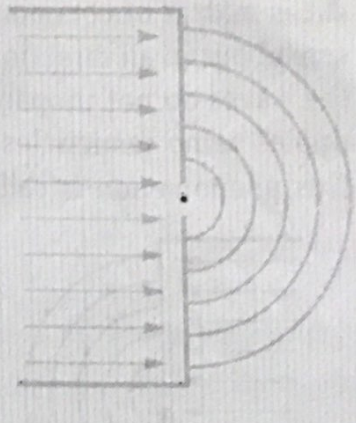


"No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them".

– Richard P. Feynman

CHAPTER

2

Diffraction of Light Waves

It is matter of common experience that the sound waves or water escaping through a small hole spread out in all directions as if they have originated at the hole. After passing through the hole all the waves do not propagate in its original direction but a part of it is bent, the wavefronts for the ripples being semicircles with their centres situated at the centre of the hole as shown in Fig. 2.1. This phenomenon is called **diffraction** and is an important characteristic of wave motions.

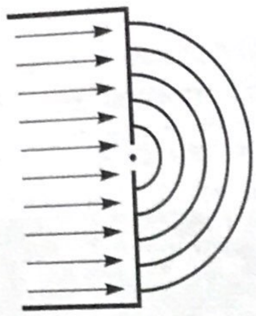


Fig. 2.1 An illustration for diffraction of light waves.

In a similar way when a beam of light passes through a small opening, it also deviates from its rectilinear path and does bend round the corners as other types of waves do, but the amount of this bending is extremely small. The small bending in case of light is due to its very small wavelength of the order of 6000 Å. The wavelength of ordinary sound waves is approximately 6 cm. Consequently the diffraction in case of light is 10^{-5} fold less, a small obstacle will be required to achieve the same effect in sound.

The phenomenon in the case of light was first discovered by *Francesco Grimaldi* in the *1600*. He found that with a point source of light, the shadow of an obstacle was bigger than that given by the geometrical construction and also that the shadow formed by a small object is not sharp and well defined as is expected from the rectilinear propagation of light on the basis of *corpuscular theory*. Light bends round the corners of these obstacles and spreads to some extent into the region of geometrical shadow.

DEFINITION

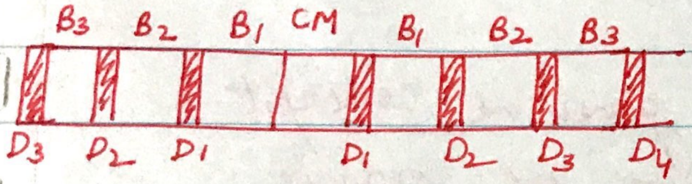
This bending of light round the corners of an obstacle (of which the size is comparable to the wavelength of light) or the encroachment of light within the geometrical shadow is called diffraction.

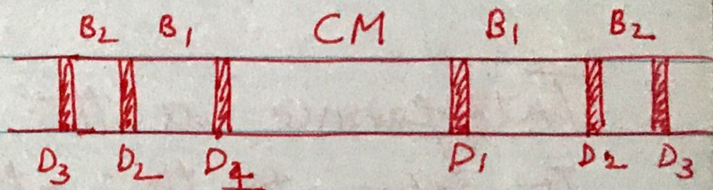
Diffraction of light waves may be observed by looking through a narrow slit between two fingers at a distance light source or by looking at a street light through a cloth umbrella. Until the crack between two fingers is half a cm, the long thin light source through it would appear normal but as the crack is narrowed to about 1/10th of a mm, the source would appear spread out in a direction perpendicular to the fingers. It indicates that the light is diffracted by the opening.

Although diffraction and interference occur simultaneously in nature, but they have following differences:

S.No.	Interference	Diffraction
1.	Interference is the result of superposition of secondary waves starting from two different wavefronts originating from two coherent sources.	Diffraction is the result of superposition of secondary waves starting from different parts of the same wavefront..
2.	All bright and dark fringes are of equal width.	The width of central bright fringe is much more than that of any other secondary maximum.
3.	All bright fringes are of same intensity.	Intensity of bright fringes decreases as we move away from central bright fringe on either side.
4.	Regions of dark fringes are perfectly dark. So there is a good contrast between bright and dark fringes.	Region of dark fringes are not perfectly dark. So there is a poor contrast between bright and dark fringes.
5.	At an angle λ/d, we get a bright fringe in the interference pattern of two narrow slits separated by a distance d.	At an angle of λ/d, we get the first dark fringes in the diffraction pattern of a single slit width d.

Difference in interference pattern & diffraction pattern





equally spaced & equal width of fringe pattern.

Interference

Here intensity of $B_1 > B_2 > B_3$ & $D_1 > D_2 > D_3$.

and fringe patterns are not equally spaced.

