Absent spetra is for one wavelength source.

Absent spectra (some Principal Maximas Missing).

Sometimes in the spectra obtained by grating, certain order of spectra are absent.

for eg → 1^{st} order spectra is visible
II^{nd} " " " invisible
& again III^{rd} " " " visible.

The intensity expression for N-slits is

$$I = I_0 \underbrace{\frac{\sin^2\alpha}{\alpha^2}}_{\text{diffraction through single slit}} \underbrace{\frac{\sin^2 N\beta}{\sin^2\beta}}_{\text{interference of all the diffracted waves.}}$$

⇒ in n-slit, there exist a contribution of single slit diffraction & plus the interference of all the diffracted rays through N-slits.

If we are viewing absence of spectra

⇒ ~~Cond~~ condition of minima for single slit.

ie $a \sin\theta = m\lambda$. —— (1) $m = 1, 2, 3 \ldots$ ~~N~~ excluding zero $m \neq 0$.

will ~~be~~ overlap with principal maxima condition due to N-slits..

ie $(a+b) \sin\theta = n\lambda$. —— (2)
$n = 0, 1, 2, 3.$

Thus if the two conditions (1) & (2) overlap each other, then the spectra will be missing.

Now why does this happens?
The condition for the overlapping of the above two equations (w/c results in absent spectra) is :—

$$\frac{(a+b)\sin\theta = n\lambda}{a\sin\theta \quad m\lambda.} \Rightarrow \frac{a+b}{a} = \frac{n}{m}.$$

Case 1 if $a=b$ [i.e. if opacity = transparency].

then $n = 2m$.
Now $m = 1, 2, 3 \ldots$

$\Rightarrow n = 2, 4, 6 \ldots$ [this order of spectra will be missing]
(P. Maxima will be missing).

*
Case 2 if $b = 2a$ [width of ~~transparency~~ opacity is double of ~~opacity~~ transparency].

This condition is satisfied in Labs

If $m = 1, 2, 3$

~~$n =$~~ $3a/a = n/m \Rightarrow n = 3m$

$\Rightarrow n = 3, 6, 9 \ldots$ (missing order).

Calculation of

1. Number of order of spectra in Given Grating

$$(a+b)\sin\theta = n\lambda. \quad \text{Grating Law}$$

$$\Rightarrow n = \frac{(a+b)\sin\theta}{\lambda}$$

$(a+b) \rightarrow$ grating element, $= \frac{1}{N}$ per inch

$= \frac{2.54}{N}$ per cm.

N = no. of lines per inch in grating.

If $\theta = 90°$

$\Rightarrow n \rightarrow n_{max} = \frac{(a+b)}{\lambda}$.] At this Condition Maximum no. of spectras are visible

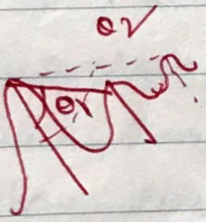
---

Q. Why third order spectra is invisible in diffraction grating experiment

Ans $n = \frac{(a+b)\sin\theta}{\lambda}$ for Labs $\lambda = 6000A°$

$N = 15000$ lines/inch.

$\Rightarrow a+b = \frac{1}{N} = \frac{1}{15000} \times 2.54$ cm.



$$\theta_1 = \frac{1 \times 6000A°}{a+b}$$

$\approx 20°$

$\theta_2 = 40°$ (for $n=2$).

and for $n=3$, $\sin\theta_3 > 1$

$\Rightarrow n=3$ Not visible.

