qvxjlalzz

November 28, 2023

[1]:
```
#Load Required Libraries
import sys
import sklearn
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

import warnings
warnings.filterwarnings('ignore')
```

[2]:
```
insurance_data = pd.read_csv("insurance.csv")
```

0.1 Question A: Summarize the data. How much data is present? What attributes/features are continuous valued? Which attributes are categorical?

[3]:
```
insurance_data.head()
```

[3]:

	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

[4]:
```
insurance_data.describe()
```

[4]:

	age	bmi	children	charges
count	1338.000000	1338.000000	1338.000000	1338.000000
mean	39.207025	30.663397	1.094918	13270.422265
std	14.049960	6.098187	1.205493	12110.011237
min	18.000000	15.960000	0.000000	1121.873900
25%	27.000000	26.296250	0.000000	4740.287150
50%	39.000000	30.400000	1.000000	9382.033000
75%	51.000000	34.693750	2.000000	16639.912515
max	64.000000	53.130000	5.000000	63770.428010

[5]:
```
insurance_data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1338 entries, 0 to 1337
Data columns (total 7 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   age       1338 non-null   int64
 1   sex       1338 non-null   object
 2   bmi       1338 non-null   float64
 3   children  1338 non-null   int64
 4   smoker    1338 non-null   object
 5   region    1338 non-null   object
 6   charges   1338 non-null   float64
dtypes: float64(2), int64(2), object(3)
memory usage: 73.3+ KB
```

[6]:
```
insurance_data.isnull().sum()
```

[6]:
```
age         0
sex         0
bmi         0
children    0
smoker      0
region      0
charges     0
dtype: int64
```

Observations :

1. There is about 1338 entries in the datasets(rows) and 7 features (columns)
2. sex, smoker and region attributes has categorical values as its datatype is object
3. None of the attributes have the null value

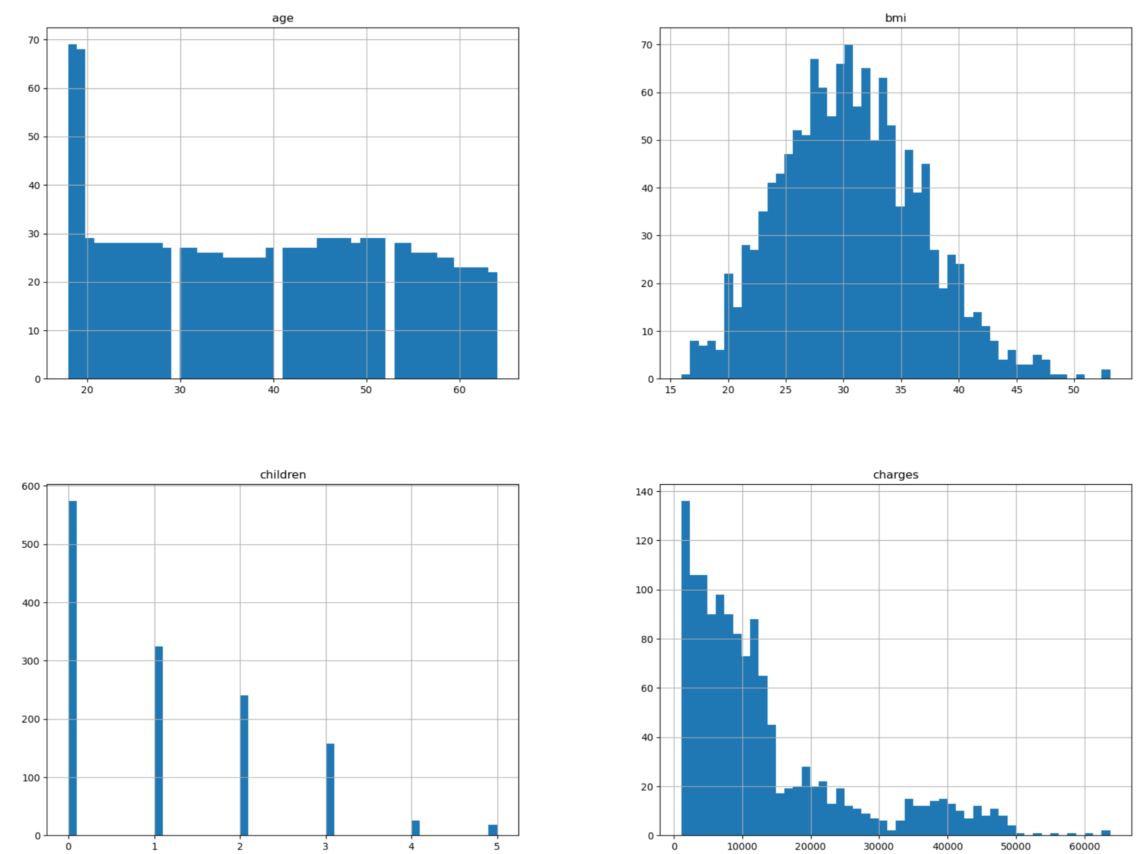
0.2 Question B: Display the statistical values for each of the attributes, along with visualizations (e.g., histogram) of the distributions for each attribute. Explain noticeable traits for key attributes. Are there any attributes that might require special treatment? If so, what special treatment might they require?

[7]:
```
insurance_data.describe(include='all')
```

[7]:
```
                age   sex          bmi     children smoker     region  \
count   1338.000000  1338  1338.000000  1338.000000   1338       1338
unique          NaN     2          NaN          NaN      2          4
top             NaN  male          NaN          NaN     no  southeast
freq            NaN   676          NaN          NaN   1064        364
```

```
mean       39.207025   NaN    30.663397     1.094918   NaN         NaN
std        14.049960   NaN     6.098187     1.205493   NaN         NaN
min        18.000000   NaN    15.960000     0.000000   NaN         NaN
25%        27.000000   NaN    26.296250     0.000000   NaN         NaN
50%        39.000000   NaN    30.400000     1.000000   NaN         NaN
75%        51.000000   NaN    34.693750     2.000000   NaN         NaN
max        64.000000   NaN    53.130000     5.000000   NaN         NaN

              charges
count     1338.000000
unique            NaN
top               NaN
freq              NaN
mean     13270.422265
std      12110.011237
min       1121.873900
25%       4740.287150
50%       9382.033000
75%      16639.912515
max      63770.428010
```

[8]:
```
insurance_data.hist(bins = 50, figsize=(20,15))
plt.show()
```



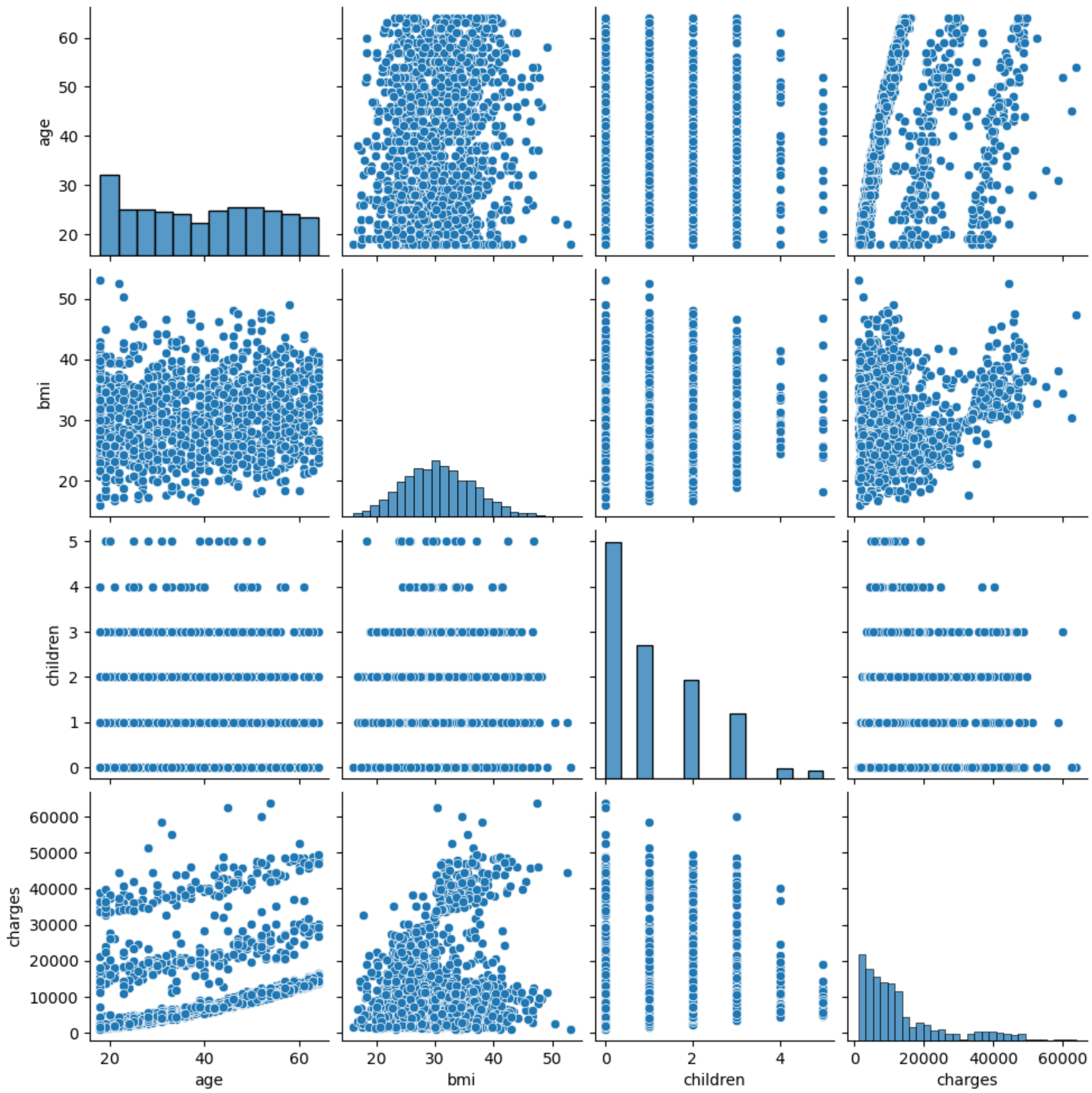
Observations:

```
Key atrributes:
    1. We have nearly uniform number of records in all age groups.
    2. The BMI shows characteristics if normal distribution.
    3. Most of records indicate that people have less than 1 children.
    4. Most peoples Insurance charges ranges between 0 to 15000.
The atrributes that require special treatment are sex, smoker and region attributes as they co
```

0.3 Question C: Analyze and discuss the relationships between the data attributes, and between the data attributes and label. This involves computing the Pearson Correlation Coefficient (PCC) and generating scatter plots.

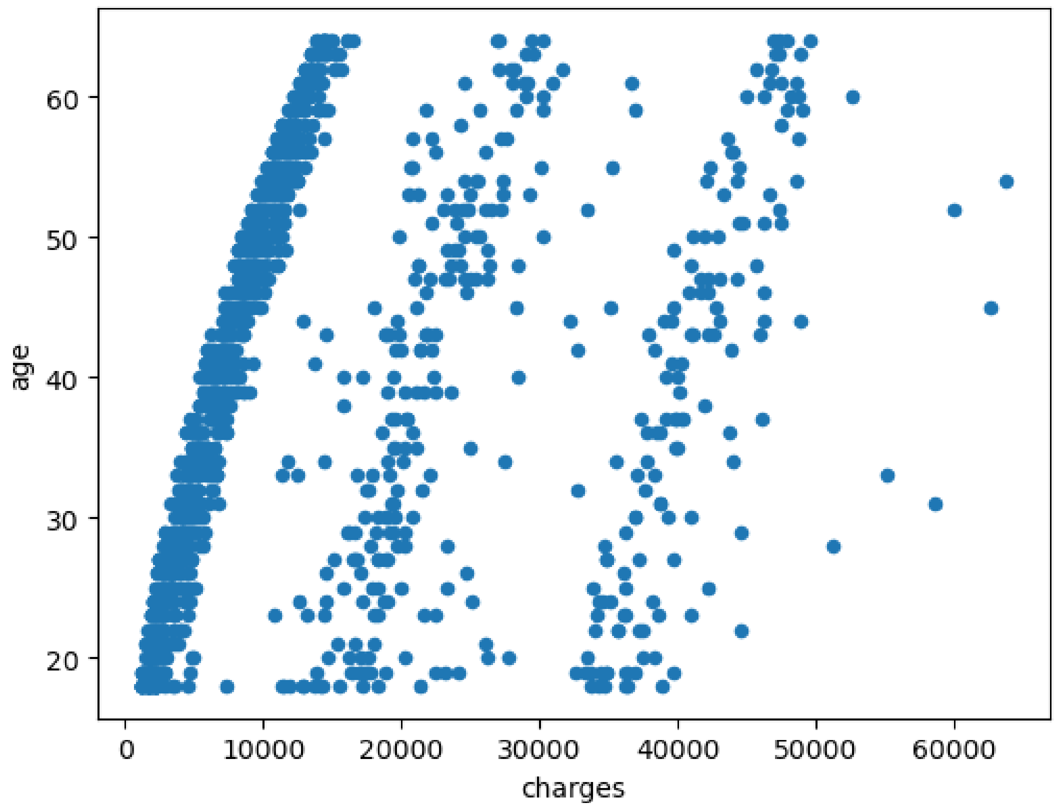
[9]:
```
# Analyzing Relationships by Pairplot
sns.pairplot(insurance_data)
```

