\}$ **Answer: (1) and (2) are equivalance relation**
C	10	Let R be an equivalence relation on a set $A = \{a, b, c, d\}$ defined as $R = \{(a, a), (b, b), (b, d), (c, c), (d, b), (d, d)\}$. Find the distinct equivalence classes of R. **Answer: Distinct equivalence classes:** $\{\mathbf{a}\}, \quad \{\mathbf{b}, \mathbf{d}\}, \quad \{\mathbf{c}\}$
C	11	Let $A = \{1, 2, 3, \ldots, 20\}$. R be an equivalence relation on a set defined as follow: $\forall x, y \in A, \quad xRy \Leftrightarrow x - y$ is divisible by 4 Find the distinct equivalence classes of R. **Answer: Distinct equivalence classes:** $\{\mathbf{1, 5, 9, 13, 17}\}, \quad \{\mathbf{2, 6, 10, 14, 18}\}, \quad \{\mathbf{3, 7, 11, 15, 19}\}$ $\{\mathbf{4, 8, 12, 16, 20}\}$

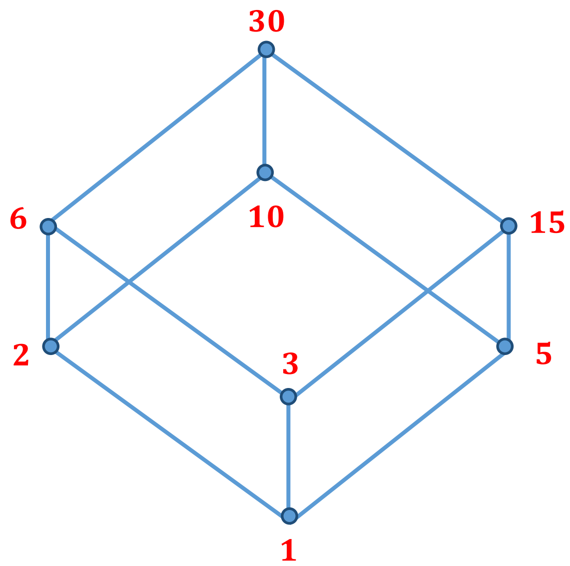
Method – 5 ⇝ Partially Ordered Relation

Examples of Method-5: Partially Ordered Relation

A	1	Which of these relations on $\{0,\ 1,\ 2,\ 3\}$ are partial orderings? **(1)** $\{(0,0),\ (1,1),\ (2,2),\ (3,3)\}$ **(2)** $\{(0,0),\ (1,1),\ (2,0),\ (2,2),\ (2,3),\ (3,2),\ (3,3)\}$ **(3)** $\{(0,0),\ (1,1),\ (1,2),\ (2,2),\ (3,3)\}$ **(4)** $\{(0,0),\ (1,1),\ (1,2),\ (1,3),\ (2,2),\ (2,3),\ (3,3)\}$ **(5)** $\{(0,0),\ (0,1),\ (0,2),\ (1,0),\ (1,1),\ (1,2),\ (2,0),\ (2,2),\ (3,3)\}$ **Answer: (1), (3) and (4) are partial orderings**
B	2	Let R be the relation on the set of people such that xRy if x and y are people and x is older than y. Show that R is not a partial ordering. **Hint: Check that R is reflexive, antisymmetry and transitive or not.**
A	3	Let X be a nonempty set. Prove that $(\mathcal{P}(X), \subseteq)$ is a poset. **Hint: Check that $\subseteq$ is reflexive, antisymmetry and transitive or not.**
B	4	Prove that $(\mathbb{Z}, \preccurlyeq)$ is not a poset where, $\preccurlyeq$ be a relation defined on $\mathbb{Z}$ as follow: $a \preccurlyeq b \Leftrightarrow a = 2b,\ \forall\, a, b \in \mathbb{Z}$
C	5	Prove that $(\mathbb{Z}, \preccurlyeq)$ is not a poset where, $\preccurlyeq$ be a relation defined on $\mathbb{Z}$ as follow: $a \preccurlyeq b \Leftrightarrow$ a is divisible by $b^2,\ \forall\, a, b \in \mathbb{Z}$

