Index

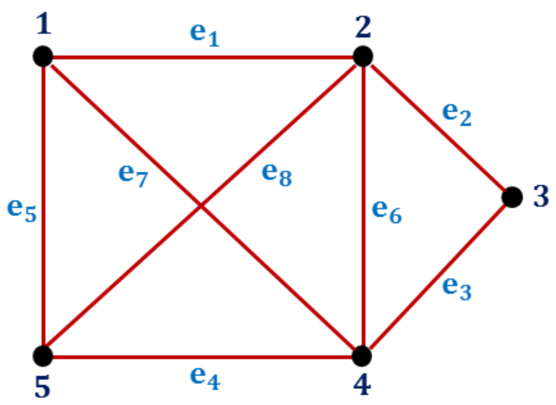
Unit – 4 ⇝ Graph Theory – I

Method 1 ⇝ Basic Terminologies

<table>
<tr><td>A</td><td>1</td><td>Define the following terms with example:

Graph
Vertex
Edge
Self – Loop
Adjacent Vertices
Parallel Edges
Incident Edge
Isolated Vertex
Null Graph
Directed Edge
Directed Graph
Undirected Edge
Undirected Graph
Mixed Graph

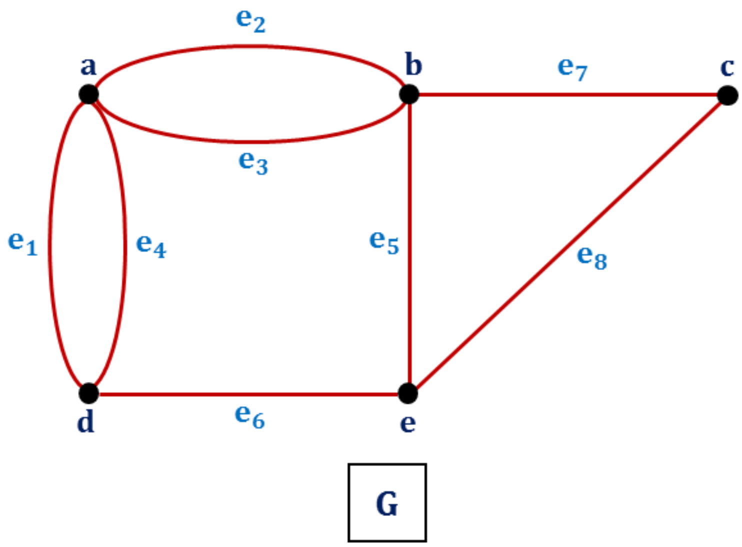
Answer: Refer Theory</td></tr>
<tr><td>A</td><td>2</td><td>Draw the undirected graph $G = (V, E, \phi)$, where $V = \{ 1, 2, 3, 4, 5 \}$, $E = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 \}$ and the incidence relation ϕ is given as: $e_1 = (1, 2), e_2 = (2, 3), e_3 = (3, 4), e_4 = (4, 5), e_5 = (5, 1), e_6 = (2, 4)$, $e_7 = (1, 4), e_8 = (5, 2)$. From the graph, answer the following equations:
(1) Is there any loop in G? If yes, then mention it.
(2) Are there any parallel edges in G? If yes, then mention it.

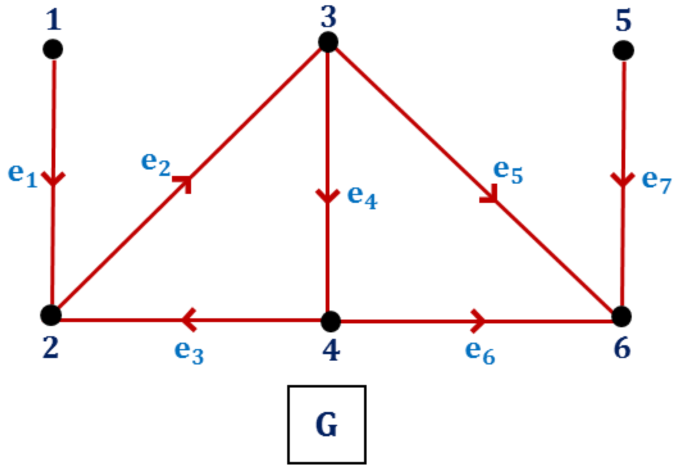
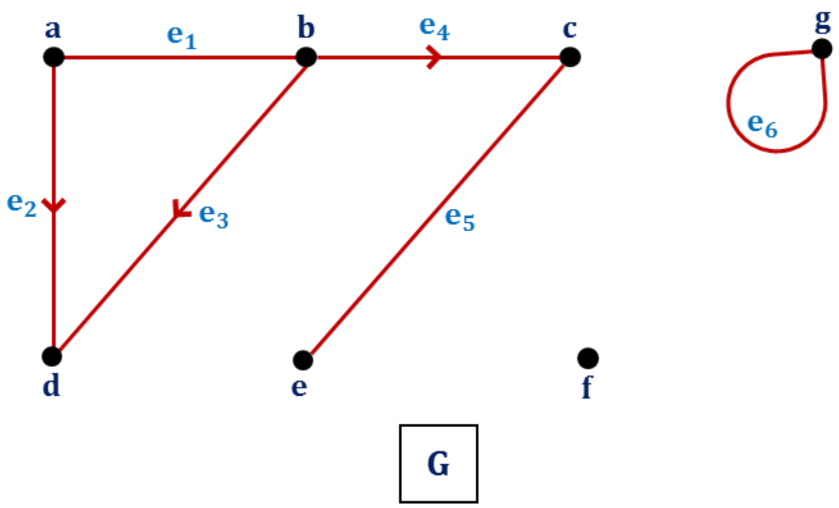
Answer:


G

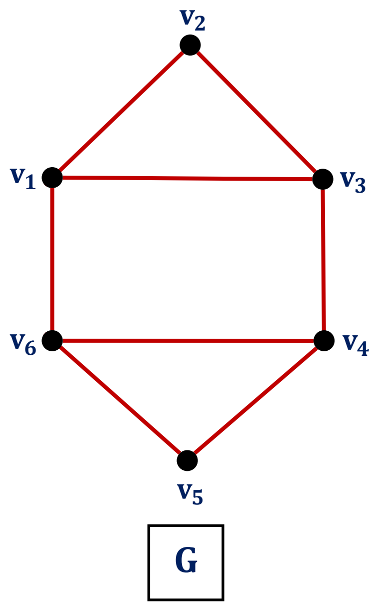
(1) There is no loop in G.

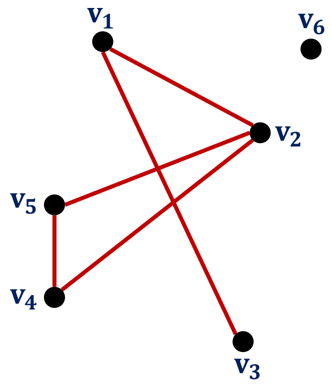
(2) There is no parallel edge in G.</td></tr>
</table>

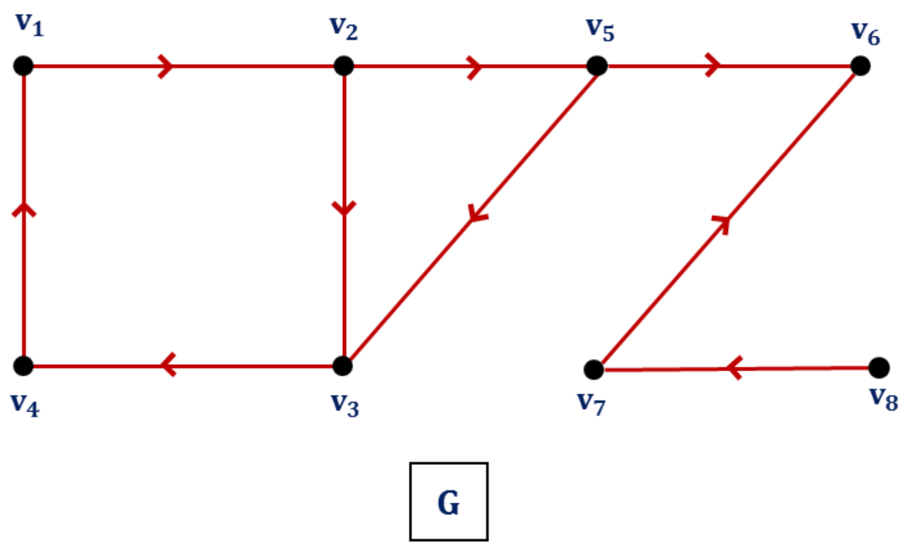
B	3	Draw the undirected graph $G = (V, E, \phi)$, where $V = \{ a, b, c, d, e \}$, $E = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 \}$ and the incidence relation ϕ is given as: $e_1 = (a, d), e_2 = (a, b), e_3 = (a, b), e_4 = (a, d), e_5 = (b, e), e_6 = (d, e)$, $e_7 = (b, c), e_8 = (c, e)$. From the graph, answer the following equations: (1) Is there any loop in G? If yes, then mention it. (2) Are there any parallel edges in G? If yes, then mention it. **Answer:**  **(1) There is no loop in G.** **(2) Yes, edges e_1 and e_4, e_2 and e_3 are parallel.**