[9]:
```
<seaborn.axisgrid.PairGrid at 0x7fa4170f4790>
```



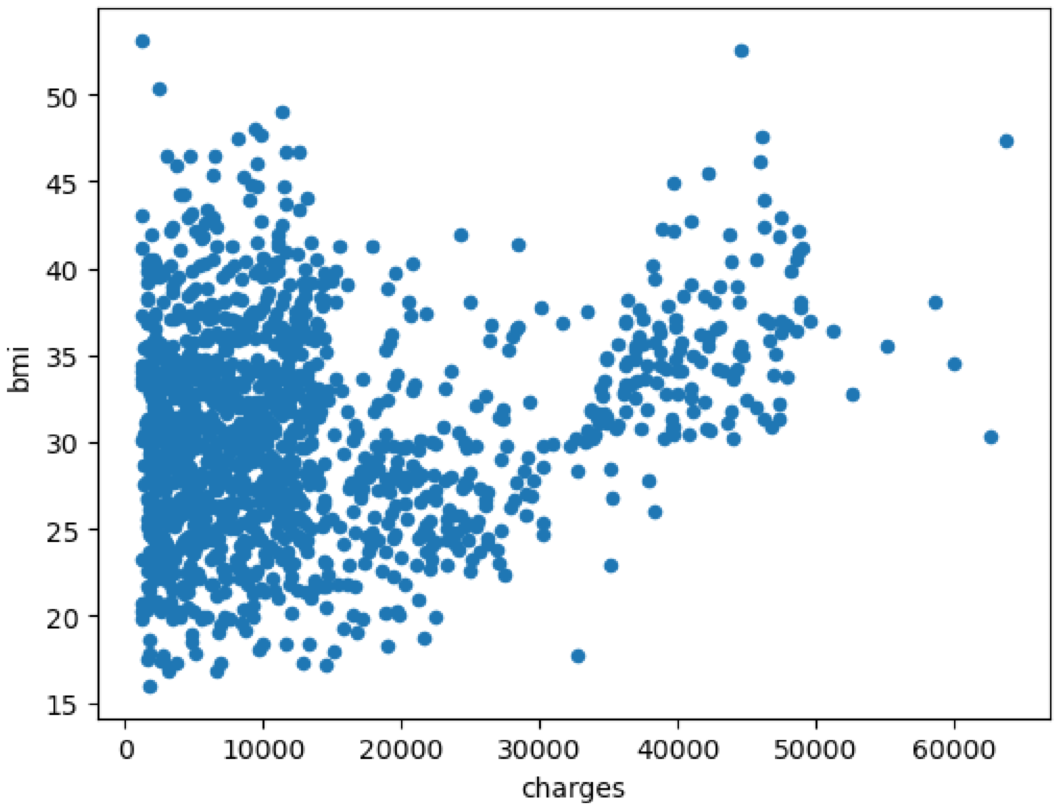

```
[10]: # Analysing Relationship between data attributes and label
insurance_data.plot.scatter(x='charges', y= 'age')
```

[10]: <AxesSubplot: xlabel='charges', ylabel='age'>



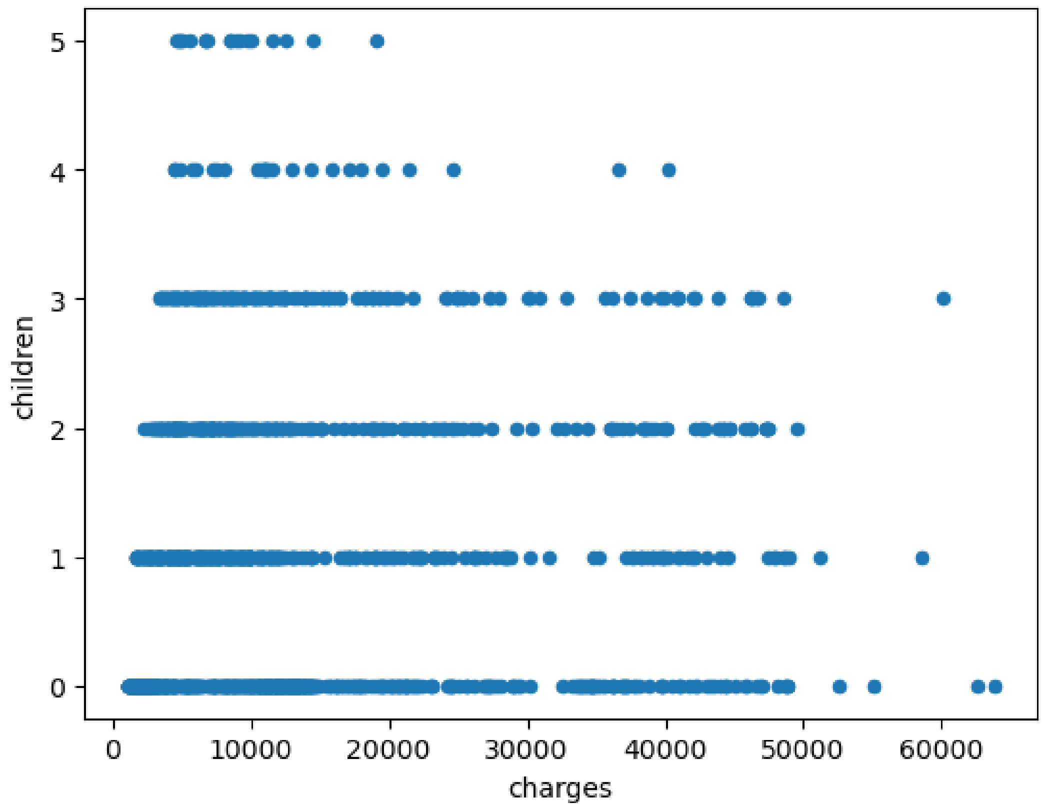
[11]:
```
# Analysing Relationship between data attributes and label
insurance_data.plot.scatter(x='charges', y= 'bmi')
```

[11]: <AxesSubplot: xlabel='charges', ylabel='bmi'>



[12]:
```
# Analysing Relationship between data attributes and label
insurance_data.plot.scatter(x='charges', y= 'children')
```

[12]: <AxesSubplot: xlabel='charges', ylabel='children'>



[13]:
```
#Computing PCC
insurance_data.corr(method ='pearson')
```

[13]:

	age	bmi	children	charges
age	1.000000	0.109272	0.042469	0.299008
bmi	0.109272	1.000000	0.012759	0.198341
children	0.042469	0.012759	1.000000	0.067998
charges	0.299008	0.198341	0.067998	1.000000

0.3.1 Performing data cleaning

1. As there are no null values there is no need for substitution.
2. We will scale everything into the same scale(except charges).
3. And, finally we will separate label charges from the data.

[14]:
```
from sklearn.pipeline import Pipeline
from sklearn.preprocessing import StandardScaler
from sklearn.impute import SimpleImputer
from sklearn.pipeline import make_pipeline
from sklearn.compose import ColumnTransformer
from sklearn.preprocessing import OrdinalEncoder
```

[15]:
```
num_attribs = ['age', 'bmi', 'children','charges']

num_pipeline = make_pipeline(StandardScaler(),SimpleImputer(strategy="median"))
```

[16]:
```
cat_attribs = ["sex","smoker","region"]

cat_pipeline = make_pipeline(
    SimpleImputer(strategy="most_frequent"),
    OrdinalEncoder())
```

[17]:
```
preprocessing = ColumnTransformer([
    ("num", num_pipeline, num_attribs),
    ("cat", cat_pipeline, cat_attribs),
])
```

[18]:
```
features_prepared = preprocessing.fit_transform(insurance_data)
```

[19]:
```
features = pd.DataFrame(
    features_prepared, columns=preprocessing.get_feature_names_out(),
    index=insurance_data.index)
```

[20]:
```
labels = pd.DataFrame(insurance_data[['charges']])
```

[21]:
```
features.describe() , labels.describe()
```

[21]:
```
(            num__age      num__bmi  num__children  num__charges     cat__sex  \
 count  1.338000e+03  1.338000e+03   1.338000e+03  1.338000e+03  1338.000000
 mean  -1.805565e-16 -2.124194e-16  -5.576008e-17 -8.098488e-17     0.505232
 std    1.000374e+00  1.000374e+00   1.000374e+00  1.000374e+00     0.500160
 min   -1.509965e+00 -2.412011e+00  -9.086137e-01 -1.003557e+00     0.000000
 25%   -8.691547e-01 -7.164063e-01  -9.086137e-01 -7.046504e-01     0.000000
 50%   -1.474046e-02 -4.320880e-02  -7.876719e-02 -3.212089e-01     1.000000
 75%    8.396738e-01  6.611572e-01   7.510793e-01  2.783441e-01     1.000000
 max    1.765289e+00  3.685522e+00   3.240619e+00  4.171663e+00     1.000000

        cat__smoker  cat__region
 count  1338.000000  1338.000000
 mean      0.204783     1.515695
 std       0.403694     1.104885
 min       0.000000     0.000000
 25%       0.000000     1.000000
 50%       0.000000     2.000000
 75%       0.000000     2.000000
 max       1.000000     3.000000  ,
             charges
 count   1338.000000
 mean   13270.422265
```

```
std     12110.011237
min      1121.873900
25%      4740.287150
50%      9382.033000
75%     16639.912515
max     63770.428010)
```

0.4 Question D: Select 20% of the data for testing. Describe how you did that and verify that your test portion of the data is representative of the entire dataset.

To select 20% data for testing, we will use the train_test_split() method from sklearn.model_selection class, with test_size = 0.2. Here, 0.2 indicates 20% of total dataset.

[22]:
```
from sklearn.model_selection import train_test_split
phi_train, phi_test, t_train, t_test = train_test_split(features, labels,␣
↪test_size=0.2, shuffle=True, random_state=42)
```

[23]:
```
phi_train.describe()
```

[23]:
```
          num__age     num__bmi  num__children  num__charges     cat__sex  \
count  1070.000000  1070.000000    1070.000000   1070.000000  1070.000000
mean      0.010679    -0.016897       0.010422      0.006251     0.512150
std       1.002083     0.991384       1.009080      0.992898     0.500086
min      -1.509965    -2.412011      -0.908614     -1.003557     0.000000
25%      -0.869155    -0.731375      -0.908614     -0.691650     0.000000
50%       0.020860    -0.074377      -0.078767     -0.305232     1.000000
75%       0.839674     0.628758       0.751079      0.287162     1.000000
max       1.765289     3.685522       3.240619      4.074389     1.000000

        cat__smoker  cat__region
count  1070.000000  1070.000000
mean      0.205607     1.508411
std       0.404334     1.115175
min       0.000000     0.000000
25%       0.000000     1.000000
50%       0.000000     2.000000
75%       0.000000     2.000000
max       1.000000     3.000000
```

[24]:
```
phi_test.describe()
```

[24]:
```
         num__age    num__bmi  num__children  num__charges    cat__sex  \
count  268.000000  268.000000     268.000000    268.000000  268.000000
mean    -0.042636    0.067460      -0.041610     -0.024956    0.477612
std      0.994241    1.034637       0.965548      1.031202    0.500433
min     -1.509965   -2.271753      -0.908614     -1.002762    0.000000
```

```
25%     -0.940356  -0.654890     -0.908614     -0.741951    0.000000
50%     -0.085942   0.019128     -0.078767     -0.395072    0.000000
75%      0.768473   0.744205      0.751079      0.255868    1.000000
max      1.765289   3.595298      3.240619      4.171663    1.000000

       cat__smoker  cat__region
count   268.000000   268.000000
mean      0.201493     1.544776
std       0.401866     1.064340
min       0.000000     0.000000
25%       0.000000     1.000000
50%       0.000000     2.000000
75%       0.000000     2.000000
max       1.000000     3.000000
```

Observation:

1. The data given by phi_train and phi_test, the mean, standard deviation and the quartile ranges are almost similar. This means the both train and test datasets are derived from the same distribution. Thus, are representative of the entire dataset.

0.5 Question E: Linear Regression

0.5.1 E - Part 1: Linear Model using K-Fold with Normal form (train and val loss).

[25]:
```
from sklearn.model_selection import KFold, learning_curve, cross_validate
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, r2_score, mean_absolute_error
```

[26]:
```
reg = LinearRegression().fit(phi_train, t_train)

reg_cv = cross_validate(reg,
                        phi_train,
                        t_train,
                        ␣
 ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                        cv=4,
                        return_train_score=True)
```

[27]:
```
# let's look at the output from k fold
print("Linear Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(reg_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
```

```
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['test_neg_mean_absolute_error'])))
```

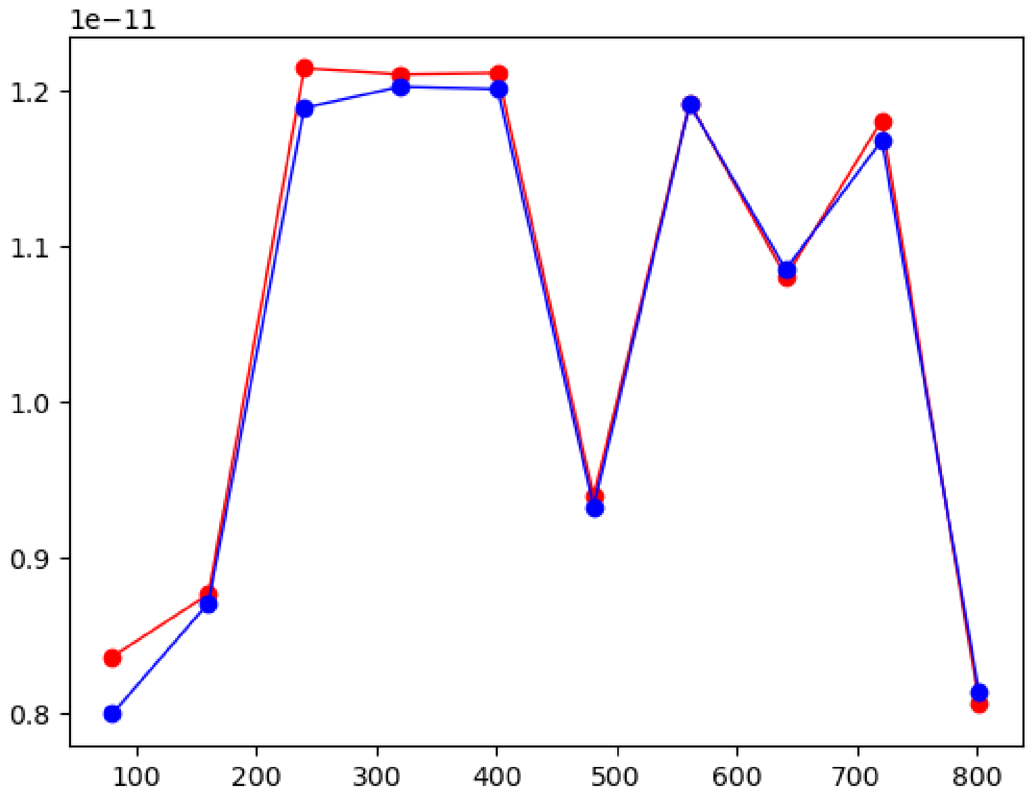
```
Linear Regression:

 For metric R^2 :
         Training loss: 1.000
         Validation loss: 1.000

 For metric Root mean Squared Error(RMSE) :
         Training loss: 0.000
         Validation loss: 0.000

 For metric Mean Absolute Error(MAE) :
         Training loss: 0.000
         Validation loss: 0.000
```