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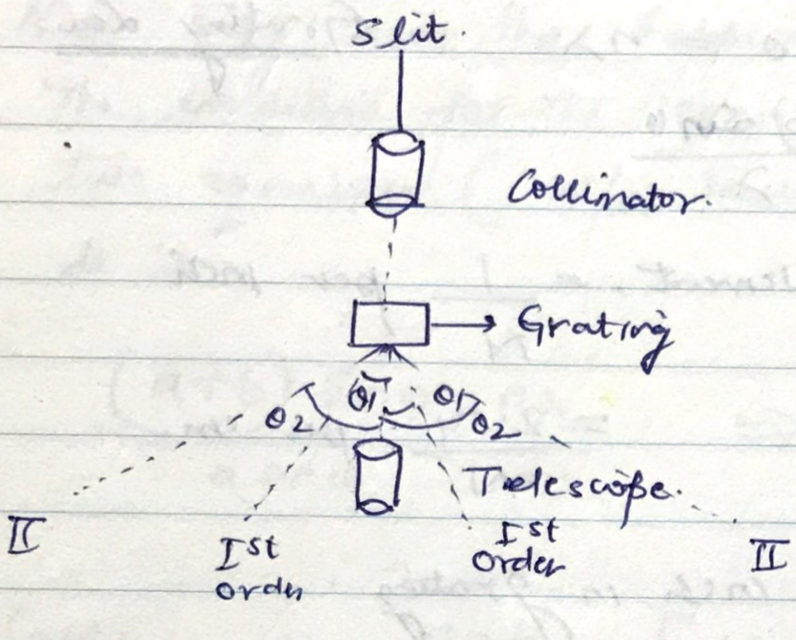
Determination of λ of Light by diffraction Grating.



$$(a+b)\sin\theta = n\lambda$$

Maximum no. of spectra visible

only 2 spectras are visible

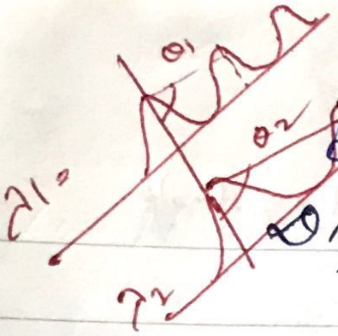
$$\sin\theta_1 = \frac{1\times\lambda}{a+b}$$

$$= 6\times10^{-5}/2.54/15$$

$$\approx 20^\circ$$

$\sin\theta_2 = 40^\circ$

$\sin\theta_3 > 1$



Angular spacing.

Dispersive Power of Grating.

(how much colours are dispersed. angular spacing is obtained b/w P. maxima when λ is changed)

It is defined as the change in the angle of diffraction corresponding to the unit change in wavelength of light.

Let θ = angle of diffraction for λ.
$(\theta + d\theta)$ " " " $(\lambda + d\lambda)$

$\Rightarrow$ dispersive power $= \dfrac{d\theta}{d\lambda}$.

Now $(a+b)\sin\theta = n\lambda$. $\longrightarrow$ GRATING LAW or position of P. Max.

diffrentiating w.r.t λ.

$$\Rightarrow (a+b)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$\Rightarrow \frac{d\theta}{d\lambda} = n \times \frac{1}{(a+b)} \times \frac{1}{\cos\theta}.$$

$\Rightarrow$ Case 1 $\quad d\theta/d\lambda \propto n$. (Order of spectrum).

Thus angular spacing for IInd order spectral line is double than 1st order spectral line - ie IInd order spectral line is twice as wide than Ist order

~~Case~~

$\frac{1}{(a+b)}$ = no. of lines per cm length of grating

$a+b = \frac{1}{N \text{ (total no. of lines)}}$

Case 2. $\frac{d\theta}{d\lambda} \propto \frac{1}{(a+b)}$. $\Rightarrow N = \frac{1}{a+b}$.

Dispersive power or angular dispersion is more for grating having large no. of lines per ~~inch~~ cm.

Case 3 $d\theta/d\lambda \propto \frac{1}{\cos\theta}$.

(a) If $\theta = 0^\circ \Rightarrow \cos\theta = 1 \Rightarrow$ max value.
$\Rightarrow$ angular dispersion is minimum.

~~If $\theta = 1 \Rightarrow \cos\theta = 0$ If θ is small $\Rightarrow$~~
~~$\Rightarrow d\theta/d\lambda$ is large.~~

(b) If θ is large $\Rightarrow \cos\theta$ is small $\Rightarrow$ higher is angular dispersion.

$\Rightarrow d\theta/d\lambda$ increases with θ.