C	4	Draw the directed graph $G = \langle V, E \rangle$, where $V = \{ 1, 2, 3, 4, 5, 6 \}$, $E = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7 \}$ and the incidence relation is given as: $e_1 = \langle 1, 2 \rangle, e_2 = \langle 2, 3 \rangle, e_3 = \langle 4, 2 \rangle, e_4 = \langle 3, 4 \rangle, e_5 = \langle 3, 6 \rangle, e_6 = \langle 4, 6 \rangle,$ $e_7 = \langle 5, 6 \rangle$. **Answer**: 
C	5	Draw the mixed graph G, where the vertex set $V = \{ a, b, c, d, e, f, g \}$, edge set $E = \{ e_1, e_2, e_3, e_4, e_5, e_6 \}$ and the incidence relation is given as: $e_1 = (a, b), e_2 = \langle a, d \rangle, e_3 = \langle b, d \rangle, e_4 = \langle b, c \rangle, e_5 = (e, c), e_6 = (g, g)$. **Answer**: 

Method 2 ⇝ First and Second Theorem of Graph Theory

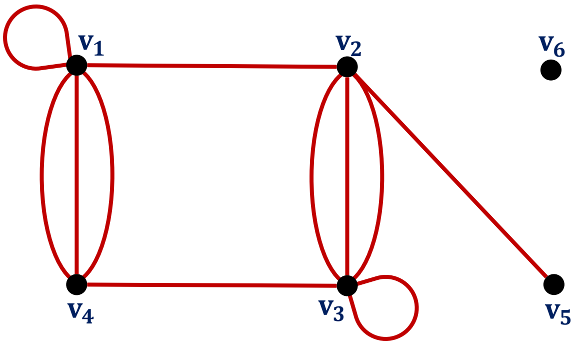
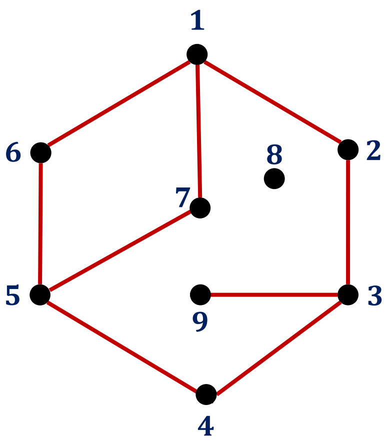
A	1	Define the following terms: Order of a Graph · Size of a Graph · Degree of a Vertex · Odd Vertex · Even Vertex Isolated Vertex · Pendent Vertex · Out – Degree of a Vertex · In – Degree of a Vertex · Total Degree of a Vertex **Answer: Refer Theory**
A	2	Find size, order and degree of each vertex of the given graph.  **Answer:** $\lvert V(G) \rvert = 6$, $\lvert E(G) \rvert = 8$, $d(v_1) = 3$, $d(v_2) = 2$, $d(v_3) = 3$, $d(v_4) = 3$, $d(v_5) = 2$, $d(v_6) = 3$
A	3	State First and Second theorem of Graph Theory. **Answer: Refer Theory**
A	4	How many vertices are necessary to construct a graph with exactly 6 edges in which all vertices have degree 2? **Answer: 6**

A	5	Determine even vertices, odd vertices, isolated vertices and pendent vertices of given graph.  $\boxed{G}$ **Answer: Even Vertices ⇝ $v_1, v_4, v_5, v_6,$ Odd Vertices ⇝ v_2, v_3** **Pendent Vertices ⇝ $v_3,$ Isolated Vertices ⇝ v_6**
A	6	Use the First or Second theorem of Graph Theory to answer the following statements: (1) Does there exists a graph in which 3 vertices of degree 5? Justify. (2) Does there exists a graph in which 2 vertices of degree 4? Justify **Answer: (1) No, because number of odd vertices must be even.** **(2) Yes, from second theorem of Graph Theory**

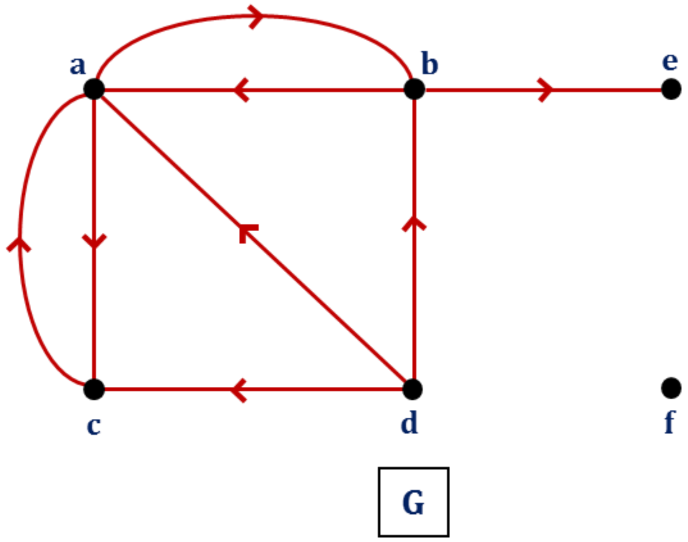
A	7	Find in – degree, out – degree and total degree of each vertex for the following graph:  **Answer**:

In – degree	Out – degree	Total degree
$d^-(v_1) = 1$	$d^+(v_1) = 1$	$d(v_1) = 2$
$d^-(v_2) = 1$	$d^+(v_2) = 2$	$d(v_2) = 3$
$d^-(v_3) = 2$	$d^+(v_3) = 1$	$d(v_3) = 3$
$d^-(v_4) = 1$	$d^+(v_4) = 1$	$d(v_4) = 2$
$d^-(v_5) = 1$	$d^+(v_5) = 2$	$d(v_5) = 3$
$d^-(v_6) = 2$	$d^+(v_6) = 0$	$d(v_6) = 2$
$d^-(v_7) = 1$	$d^+(v_7) = 1$	$d(v_7) = 2$
$d^-(v_8) = 0$	$d^+(v_8) = 1$	$d(v_8) = 1$

B	13	Can we construct a graph with 13 vertices such that all vertices have degree 3? Justify. **Answer**: **No because number of odd vertices must be even.**

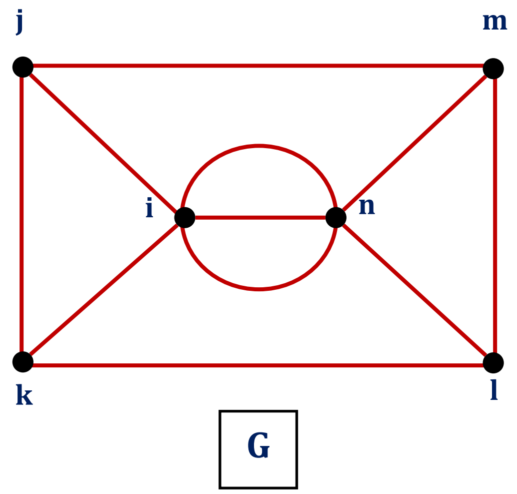
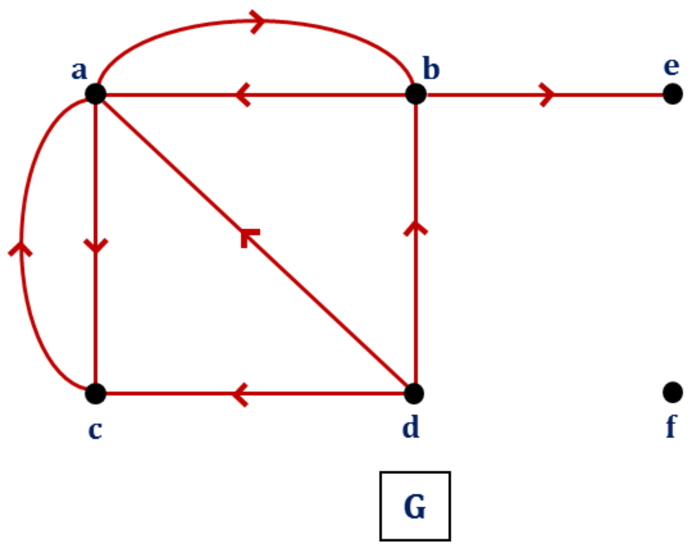
B	8	Determine even vertices, odd vertices, isolated vertices and pendent vertices of given graph.  **G** **Answer**: **Even Vertices** ⇝ v_1, v_3, v_4, v_6, **Odd Vertices** ⇝ v_2, v_5 **Pendent Vertices** ⇝ v_5, **Isolated Vertices** ⇝ v_6
B	9	Determine size, order and degree of each vertex of the given graph. Also classify vertices into even vertices, odd vertices, isolated vertices and pendent vertices.  **G** **Answer**: $\|V(G)\| = 9$, $\|E(G)\| = 9$, $d(1) = 3$, $d(2) = 2$, $d(3) = 3$, $d(4) = 2$, $d(5) = 3$, $d(6) = 2$, $d(7) = 2$, $d(8) = 0$, $d(9) = 1$

B	10	Find indegree, outdegree and total degree of each vertex for the following graph. Also, verify the degree sum formula.