2.1 TYPES OF DIFFRACTION OF LIGHT WAVES

The diffraction phenomenon is usually divided into *two* categories

- Fresnel Type
- Fraunhofer Type

1. Fresnel's Type of Diffraction

In the Fresnel class of diffraction the source of the light and the screen are generally at a *finite distance* from the diffracting aperture as shown in Fig. 2.2. There is no lens or mirror in the optical system.

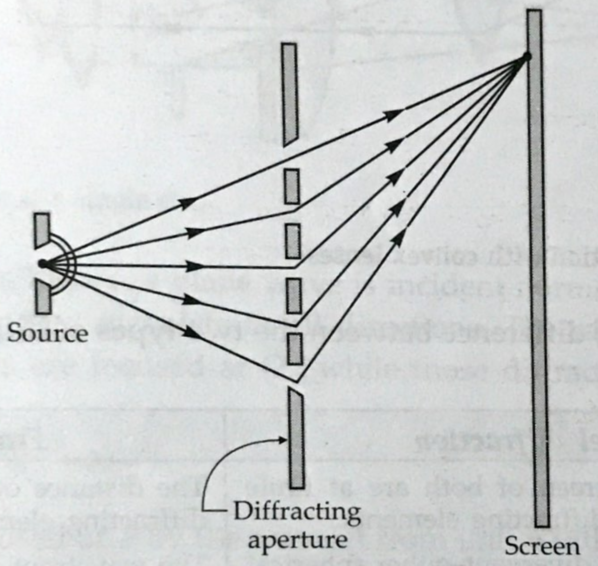


Fig. 2.2 Fresnel diffraction.

2. Fraunhofer's Type of Diffraction

In the Fraunhofer class of diffraction, the source of light and screen are at infinite distance from the diffracting aperture as shown in Fig. 2.3.

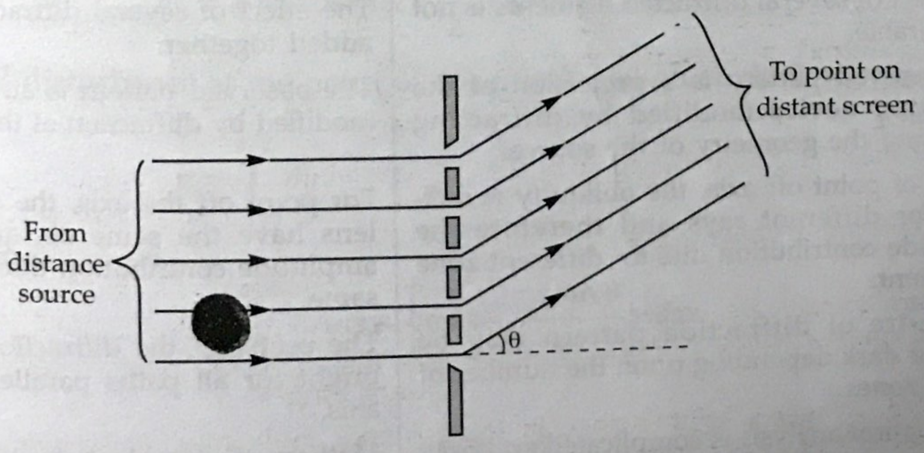


Fig. 2.3 Fraunhofer diffraction.

Fraunhofer diffraction can be easily achieved by placing the source on the focal plane of the convex lens and placing the screen on the focal plane of another convex lens. This arrangement is visualised in Fig. 2.4.