Now in Vibgyor spectrum
θ red > θ violet?
bcz λ Red < λ violet
(bcz θ increases with λ)

{also $\theta \uparrow$ with λ.
$\Rightarrow$ Grating spectra spread much more in Red Region than violet Region}

$\Rightarrow$ Angular dispersion in Red > Violet.

* {Thus grating spectra in Red Region spread much more than in violet Region}

Resolving power

Resolving power = ~~The~~ The ability of an optical instrument to produce distinctly separate images of two very close objects is called resolving power.

For eg in a grating exp, the naked eye can't resolve the two spectral lines. Hence we use telescope or in biological samples we use microscope.

Limitation of Naked eye

An eye can see two separate objects if the angle subtended by the object at the eye is greater than about $1'$.

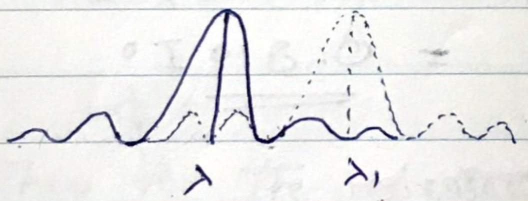
Thus $1'$ is the Resolving limit of eye

If the angle subtended is $< 1'$ then the eye can't see two objects separate.

Rayleigh's limit of Resolution

if we have two bright ~~so~~ point sources then two patterns or two spectral lines are regarded as separate or resolved if the central maximum of one (λ_1) falls up on the first minimum of other (λ_2)

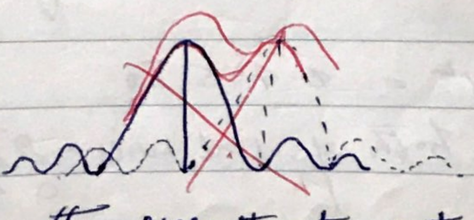
Case 1 If the diff. b/w two sources having wavelength ~~a~~ λ & λ_1 is very large then the spectral lines are distinctly separate.

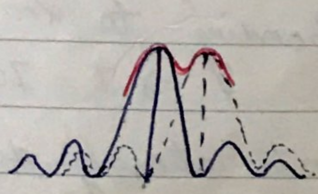


Here the ang. separation b/w two P Max's is large.

Case 2 If the diff b/w λ & λ_1 is small, then the P. Maximae of two spectral lines lie close to each other.

i.e the P. Max of 1'st spectral line lie on first minima of IInd " " ".





Hence the resultant intensity curve has double hump & a dip.

The two separate intensity curves intersect in the region with $\phi = \pi/2$.

Thus by this resultant intensity curve, Rayleigh found that in this condition the two spectral lines are just resolved.

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$ → single slit Relation is used. Not n-slit.

phase diff in this case $\alpha = \pi/2 \Rightarrow \boxed{I = 0.405 I_0}$

Thus the intensity of each pattern is 0.405 times the Max. intensity.

or The Resultant intensity is $0.405 \times 2I_0$
$= \underline{\underline{0.810 I_0}}$.

This is Rayleigh's Criterion
↓
that is the two spectral lines are just resolved when the intensity at the dip (or resultant intensity) = 0.810 times the max. intensity of either maxima.

The two separate curves intersect in the region corresponding to $\alpha = \pi/2$.

$\Rightarrow I = I_0 \sin^2\alpha/\alpha^2 = 0.405 I_0$.

Resultant intensity due to both spectral lines $= 2I_0 \times 0.405$
$= 0.810 I_0$.

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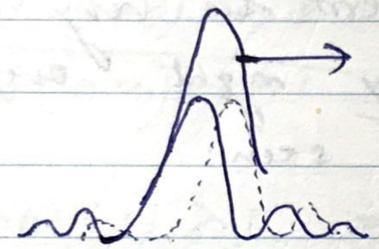
Case 3 → If the diff b/w λ & λ' is very ~~large~~ small. then the

P Maxima's of two spectral lines are very close to each other such that there is an overlapping of the intensity curves.



