Now from the condition of minima.

eq (8)

$\sin\theta = \pm \frac{\lambda}{a}$

Here $m=1$.

$a \sin\theta = \pm m\lambda$

for 1^{st} minima, $m = 1$

$\Rightarrow \pm \frac{\lambda}{a} = \frac{y}{D}$

$\Rightarrow \boxed{y = \pm \frac{\lambda D}{a}}$

also $f = D$.

$\Rightarrow \boxed{y = \pm \frac{\lambda f}{a}}$

Now width of Principal Maxima $= 2y$

$\Rightarrow$ width $= \boxed{\frac{2 \times \lambda f}{a}}$

$\Rightarrow$ $\hookrightarrow \propto \lambda$

$\propto \frac{1}{a}$

for conceptual Question

Effect of slit width

Now from eq (8) $\boxed{\sin\theta = \pm \frac{\lambda}{a}}$

$\Rightarrow$ ~~If θ for a given λ, θ small~~

~~$\Rightarrow \sin\theta =$ small~~

* ~~Since~~ $\theta =$ angular may width.

Numericals will form on this

Thus if for given λ.

Case 1 — a is large

$\Rightarrow$ $\sin\theta$ = small

$\Rightarrow$ θ = small.

$\Rightarrow$ Maxima & minima lie close to central maxima.
(Principal)

Case 2 — If a is small
i.e. narrow slit

$\Rightarrow$ $\sin\theta$ = Large

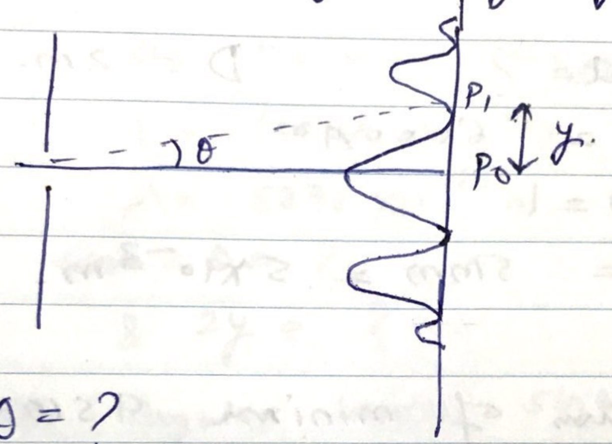
$\Rightarrow$ θ = "

$\Rightarrow$ Maxima & minima are quite distant & clear.

Numericals

Q1 Find the ~~Ray~~ angular width of central bright maximum in Fraunhofer diffraction pattern of a slit of width 12×10^{-7} m when the slit is illuminated by a light of $\lambda = 6000 A^\circ$

Ans



$\Rightarrow \quad \theta = ?$

$a = 12 \times 10^{-7}$ m.

$\lambda = 6000 A^\circ$

$\Rightarrow \quad \sin\theta = \frac{\lambda}{a} = 1/2 \quad \Rightarrow \quad \theta = 30^\circ$

Q2 In F. Diffraction, due to narrow slit, a screen is placed 2m away from the lens to obtain pattern. If the slit width is 0.2 mm & the first minima lies 5mm on either side of central maxima, find λ.

$D = 2m, \quad a = 0.2\,mm = 0.2 \times 10^{-3}\,mm.$

$y = 5\,mm = 5 \times 10^{-3}\,m.$

$y = \frac{\lambda D}{a} \Rightarrow \lambda = \frac{y \times a}{D} = \frac{5 \times 10^{-3} \times 0.2 \times 10^{-3}}{2}$

$\lambda = 5000 A^\circ.$

Q3 A screen is placed, 2m away from a narrow slit w/c is illuminated with light of $\lambda = 6000 A^{\circ}$. If the first minima lies 5mm on either side of central maxima. Calculate slit width.

Ans

$a = ?$ $D = 2m.$

$\lambda = 6000 A^{\circ}$

$m = 1$

$y\ \cancel{x} = 5mm = 5 \times 10^{-3} m.$

Now condition of minima $a \sin\theta = m\lambda$.

for $\sin\theta = \frac{y}{D}.$ {acc^n to diag of slit width of principal maxima}

$$\sin\theta = \frac{5 \times 10^{-3}}{2}$$

$$\Rightarrow a = \frac{1 \times 6000 \times 10^{-10}}{5 \times 10^{-3}/2}$$

$$= \boxed{2.4 \times 10^{-4} m}$$

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Q4 A parallel beam of light ($\lambda = 5890 \times 10^{-10}$ m) is incident perpendicularly on a slit of width 0.1 mm. Calculate angular width & linear width of central maximum formed on a screen 100 cm away

Sol

$a = 0.1 \text{ mm} = 0.1 \times 10^{-3}$ m.