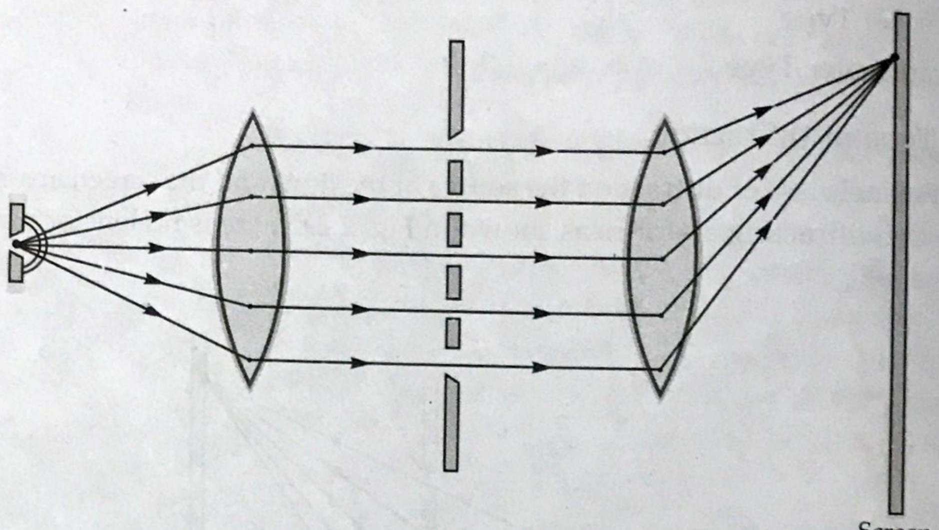


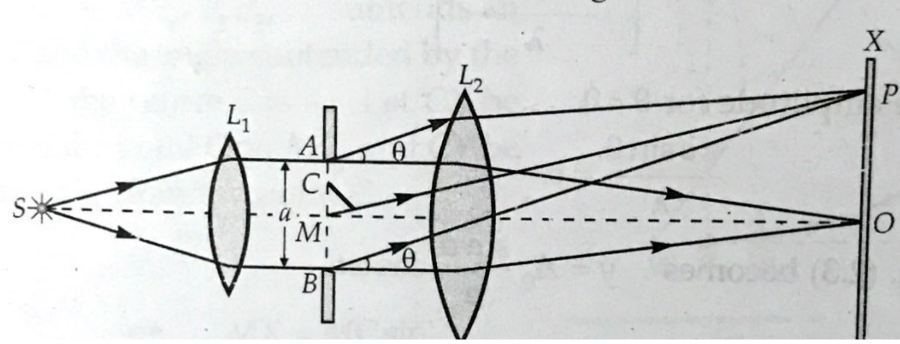
Fig. 2.4 Fraunhofer diffraction with convex lenses.

The main points of difference between the two types of diffraction have been summarized below :

S.No.	*Fresnel diffraction*	*Fraunhofer diffraction*
1.	The source and screen of both are at finite distance from the diffracting elements.	The distance of the source and screen from the diffracting elements is effectively infinite.
2.	The wavefronts are divergent–either spherical or cylindrical.	The wavefront incident on the aperture is plane which is realized by using a collimating lens.
3.	Distances are important in this class of diffraction.	The angular inclinations are important in this of diffraction.
4.	No mirror or lens is used for observation.	Diffracted light is collected by a lens in a telescope.
5.	The rays proceed directly to the axial points.	There is large number of parallel rays falling on the lens corresponding to each point on the screen.
6.	The effect of several diffracted elements is not considerable.	The effect of several diffracted elements can be added together.
7.	The observed pattern is a projection of the diffracting device modified by diffracting effects and the geometry of the source.	The observed pattern is an image of the source modified by diffraction at the diffracting devices.
8.	In case of point off axis, the obliquity is different for different rays and therefore the amplitude contribution due to different zone is different.	For point off the axis, the incident rays on the lens have the same obliquity and hence the amplitude contribution due to each zone is the same.
9.	The centre of diffraction pattern may be bright or dark depending upon the number of Fresnel zones.	The centre of the diffraction pattern is always bright for all paths parallel to the axis of the axis.
10.	Mathematical analysis is complicated and only approximated.	Mathematical analysis is rigorous and easy.