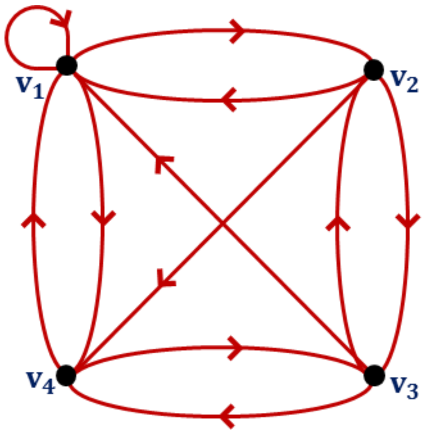
Answer:

In – degree	Out – degree	Total degree
$d^{-}(a) = 3$	$d^{+}(a) = 2$	$d(a) = 5$
$d^{-}(b) = 2$	$d^{+}(b) = 2$	$d(b) = 4$
$d^{-}(c) = 2$	$d^{+}(c) = 1$	$d(c) = 3$
$d^{-}(d) = 0$	$d^{+}(d) = 3$	$d(d) = 3$
$d^{-}(e) = 1$	$d^{+}(e) = 0$	$d(e) = 1$
$d^{-}(f) = 0$	$d^{+}(f) = 0$	$d(f) = 0$

B	11	Verify Handshaking theorem for the following graph. 
B	12	Verify First and Second theorem of Graph Theory for the following graph. 

C | 14

Find indegree, outdegree and total degree of each vertex for the following graph. Also, verify the degree sum formula.



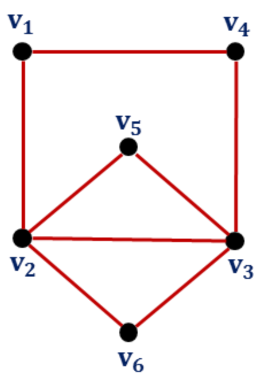
Answer:

In – degree	**Out – degree**	**Total degree**
$d^-(v_1) = 4$	$d^+(v_1) = 3$	$d(v_1) = 7$
$d^-(v_2) = 2$	$d^+(v_2) = 3$	$d(v_2) = 5$
$d^-(v_3) = 2$	$d^+(v_3) = 3$	$d(v_3) = 5$
$d^-(v_4) = 3$	$d^+(v_4) = 2$	$d(v_4) = 5$

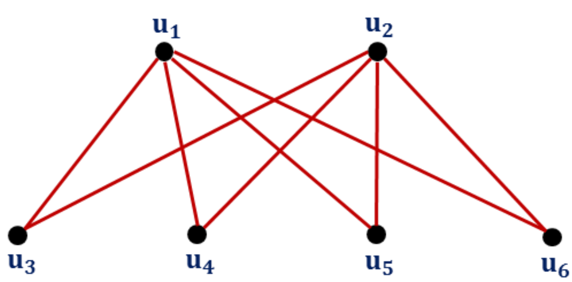
C | 15

Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw one such graph.

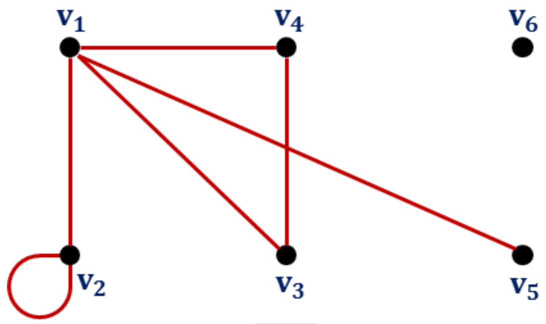
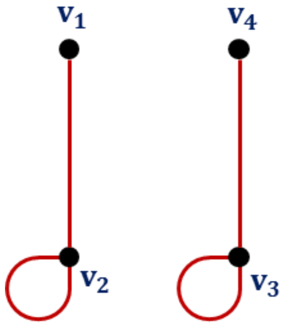
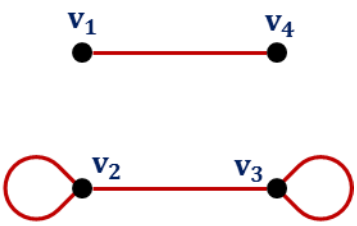
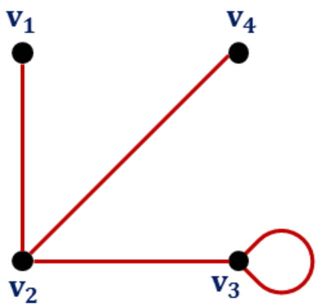
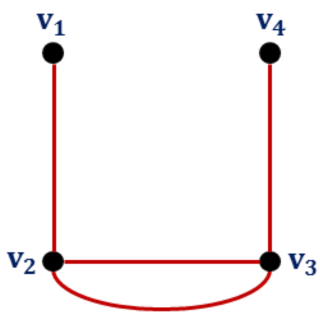
Answer: edegs $e = 8$



G_1



G_2

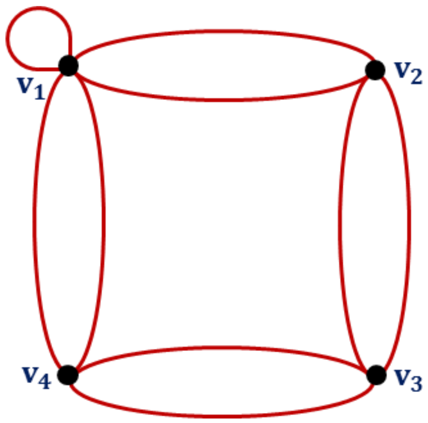
C	16	Construct a simple graph with 6 vertices which has degree 4, 3, 2, 2, 1, 0. **Answer**:  G
C	17	Draw at least three graphs having four vertices of degree 1, 1, 3 and 3. **Answer**:  G_1  G_2  G_3  G_4

Method 3 ⇝ Special Types of Graphs

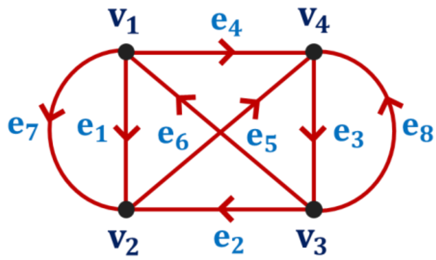
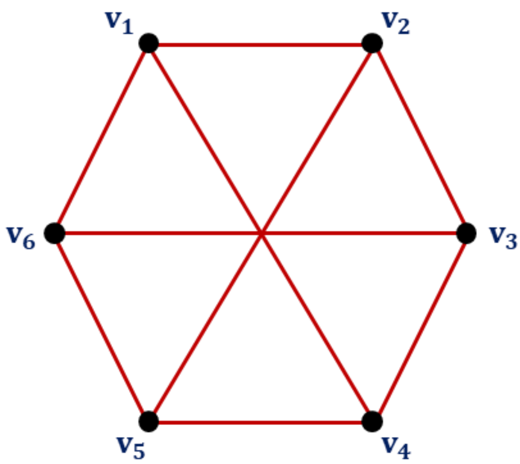
<table>
<tr><td>A</td><td>1</td><td>Define the following terms with example:

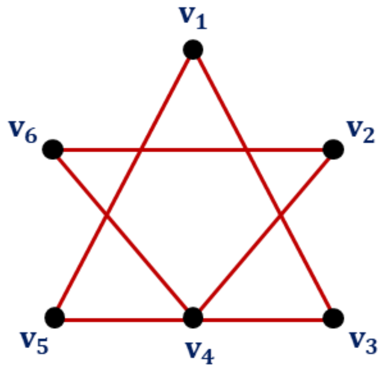
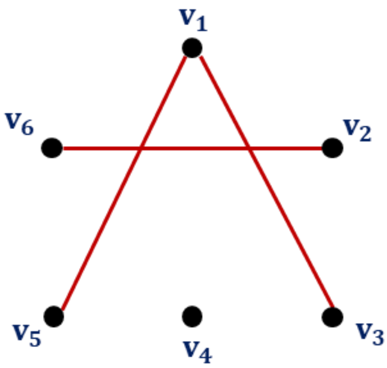
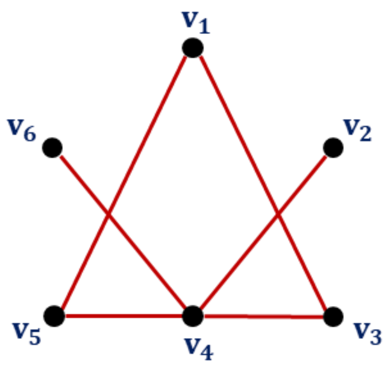
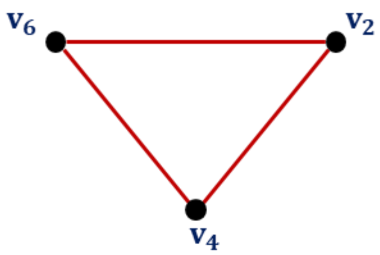
Undirected Multigraph | Cycle Graph
Directed Multigraph | Bipartite Graph
Simple Graph | Complete Bipartite Graph
Regular Graph | Subgraph
Complete Graph | Vertex and Edge Deleted Subgraph

Answer: Refer Theory</td></tr>
<tr><td>A</td><td>2</td><td>Answer the following questions for the undirected graph G:
(1). Check whether the graph is simple or not. Justify it.
(2). Check whether the graph is multigraph or not. Justify it.
(3). Check whether the graph is mixed or not. Justify it.