Training and validation loss as a function of training set size

[28]:
```
train_sizes, train_scores, valid_scores = learning_curve(reg,
                                                         phi_train,
                                                         t_train,
                                                         train_sizes=np.
 ↪linspace(0.1, 1.0,10),
                                                         cv=4,
                                                        ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

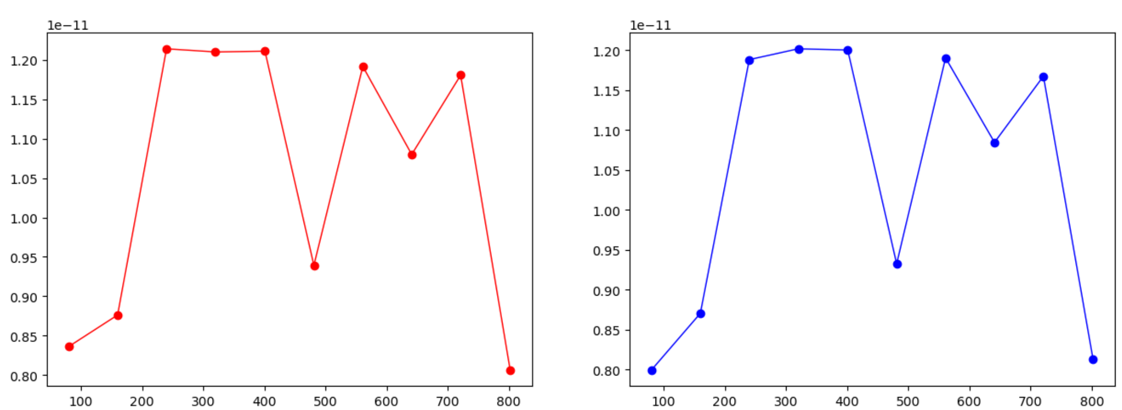
[29]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[29]:
```
[<matplotlib.lines.Line2D at 0x7fa414d27f10>]
```



[30]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[30]: [<matplotlib.lines.Line2D at 0x7fa414beae90>]



0.5.2 E - Part 1: Linear Model using K-Fold with SGD (train and val loss).

[31]:
```
from sklearn.linear_model import SGDRegressor
```

[32]:
```
sgd_reg = SGDRegressor(max_iter=1000,
                       tol=1e-5,
                       eta0=0.01,
                       n_iter_no_change=100,
                       random_state=42)

sgd_reg.fit(phi_train, t_train)

sgd_reg_cv = cross_validate(sgd_reg,
                        phi_train,
                        t_train,
                        ␣
 ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                        cv=4,
                        return_train_score=True)
```

[33]:
```
# let's look at the output from k fold
print("Stochastic Gradient Descent Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(sgd_reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(sgd_reg_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(sgd_reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(sgd_reg_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(sgd_reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(sgd_reg_cv['test_neg_mean_absolute_error'])))
```

```
Stochastic Gradient Descent Regression:

 For metric R^2 :
         Training loss: 1.000
         Validation loss: 1.000

 For metric Root mean Squared Error(RMSE) :
```

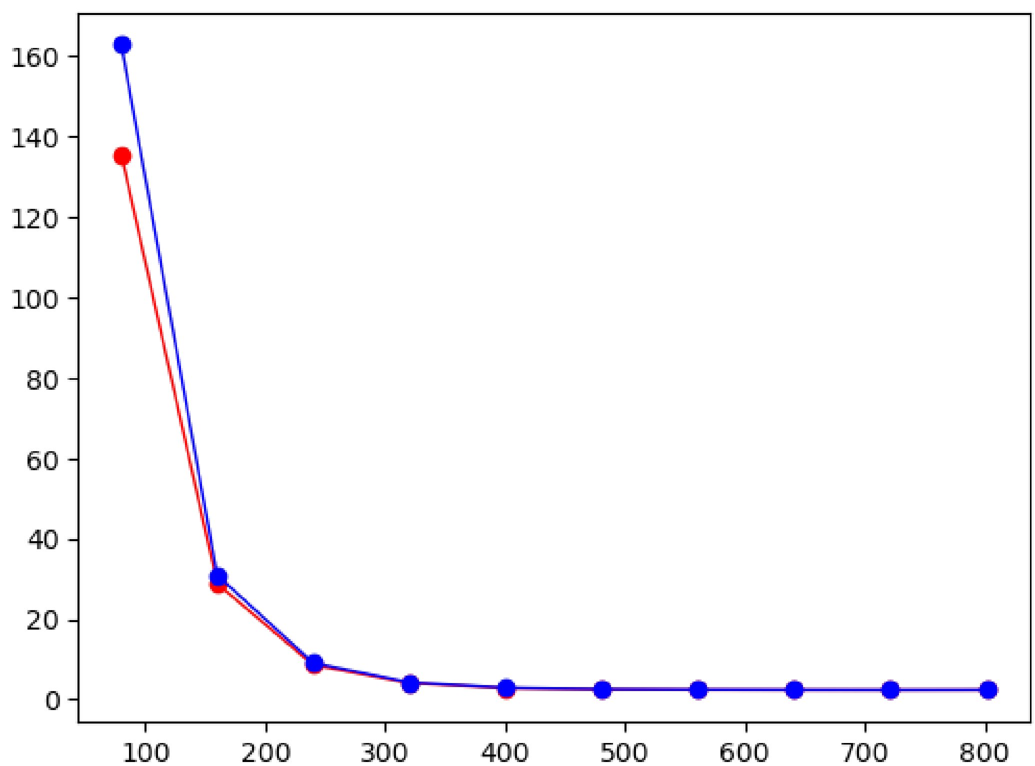
```
        Training loss: 2.396
        Validation loss: 2.418

For metric Mean Absolute Error(MAE) :
        Training loss: 1.653
        Validation loss: 1.669
```

Training and validation loss as a function of training set size

[34]:
```
train_sizes, train_scores, valid_scores = learning_curve( sgd_reg,
                                                          phi_train,
                                                          t_train,
                                                          train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                          cv=4,
                                                         ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

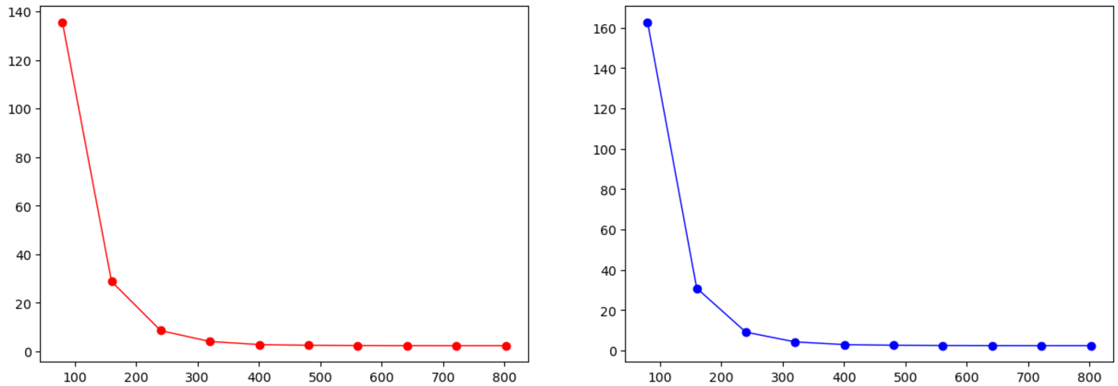
[35]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[35]: [<matplotlib.lines.Line2D at 0x7fa414105d50>]



[36]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[36]:
```
[<matplotlib.lines.Line2D at 0x7fa4141c2ce0>]
```



0.5.3 E - Part 2: Ridge Regression

[1]:
```
from sklearn.linear_model import Ridge
```

[38]:
```
ridge_reg = Ridge(alpha=0.1, solver="cholesky")

ridge_reg.fit(phi_train, t_train)

ridge_reg_cv = cross_validate(ridge_reg,
                              phi_train,
                              t_train,
                              ␣
  ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                              cv=4,
                              return_train_score=True)
```

[39]:
```
# let's look at the output from k fold
print("Ridge Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(ridge_reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(ridge_reg_cv['test_r2'])))
```

```
print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['test_neg_mean_absolute_error'])))
```

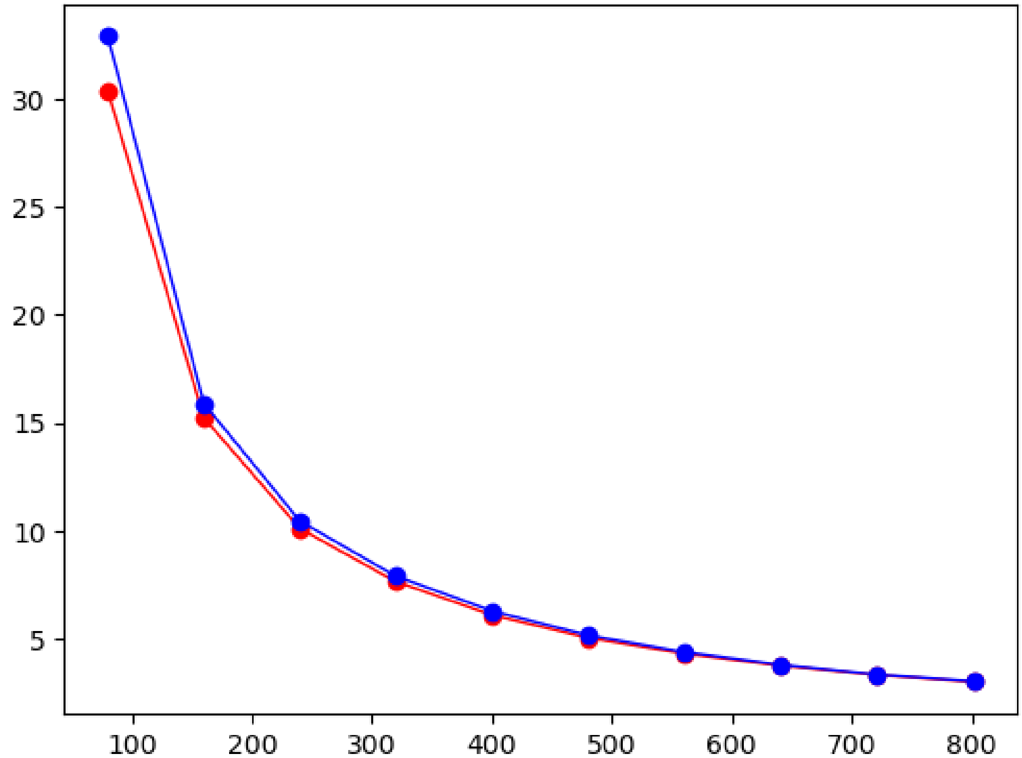
```
Ridge Regression:

 For metric R^2 :
         Training loss: 1.000
         Validation loss: 1.000

 For metric Root mean Squared Error(RMSE) :
         Training loss: 2.988
         Validation loss: 3.016

 For metric Mean Absolute Error(MAE) :
         Training loss: 2.059
         Validation loss: 2.079
```

Training and validation loss as a function of training set size

[40]:
```
train_sizes, train_scores, valid_scores = learning_curve(ridge_reg,
                                                         phi_train,
                                                         t_train,
                                                         train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                         cv=4,
                                                        ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

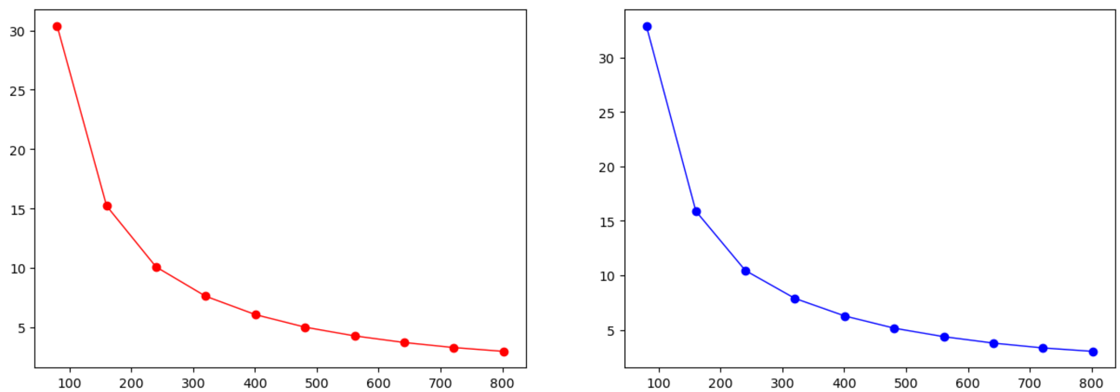
[41]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[41]: [<matplotlib.lines.Line2D at 0x7fa415a12ce0>]



[42]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[42]:
```
[&lt;matplotlib.lines.Line2D at 0x7fa40c64c700&gt;]
```



0.5.4 E - Part 2: Lasso Regression

[49]:
```
from sklearn.linear_model import Lasso
```

[50]:
```
lasso_reg = Lasso(alpha=0.1)

lasso_reg.fit(phi_train, t_train)

t_pred = lasso_reg.predict(phi_test)

lasso_reg_cv = cross_validate(lasso_reg,
                              phi_train,
                              t_train,
                              ␣
  ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                              cv=4,
                              return_train_score=True)
```

[51]:
```
# let's look at the output from k fold
print("Lasso Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(lasso_reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(lasso_reg_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
  ↪mean(lasso_reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
  ↪mean(lasso_reg_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
  ↪mean(lasso_reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
  ↪mean(lasso_reg_cv['test_neg_mean_absolute_error'])))
```

```
Lasso Regression:

 For metric R^2 :
         Training loss: 1.000
         Validation loss: 1.000

 For metric Root mean Squared Error(RMSE) :
         Training loss: 0.601
         Validation loss: 0.607
```

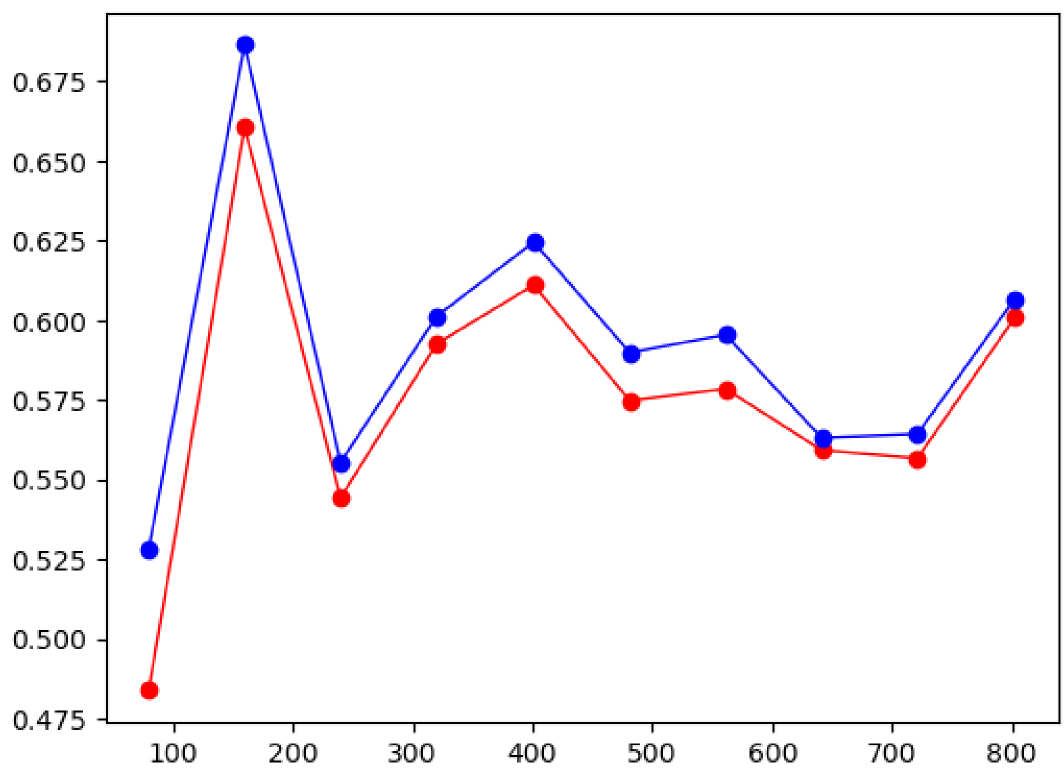
```
For metric Mean Absolute Error(MAE) :
        Training loss: 0.440
        Validation loss: 0.444
```

Training and validation loss as a function of training set size

[52]:
```
train_sizes, train_scores, valid_scores = learning_curve(lasso_reg,
                                                         phi_train,
                                                         t_train,
                                                         train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                         cv=4,
                                                         ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

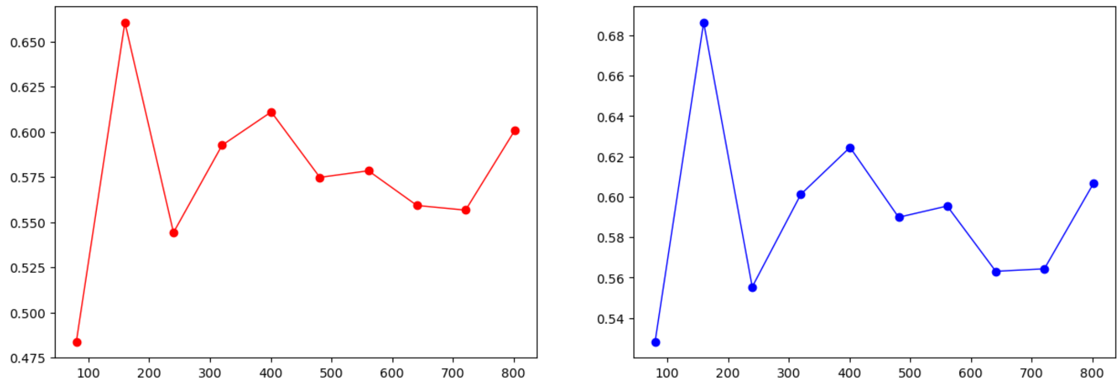
[53]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[53]: [<matplotlib.lines.Line2D at 0x7fa40c4a5840>]



[54]:
```python
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[54]: [<matplotlib.lines.Line2D at 0x7fa40c36ec50>]



0.5.5 E - Part 2: Elastic Net Regression

[62]:
```python
from sklearn.linear_model import ElasticNet
```

[63]:
```python
elastic_net = ElasticNet(alpha=0.1, l1_ratio=0.5)

elastic_net.fit(phi_train, t_train)

t_pred = elastic_net.predict(phi_test)

elastic_net_cv = cross_validate(elastic_net,
                               phi_train,
                               t_train,
                               ␣
 ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                               cv=4,
                               return_train_score=True)
```

[64]:
```python
# let's look at the output from k fold
print("Elastic Net Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(elastic_net_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(elastic_net_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
```

```
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(elastic_net_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(elastic_net_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(elastic_net_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(elastic_net_cv['test_neg_mean_absolute_error'])))
```

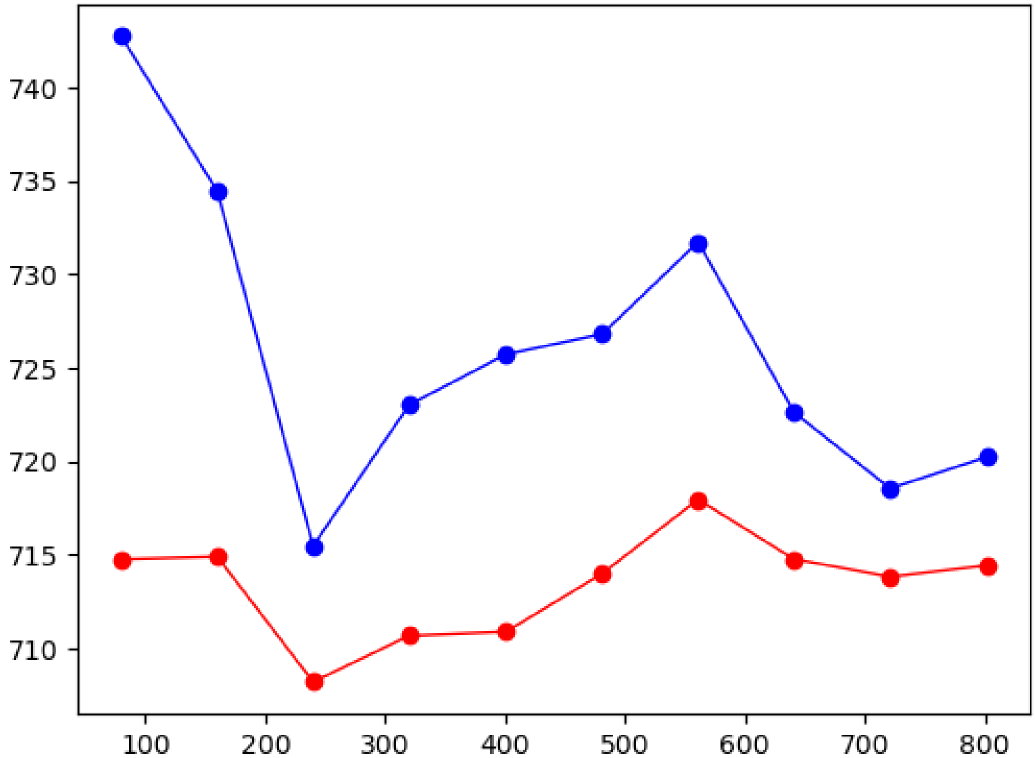
```
Elastic Net Regression:

 For metric R^2 :
         Training loss: 0.996
         Validation loss: 0.996

 For metric Root mean Squared Error(RMSE) :
         Training loss: 714.499
         Validation loss: 720.467

 For metric Mean Absolute Error(MAE) :
         Training loss: 519.085
         Validation loss: 523.041
```

Training and validation loss as a function of training set size

[65]:
```
train_sizes, train_scores, valid_scores = learning_curve(elastic_net,
                                                         phi_train,
                                                         t_train,
                                                         train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                         cv=4,
                                                        ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

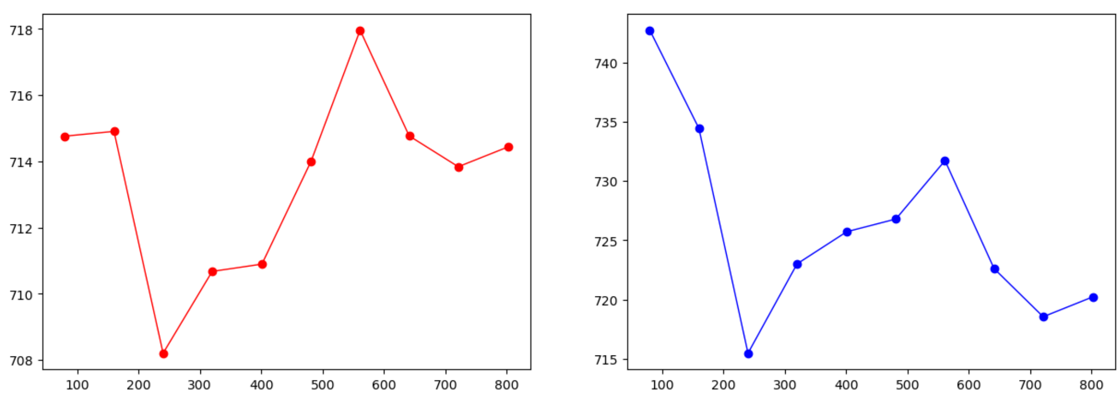
[66]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[66]: [<matplotlib.lines.Line2D at 0x7fa40c1d3d60>]



[67]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[67]:
```
[<matplotlib.lines.Line2D at 0x7fa407fb5150>]
```



0.5.6 E - Part 3: Hyperparameter Tuning for Ridge Regression

[43]:
```
from sklearn.linear_model import RidgeCV
```

[44]:
```
ridge_cv = RidgeCV(alphas=np.arange(0.01,2,0.01)).fit(phi_train, t_train)
ridge_cv.score(phi_train, t_train)
```

[44]: 0.9999999996515413

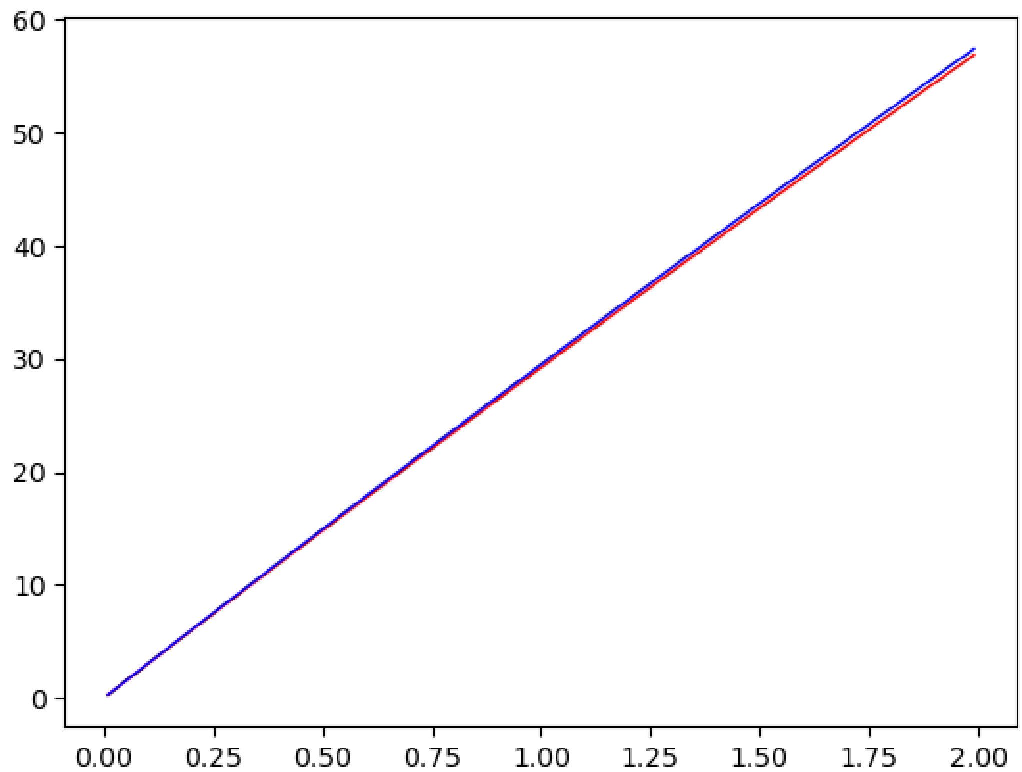
[45]:
```
ridge_cv.alpha_
```

[45]: 0.01

[46]:
```
ridge_reg_training_loss = []
ridge_reg_validation_loss = []
for aplha_value in np.arange(0.01,2,0.01):
    ridge_reg = Ridge(alpha=aplha_value, solver="cholesky")
    ridge_reg_cv = cross_validate(ridge_reg,
                                  phi_train,
                                  t_train,
                                  scoring='neg_root_mean_squared_error',
                                  cv=4,
                                  return_train_score=True)

    ridge_reg_training_loss.append(-np.mean(ridge_reg_cv['train_score']))
    ridge_reg_validation_loss.append(-np.mean(ridge_reg_cv['test_score']))
```