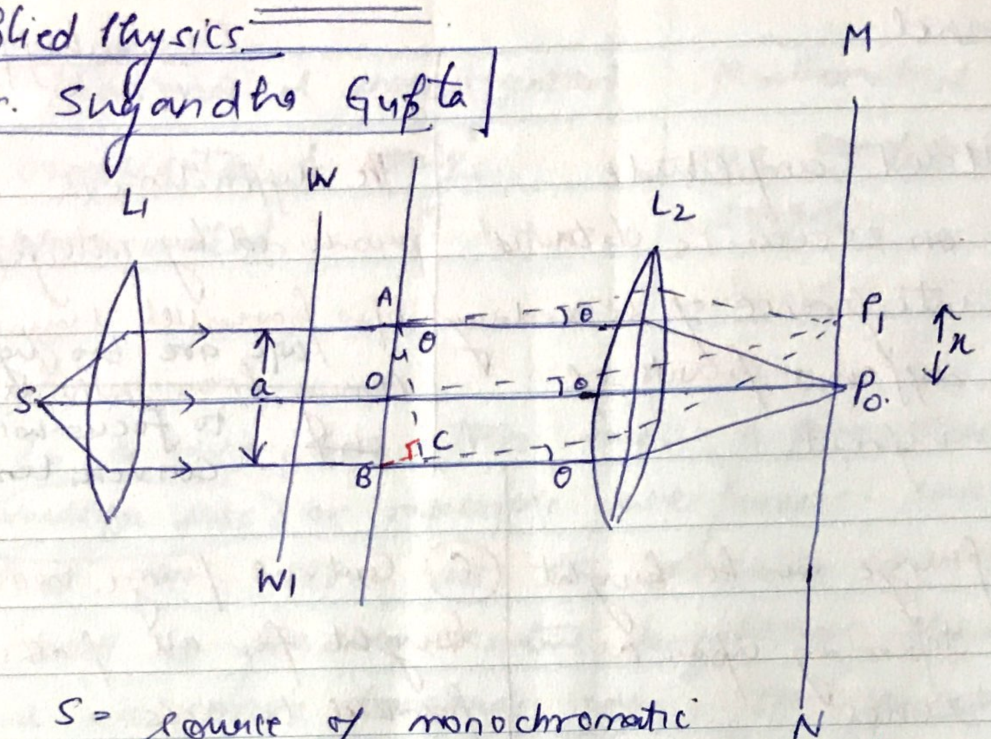
2.2 FRAUNHOFER'S DIFFRACTION AT A SINGLE SLIT

A slit is a rectangular aperture whose length is large compared to its breadth. Let a parallel beam of monochromatic light of wavelength λ be incident normally upon a narrow slit of $AB=a$, as visualised in Fig. 2.5. Let the diffracted light be focused by a convex lens L_2. The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on either side of central bright band.



FRANHOFER DIFFRACTION AT SINGLE SLIT

Applied Physics
Dr. Sugandha Gupta



S = source of monochromatic light. placed at focal length of lens L_1

L_1 = collimating lens. sending the parallel waves.

a = width of slit

WW_1 = plane wave due to parallel rays.

⇒ acc^n to Huygen principle each point on this wavefront is a source of sec. disturbance.

MN = screen. collecting the diffracting beam with the help of L_2 lens bcz screen is also at infinite distance from slit.

P_0 = central bright spot position.

✓ This is because secondary waves reaching at P_0 are all equidistant from O. (centre). Hence they travel same distance to reach P_0. Thus ~~phase~~ Path diff = zero ⇒ Bright

P_1 = a point on the screen at w/c diffracted rays are produced at angle θ.

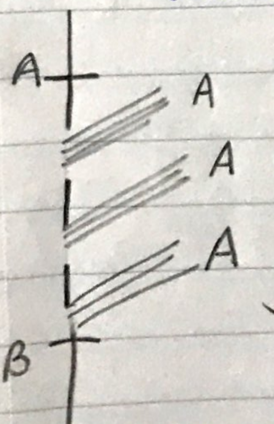
AC is a normal drawn on BP_1

$\Rightarrow$ BC is the path difference $= AB \sin\theta = a \sin\theta$

<u>Note</u>: We consider infinite point sources of secondary wavelets on the wavefront b/w point A & B of the slit. AB is divided into '<u>n</u>' equal parts. Thus each part act as a source of secondary wavelet. & each slit sends wave of equal amplitude = A

$\Rightarrow$ <u>A</u> = Amplitudes of all the diffracted waves reaching the point P_1 will be <u>equal</u> but the phase diff. will vary from 0 to $\frac{2\pi}{\lambda} a \sin\theta$

$\Rightarrow$ Phase diff. b/w the two <u>consecutive</u> rays reaching point P_1 is



n equal parts

$$\phi = \frac{\frac{2\pi}{\lambda} a \sin\theta}{n}$$

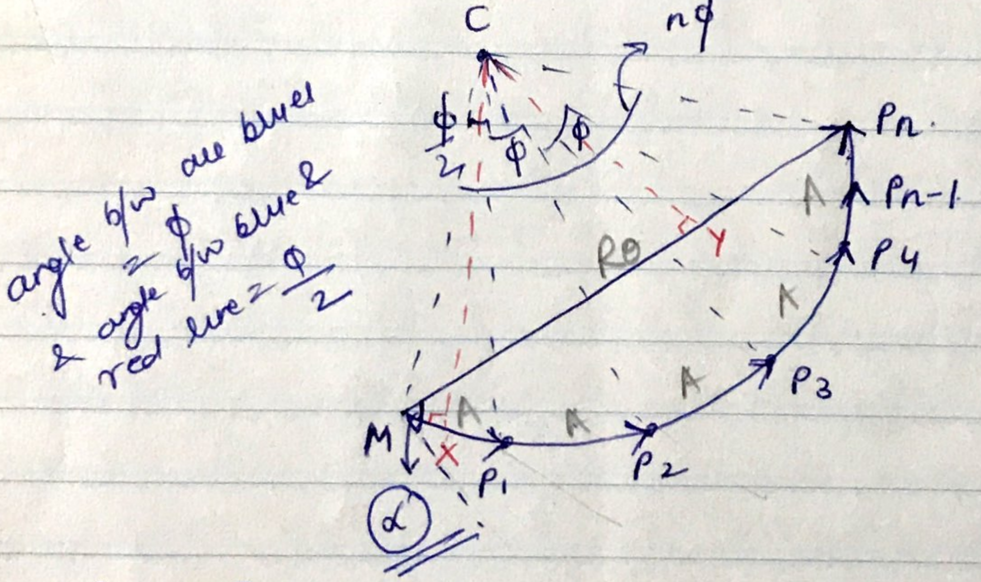
$$\phi = \frac{1}{n} \frac{2\pi}{\lambda} a \sin\theta. \quad \text{—} (1)$$

$\Rightarrow$ We have n waves all of amplitude A & ϕ = phase diff. b/w two consecutive waves.