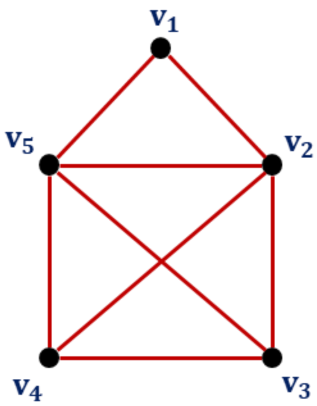

G

Answer: (1) Graph G is simple graph.
(2) Graph G is not multigraph.
(3) Graph G is not mixed graph.</td></tr>
</table>

A	3	Answer the following questions for the directed graph G: (1). Check whether the graph is simple or not. Justify it. (2). Check whether the graph is multigraph or not. Justify it. (3). Check whether the graph is mixed or not. Justify it.  G **Answer:** (1) **Graph G is not simple graph.** (2) **Graph G is multigraph.** (3) **Graph G is not mixed graph.**
A	4	Draw a 3 – regular graph having 6 vertices, if possible. **Answer:**  G
A	5	Determine the total number of edges of the following graphs: (1). C_{15} (2). K_{23} (3). $K_{12,11}$ **Answer:** (1) **15** (2) **253** (3) **132**

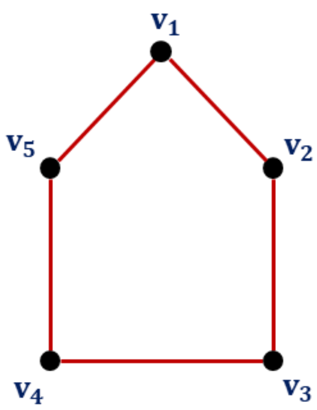
A	6	Find any three subgraphs of the following graph.  G **Answer**:  H_1  H_2  H_3

A | 7 | Draw the six subgraphs of the following graph.

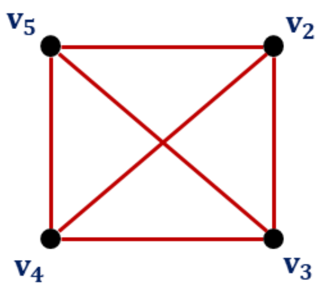


G

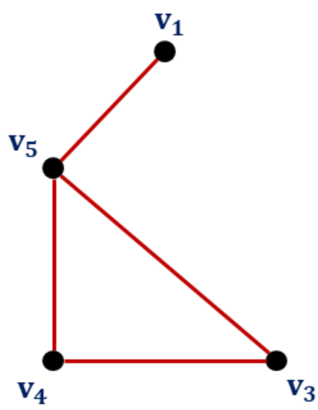
Answer:



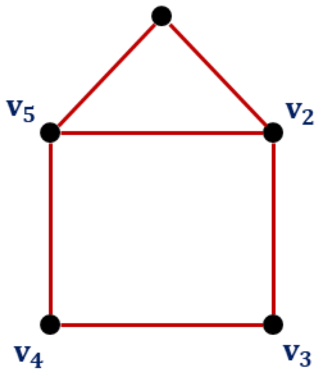
H_1



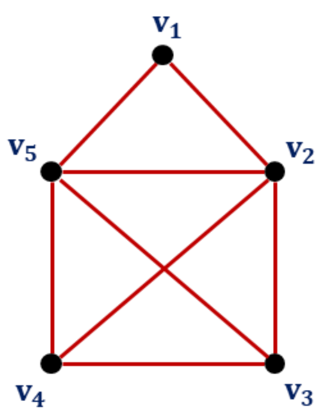
H_2



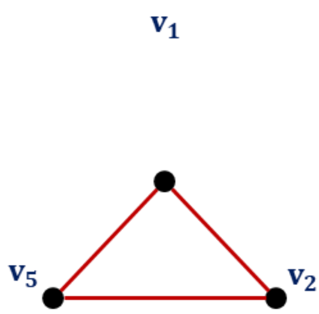
H_3



H_4



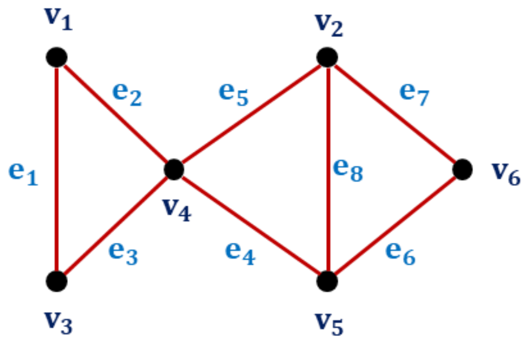
H_5



H_6

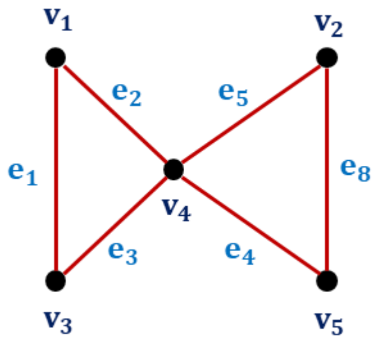
<table>
<tr>
<td>A</td>
<td>8</td>
<td>

Draw the graph $G \setminus \{ v_6 \}, G \setminus \{ v_6, v_2 \}$ and $G \setminus \{ e_6, e_7 \}$ for the following graph G:

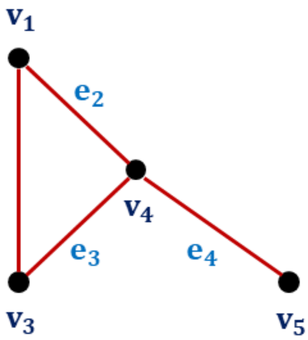


$\mathbf{G}$

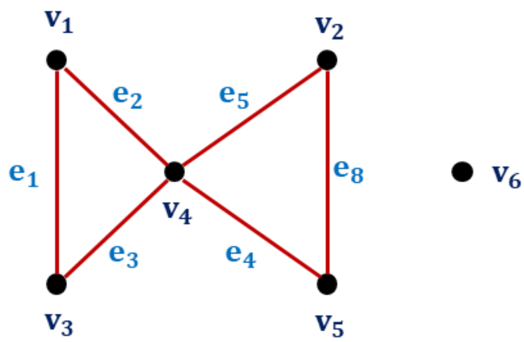
Answer:



$\mathbf{G - \{v_6\}}$



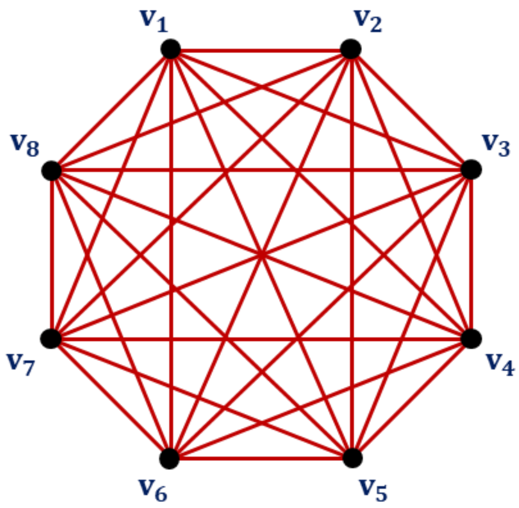
$\mathbf{G - \{v_6, v_2\}}$



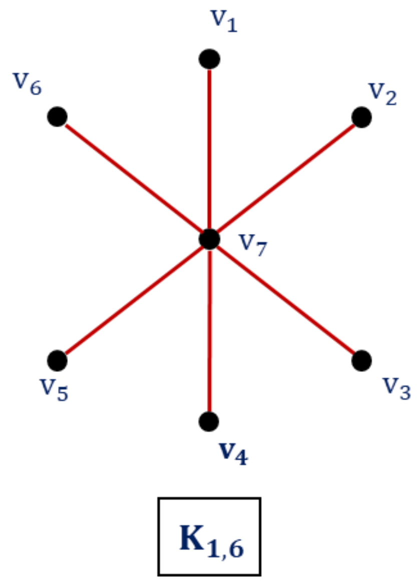
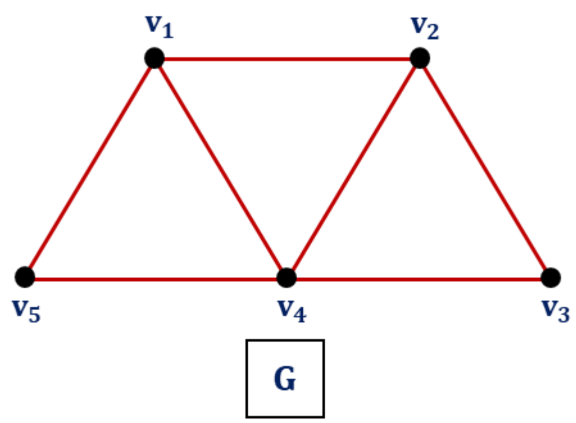
$\mathbf{G - \{e_6, e_7\}}$