Method 6 ⇝ Hasse–diagram

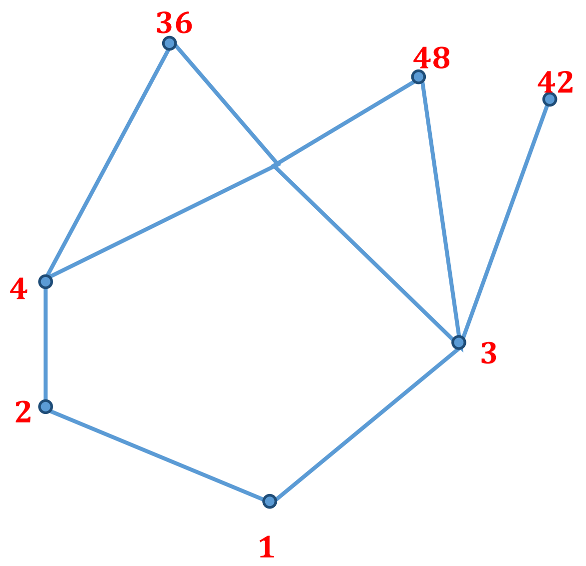
Examples of Method-6: Hasse–diagram

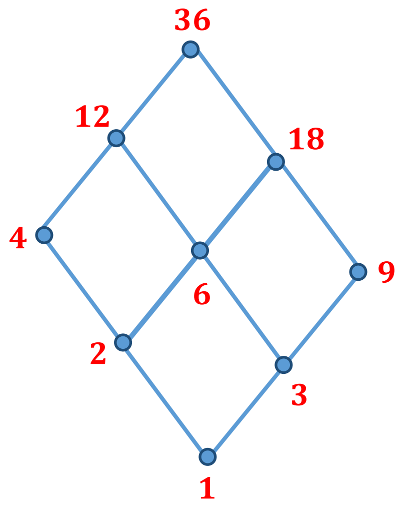
B	1	Draw the Hasse–diagram for poset $\langle A, \le \rangle$. Find cover of each element of set A if possible. $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$. "$a \le b$" if "a divides b" . **Answer : Cover of 1 $= 2, 3, 5$ Cover of 2 $= 6, 10$** **Cover of 3 $= 6, 15$ Cover of 5 $= 10, 15$** **Cover of 6 $= 30$ Cover of 10 $= 30$** **Cover of 15 $= 30$** 

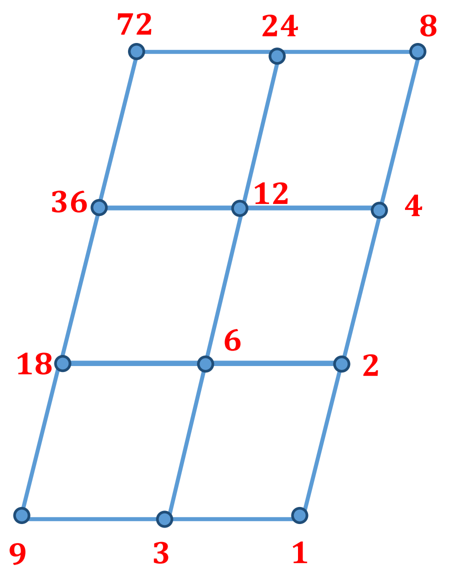
A	2	Draw the Hasse–diagram for poset $\langle A, \le \rangle$. Find cover of each element of set A if possible. $A = \{1, 2, 3, 4, 36, 42, 48\}$. "a ≤ b" if "a divides b".