$D = 100$ cm $= 1$ m.

$\lambda = 5890 \times 10^{-10}$ m.

~~sin θ~~ $2\theta = ?$ bcz θ = half ang. width.

& $2y = ?$

$\Rightarrow$ Angular width $\sin\theta = \lambda / a$

$$\sin\theta = \frac{5890 \times 10^{-10}}{0.1 \times 10^{-3}} = 5.89 \times 10^{-3}$$

Now accⁿ to figure, for small θ, $\sin\theta \approx \theta$

$\Rightarrow \theta = 5.89 \times 10^{-3}$ radians.

$\Rightarrow 2\theta = 11.78 \times 10^{-3}$ Radians.

also width $= 2y$ & $y = \frac{\lambda D}{a}$

$$= \frac{5890 \times 10^{-10} \times 1}{0.1 \times 10^{-3}} = 5890 \times 10^{-5}$$

$\Rightarrow 2y = 5890 \times 10^{-5} = 11780 \times 10^{-5}$ m.

H.W

Q5 Monochromatic light is incident on a slit of width 0.012 mm. The angular positions of first bright light is 5.2°. Calculate λ.

Ans We know condition of Maxima is

$$a \sin\theta = (2n+1)\frac{\lambda}{2}$$

Now $n=1 \Rightarrow a\sin\theta = \frac{3\lambda}{2}$

$\theta = 5.2°$

$= 5.2 \times \frac{\pi}{180}$ radians

Now $\sin\theta \approx \theta$ (for small θ)

$$\Rightarrow a \times \frac{5.2 \times \pi}{180} = \frac{3 \times \lambda}{2}$$

$$\Rightarrow \frac{0.012 \times 10^{-3} \times 5.2 \times 22 \times 2}{7 \times 180 \times 3}.$$

$$= \frac{2.7456 \times 10^{-3}}{3780} = 0.0007263 \times 10^{-3}$$

$7263 \times 10^{-3} \times 10^{-7} = 726.3 \times 10^{-9}$ m

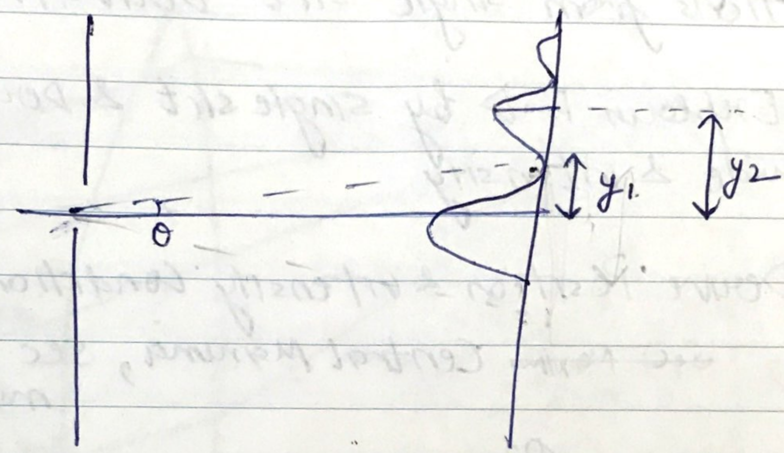
$= \boxed{726.3 \text{ nm}}$

*

Q6.

Diffraction pattern on single slit width of 0.5 cm is formed by a lens of focal length = 40 cm. Calculate the distance b/w first dark fringe & first bright fringe from the axis.

$\lambda = 4890\ A°$



$\Rightarrow$ Acc^n to question, we have to calculate $y_2 - y_1$.