Hence the spectral lines are not resolved.

imp

Thus for the spectral line ~~to~~ to resolve. the principal maxima of one must lie on sec. minima of other. Thus the angular separation b/w the principal maxima of two spectral line (of a given order) should be equal to half angular width of either principal maxima

Resolving power of a Grating

Let we have light of two wavelengths λ_1 & λ_2.

$\Rightarrow$ We will get spectral lines from λ_1 & λ_2. bcz each wavelength will produce its own dyffraction pattern.

Now if these dyffraction patterns lie very close to each other then they might overlap & ~~we~~ could not separetely seen.

Acc^n to Rayleighs criterion, the two ~~separate~~ spectral lines are just resolved (or separately seen) if P.M due to ~~λ_2~~ falls on first S.C Minima due to ~~λ_2~~ λ_1 (or vice versa)

Now sec. Minima due to λ_1 is

$$N(a+b)\sin\theta = m\lambda_1 \quad -(1)$$

and if we talk about first sec minima adjacent to n^{th} P. Max.

$$\Rightarrow N(a+b)\sin\underbrace{\theta_n + d\theta_n}_{} = (nN+1)\lambda_1$$

$$N(a+b)\sin\theta' = (nN+1)\lambda_1 \quad -(2)$$

Now Acc^n to Rayleighs P.M of λ_2 ~~is~~ should lie on this angle.

$$(a+b)\sin\theta' = n\lambda_2. \quad -(3)$$

total no. of lines on grating

Multiplying N on both sides

$$\Rightarrow N(a+b)\sin\theta' = nN\lambda_2 \quad \text{--- (4)}$$

~~from~~ Acc^n to Rayleigh's criterion

eq (2) = (4)

$$\Rightarrow \frac{\cancel{N(a+b)\sin\theta'}}{\cancel{N(a+b)\sin\theta'}} = \frac{(nN+1)\lambda_1}{nN\lambda_2}$$

$$\Rightarrow (nN+1)\lambda_1 = nN\lambda_2$$

Let $\lambda_1 = \lambda$ & $\lambda_2 = \lambda + d\lambda$

$$\Rightarrow (nN+1)\lambda = nN(\lambda + d\lambda) \quad \text{solving}$$

$$\Rightarrow \lambda = nNd\lambda$$

Now Resolving ~~Powde~~ Power

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$

Thus R.P = product of n (order) & total no. of ~~line~~ lines.

from Grating law

$$\boxed{\frac{\lambda}{d\lambda} = N \times \frac{(a+b)\sin\theta}{\lambda}}$$

& $N(a+b)$ = total Width of Grating

$$\Rightarrow \boxed{\frac{\lambda}{d\lambda} = \frac{W\sin\theta}{\lambda}}$$

If $\theta = 90^\circ$

$\Rightarrow$ Max. Resolving Power $= \dfrac{N(a+b)}{\lambda}$

$= \dfrac{\text{Total width}}{\lambda}$

~~also dispersive power is~~

$$\cancel{\frac{d\theta}{d\lambda} = \frac{n}{a+b\cos\theta}}$$

DIFFERENCE b/w D.P & R.P

Also we know that dispersive power is

(1) $\dfrac{d\theta}{d\lambda} = \dfrac{n}{(a+b)\cos\theta}$ D.P

substituting n from dispersive power into resolving power

(2) $\Rightarrow \dfrac{\lambda}{d\lambda} = nN = \underbrace{N(a+b)}_{\text{width}}\cos\theta \dfrac{d\theta}{d\lambda}$

~~$\Rightarrow$ R.P~~ =

$\Rightarrow$ $D.P \propto n.$

(2) $\propto \frac{1}{a+b}$

But D.P doesn't depend upon width of Grating.

But $R.P \propto nN$

$\propto$ Width$/\lambda$.

(2')

~~(a+b) x width = N.~~

$(a+b) \times N =$ width

$\Rightarrow$. Therefore if the no. of lines per cm is same for two Gratings, then the dispersive Power will be same

but Resolving Power will be different.

and the grating which has larger width will have higher R.P.

(3) D.P Refers to wide separation of spectral lines

R.P " " ability of instrument to show spectral lines as separate