Answer : Cover of $1 = 2, 3$ **Cover of $3 = 36, 42, 48$**

Cover of $2 = 4, 42$ **Cover of $4 = 36, 48$**

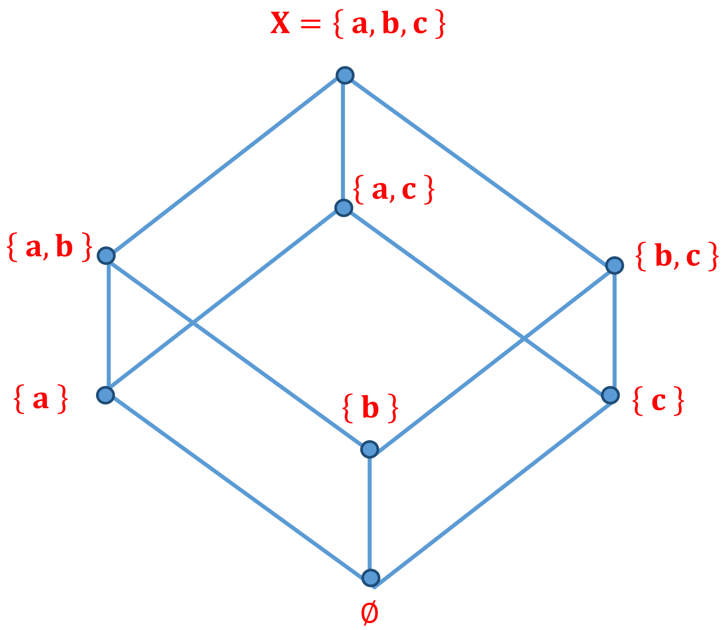


C	3	Draw the Hasse–diagram for poset $\langle A, \le \rangle$, "$a \le b$" if "a divides b" for set $A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$. Find cover of each element of set A if possible. **Answer : Cover of 1 = 2, 3** **Cover of 2 = 4, 6** **Cover of 3 = 6, 9** **Cover of 4 = 12** **Cover of 6 = 12, 18** **Cover of 9 = 18** **Cover of 12 = 36** **Cover of 18 = 36** 

C	4	Draw the Hasse–diagram for poset $\langle A, \leq \rangle$, "$a \leq b$" if "a divides b" for set $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$. Find cover of each element of set A if possible. **Answer : Cover of 1 = 2, 3** **Cover of 2 = 4, 6** **Cover of 3 = 6, 9** **Cover of 4 = 8, 12** **Cover of 6 = 12, 18** **Cover of 8 = 24** **Cover of 9 = 18** **Cover of 12 = 24, 36** **Cover of 18 = 36** **Cover of 24= 72.** **Cover of 36= 72** 

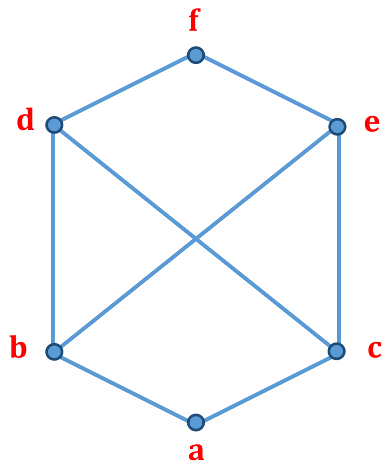
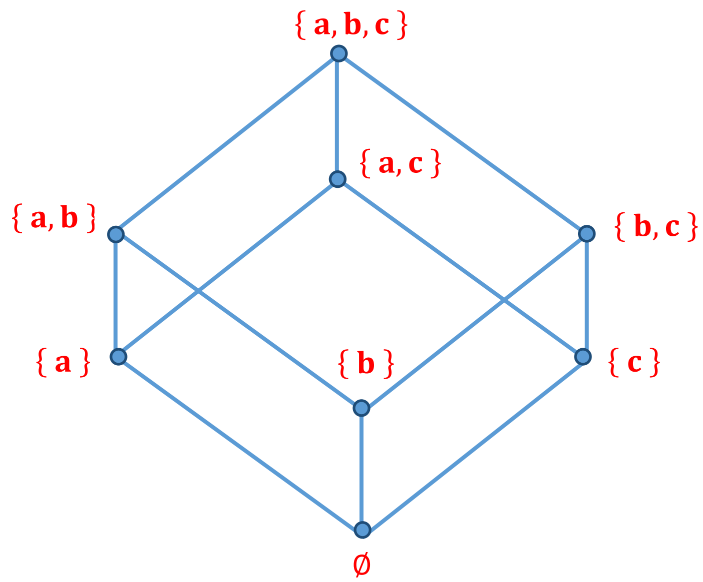
Method 7 ⇝ Totally Ordered Set