Now for minima, $\boxed{a \sin\theta = n\lambda}$ $n=1$

also from diag $\sin\theta = \frac{y_1}{D} = \frac{y_1}{f}$

$$\Rightarrow a \times \frac{y_1}{f} = 1 \times \lambda$$

$$\Rightarrow y_1 = \frac{1 \times 4890 \times 10^{-10} \times 40 \times 10^{-2}}{5 \times 10^{-3}}$$

$$= 3.912 \times 10^{-5}\ m$$

also for Maxima, $\boxed{a \sin\theta = (2n+1)\frac{\lambda}{2}}$

$$\Rightarrow a \sin\theta = \frac{3\lambda}{2} \qquad n=1$$

& Here $\sin\theta = \frac{y_2}{D} = \frac{y_2}{f}$

$\Rightarrow 9 \times \frac{y_2}{f} = \frac{3\lambda}{2}$

$\Rightarrow y_2 = 5.868 \times 10^{-5}$

$\Rightarrow y_2 - y_1 = 1.596 \times 10^{-5} m.$

Questions from single slit Derivation.

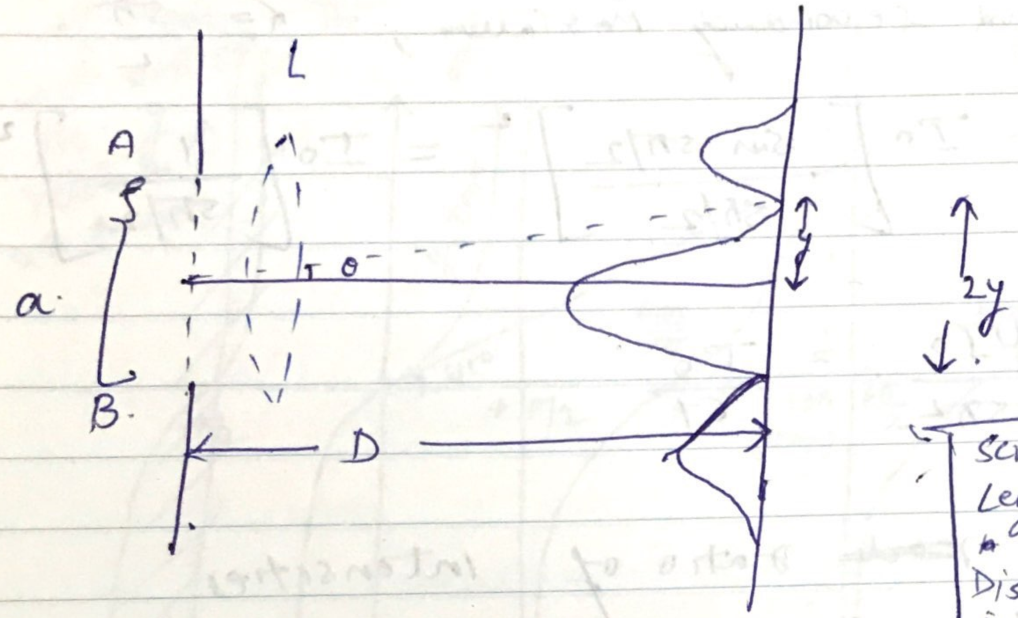
Q1 Explain F.D by single slit & Derive Resultant amplitude & intensity

Q2 Derive Position & intensity conditions for ~~Sec. Maxima~~ Central Maxima, sec. minima & maxima.

Q3 ~~Deri~~ Draw intensity Graph for F.D single slit.

Q5 } Q6 Intensity ~~separation~~

Width of Central Maxima



In correct diag drawn in devraj

Screen is placed at focal Length of Lens. & ~~dist~~ Lens is near to slit. Distance b/w slit & Lens is D
$\Rightarrow f$ can be taken $= D$.

a =

D =

Now a Lens is placed b/w slit & screen such that Lens is very near to slit. & there exist infinite distance b/w slit & screen.

$\Rightarrow f$ (focal length of Lens) $\approx D$

also $\tan\theta = \dfrac{y}{D}$

θ is the angular half width & 2θ = angular width of C.M

θ is very small

$\Rightarrow \tan\theta \approx \theta \approx \sin\theta$

$= \dfrac{y}{D}$

2.6 PLANE DIFFRACTION GRATING

An arrangement consisting of a large number of equidistant parallel rectangular slits of equal width separated by equal opaque portions is known as a diffraction grating. It may be constructed by ruling equidistant parallel lines with a fine diamond point on an optically plane glass plate, forming a *plane transmission grating*. The ruled lines are opaque to light while the space in between any two lines is transparent to light. The lines varying from 12,000 to 30,000 to an inch are drawn and the ruled surface varies from two to six inches. The transmission gratings are plane and are ruled on glass surfaces. If lines are drawn on a plane or concave reflecting surface; then light is reflected from the positions of mirrors in between any two lines and it forms a *plane* or *concave reflection grating*. The lines act like opaque wires and are called opacities. While the clear spaces between the lines are known as transparencies. If width of the transparency and opacity be 'a' and 'b' respectively, then distance $(a + b)$ is called a *Grating constant* or Grating element and usually denoted by 'g'.