∴ The aim is to find the Resultant amplitude at Point P_1 by vector Method. (see figure).

Hence we draw a vector polygon in w/c the phase diff varies from

$$0 \longrightarrow \frac{2\pi}{\lambda} a \sin\theta$$



MP_1, P_1P_2, P_2P_3 .... P_n are the vectors w/c represents the sides of polygon. such that each vector has same magnitude A. & angle b/w consecutive vectors is ϕ.

MPₙ is the Resultant vector (Joining the origin of first vector & termination pt of last vector) of all the vectors. having Magnitude R_0.

CX is a normal on MP_1 & CY is a normal on MP_n.

$\Rightarrow \Delta\ CXM$

$\angle C = \phi/2 \Rightarrow \sin\frac{\phi}{2} = \frac{MX}{MC}$

$\Rightarrow MX = MC \sin\phi/2$

Now Accn to the figure, $MX = \frac{1}{2} MP_1 = \frac{A}{2}$.

$\Rightarrow \frac{A}{2} = MC \sin\frac{\phi}{2}$ ——— (2)

Similarly from Δ CYM.

$\frac{MY}{MC} = \sin\frac{n\phi}{2} \Rightarrow MY = MC \sin\frac{n\phi}{2}$

from figure $MY = \frac{1}{2} MP_n = \frac{1}{2} R_\theta$

$\Rightarrow \frac{R_\theta}{2} = MC \sin\frac{n\phi}{2}$ ——— (3)

from (3) ÷ (2)

$\Rightarrow \frac{R_\theta/2}{A/2} = \frac{MC \sin\frac{n\phi}{2}}{MC \sin\phi/2} \Rightarrow R_\theta = A\left[\frac{\sin(n\phi)/2}{\sin\phi/2}\right]$

Now put eq (1) in above

$$\Rightarrow R_\theta = \frac{A \sin\frac{\pi a \sin\theta}{\lambda}}{\sin\frac{\pi a \sin\theta}{n\lambda}}.$$

Let $\frac{\pi a \sin\theta}{\lambda} = \alpha$. —— (4)

$\Rightarrow R_\theta = A \frac{\sin\alpha}{\sin(\alpha/n)}$ =

Now n is very large $\Rightarrow \frac{\alpha}{n}$ quantity will be very small

$\Rightarrow \sin(\alpha/n) \simeq \alpha/n$. (for very small angles)

$\Rightarrow R_\theta = \frac{A \sin\alpha}{\alpha/n} = A n \frac{\sin\alpha}{\alpha}$ —— (5)

Now from ~~fig~~ eq (1)

if $\theta = 0 \Rightarrow \phi = 0$

& from eq (4) if. $\theta = 0 \Rightarrow \alpha = 0$

so $\lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$

from eq (5) $\Rightarrow$ $R_\theta \to R_0$ (R_θ will be written as R_0 (max. value))

from eq (5)

$R_0 = nA$.

Hence eq (5) can now be written as-

$$\boxed{R_\theta = R_0 \frac{\sin\alpha}{\alpha}}$$ —— (6).

This gives us the resultant amplitude at θ diffracting angle at P_1.

$\therefore$ Resultant intensity

$$I \propto R_\theta^2 \Rightarrow I = K R_\theta^2$$

$$= \underbrace{K R_0^2}_{I_0} \left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\Rightarrow \boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2}} \quad \text{--- (7)}$$

Q3 → Describe ~~Derive~~ the Fraunhoffer diffraction at single slit & Derive the Resultant amplitude & Resultant intensity

Case 1 Position of central Maxima

At point P_0 on screen $\alpha = 0$ (9)

Q4 Position & intensity of C.M.

$$\Rightarrow \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1 \Rightarrow I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

$\Rightarrow I = I_0 \times 1$. from eq (7) (5).

$\Rightarrow$ intensity at central point is maximum. bcz all the waves reach at P_0 are in phase.

Q3 **Case 2 Position of ~~central~~ secondary minima**

Now acc^n to eq. (7) for minima I should be equal to zero (minimum)

$$\Rightarrow \frac{\sin\alpha}{\alpha} = 0.$$

However α can't be zero bcz we are talking about point P_1 where α exists.