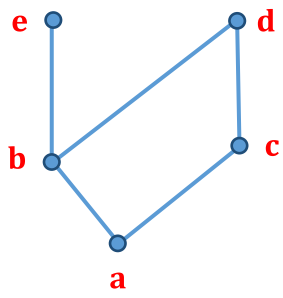
Examples of Method-7: Totally Ordered Set

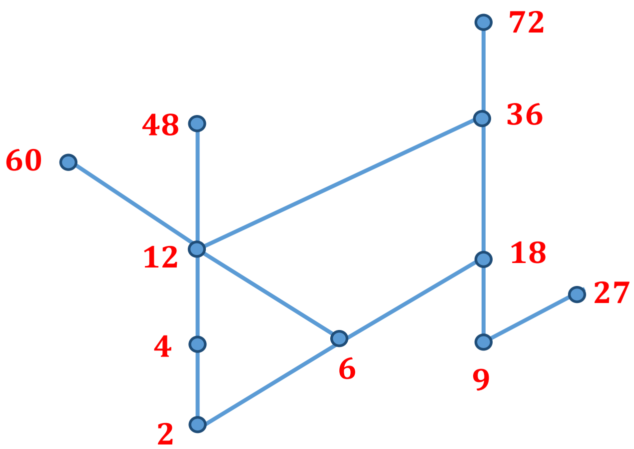
B	1	Prove that $\langle \{\{a\}\}, \subseteq \rangle$ is a toset.
C	2	Show that $\langle \{ 1, 3, 3^2, 3^3, 3^4, 3^5, \ldots \}, D \rangle$ is toset.
A	3	From following diagram find any two Chains and AntiChains.  **Answer** : **Chains** $= \{\{ a \}, \{ a, b \}, \{a, b, c\} \}, \{\{ b, c \}, \{ a, b, c \}\}$ **AntiChains** $= \{\{ a \}, \{ b \}\}, \{\{ b \}, \{ c \}\}$

Method 8(a) ⇝Least Member and Greatest Member

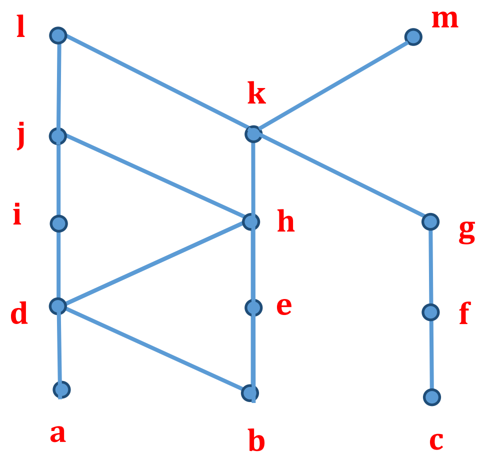
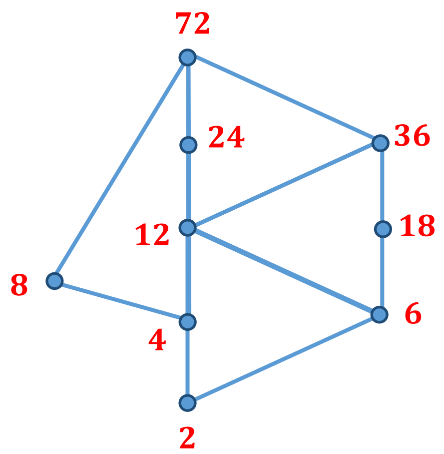
Examples of Method-8(a): Least Member and Greatest Member