There are only a few originally ruled gratings. For practical purposes, replica gratings are made from the original grating. For it, a solution of celluloid dissolved in a volatile solvent is poured over the surface of a ruled grating and allowed to harden. This plastic film, when removed from the grating surface, is found to retain an impression of the rulings of the original grating and is fixed between two glass plates if a transmission grating is required or against a silvered surface to obtain a reflection grating.

2.6.1 Theory of Diffraction Grating

Let a parallel beam of monochromatic light of wavelength λ be incident normally on N parallel slits each of width 'a' and separated by a distance b. Such an arrangement providing a large number of parallel equidistant slits is known as diffraction grating. All the rays issuing from the spaces reach

P_0, Fig. 2.8, on the screen in phase with one another, reinforce each other and provide a central maximum. A part of the light gets diffracted in various directions, those diffracted at an angle θ with the initial direction reach P_1 on passing through a convex lens L in different phases. As a result dark and bright bands on both sides of the central maximum are obtained.

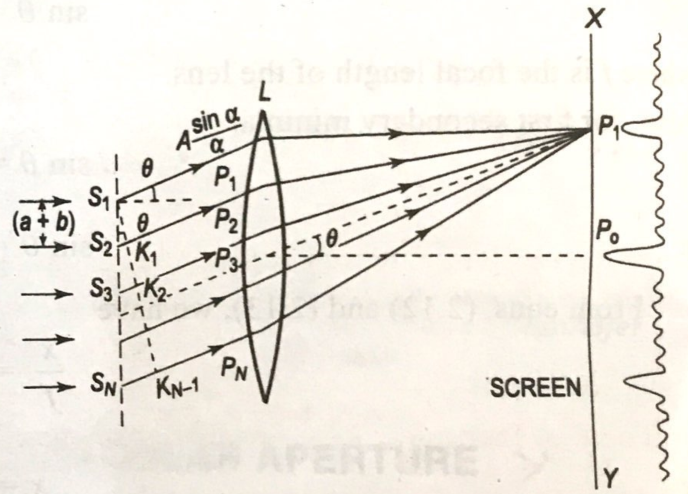
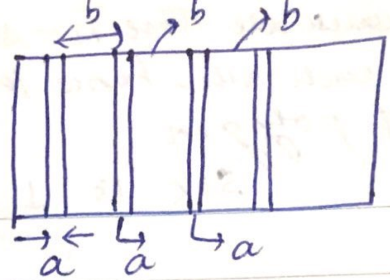


Fig. 2.8 *Diffraction of plane waves at plane diffraction grating*

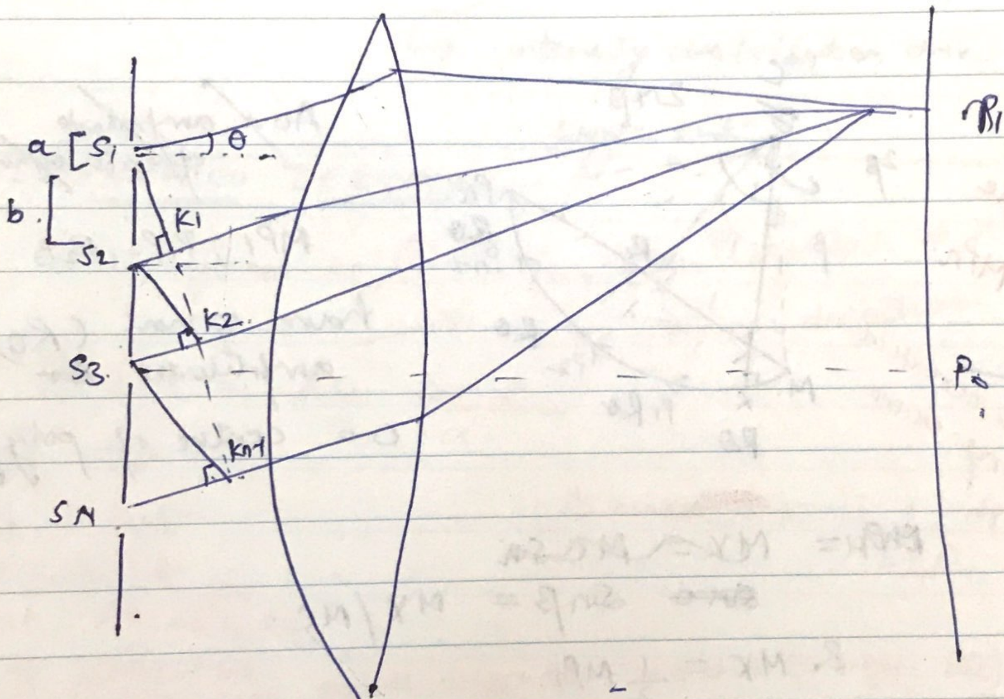
According to Huygens principle, as soon as the plane wavefront is incident on the slits, all points in each slit become the sources of secondary disturbances in all directions. By the theory of Fraunhofer diffraction at a single slit, the secondary waves from all points in a slit diffracted in a direction θ are equivalent to a single wave of amplitude