</td>
</tr>
</table>

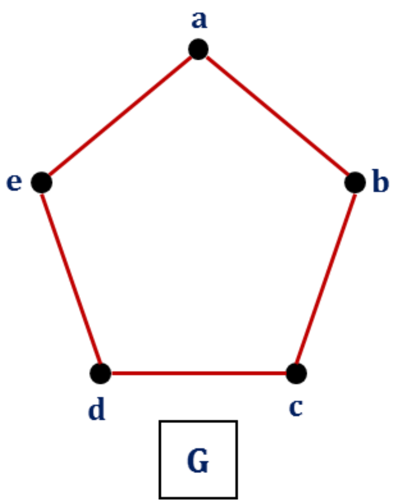
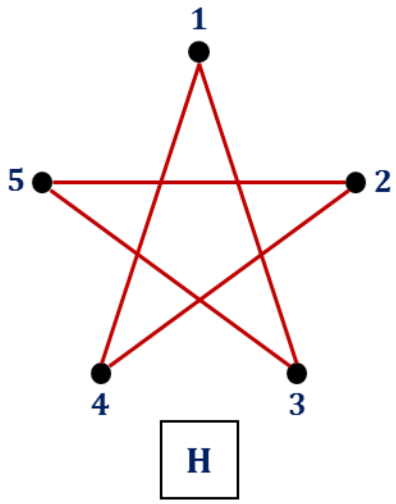
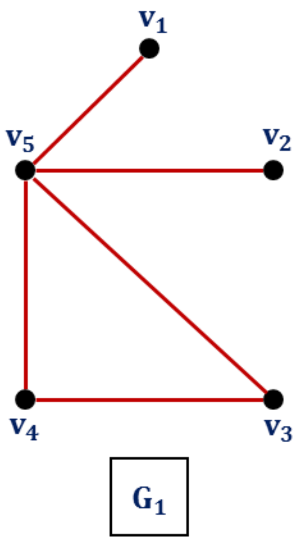
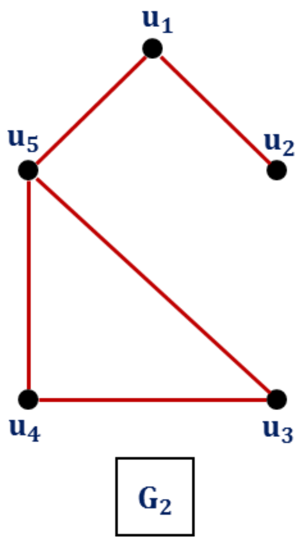
<table>
<tr>
<td>B</td>
<td>9</td>
<td>Draw a complete graph K_8 and answer the following questions:
(1). Check whether K_8 is regular graph or not. Justify it.
(2). Find the total number of edges of K_8.
(3). Find the total number of subgraphs of K_8.

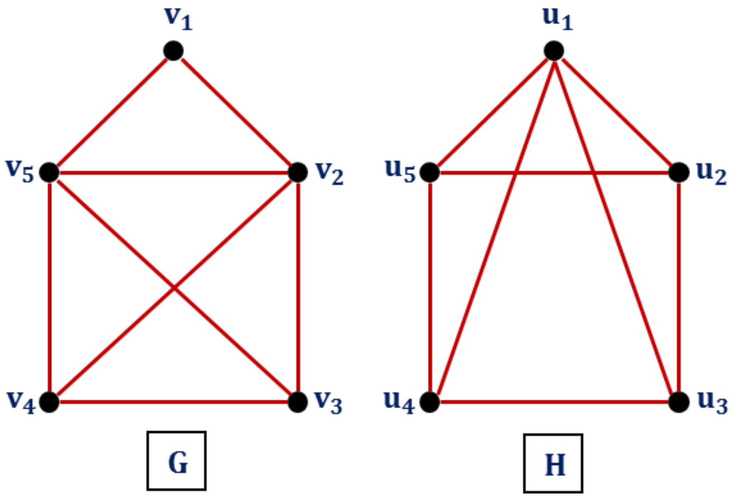
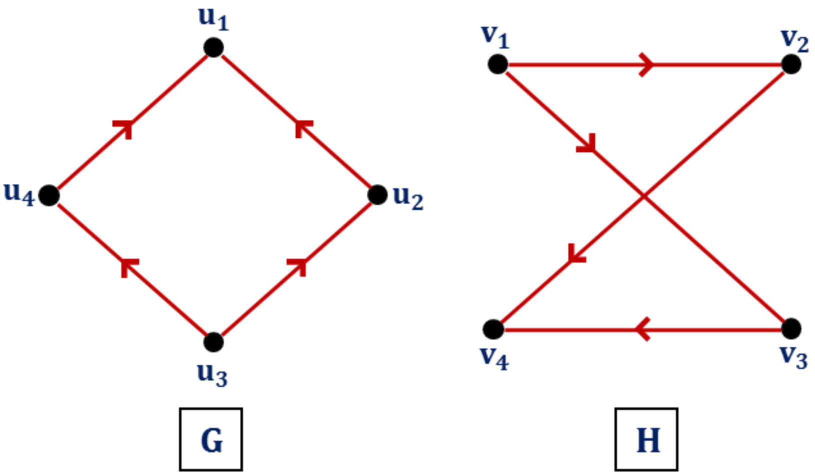
Answer:


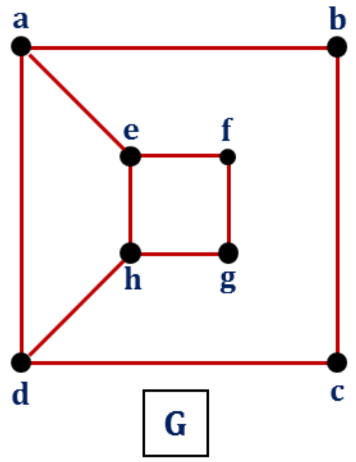
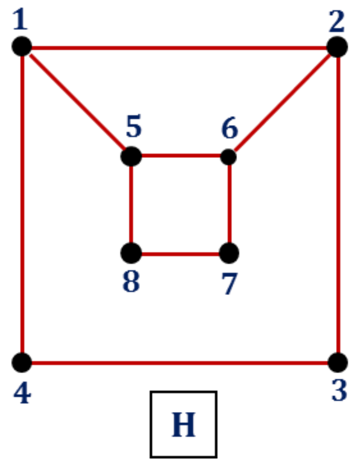
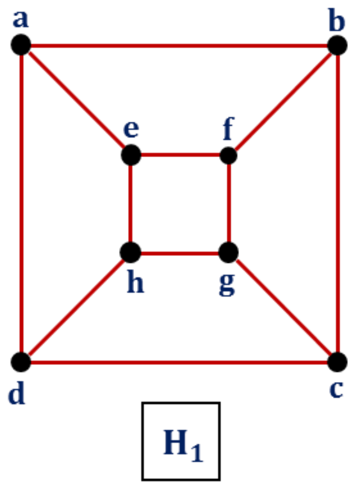
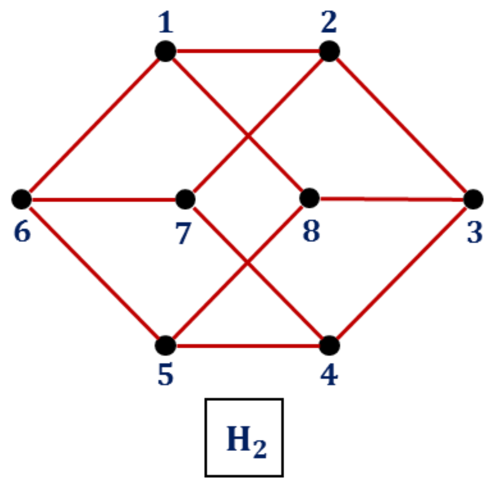
$\boxed{K_8}$
$\mathbf{(1)\ K_8 \text{ is } 7 - \text{regular graph}} \qquad \mathbf{(2)\ 28} \qquad \mathbf{(3)\ 2^{35}}$</td>
</tr>
</table>

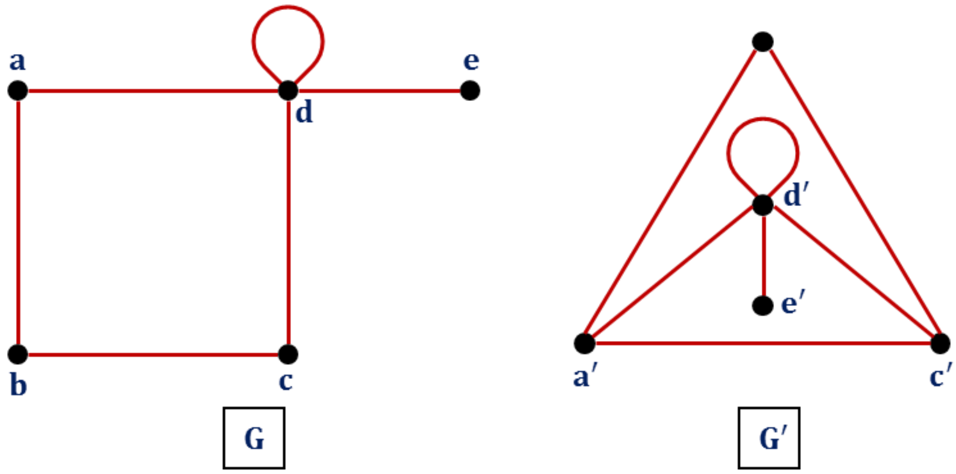
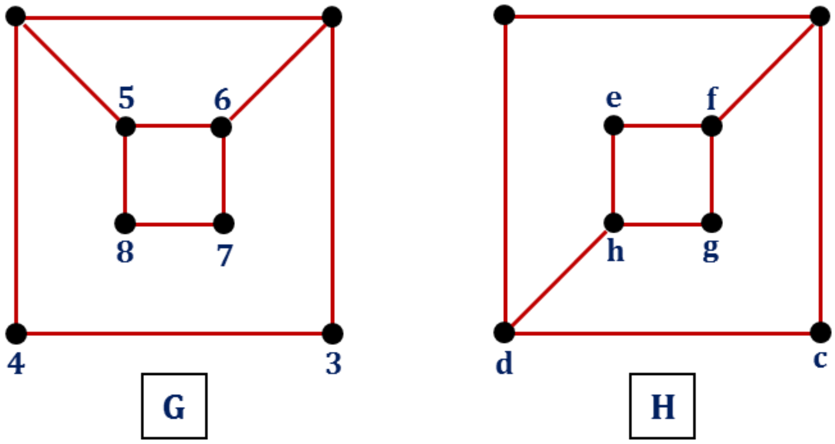
B	10	Draw a complete bipartite graph $K_{1,6}$ and answer the following questions: (1). Check whether $K_{1,6}$ is regular graph or not. Justify it. (2). Find the total number of edges of $K_{1,6}$ (3). Find the total number of subgraphs of $K_{1,6}$. **Answer**:  **(1) $K_{1,6}$ is not regular graph (2) 6 (3) 2^{12}**
B	11	Is it possible to draw a 5 – regular graph having 17 vertices? **Answer: Not possible**
B	12	Check whether the given graph is complete or not? Justify.  **Answer: Not a complete graph**