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$\Rightarrow \sin\alpha = 0$

$\Rightarrow \alpha = \pm m\pi \qquad m \neq 0$

$m = 1, 2, 3, 4 \ldots$

Now acc^n to eq (4)

$$\frac{\pi a \sin\theta}{\lambda} = \alpha$$

$\therefore \underline{\quad} = \pm m\pi$

$\Rightarrow \boxed{a\sin\theta = \pm m\lambda} \longrightarrow (8)$ → Condition of minima

→ Numericals

where $m = 1, 2, 3 \ldots$ gives the direction of 1st, second, third minimas respectively.

Here $m \neq 0$. bcz for that $\theta = 0$ and that will correspond to principal maxima.

~~Positions of~~

POSITIONS & INTENSITIES OF SECONDARY MAXIMAS

Now from eq (8) we can understand that condition of Maxima will be obtained by adding a factor of $\frac{\lambda}{2}$. bcz minima's & maxima's equations are complementary by the factor of $\frac{\lambda}{2}$

Do not do from Devraj

$$\Rightarrow \quad a\sin\theta = \pm m\lambda + \frac{\lambda}{2}.$$

$$= \pm\left(m+\frac{1}{2}\right)\lambda.$$

Now from eq (4) $\Rightarrow$ $\alpha = \frac{\pi a \sin\theta}{\lambda}.$

$$= \pm\frac{\pi}{\lambda}\left(m+\frac{1}{2}\right)\lambda. = \pm\pi\left(m+\frac{1}{2}\right)$$

for various values of $m = 1, 2, 3$

$$\Rightarrow \quad \alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, + \text{-----}$$

$\Rightarrow$ from eq (7)

$$I_1 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2 = I_0\left(\frac{-1}{3\pi/2}\right)^2$$

$\downarrow$

first secondary Maxima.

$$= \frac{4}{9\pi^2} I_0 = \frac{I_0}{22}.$$

$$I_2 = I_0\left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 = I_0\left(\frac{2}{5\pi}\right)^2 = \frac{I_0}{61}$$

$\downarrow$

Second secondary Maxima

$\Rightarrow$ This shows that intensities of secondary Maximas falls of rapidly.

Thus the Ratio's of intensities are

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}$$

or

$$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121}.$$

What is angle α

consider a ray (from single slit). reaching at P_1.

Now we have considered n-slits. so the resultant of n slits will reach at P_1

$\Rightarrow$ angle b/w the two ~~way~~ waves is α. (ie initial vector. wave. & resultant wave vector)

& 2α = angle b/w initial wave vector. & final wave vector. (from A & B) from S.I. Ganga

Positions of Secondary Maxima & Minimas

In b/w principal maxima [w/c occurs for $\alpha = 0$ & $\theta = 0$] & secondary minima's there exist weak secondary maximas.

Position of Secondary Maxima.

Diffrentiate eq (7) w.r.t α.

$\frac{dI}{d\alpha}$ ie intensity is diffrentiating w.r.t α / α is related to θ. & θ is related to ϕ.

ϕ is related to path diff.

Hence if we diffrente intensity w.r.t. position (ie path diff.) we will get positions of maximas & minimas

$$\frac{dI}{d\alpha} = \quad I = I_0 \frac{\sin^2\alpha}{\alpha^2} \quad \text{—— (7)}$$

$$\frac{dI}{d\alpha} = 2 I_0 \frac{\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right]$$

so for Maxima's & minimas

$dI/d\alpha = 0$.

$\Rightarrow \quad \sin\alpha = 0$ —— (1) bcz $\alpha \neq 0$ otherwise it will give position of Principal Maximas.

(2) $\quad \alpha\cos\alpha - \sin\alpha = 0$. —— (2).

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Now eq (1) $\sin\alpha = 0$ gives the position of secondary minima.

$\Rightarrow \alpha = \pm m\pi$.

$m = 1, 2, 3, 4 \ldots\ldots$

Accⁿ to eq (4)

$\alpha = \dfrac{\pi a \sin\theta}{\lambda}$

$= \pm m\pi$.

Condition of minima will be used in solving numericals.

* $\Rightarrow$ $\boxed{a \sin\theta = \pm m\lambda}$ —— (8).

$m = 1, 2, 3 \ldots$

gives the direction of 1^{st}, II^{nd}, III^{rd} minimas.

Here $m \neq 0$. bcz for $m = 0 \Rightarrow \theta = 0$

$\Rightarrow$ principle Maxima.

Condition (2) $\alpha\cos\alpha - \sin\alpha = 0$

$\Rightarrow y = \alpha$ & $y = \tan\alpha$

The values of α satisfying above eqⁿ are obtaining by plotting two curves $y = \alpha$ & $y = \tan\alpha$ on same graph. The point of intersectⁿ of two curves gives the value of α.