Q Grating element $=(a+b)$.

Grating



Its an arrangement of large no. of equidistant parallel rectangular slits of equal width separated by equal opaque portions.



Now the Resultant amplitude $R_\theta = R_0 \frac{\sin\alpha}{\alpha}$ acc^n to theory of single slit, the secondary waves from all points of the slits diffracted at angle θ is equivalent to single wave of amplitude

$$A_\theta = A \frac{\sin\alpha}{\alpha}.$$

amplitude of wave at Principal maxima

Resultant amplitude due to single slit

A_θ - amplitude of each wave.

& $\alpha = \frac{\pi a \sin\theta}{\lambda}$ eq (4).

from eq ⑥. $\longrightarrow$ maximum amplitude $= nA$.
in normal direction.

$$R_\theta = R_0 \frac{\sin\alpha}{\alpha}$$

$\downarrow$

Resultant amplitude due to ~~2~~ single slits in θ direction
(i.e Resultant amp. of all diffracted waves due to single slit).

Now. S_1K_1 is $\perp$ drawn on S_2P.

$\Rightarrow$ path diff $(S_2K_1) = (a+b)\sin\theta$

Let phase diff $= \frac{2\pi}{\lambda}(a+b)\sin\theta$

$\downarrow$

[In single slit 'ϕ' is phase diffn] (2β).

In Δ MCX. $\sin\beta = \frac{MX}{MC}$.

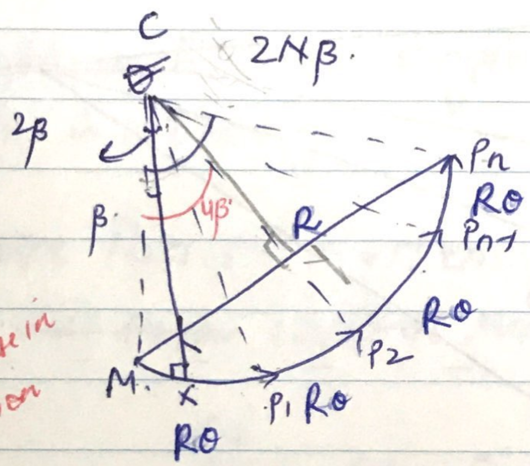
$\Rightarrow$ ~~MC~~ $\cdot$ $MX = MC\sin\beta$.

also Accn to figure, $MX = \frac{1}{2}MP_1$.

$\Rightarrow \frac{1}{2}MP_1 = MC\sin\beta \Rightarrow \boxed{MP_1 = 2MC\sin\beta.}$ — ①

2Nβ is angle b/w MP_1 & MP_n.

Note: The phase increase in ... tic progression



~~Ao = amplitude of each wave~~

MP_1, P_1P_2, P_2P_3

have equal (R_0) amplitudes ~~Ao~~.

O = centre of polygon.

Similarly $MP_N = 2MC \sin N\beta.$

Substitute the value of $2MC$ from (1)

$$\Rightarrow MP_N = \frac{MP_1}{\sin\beta} \sin N\beta.$$

Again modified version is done on last page (after all chapters)

$$= MP_1 \frac{\sin N\beta}{\sin\beta}.$$

Represents wave vector due to single slit.

of waves diffracted at angle θ.

$$= R_0 \frac{\sin N\beta}{\sin\beta}$$

Resultant amp. of N diffracted waves due to N slits.

$$\boxed{R = \; = R_0 \frac{\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}}$$

$$\& \; I = \underbrace{R_0^2 \frac{\sin^2\alpha}{\alpha^2}} \underbrace{\frac{\sin^2 N\beta}{\sin^2\beta}}$$

Intensity distribution due to diffraction at single slit.

interference b/w N diffracted waves due to N-slits.

