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Unit – 1 ⇝ Number Theory and Counting

Unit – 1.1 ⇝ Number Theory

Method – 1 ⇝ Principle of Mathematical Induction

Examples of Method-1: Principle of Mathematical Induction

A	1	Using mathematical induction to prove that, $(1 \cdot 2 \cdot 3) + (2 \cdot 3 \cdot 4) + (3 \cdot 4 \cdot 5) + \cdots + \big(n \cdot (n+1) \cdot (n+2)\big) = \frac{n(n+1)(n+2)(n+3)}{4}$; $n \in \mathbb{N}$.
B	2	For any real number i, and any integer $n \geq 2$, $\sum_{i=1}^{n-1} i\,(i+1) = \frac{n(n-1)(n+1)}{3}$.
C	3	Prove that proposition P, the sum of the cubes of the first n positive integers is $\frac{n^2(n+1)^2}{4}$.
A	4	Prove statement $7^n - 1$ is divisible by 6, for all integers $n \geq 0$ using mathematical induction.
B	5	Prove statement $2^{2n} - 1$ is divisible by 3, for all integers $n \geq 0$ using mathematical induction.
A	6	Prove the inequality $1 + 3n \leq 4^n$, for all integers $n \geq 0$, using principal of mathematical induction.
B	7	Prove the inequality $2^n < (n+2)!$, for all integers $n \geq 0$, using principal of mathematical induction.

Method – 2 ⇝ Congruence Relation

Examples of Method-2: Congruence Relation

A	1	Which of the following is true? **(1)** $472 \equiv 368 (\text{mod } 26)$ **(2)** $793 \equiv 682 (\text{mod } 9)$ **Answer**: **(1)**
A	2	Which of the following is true? **(1)** $269 \equiv 413 (\text{mod } 12)$ **(2)** $445 \equiv 536 (\text{mod } 18)$ **Answer**: **(1)**
A	3	Find the smallest positive integer which is congruent modulo m = 7 to each of the following numbers: **(1)** 386 **(2)** −192 **Answer**: **(1) 1 (2) 4**
B	4	Find the smallest integer from set $\{-3, -2, -1, 0, 1, 2, 3\}$ which is congruent modulo m = 6 to each of the following numbers: **(1)** 257 **(2)** −466 **Answer**: **(1) − 1 (2) 2**

Method – 3 ⇝ Encrypting and Decrypting a Message using Caesar Cipher Method

Examples of Method-3: Encrypting and Decrypting a Message using Caesar Cipher Method

A	1	Encrypt given message **"JAVATPOINT"** using Caesar Cipher method with key value k = 3. **Answer**: "**MDYDWSRLQW**"
A	2	Convert given ciphertext **"GSQTYXIV"** into plaintext by using key value k = 4. **Answer**: **"COMPUTER"**
B	3	Decrypt given message **"JGXYNGT ATOBKXYOZE"** using Caesar Cipher method with key value k = 6. **Answer**: "**DARSHAN UNIVERSITY**"

Method – 4 ⇝ Encrypting and Decrypting a Message using RSA Cryptosystem

Examples of Method-4: Encrypting and Decrypting a Message using RSA Cryptosystem

A	1	Encrypt a message $m = 8$ using RSA algorithm with $p = 7, q = 11, e = 7$. **Answer: $c = 57$**
A	2	Encrypt a message $m = 13$ using RSA algorithm with $p = 11,\ q = 3,\ e = 3$. **Answer: $c = 19$**
B	3	In RSA algorithm if $p = 11, q = 23, e = 13$, then what is value of d? **Answer: $d = 17$**
B	4	In a system RSA algorithm with $p = 5, q = 9$ is implemented for data security. What is value of decryption key, if the value of encryption key is 27? **Answer: $d = 19$**
B	5	Decrypt a message $c = 4$ with value $p = 3,\ q = 11,\ e = 7$. **Answer: $m = 31$**
C	6	Using RSA algorithm find plain text for cipher text of 6 provided $p = 13$, $q = 11,\ e = 29$. **Answer: $m = 2$**

Unit – 1.2 ⇝ Counting

Method – 5 ⇝ Basics of Counting

Examples of Method-5: Basics of Counting

A	1	Find the number of 3 letter words, with or without meaning, which can be formed out of the letters of the word PYTHON, where the repetition of the letters is not allowed. **Answer: 120**
A	2	From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position? **Answer: 56**
A	3	Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco? **Answer: 42**
A	4	A student can choose a computer project from one of three lists. The three lists contain 23, 15 and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from? **Answer: 57**
B	5	Using the digits 0, 1, 2, 3, 4, 5, and 6, how many 3-digit odd numbers can be formed if the digits can be repeated? **Answer: 60**

B	6	How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once? **Answer: 336**
C	7	How many 4-digit numbers are there less than 7000 without repetition of digits? **Answer: 3024**
B	8	How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated? **Answer: 120**
A	9	How many 4-letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated? **Answer: 5040**
C	10	How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits? **Answer: 120**
C	11	Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available. **Answer: 320**

Method – 6 ⇝ Pigeonhole Principle

Examples of Method-6: Pigeonhole Principle

A	1	There are 38 different time slots during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed? **Answer: 18 different rooms will be needed**
A	2	What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state? **Answer: Minimum 4951 students**
B	3	Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.
C	4	**(1)** How many cards must be selected from a standard deck of 52 cards to guarantee that at least two cards of the same kind are selected? **(2)** How many must be selected from a standard deck of 52 cards to guarantee that at least three hearts are selected? **Answer: (1) 14 (2) 42**
C	5	A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark. **(1)** How many socks must he take out to be sure that he has at least two socks of the same color? **(2)** How many socks must he take out to be sure that he has at least two black socks? **Answer: (1) 3 socks (2) 14 Socks**

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