Now $y = \alpha$ is a straight line. —— (a).

and now for $y = \tan\alpha$.

(b) $\alpha = 0 \Rightarrow \tan\alpha = 0$.

(c) $\alpha = \frac{\pi}{2} \Rightarrow \tan\frac{\pi}{2} = \pm\infty$.

(d) $\alpha = \pm\pi \Rightarrow \tan\pm\pi = 0$.

(e) $\alpha = \pm\frac{3\pi}{2} \Rightarrow \tan\pm\frac{3\pi}{2} = \pm\infty$.

(f) $\alpha = \pm 2\pi \Rightarrow \tan\pm 2\pi = 0$.

(g) $\alpha = \pm\frac{5\pi}{2} \Rightarrow \tan\pm\frac{5\pi}{2} = \pm\infty$.

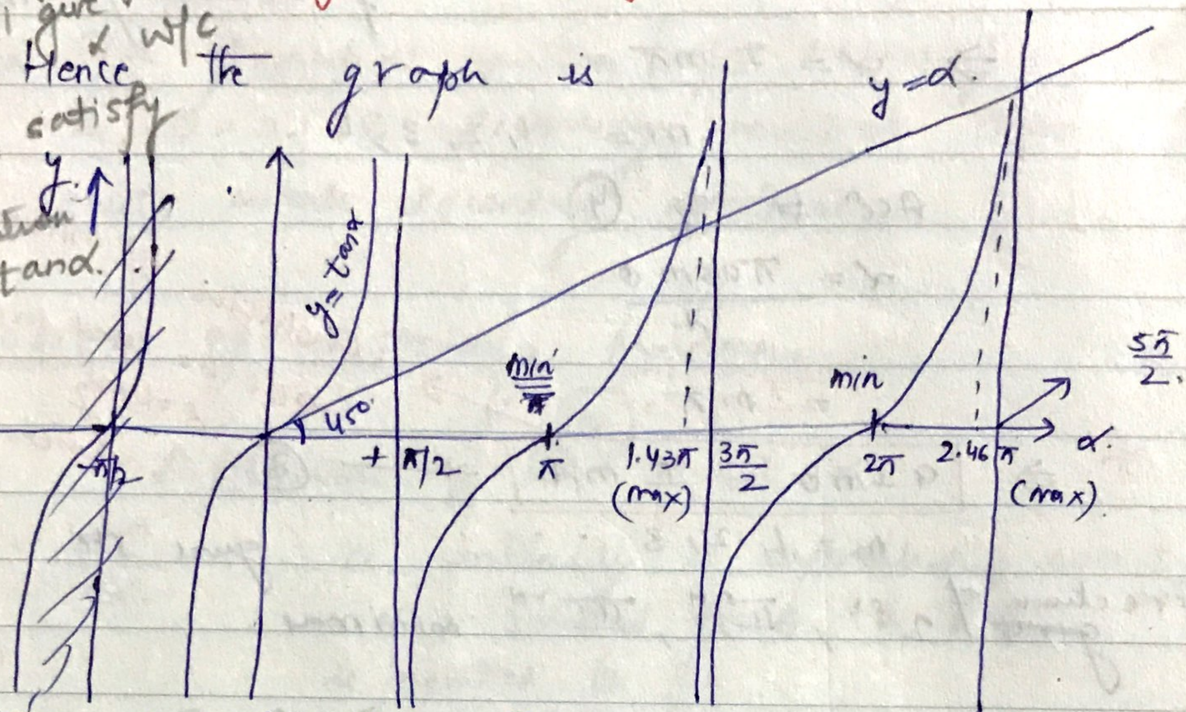
$y = \alpha$ is a straight line making an angle of $45°$ with α.
& $y = \tan\alpha$ is a family of discontinous curves with assymtotes at $\alpha = \pi$.

intersection of line ($y=\alpha$ & $y=\tan\alpha$) cut at $\alpha = (2n+1)\pi/2$ where $n \neq 0$ & $n = 1, 2, 3$

point of intersection of $y=\alpha$ & $y=\tan\alpha$ will give value This gives the value of α.

of α w/c will satisfy the relation $\alpha = \tan\alpha$.

Hence the graph is



(The point of intersection of line & curve gives value of α). $\alpha \approx \cancel{0}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

Similar graph can be drawn on −ve side

or $\alpha = \cancel{1.43\pi}\ \cancel{0},\ 1.43\pi,\ 2.46\pi$.

Thus substituting these values in eq

(a) For $\alpha = 0$ $\quad I = I_0$ [Principal Maxima].