(bcz each slit act as a coherent source)

Principal Maxima due to N- slits

The generalised expression due to N- slits for intensity is :-

$$I = R_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{---(1)}$$

In single slit, we had $I = R_0^2 \frac{\sin^2\alpha}{\alpha^2}$.

$\theta = 0$
$\Rightarrow \alpha = 0$

for principal Maxima we did $\alpha \to 0$

$$\Rightarrow \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$$

$$\Rightarrow I = R_0^2$$

& this R_0^2 can be written as I_0. (Max. intensity).

$\boxed{I = I_0}$

so here in eq (1) if we are taking about principal Maxima, we have to keep

$\theta \to 0$
$\Rightarrow \beta \to 0$

$\sin\beta = 0$ & $\sin N\beta = 0$

$$\Rightarrow \beta = \pm \cancel{m} n\pi$$

Here $\cancel{m}\, n = 0, 1, 2, 3 \ldots$

$$\Rightarrow \frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$$ Thus we have arrived on Indeterminate form

Thus applying L' Hospital Rule.

$$\lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}\sin N\beta}{\frac{d}{d\beta}\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{N \times \cos N\beta}{\cos\beta}$$

$$= \frac{N \cos N(\pm n\pi)}{\cos(\pm n\pi)} = \pm N.$$

$\cos n\pi = 1$

intensity of Principal Maximum

$$\boxed{I = R_0^2 \frac{\sin^2\alpha}{\alpha^2} \times N^2}$$

Thus brightness of principal maxima — (2) increases with no. of slits.
or $I = I_0 \times N^2$

This is the intensity of principal Maxima

POSITION OF PRINCIPAL MAXIMA

We have arrived on this Relation bcz of $\beta = \pm \cancel{m}n\pi$ — (3)

from the phase diff eq $2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta$

$$\Rightarrow \beta = \cancel{2\pi}\frac{\pi}{\lambda}(a+b)\sin\theta \quad \text{— (4)}$$

From (3) & (4)

$$\Rightarrow \frac{\pi}{\lambda}(a+b)\sin\theta = \pm \cancel{m}n\pi$$

Position of Principal maximum $\Rightarrow$ $\boxed{(a+b)\sin\theta = \pm \cancel{m}n\lambda}$ — (5) GRATING LAW

$\Rightarrow n\cancel{m} = 0, \theta = 0$ gives zero order principal maximum. (Central maxima)

$n\cancel{m} = \pm 1$ " first order principal "

$n\cancel{m} = \pm 2$ " second " principal "

Important Points / observations

(1) Equation (5) contains $\pm$ sign.

$\Rightarrow$ ~~for m=1~~, we get two principal maxima's on either side of zero-order ~~maxima's~~ maximum.

~~Similarly for m=2, we get four principal maxima's on either side of zero order maximum~~

② In equation ⑤, N is ~~not~~ not present.

⇒ position of principal maxima is independent of no. of slits present.

③ Also in eq ⑤, $L.H.S = (a+b)\sin\theta$.

and this $(a+b)\sin\theta$ is known as path diff. acc to figure.

⇒ for maxima path diff. $[(a+b)\sin\theta]$ is the multiple of whole number of λ.

⇒ for first order maxima, path diff $= \lambda$.

" II^{nd} " " , " " $= 2\lambda$.

Positions of Maximas & Minimas

$$\frac{dI}{d\beta} = \frac{A_0^2 \sin^2\alpha}{\alpha^2} \times 2\underbrace{\left[\frac{\sin N\beta}{\sin\beta}\right]}_{\text{minima}} \underbrace{\left[\frac{N\cos\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right]}_{\text{maxima}}$$

for Maximas & Minima

$$\frac{dI}{d\beta} = 0.$$

for Minima we will consider $\frac{\sin N\beta}{\sin\beta}$ term.