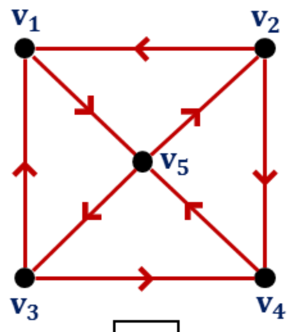
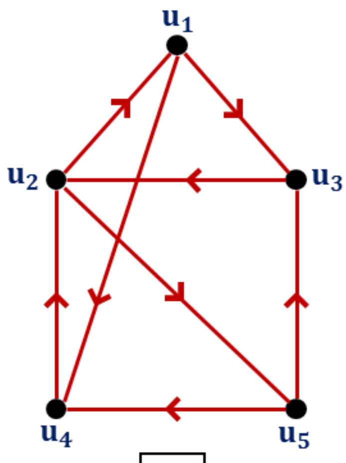
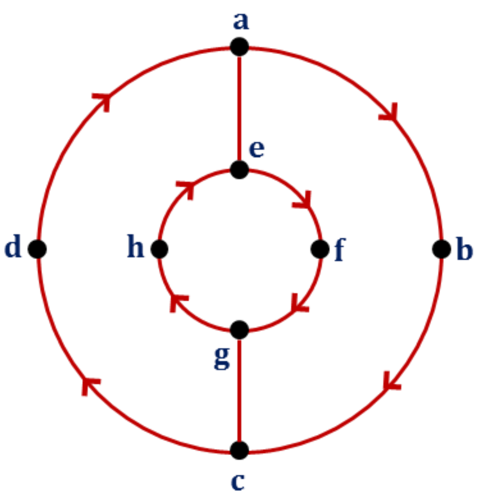
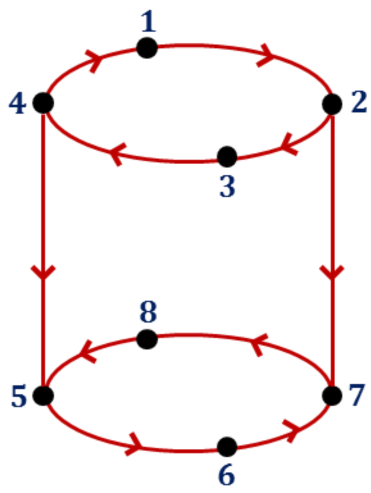
Method 4 ⇝ Graph Isomorphism

A	1	Define Isomorphism of Directed and Undirected graph. **Answer: Refer Theory**
A	2	Check whether the following graphs G & H are isomorphic or not?  **Answer: G ≅ H**
A	3	Check whether the following graphs G_1 & G_2 are isomorphic or not?  **Answer: $G_1 \ncong G_2$**

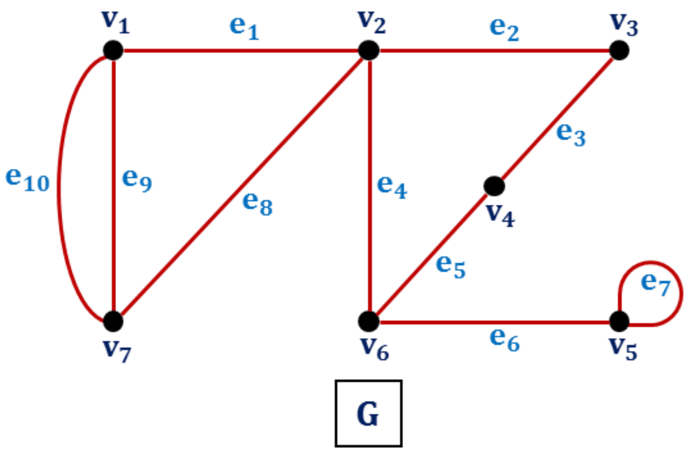
A	4	Check whether the following graphs G & H are isomorphic or not?  **Answer**: **G $\ncong$ H**
A	5	Check whether the following graphs G & H are isomorphic or not?  **Answer**: **G $\cong$ H**

B	6	Check whether the following graphs G & H are isomorphic or not?   **Answer: $G \cong H$**
B	7	Check whether the following graphs H_1 & H_2 are isomorphic or not?   **Answer: $H_1 \cong H_2$**

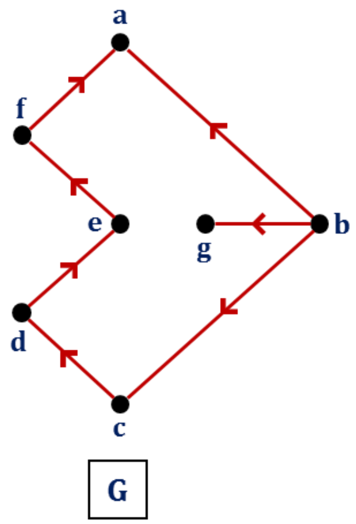
C	8	Show that the following graphs are isomorphic. 
C	9	Check whether the following graphs G & H are isomorphic or not?  **Answer**: **G $\ncong$ H**

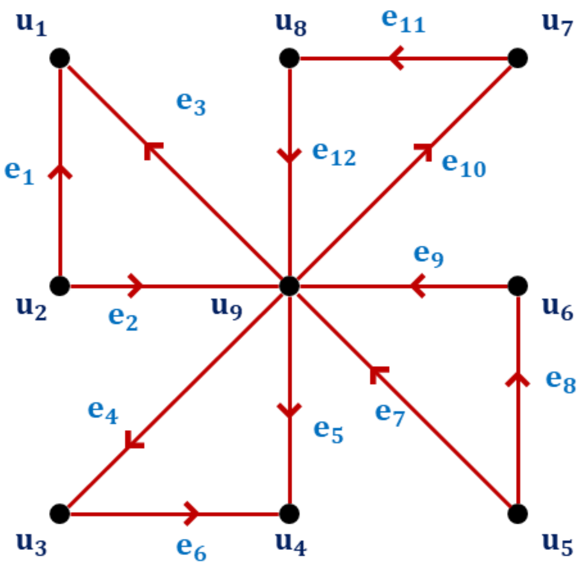
C	10	Show that the following graphs are isomorphic.  G  H
C	11	Check whether the following graphs G_1 & G_2 are isomorphic or not?  G_1  G_2 **Answer**: $G_1 \cong G_2$

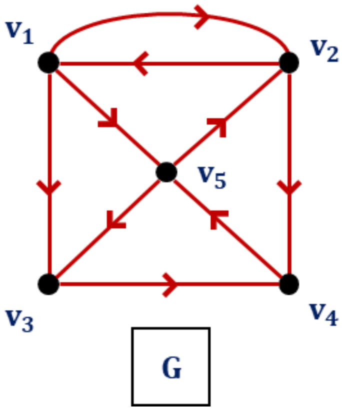
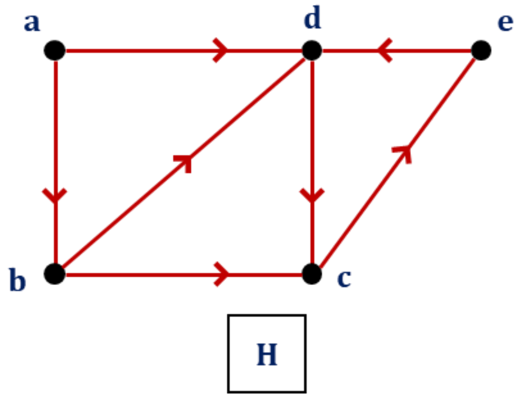
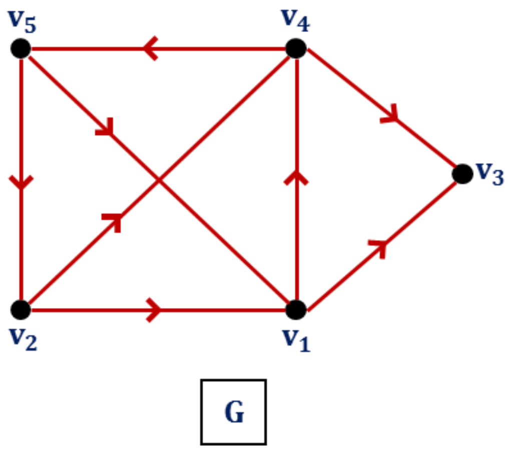
Method 5 ⇝ Connectivity