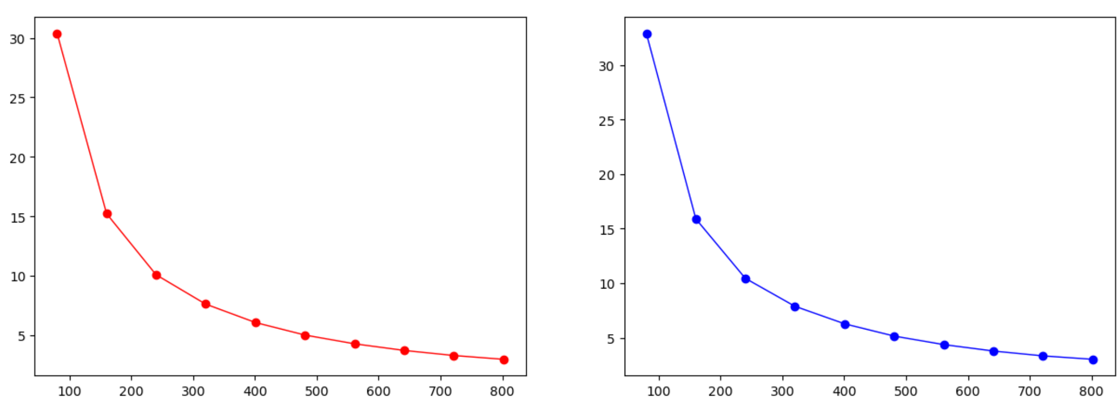
[47]:
```
plt.plot(np.arange(0.01,2,0.01), ridge_reg_training_loss, "r-", linewidth=1,␣
 ↪label="train")
plt.plot(np.arange(0.01,2,0.01), ridge_reg_validation_loss, "b-", linewidth=1,␣
 ↪label="valid")
```

[47]: [<matplotlib.lines.Line2D at 0x7fa40c52a1d0>]



[48]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[48]: [<matplotlib.lines.Line2D at 0x7fa40c5da9e0>]



0.5.7 E - Part 3: Hyperparameter Tuning for Lasso Regression

[55]:
```
from sklearn.linear_model import LassoCV
```

[56]:
```
lasso_cv = LassoCV(alphas=np.arange(0.01,2,0.01)).fit(phi_train, t_train)
lasso_cv.score(phi_train, t_train)
```

[56]: 0.9999999954967531

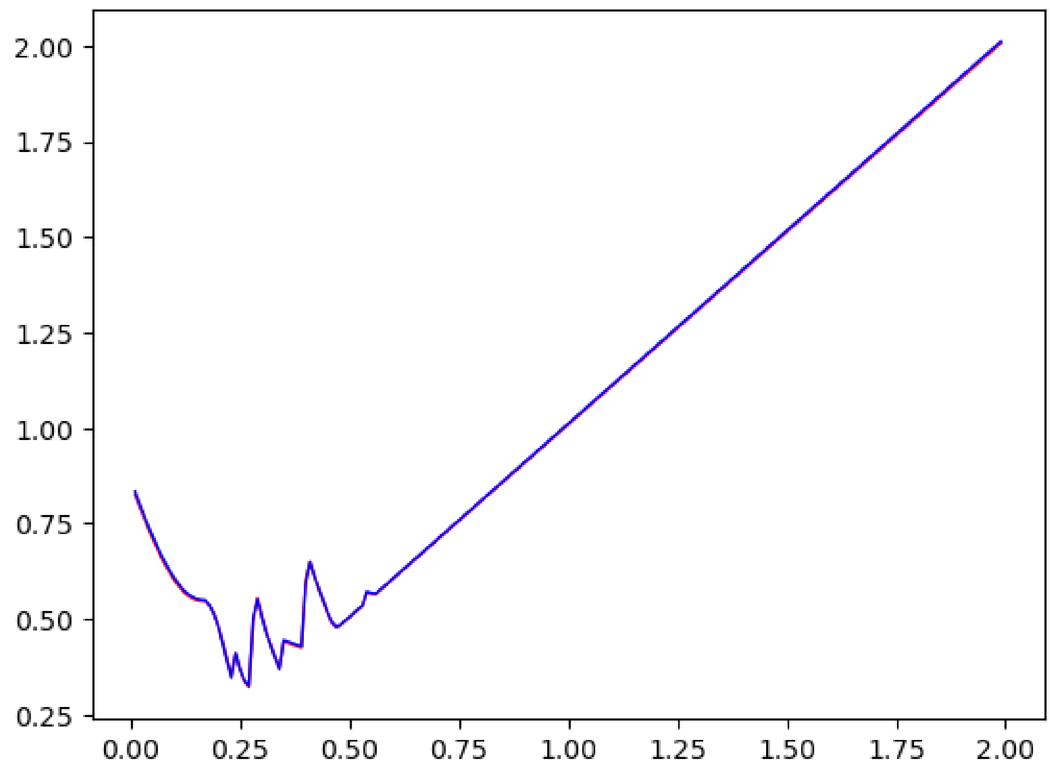
[57]:
```
# The amount of penalization chosen by cross validation.
lasso_cv.alpha_
```

[57]: 0.01

[58]:
```
lasso_reg_training_loss = []
lasso_reg_validation_loss = []
for aplha_value in np.arange(0.01,2,0.01):
    lasso_reg = Lasso(alpha=aplha_value)
    lasso_reg_cv = cross_validate(lasso_reg,
                                  phi_train,
                                  t_train,
                                  scoring='neg_root_mean_squared_error',
                                  cv=4,
                                  return_train_score=True)

    lasso_reg_training_loss.append(-np.mean(lasso_reg_cv['train_score']))
    lasso_reg_validation_loss.append(-np.mean(lasso_reg_cv['test_score']))
```

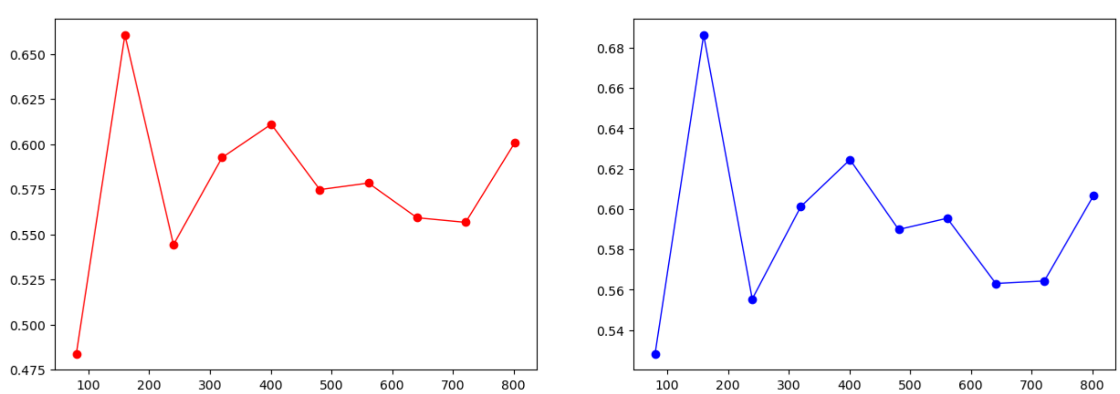
[59]:
```
plt.plot(np.arange(0.01,2,0.01), lasso_reg_training_loss, "r-", linewidth=1,␣
 ↪label="train")
plt.plot(np.arange(0.01,2,0.01), lasso_reg_validation_loss, "b-", linewidth=1,␣
 ↪label="valid")
```

[59]: [<matplotlib.lines.Line2D at 0x7fa40c25cee0>]



[60]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[60]:
```
[<matplotlib.lines.Line2D at 0x7fa40c107070>]
```



[61]:
```python
print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.mean(lasso_reg_cv['train_score'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(lasso_reg_cv['test_score'])))
```

```
 For metric Root mean Squared Error(RMSE) :
	 Training loss: 2.007
	 Validation loss: 2.012
```

1 ElasticNet with different penalty terms :

The ElasticNet mixing parameter, with 0 <= l1_ratio <= 1.

For l1_ratio = 0 the penalty is an L2 penalty.

For l1_ratio = 1 it is an L1 penalty.

For 0 < l1_ratio < 1, the penalty is a combination of L1 and L2.

1.0.1 E - Part 3: Hyperparameter Tuning for ElasticNet Regression

[68]:
```python
from sklearn.linear_model import ElasticNetCV
```

[69]:
```python
elastic_net_cv = ElasticNetCV(alphas=np.arange(0.01,2,0.01), l1_ratio=np.
 ↪arange(0.0,1.1,0.1)).fit(phi_train, t_train)
elastic_net_cv.score(phi_train, t_train)
```

[69]:
```
0.9999999954967531
```

[70]:
```python
elastic_net_cv.alpha_ , elastic_net_cv.l1_ratio_
```

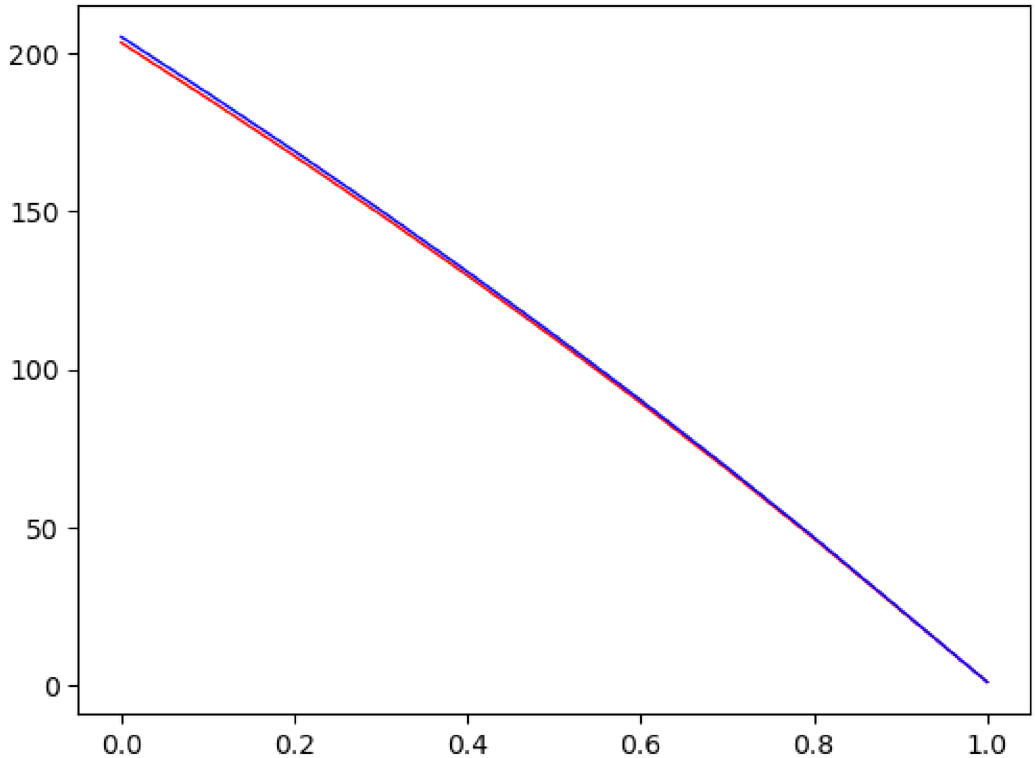
[70]:
```
(0.01, 1.0)
```

Training and validation loss as a function of training iterations.

[71]:
```python
elastic_net_training_loss = []
elastic_net_validation_loss = []
for l1_ratio_value in np.arange(0.0,1.1,0.1):
    elastic_net = ElasticNet(alpha=0.01, l1_ratio=l1_ratio_value)
    elastic_net_cv = cross_validate(elastic_net,
                                    phi_train,
                                    t_train,
                                    scoring='neg_root_mean_squared_error',
                                    cv=4,
                                    return_train_score=True)

    elastic_net_training_loss.append(-np.mean(elastic_net_cv['train_score']))
    elastic_net_validation_loss.append(-np.mean(elastic_net_cv['test_score']))
```

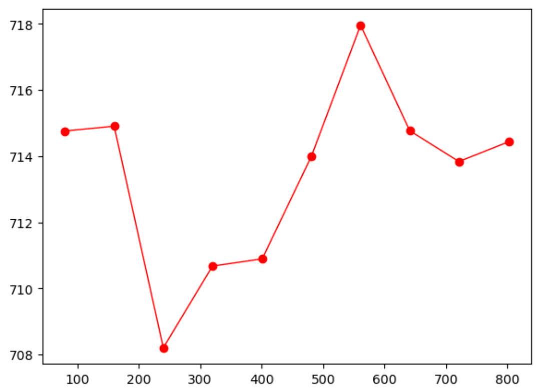
[72]:
```
plt.plot(np.arange(0.0,1.1,0.1), elastic_net_training_loss, "r-", linewidth=1,␣
 ↪label="train")
plt.plot(np.arange(0.0,1.1,0.1), elastic_net_validation_loss, "b-",␣
 ↪linewidth=1, label="valid")
```

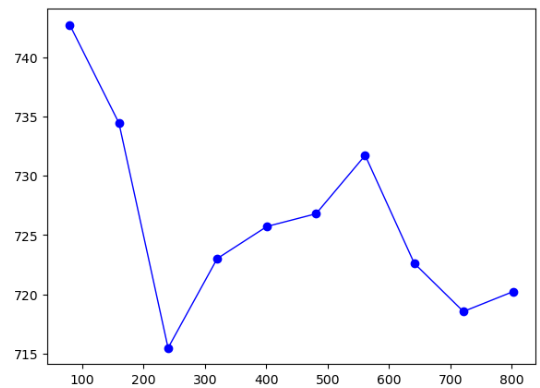
[72]: [<matplotlib.lines.Line2D at 0x7fa407e6f9a0>]



[73]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[73]: [<matplotlib.lines.Line2D at 0x7fa407e81540>]





1.0.2 Task E: Observations:

As we can observe, there is very little diffrence between training loss and testing loss of all the linear regression models, when a cross validation method used with 3 parameters 2, Root-Mean Squared Error, and Mean Absolute error. This suggests that the Linear regression model is overfitting but with a very little margin.