Now $\sin\beta \neq 0$ bcz then we will reach to principal maxima condition.

$$\Rightarrow \frac{\sin N\beta}{\sin\beta} = 0 \Rightarrow \boxed{I = 0}$$

(6)

for minima intensity $= 0$

Position of Minima

$$\sin N\beta = 0 \Rightarrow N\beta = \pm m\pi$$

$$\Rightarrow N \times \frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi.$$

(7)

$$\Rightarrow \boxed{N(a+b)\sin\theta = \pm m\lambda.}$$

Position of Minima

$m \neq 0$

$m = 1, 2, 3, \ldots$

Now in $N(a+b)\sin\theta = \pm m\lambda$.

If $m = 0$ $\longrightarrow$ this will give condⁿ of zero order principal maxima.
$\Rightarrow \theta = 0°$

If $m = N$ $\rightarrow$ this will give " " I^{st} order principal maxima.

If $m = 2N$ $\longrightarrow$ " " " " " II^{nd} " " ".

Similarly for nN.

$\Rightarrow$ the integer 'm' can't have value $0, N, 2N, 3N \ldots nN$.
b'cz these values will corresponds to principal maxima.

$\Rightarrow$ m will have values $m = 1, 2, 3 \ldots (N-1)$ only
w/c will give position of secondary minima.

Thus	$m = 0$	$(N-1)$	N	$(N+1)$	$2N$	...
	P. Maxima	S. Minima.	P. Max. of I^{st} order	Sec. Minima	P. Max	

Thus b/w two consecutive principal maxima, there are $(N-1)$ minima's w/c are equispaced.

Intensity of secondary maxima

$$N \cos N\beta \sin\beta - \sin N\beta \cos\beta = 0. \quad \& \quad \sin\beta \neq 0, \ \sin^2\beta \neq 0$$

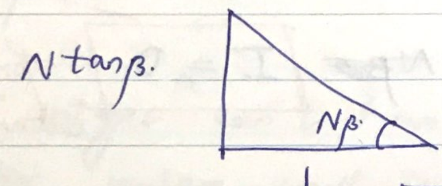
[This will give principal maxima]

$$\Rightarrow N \cos N\beta \sin\beta = \sin N\beta \cos\beta.$$

$$\Rightarrow \tan N\beta = N \tan\beta.$$

$$\Rightarrow \tan N\beta = \frac{N \tan\beta}{1} = \frac{\text{Perpendicular}}{\text{Base}}$$

Thus this can be represented by a right angle Δ.



$$\Rightarrow \text{hypotenuse} = \sqrt{1^2 + N^2 \tan^2\beta}$$

Thus we can find the value of $\sin^2 N\beta$ w/c is present in intensity Relation.

$$\Rightarrow \sin N\beta = \frac{P}{H} = \frac{N \tan\beta}{\sqrt{1 + N^2 \tan^2\beta}}$$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2 \tan^2\beta \,/\, (1 + N^2 \tan^2\beta)}{\sin^2\beta}$$

$$= \frac{N^2 \left(\frac{\cancel{\sin^2\beta}}{\cos^2\beta}\right) \Big/ \left(1 + N^2 \tan^2\beta\right)}{\cancel{\sin^2\beta}}$$

$$= \frac{N^2}{\cos^2\beta \left(1 + N^2 \tan^2\beta\right)} = \frac{N^2}{\cos^2\beta + N^2 \sin^2\beta}.$$

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$$= \frac{N^2}{1 - \sin^2\beta + N^2 \sin^2\beta} = \frac{N^2}{1 + (N^2-1)\sin^2\beta}$$

Thus substituting the value of $\frac{\sin^2 N\beta}{\sin^2\beta}$ in eq (1)

$$I = R_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\Rightarrow \boxed{I = R_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{N^2}{1+(N^2-1)\sin^2\beta}} \quad (8)$$

intensity for sec Max

From eq (8) I of sec. Max $\propto \frac{N^2}{1+(N^2-1)\sin^2\beta}$

& From eq (2) I of princ max $\propto N^2$

$$\Rightarrow \frac{I\ (\text{sec. Max})}{I\ (\text{princ Max})} = \frac{\cancel{N^2}}{1+(N^2-1)\sin^2\beta} \times \frac{1}{\cancel{N^2}}$$

Ans of Q7

Thus when N increases, intensity of sec. Max. decreases & intensity of Principal Maxima increases

Thus secondary Maxima's ~~the~~ doesn't have importance if we have large no. of slits.

& ~~we~~ in diffraction grating, we see complete darkness b/w two principal maximas.