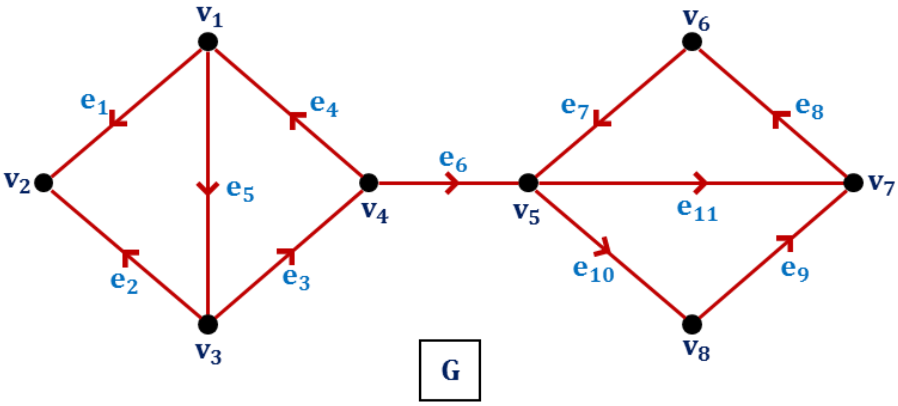
<table>
<tr><td>A</td><td>1</td><td>Define the following terms with example:
<table>
<tr><td>Path of Graph
Simple Path
Elementary Path
Circuit
Elementary Cycle
Trail
Connected Graph</td><td>Strongly Connected Graph
Unilaterally Connected Graph
Weakly Connected Graph
Strongly Connected Components
Unilaterally Connected Components
Weakly Connected Components
Maximal Connected Component</td></tr>
</table>
Answer: Refer Theory</td></tr>
<tr><td>A</td><td>2</td><td>Give six elementary paths for the following graph:


Answer: $P_1 = v_1e_1v_2$
$P_4 = v_1e_9v_7e_8v_2e_2v_3$
$P_5 = v_1e_7v_7e_8v_2e_2v_3e_3v_4$
$P_7 = v_1e_7v_7e_8v_2e_2v_3e_3v_4e_5v_6e_6v_5$
$P_2 = v_4e_5v_6e_6v_5$
$P_3 = v_7e_9v_1e_1v_2e_3v_6$</td></tr>
</table>

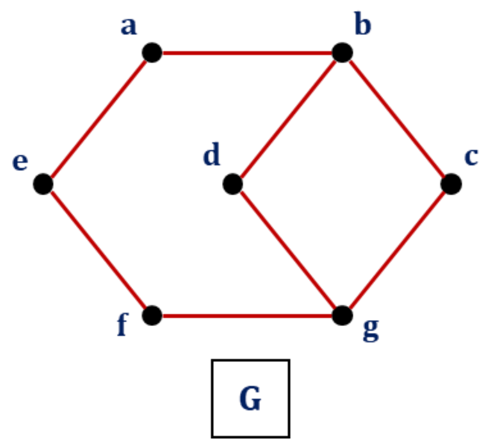
A	3	Determine whether the following graph is Strongly Connected, Unilaterally Connected or Weakly Connected. Also, find its components.  **Answer: G is weakly connected graph** **SCC : { a }, { b }, { c }, { d }, { e }, { f }, { g }** **UCC : { a, b, c, d, e, f }, { g }** **WCC : { a, b, c, d, e, f, g }**

B	4	From the given directed graph determine (1) path of length 3, 4, 5 and 6 (2) closed path of length 3 (3) an elementary path of length 6 (4) a simple but not elementary path of length 6  **G** **Answer**: **(1)** $P_3 = u_5e_8u_6e_9u_9e_3u_1$, $P_4 = u_9e_{10}u_7e_{11}u_8e_{12}u_9e_5u_4$, $P_5 = u_2e_2u_9e_{10}u_7e_{11}u_8e_{12}u_9e_5u_4$ $P_6 = u_5e_8u_6e_9u_9e_{10}u_7e_{11}u_8e_{12}u_9e_5u_4$ **(2)** $P_3 = u_9e_{10}u_7e_{11}u_8e_{12}u_9$ **(3)** **There does not exist an elementary path of length 6** **(4)** $P_6 = u_5e_8u_6e_9u_9e_{10}u_7e_{11}u_8e_{12}u_9e_5u_4$

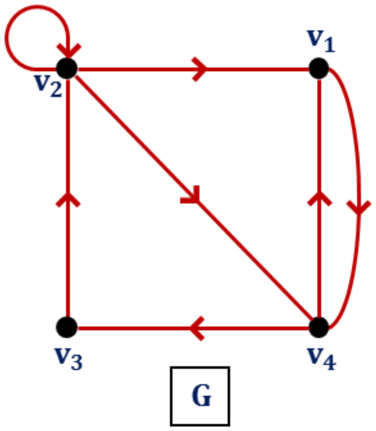
B	5	Determine whether the following graph is Strongly Connected, Unilaterally Connected or Weakly Connected. Also, find its components.   **Answer: G is strongly connected graph** **SCC & UCC & WCC : { v_1, v_2, v_3, v_4, v_5 }** **H is unilaterally connected graph** **SCC : { c, d, e }, { a }, { b }** **UCC & WCC : { a, b, c, d, e }**
B	6	Determine whether the following graph is Strongly Connected, Unilaterally Connected or Weakly Connected. Also, find its components.  **Answer: G is unilaterally connected graph** **SCC : { v_1, v_2, v_4, v_5 }, { v_3 }** **UCC & WCC : { a, b, c, d, e }**

C	7	From the given directed graph determine (1) circuit of length 3, 4, 7 (2) elementary cycle length 4 (3) trail of length 4 (4) an elementary as well as simple cycle (5) simple but not elementary cycle (6) neither elementary nor simple cycle  **Answer:** (1) $C_3 = v_1e_5v_3e_3v_4e_4v_1$, $C_4 = v_5e_{10}v_8e_9v_7e_8v_6e_7v_5$, $C_7 = v_5e_{10}v_8e_9v_7e_8v_6e_7v_5e_{11}v_7e_8v_6e_7v_5$ (2) $C_4 = v_5e_{10}v_8e_9v_7e_8v_6e_7v_5$ (3) $C_4 = v_5e_{10}v_8e_9v_7e_8v_6e_7v_5$ (4) $C_4 = v_5e_{11}v_7e_8v_6e_7v_5$ (5) **There is no such cycle in given graph.** (6) $C_7 = v_5e_{10}v_8e_9v_7e_8v_6e_7v_5e_{11}v_7e_8v_6e_7v_5$

Method 6 ⇝ Matrix Representation of Graphs

A	1	Define the following terms with example: (1) Adjacency Matrix for Undirected Graph (2) Adjacency Matrix for Directed Graph (3) Incidence Matrix for Undirected Graph (4) Incidence Matrix for Directed Graph (5) Path Matrix for Directed Graph **Answer: Refer Theory**
A	2	Determine the adjacency matrix for the following graph.  **Answer:** $$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

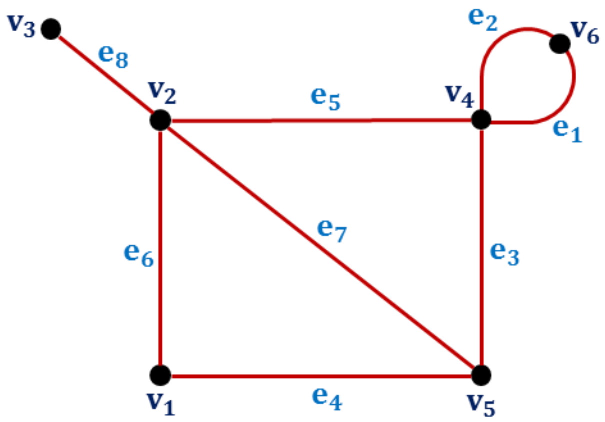
A	3	Determine the adjacency matrix for the following graph.



Answer:

$$A = \begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

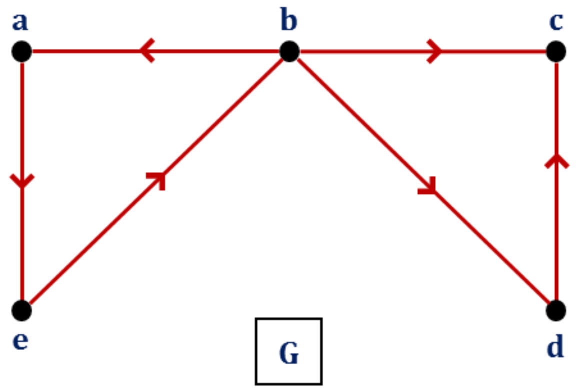
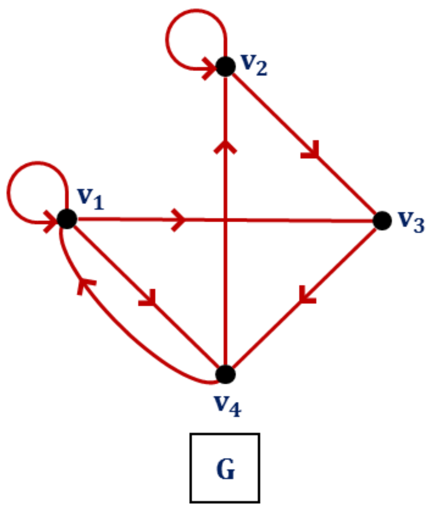
A 4 Determine the incidence matrix for the following graph:

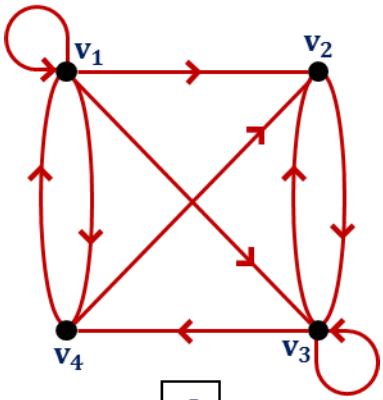


G

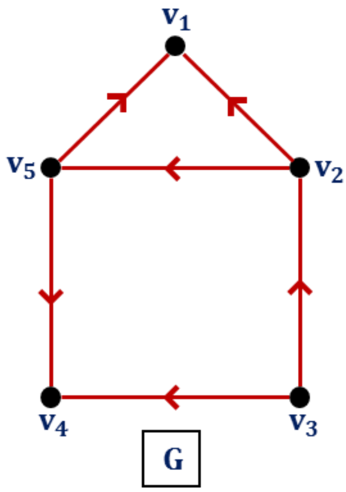
Answer:

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

A	5	Determine the path matrix for the following digraph:  **Answer**: $$P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$
B	6	Determine the path matrix for the following digraph:  **Answer**: $$P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

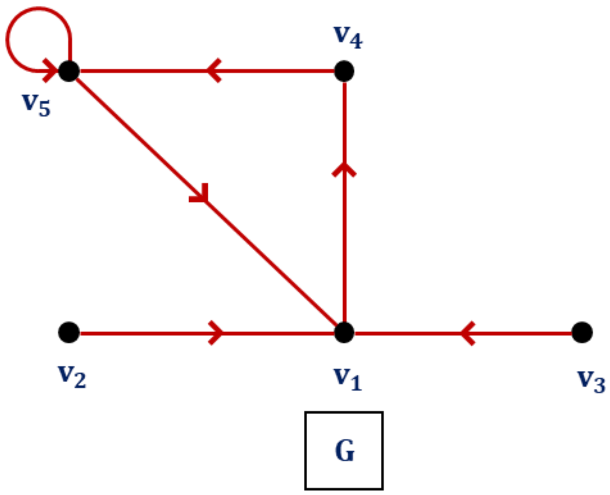
B	7	Determine the adjacency matrix for the following graph:  $\boxed{G}$ **Answer**: $$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

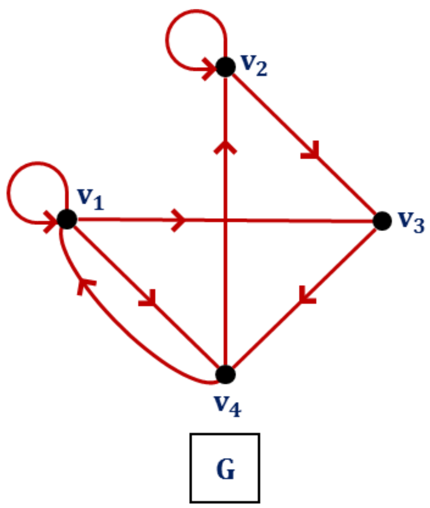
B	8	Determine the adjacency matrix for the following graph:



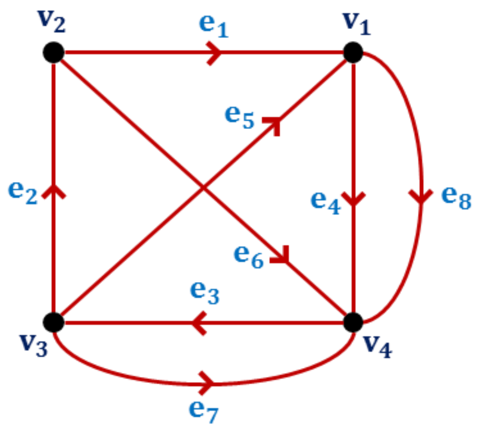
Answer:

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C	9	Draw the digraph having adjacency matrix as follows: $$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$ **Answer**: 

C	10	Draw the digraph having adjacency matrix as follows: $$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$ **Answer**: 

B	11	Determine the incidence matrix for the following digraph.:

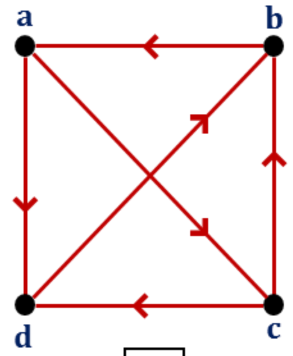


G

Answer:

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

B	12	Determine the path matrix for the following digraph:



G

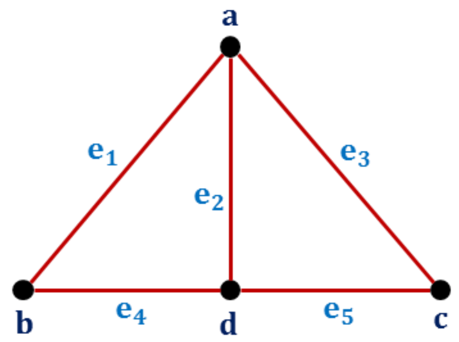
Answer:

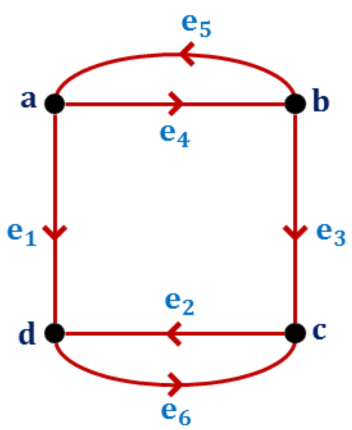
$$P = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

C	13	Draw the undirected graph having incidence matrix as follows:

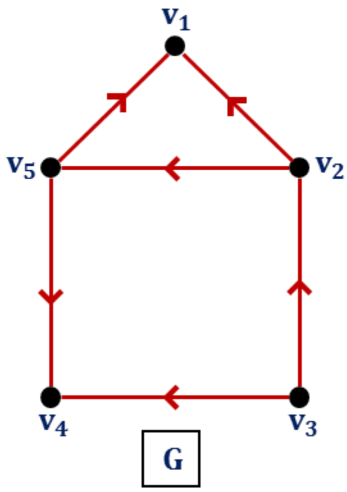
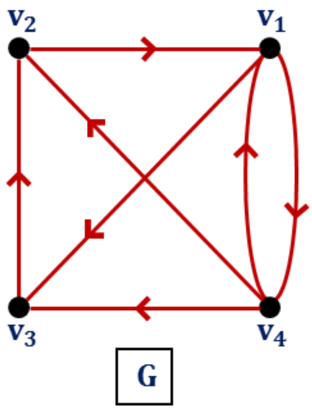
$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

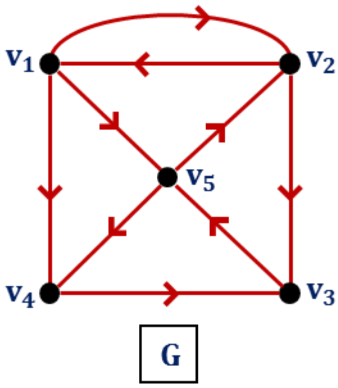
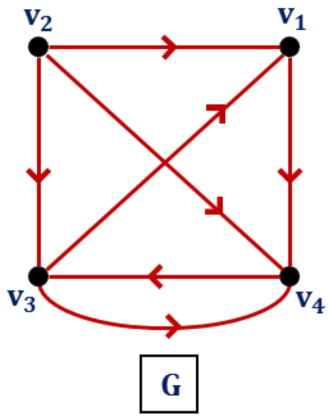
Answer:

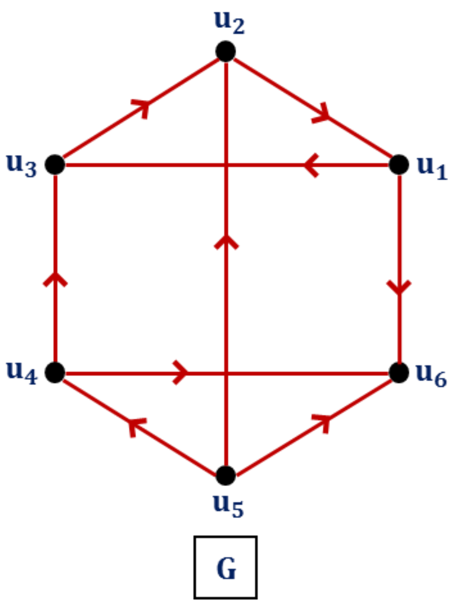


C	14	Draw the directed graph having incidence matrix as follows: $$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$ **Answer**: 

Method 7 ⇝ Warshall's Algorithm

A	1	Produce a path matrix of a following graph by using Warshall's algorithm:  **Answer**: $P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$
A	2	Apply Warshall's algorithm to produce a path matrix for the given graph.  **Answer**: $P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

B	3	Apply Warshall's algorithm to produce a path matrix for the given graph.  **Answer:** $P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$
C	4	Produce a path matrix of a following graph by using Warshall's algorithm:  **Answer:** $P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

C	5	Produce a path matrix of a following graph by using Warshall's algorithm:  **Answer**: $\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$