1.1 Question F: Polynomial Regression

1.1.1 F - Part 1: Polynomial Model using K-Fold with Normal form (train and val loss).

[74]:
```
from sklearn.preprocessing import PolynomialFeatures
poly_features = PolynomialFeatures(degree=2, include_bias=False)
```

[75]:
```
poly_phi_train = poly_features.fit_transform(phi_train)
poly_phi_test = poly_features.fit_transform(phi_test)
```

[76]:
```
reg = LinearRegression().fit(poly_phi_train, t_train)

reg_cv = cross_validate(reg,
                        poly_phi_train,
                        t_train,
                        ␣
  ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                        cv=4,
                        return_train_score=True)
```

[77]:
```
# let's look at the output from k fold
print("Linear Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(reg_cv['test_r2'])))
```

```
print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(reg_cv['test_neg_mean_absolute_error'])))
```

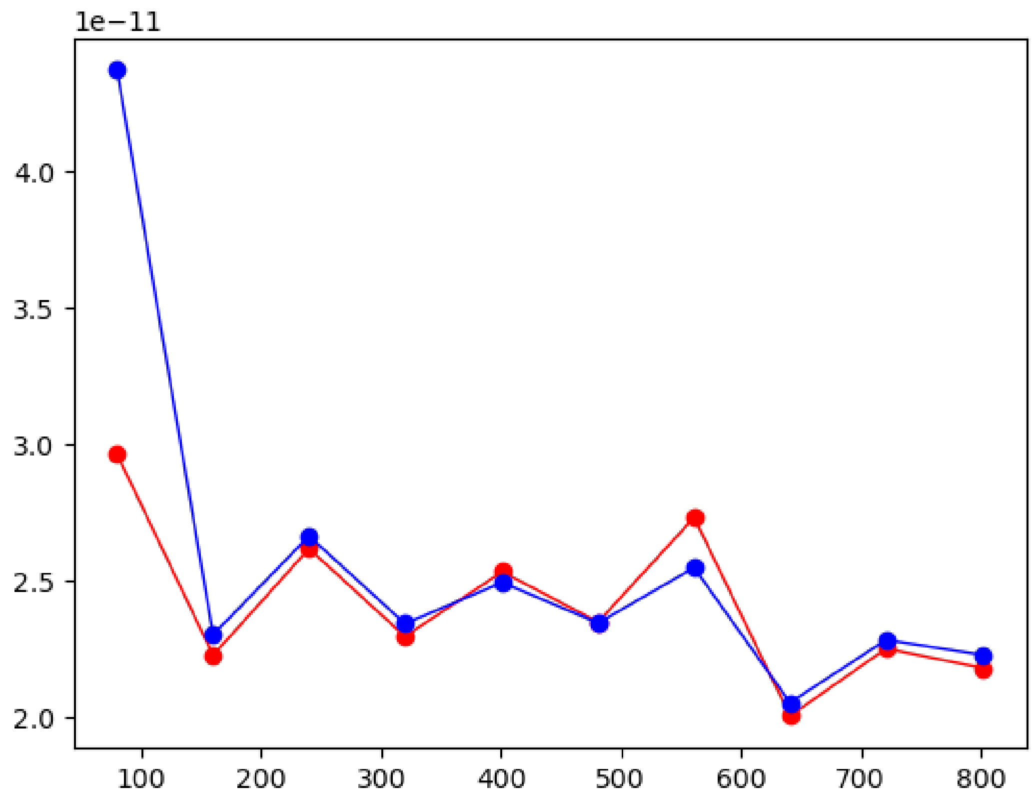
```
Linear Regression:

 For metric R^2 :
        Training loss: 1.000
        Validation loss: 1.000

 For metric Root mean Squared Error(RMSE) :
        Training loss: 0.000
        Validation loss: 0.000

 For metric Mean Absolute Error(MAE) :
        Training loss: 0.000
        Validation loss: 0.000
```

Training and validation loss as a function of training set size

[78]:
```
train_sizes, train_scores, valid_scores = learning_curve(reg,
                                                         poly_phi_train,
                                                         t_train,
                                                         train_sizes=np.
 ↪linspace(0.1, 1.0,10),
                                                         cv=4,
                                                         ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

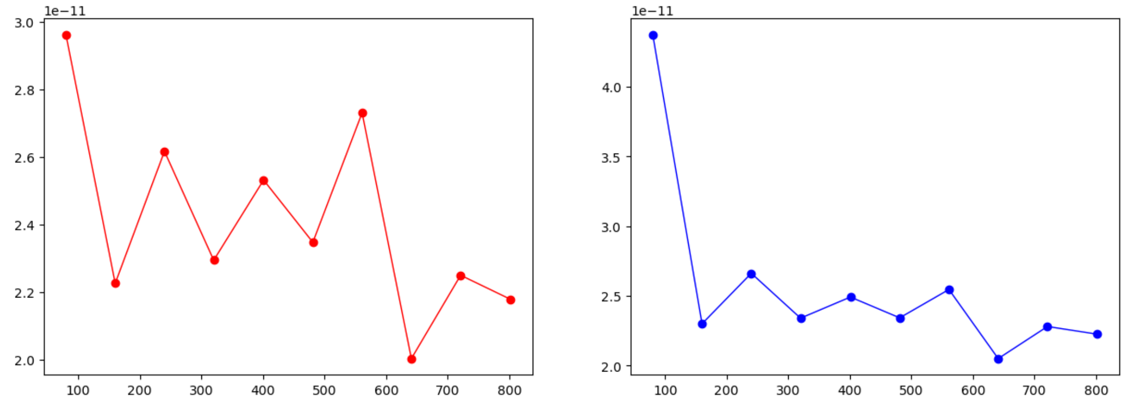
[79]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[79]: [<matplotlib.lines.Line2D at 0x7fa407dc2320>]



[80]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[80]: [<matplotlib.lines.Line2D at 0x7fa40c4659c0>]



1.1.2 F - Part 1: Polynomial Model using K-Fold with SGD (train and val loss).

[81]:
```
sgd_reg = SGDRegressor(max_iter=1000,
                       tol=1e-5,
                       eta0=0.01,
                       alpha = 0.01,
                       n_iter_no_change=100,
                       random_state=42)

sgd_reg.fit(poly_phi_train, t_train)

sgd_reg_cv = cross_validate(sgd_reg,
                        poly_phi_train,
                        t_train,
                       ␣
  ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                        cv=4,
                        return_train_score=True)
```

[82]:
```
# let's look at the output from k fold
print("Stochastic Gradient Descent Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(sgd_reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(sgd_reg_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
  ↪mean(sgd_reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
  ↪mean(sgd_reg_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
  ↪mean(sgd_reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
  ↪mean(sgd_reg_cv['test_neg_mean_absolute_error'])))
```

```
Stochastic Gradient Descent Regression:

 For metric R^2 :
         Training loss: 0.999
         Validation loss: 0.999

 For metric Root mean Squared Error(RMSE) :
         Training loss: 387.034
```

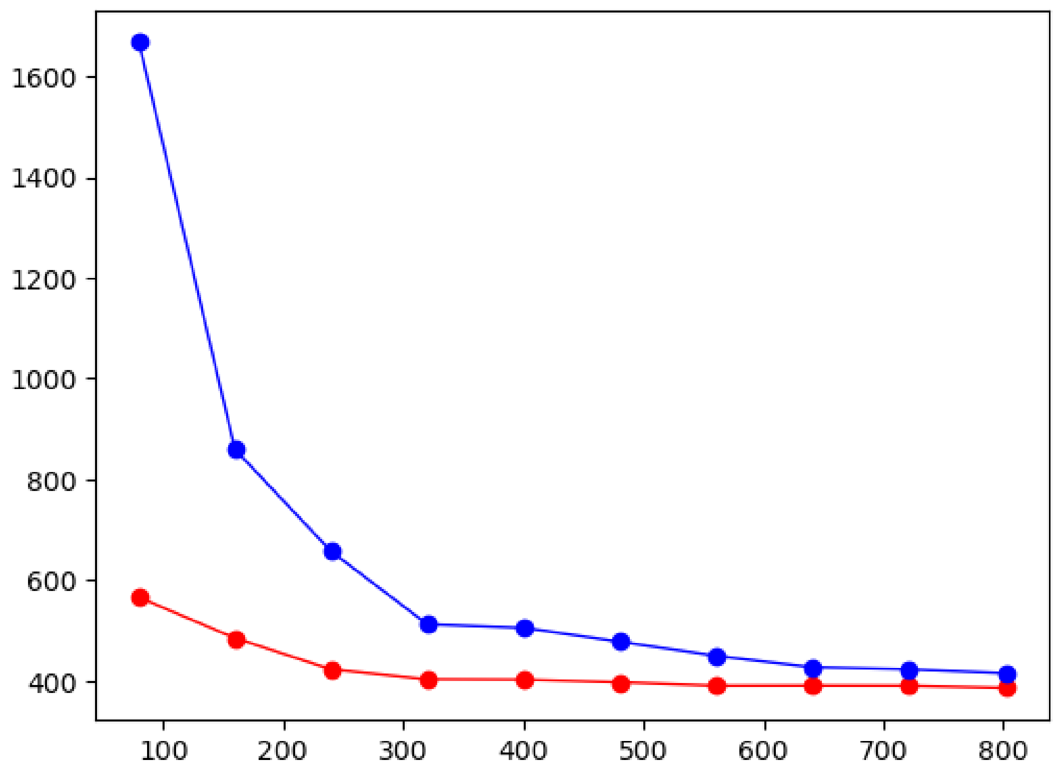
```
        Validation loss: 415.504

For metric Mean Absolute Error(MAE) :
        Training loss: 236.785
        Validation loss: 249.189
```

Training and validation loss as a function of training set size

[83]:
```
train_sizes, train_scores, valid_scores = learning_curve(sgd_reg,
                                                          poly_phi_train,
                                                          t_train,
                                                          train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                          cv=4,
                                                         ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

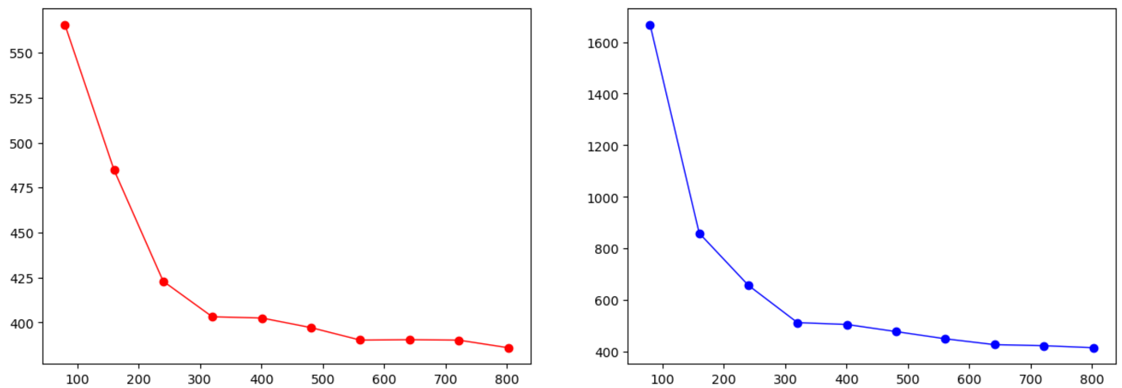
[84]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[84]: `[<matplotlib.lines.Line2D at 0x7fa407cd31f0>]`



[85]:
```python
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[85]:
```
[<matplotlib.lines.Line2D at 0x7fa407b9f160>]
```



1.1.3 F - Part 2: Ridge Regression

[86]:
```python
ridge_reg.fit(poly_phi_train, t_train)

ridge_reg_cv = cross_validate(ridge_reg,
                        poly_phi_train,
                        t_train,
                        scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                        cv=4,
                        return_train_score=True)
```

[87]:
```python
# let's look at the output from k fold
print("Ridge Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(ridge_reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(ridge_reg_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
    mean(ridge_reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
    mean(ridge_reg_cv['test_neg_root_mean_squared_error'])))
```

```
print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['test_neg_mean_absolute_error'])))
```

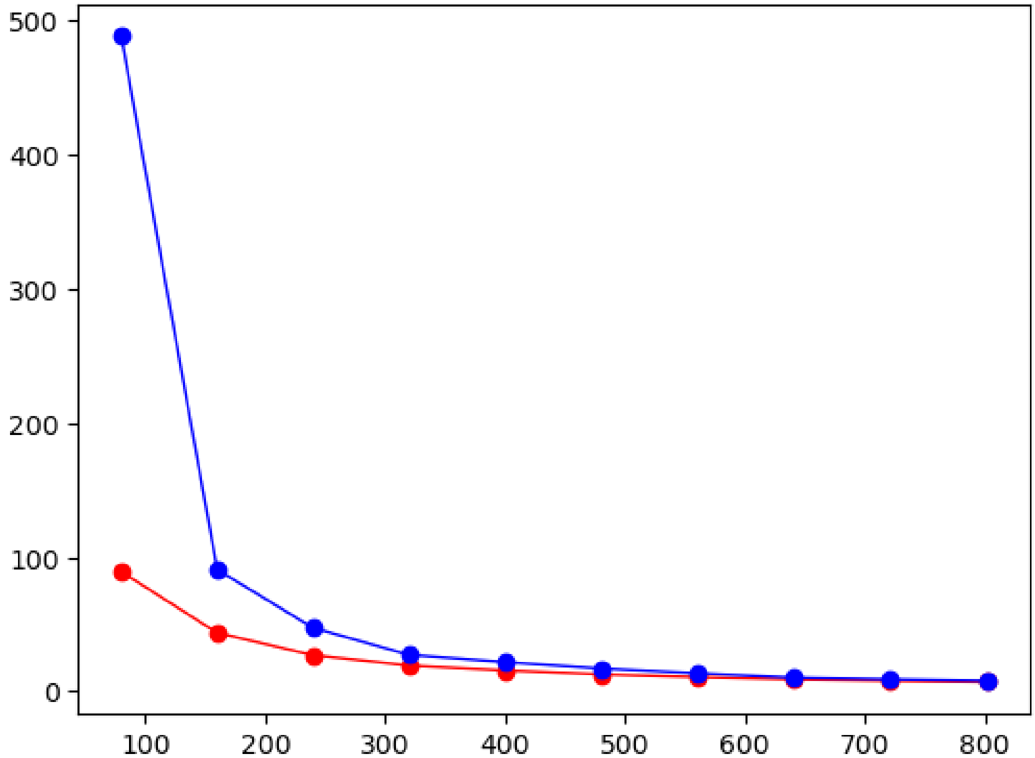
```
Ridge Regression:

 For metric R^2 :
         Training loss: 1.000
         Validation loss: 1.000

 For metric Root mean Squared Error(RMSE) :
         Training loss: 7.190
         Validation loss: 8.013

 For metric Mean Absolute Error(MAE) :
         Training loss: 4.273
         Validation loss: 4.594
```

Training and validation loss as a function of training set size

[88]:
```
train_sizes, train_scores, valid_scores = learning_curve(ridge_reg,
                                                         poly_phi_train,
                                                         t_train,
                                                         train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                         cv=4,
                                                        ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

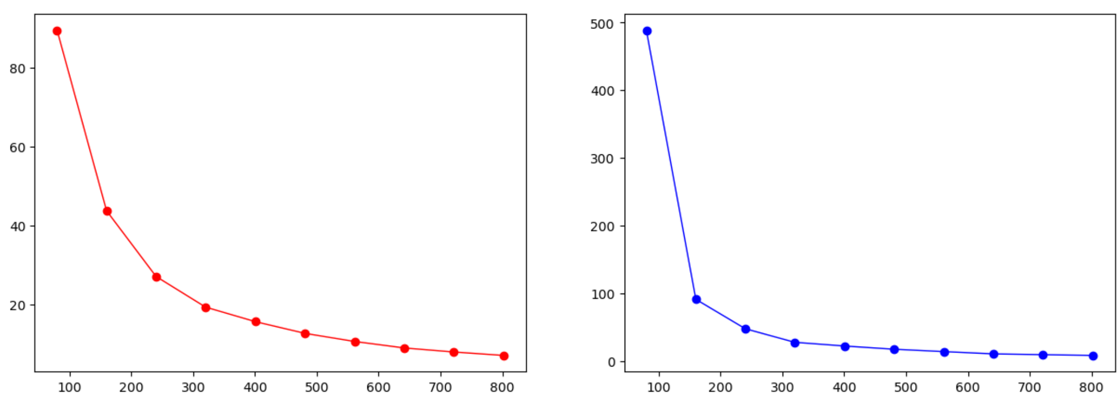
[89]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[89]:
```
[<matplotlib.lines.Line2D at 0x7fa407a6ee00>]
```



[90]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[90]:
```
[<matplotlib.lines.Line2D at 0x7fa407926170>]
```



1.1.4 F - Part 2: Lasso Regression

[96]:
```
lasso_reg.fit(poly_phi_train, t_train)

t_pred = lasso_reg.predict(poly_phi_test)

lasso_reg_cv = cross_validate(lasso_reg,
                        poly_phi_train,
                        t_train,
                        ␣
 ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                        cv=4,
                        return_train_score=True)
```

[97]:
```
# let's look at the output from k fold
print("Lasso Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(lasso_reg_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(lasso_reg_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(lasso_reg_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(lasso_reg_cv['test_neg_root_mean_squared_error'])))


print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(lasso_reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(lasso_reg_cv['test_neg_mean_absolute_error'])))
```

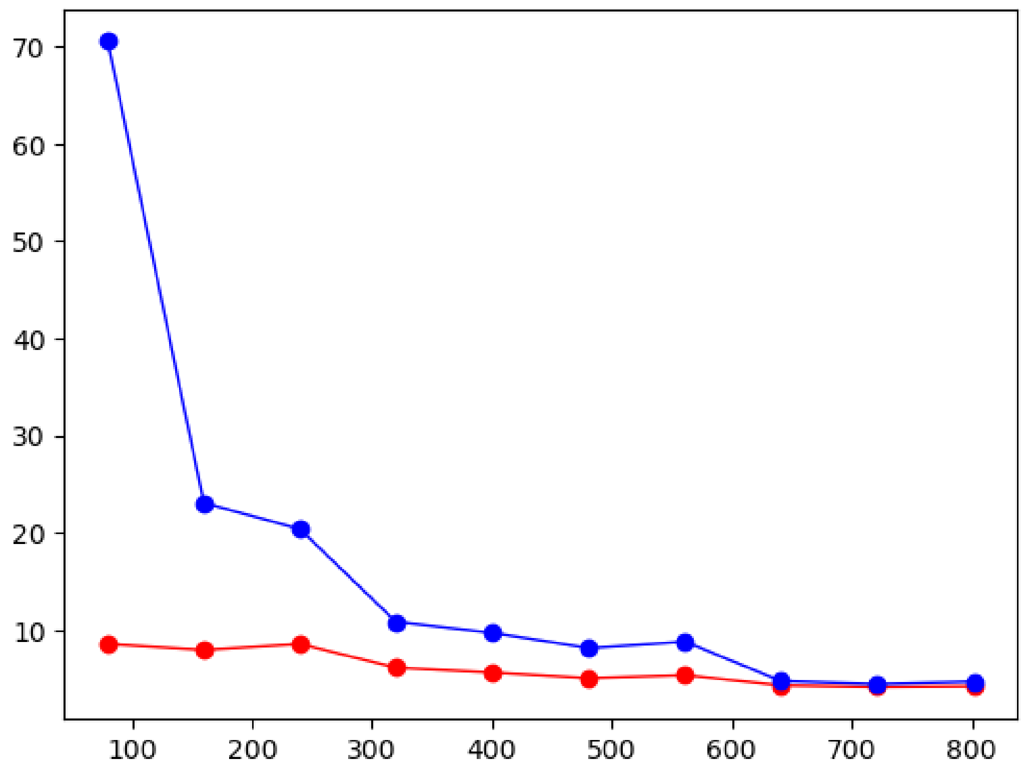
```
Lasso Regression:

 For metric R^2 :
         Training loss: 1.000
         Validation loss: 1.000

 For metric Root mean Squared Error(RMSE) :
         Training loss: 4.212
         Validation loss: 4.676

 For metric Mean Absolute Error(MAE) :
         Training loss: 2.597
         Validation loss: 2.744
```

Training and validation loss as a function of training set size

[98]:
```
train_sizes, train_scores, valid_scores = learning_curve(lasso_reg,
                                                          poly_phi_train,
                                                          t_train,
                                                          train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                          cv=4,
                                                         ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

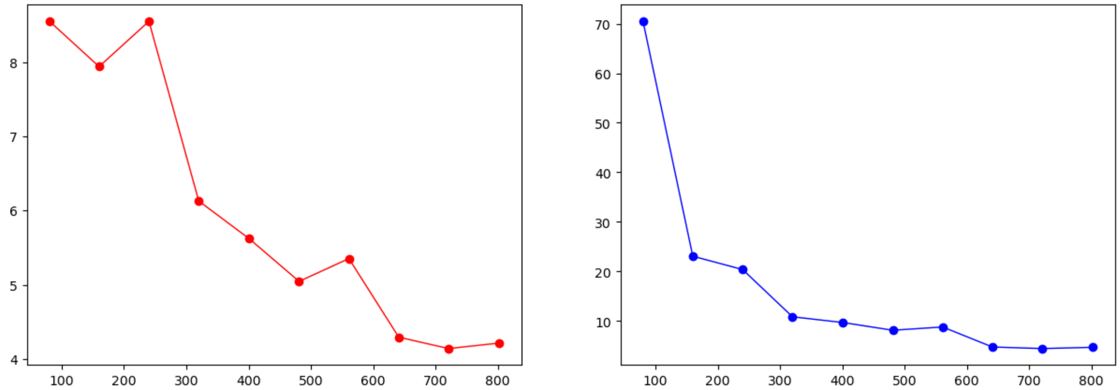
[99]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[99]: [<matplotlib.lines.Line2D at 0x7fa407771d20>]



[100]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[100]: [<matplotlib.lines.Line2D at 0x7fa40761ef50>]



1.1.5 F - Part 2: Elastic Net Regression

[106]:
```
elastic_net = ElasticNet(alpha=0.1, l1_ratio=0.5)

elastic_net.fit(poly_phi_train, t_train)

t_pred = elastic_net.predict(poly_phi_test)

elastic_net_cv = cross_validate(elastic_net,
                        poly_phi_train,
                        t_train,
                        
 ↪scoring=['r2','neg_root_mean_squared_error','neg_mean_absolute_error'],
                        cv=4,
                        return_train_score=True)
```

[107]:
```
# let's look at the output from k fold
print("Elastic Net Regression: ")
print("\n For metric R^2 :")
print("\t Training loss: {:,.3f}".format(np.mean(elastic_net_cv['train_r2'])))
print("\t Validation loss: {:,.3f}".format(np.mean(elastic_net_cv['test_r2'])))


print("\n For metric Root mean Squared Error(RMSE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(elastic_net_cv['train_neg_root_mean_squared_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(elastic_net_cv['test_neg_root_mean_squared_error'])))
```

```
print("\n For metric Mean Absolute Error(MAE) :")
print("\t Training loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['train_neg_mean_absolute_error'])))
print("\t Validation loss: {:,.3f}".format(-np.
 ↪mean(ridge_reg_cv['test_neg_mean_absolute_error'])))
```

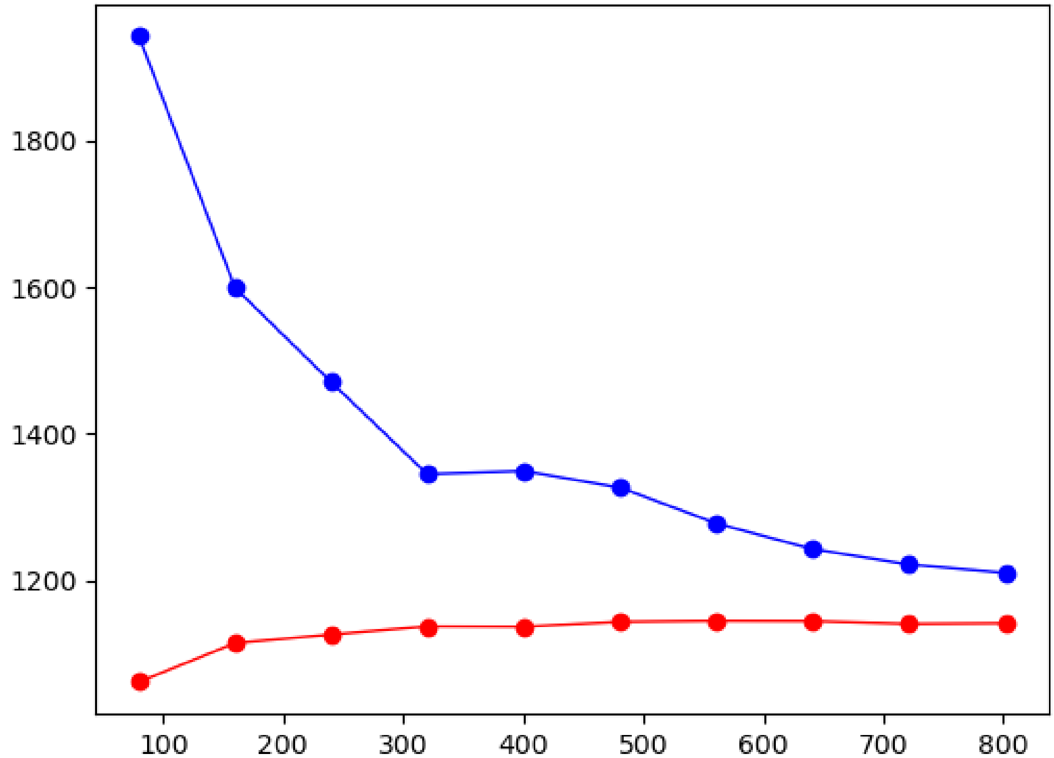
```
Elastic Net Regression:

 For metric R^2 :
	 Training loss: 0.991
	 Validation loss: 0.990

 For metric Root mean Squared Error(RMSE) :
	 Training loss: 1,141.442
	 Validation loss: 1,210.273

 For metric Mean Absolute Error(MAE) :
	 Training loss: 4.273
	 Validation loss: 4.594
```

Training and validation loss as a function of training set size

[108]:
```
train_sizes, train_scores, valid_scores = learning_curve(elastic_net,
                                                         poly_phi_train,
                                                         t_train,
                                                         train_sizes=np.
 ↪linspace(0.1, 1.0, 10),
                                                         cv=4,
                                                        ␣
 ↪scoring="neg_root_mean_squared_error")

train_errors = -train_scores.mean(axis=1)
valid_errors = -valid_scores.mean(axis=1)
```

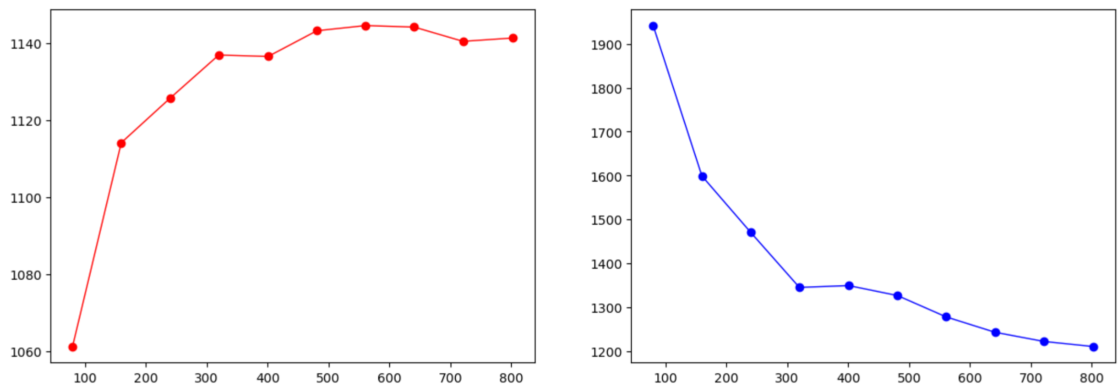
[109]:
```
plt.plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
plt.plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[109]: `[<matplotlib.lines.Line2D at 0x7fa407468790>]`



[110]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[110]: [<matplotlib.lines.Line2D at 0x7fa407314f40>]



1.1.6 F - Part 3: Hyperparameter Tuning for Ridge Regression

[91]:
```
ridge_cv = RidgeCV(alphas=np.arange(0.01,2,0.01)).fit(poly_phi_train, t_train)
ridge_cv.score(poly_phi_train, t_train)
```