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Position of secondary Maxima.

As we have already discussed, there are $(N-1)$ secondary minima b/w two consecutive principal maximas.

$\Rightarrow$ There are $(N-2)$ secondary Maximas b/w two principal maximas.

Thus when N is very large, secondary maximas become invisible!

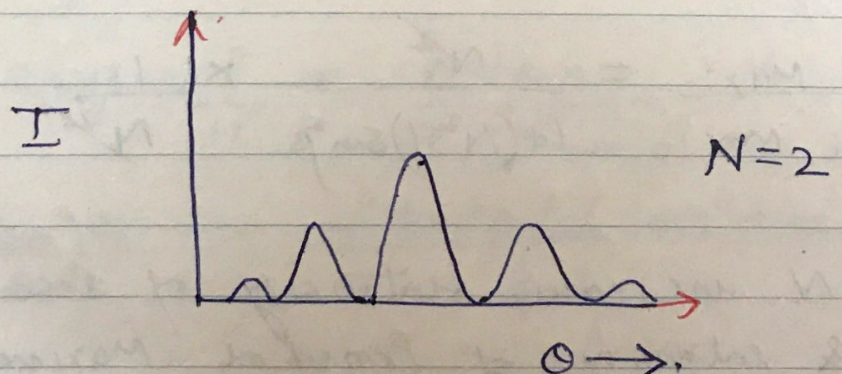
& for small N, " " can be visible

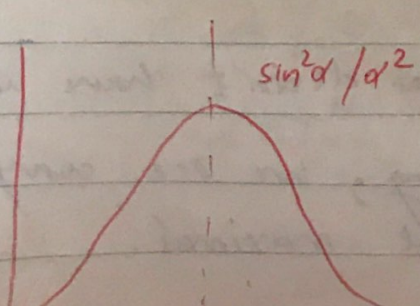
Thus in Grating, N is very large & we observed uniform darkness b/w two principal maximas, bcz minima (darkness) is present & sec. max are not present.

Case 2 When we have 2 slits

$\Rightarrow$ sec. Maximas is $N-2$

$\Rightarrow$ We will not get any sec. max.



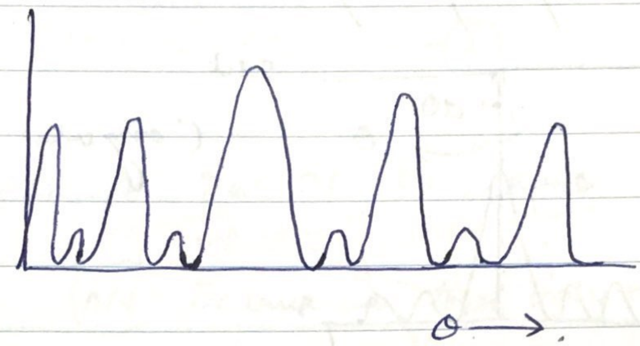


Case 1

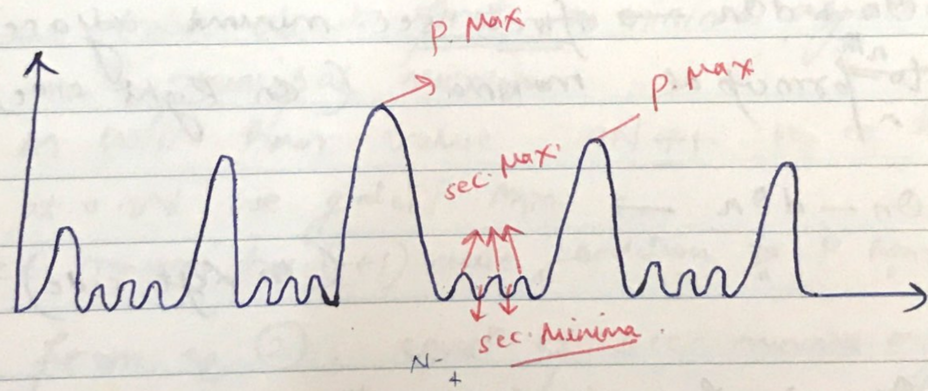
Case 2 $N = 3$.

$\Rightarrow$ sec. Maxima $N-2$ is $3-2=1$.

$\Rightarrow$ one sec. maxima in 3 - slits.



N = 5 $\Rightarrow$ 3 sec. Max.



This is known as Grating Spectra.

& The Resultant intensity variation is →