(b) First secondary Maxima. $\alpha = \frac{3\pi}{2}$

$\Rightarrow$ ~~$I = I_0$~~

$$I = I_0\left[\frac{\sin 3\pi/2}{3\pi/2}\right]^2 = I_0\left[\frac{-1}{3\pi/2}\right]^2.$$

$$I = \frac{4I_0}{9\pi^2} = \frac{I_0}{22}.$$

$\boxed{*}$ Now $\alpha = \pi a\sin\theta/\lambda$. eq (4) $\quad \alpha = (2n+1)\frac{\pi}{2}$

Condition of Maxima & will be used to solve questions

$$\Rightarrow \frac{\pi a\sin\theta}{\lambda} = (2n+1)\pi/2 \Rightarrow \boxed{a\sin\theta = (2n+1)\frac{\lambda}{2}}$$

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(c) Second secondary Maximum, $\alpha = \frac{5\pi}{2}$.

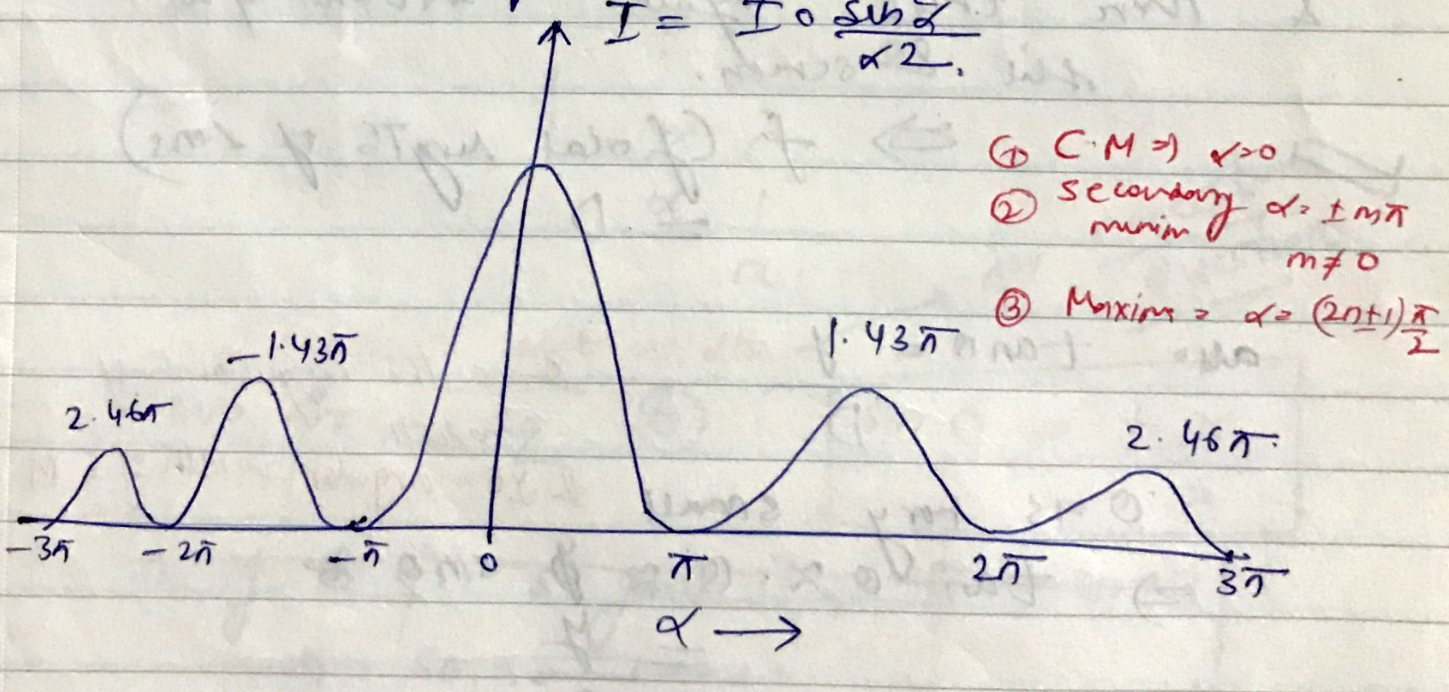
$$I = I_0 \left[\frac{\sin 5\pi/2}{5\pi/2} \right]^2 = I_0 \left[\frac{1}{5\pi/2} \right]^2.$$

$$I = \frac{4I_0}{25\pi^2} = \frac{I_0}{61}.$$

Hence the ~~ratio~~ ratio of intensities

$$\frac{1}{22} : \frac{1}{61} : \frac{1}{121}.$$

(Here intensity of central Maxima is taken unity)



Intensity distribution of Fraunhoffer diffraction of a single slit.

Minima occurs at $\pm\pi$, $\pm 2\pi$, $\pm 3\pi$

& Maxima's at $\pm 1.43\pi$, $\pm 2.46\pi$ . . . . .