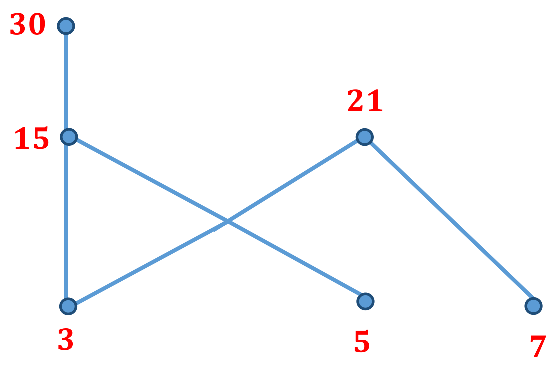
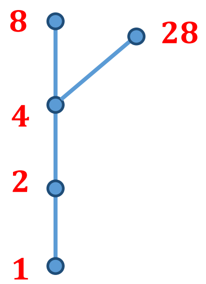
A	1	From given Hasse–diagram find Least and Greatest Member.  **Answer: Least Member – a, Greatest Member – f**
B	2	Draw the Hasse–diagram of poset $\langle P(X), \subseteq \rangle$, Where $X = \{a, b, c\}$. Find Least and Greatest Member of $P(X)$. **Answer:**  **Least Member – ∅, Greatest Member – { a, b, c }**

<table>
<tr><td>A</td><td>3</td><td>From given Hasse–diagram find Least and Greatest Member.


Answer: Least Member – a, No Greatest Member</td></tr>
<tr><td>B</td><td>4</td><td>Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Least and Greatest Member of A. Where, $A = \{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}$.


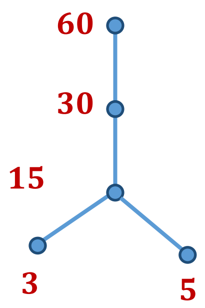
Answer:
No Least Member, No Greatest Member</td></tr>
</table>

A	5	From given Hasse–diagram find Least and Greatest Member.  **Answer: No Least Member, No Greatest Member**
B	6	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Least and Greatest Member of A. Where, $A = \{2, 4, 6, 8, 12, 18, 24, 36, 72\}$. **Answer:**  **Least Member – 2, Greatest Member – 72**

B	7	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Least and Greatest Member of A. Where, $A = \{3, \ 5, \ 7, \ 15, \ 21, \ 30\}$. **Answer**:  **No Least Member, No Greatest Member**
B	8	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Least and Greatest Member of A.Where, $A = \{1, \ 2, \ 4, \ 8, \ 28\}$. **Answer**:  **Least Member – 1, No Greatest Member**

<table>
<tr><td>B</td><td>9</td><td>Draw the Hasse–diagram of poset $\langle A, | \rangle$. Find Least and Greatest Member of Where, $A = \{3,\ 5,\ 15,\ 30,\ 60\}$.

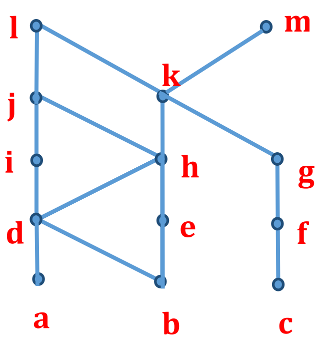
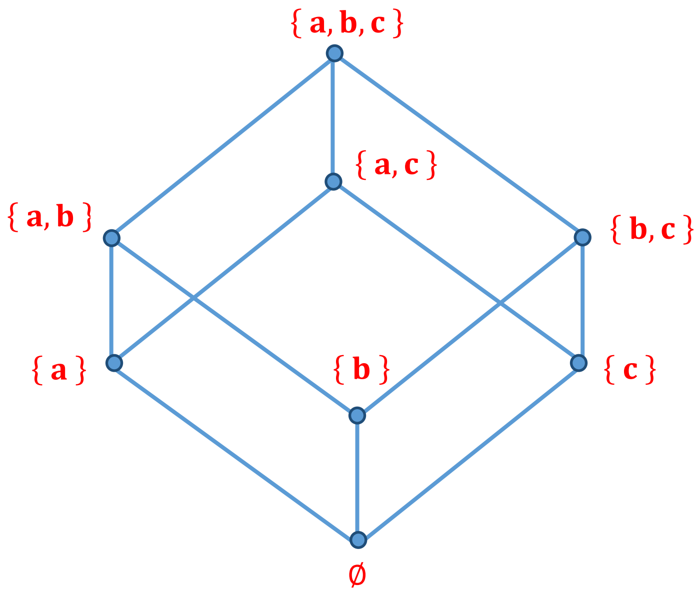
Answer:

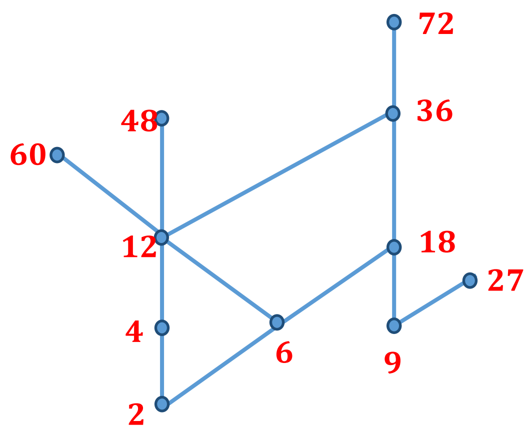
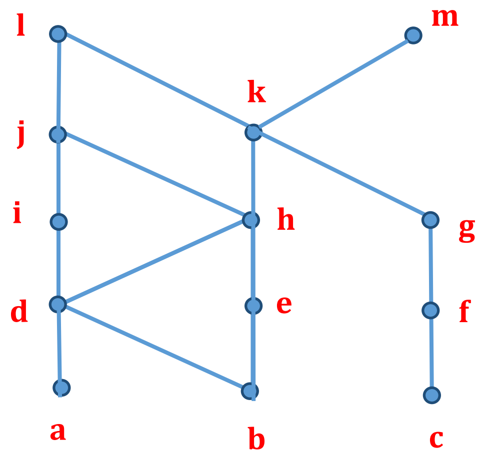


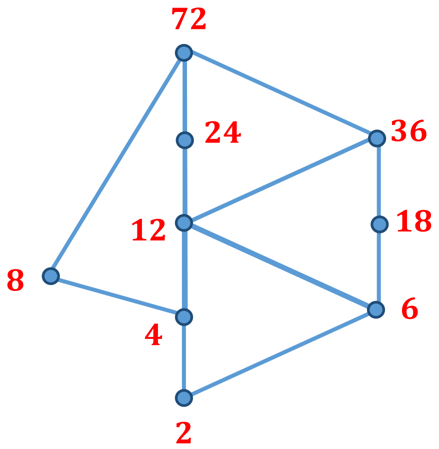
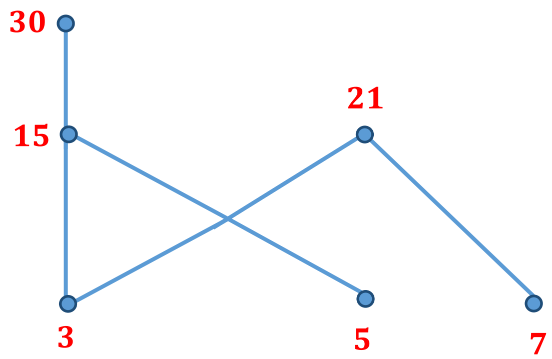
No Least Member, Greatest Member − 60</td></tr>
</table>

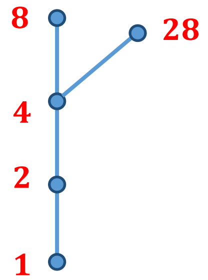
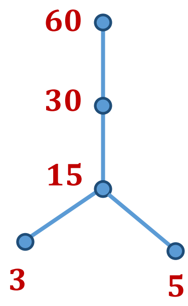
Method 8(b) ⇝ Minimal Elements and Maximal Elements

Examples of Method-8(b): Minimal and Maximal Elements

A	1	From given Hasse–diagram find Minimal and Maximal Elements.  **Answer**: **Minimal Elements – a, b, c** **Maximal Elements – l, m**
B	2	Draw the Hasse–diagram of poset $\langle P(X), \subseteq\rangle$, Where $X = \{a, b, c\}$. Find Minimal and Maximal Elements $P(X)$. **Answer**:  **Minimal Element – ∅,** **Maximal Element – { a, b, c }**

B	3	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Minimal and Maximal Elements of A.Where, $A = \{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}$. **Answer**:  **Minimal Element – 2, 9** **Maximal Element – 48, 60, 72**
A	4	From given Hasse–diagram find Minimal and Maximal Elements.  **Minimal Element – a, b, c** **Maximal Element – l, m**

B	5	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Minimal and Maximal Elements of A. Where, $A = \{2, 4, 6, 8, 12, 18, 24, 36, 72\}$. **Answer**:  **Minimal Element – 2** **Maximal Element – 72**
B	6	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Minimal and Maximal Elements of A . Where, $A = \{3, 5, 7, 15, 21, 30\}$. **Answer**:  **Minimal Element – 3, 5, 7** **Maximal Element – 30, 21**

B	7	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$.Find Minimal and Maximal Elements of A. Where, $A = \{1, 2, 4, 8, 28\}$. **Answer**:  **Minimal Element – 1** **Maximal Element – 8, 28**
B	8	Draw the Hasse–diagram of poset $\langle A, \mid \rangle$. Find Minimal and Maximal Elements of A. Where, $A = \{3, 5, 15, 30, 60\}$. **Answer**:  **Minimal Element – 3, 5** **Maximal Element – 60**

Method 8(c) ⇝ Least Upper Bound and Greatest Lower Bound

Examples of Method-8(c): Least Upper Bound and Greatest Lower Bound

B	1	Let $A = \{1, \ 2, \ 3, \ 5, \ 4, \ 9, \ 10, \ 15, \ 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Then, find LUB and GLB if exists for following sets: **(1)** $\{2, \ 3, \ 5\}$ **(2)** $\{9, \ 15\}$ **(3)** $\{10, \ 15, \ 36\}$ **(4)** $\{3, \ 4, \ 10\}$ **(5)** $\{2, \ 9, \ 5\}$ **Answer**: (see table below)

Set	GLB	LUB
$\{2, \ 3, \ 5\}$	1	Not exist
$\{9, \ 15\}$	3	Not exist
$\{10, \ 15, \ 36\}$	1	Not exist
$\{3, \ 4, \ 10\}$	1	Not exist
$\{2, \ 9, \ 5\}$	1	Not exist

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Let $A = \{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 12,\ 18,\ 24,\ 36,\ 72\}$ and the relation $\preceq$ be such that $x \preceq y$ if x divides y. Then, find LUB and GLB if exists for following sets:
(1) $\{2,\ 6,\ 9\}$
(2) $\{1,\ 4,\ 12\}$
(3) $\{12,\ 18,\ 36\}$
(4) $\{3,\ 4,\ 6\}$
(5) $\{8,\ 9,\ 12\}$

Answer:
<table>
<tr><th>Set</th><th>LUB</th><th>GLB</th></tr>
<tr><td>$\{2,\ 6,\ 9\}$</td><td>1</td><td>18</td></tr>
<tr><td>$\{1,\ 4,\ 12\}$</td><td>1</td><td>12</td></tr>
<tr><td>$\{12,\ 18,\ 36\}$</td><td>6</td><td>36</td></tr>
<tr><td>$\{3,\ 4,\ 6\}$</td><td>1</td><td>12</td></tr>
<tr><td>$\{8,\ 9,\ 12\}$</td><td>1</td><td>72</td></tr>
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* * * * * **End of the Unit** * * * * *