[91]: 0.9999999979883903

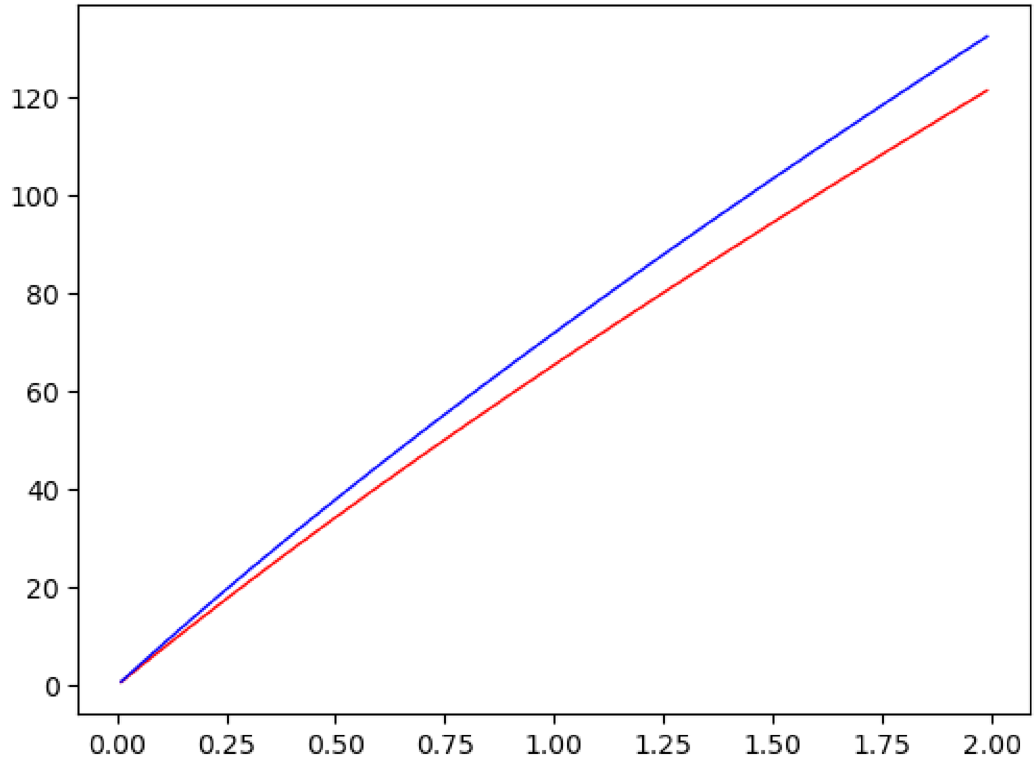
[92]:
```
ridge_cv.alpha_
```

[92]: 0.01

[93]:
```
poly_ridge_reg_training_loss = []
poly_ridge_reg_validation_loss = []
for aplha_value in np.arange(0.01,2,0.01):
    poly_ridge_reg = Ridge(alpha=aplha_value, solver="cholesky")
    poly_ridge_reg_cv = cross_validate(poly_ridge_reg,
                                   poly_phi_train,
                                   t_train,
                                   scoring='neg_root_mean_squared_error',
                                   cv=4,
                                   return_train_score=True)

    poly_ridge_reg_training_loss.append(-np.
 ↪mean(poly_ridge_reg_cv['train_score']))
    poly_ridge_reg_validation_loss.append(-np.
 ↪mean(poly_ridge_reg_cv['test_score']))
```

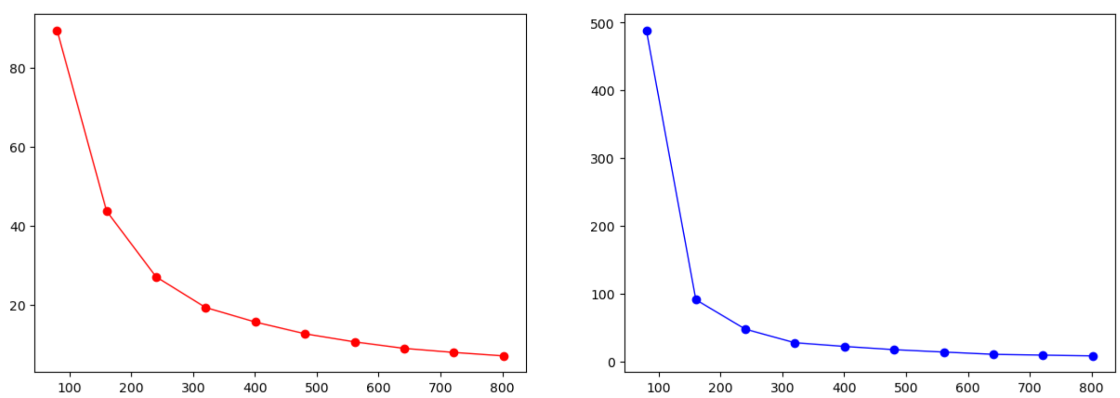
[94]:
```
plt.plot(np.arange(0.01,2,0.01), poly_ridge_reg_training_loss, "r-",␣
 ↪linewidth=1, label="train")
plt.plot(np.arange(0.01,2,0.01), poly_ridge_reg_validation_loss, "b-",␣
 ↪linewidth=1, label="valid")
```

[94]: [<matplotlib.lines.Line2D at 0x7fa40780ad10>]



[95]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[95]: [<matplotlib.lines.Line2D at 0x7fa4078dc490>]



1.1.7 F - Part 3: Hyperparameter Tuning for Lasso Regression

[101]:
```
lasso_cv = LassoCV(alphas=np.arange(0.01,2,0.01)).fit(poly_phi_train, t_train)
lasso_cv.score(poly_phi_train, t_train)
```

[101]:
```
0.9999999347773918
```

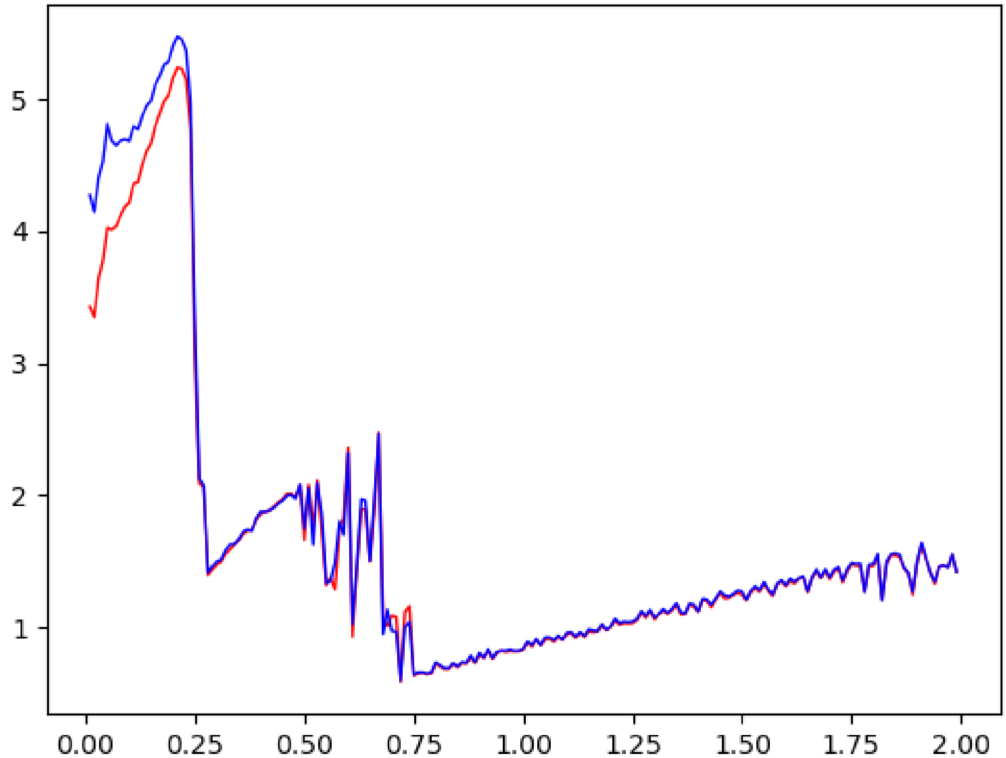
[102]:
```
# The amount of penalization chosen by cross validation.
lasso_cv.alpha_
```

[102]:
```
0.01
```

[103]:
```
poly_lasso_reg_training_loss = []
poly_lasso_reg_validation_loss = []
for aplha_value in np.arange(0.01,2,0.01):
    poly_lasso_reg = Lasso(alpha=aplha_value)
    poly_lasso_reg_cv = cross_validate(poly_lasso_reg,
                                       poly_phi_train,
                                       t_train,
                                       scoring='neg_root_mean_squared_error',
                                       cv=4,
                                       return_train_score=True)

    poly_lasso_reg_training_loss.append(-np.
 ↪mean(poly_lasso_reg_cv['train_score']))
    poly_lasso_reg_validation_loss.append(-np.
 ↪mean(poly_lasso_reg_cv['test_score']))
```

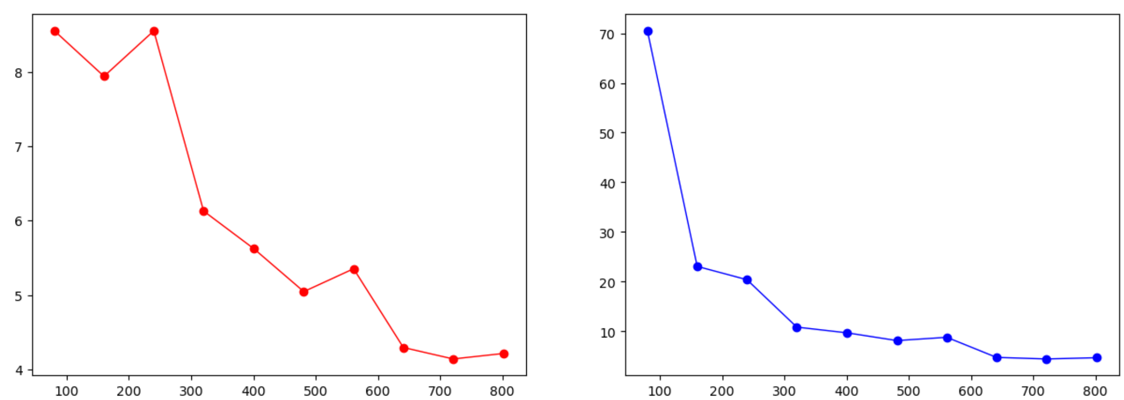
[104]:
```
plt.plot(np.arange(0.01,2,0.01), poly_lasso_reg_training_loss, "r-",␣
 ↪linewidth=1, label="train")
plt.plot(np.arange(0.01,2,0.01), poly_lasso_reg_validation_loss, "b-",␣
 ↪linewidth=1, label="valid")
```

[104]:
```
[<matplotlib.lines.Line2D at 0x7fa4076cd930>]
```



[105]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(train_sizes, train_errors, "r-o", linewidth=1, label="train")
axes[1].plot(train_sizes, valid_errors, "b-o", linewidth=1, label="valid")
```

[105]: [<matplotlib.lines.Line2D at 0x7fa407596710>]



2 ElasticNet with different penalty terms :

The ElasticNet mixing parameter, with 0 <= l1_ratio <= 1.

For l1_ratio = 0 the penalty is an L2 penalty.

For l1_ratio = 1 it is an L1 penalty.

For 0 < l1_ratio < 1, the penalty is a combination of L1 and L2.

2.0.1 F - Part 3: Hyperparameter Tuning for ElasticNet Regression

[111]:
```
elastic_net_cv = ElasticNetCV(alphas= [0.01], l1_ratio=np.arange(0.0,1.1,0.1)).
↪fit(poly_phi_train, t_train)
elastic_net_cv.score(poly_phi_train, t_train)
```

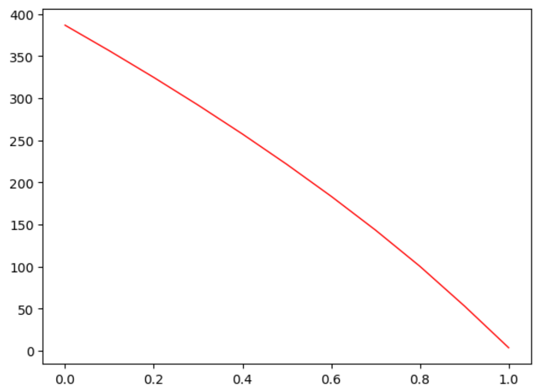
[111]: 0.9999999347773918

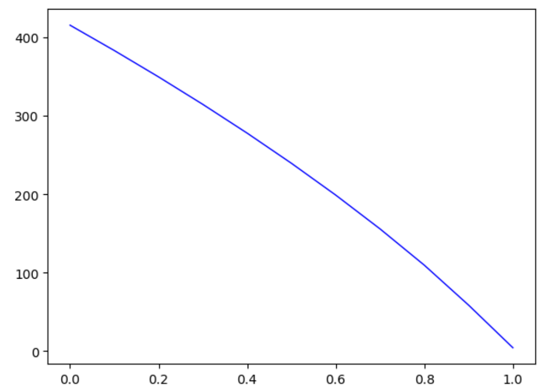
[112]:
```
elastic_net_cv.alpha_ , elastic_net_cv.l1_ratio_
```

[112]: (0.01, 1.0)

[113]:
```
poly_elastic_net_training_loss = []
poly_elastic_net_validation_loss = []
for l1_ratio_value in np.arange(0.0,1.1,0.1):
    poly_elastic_net = ElasticNet(alpha=0.01, l1_ratio=l1_ratio_value)
    poly_elastic_net_cv = cross_validate(poly_elastic_net,
                                        poly_phi_train,
                                        t_train,
                                        scoring='neg_root_mean_squared_error',
                                        cv=4,
                                        return_train_score=True)

    poly_elastic_net_training_loss.append(-np.
↪mean(poly_elastic_net_cv['train_score']))
    poly_elastic_net_validation_loss.append(-np.
↪mean(poly_elastic_net_cv['test_score']))
```

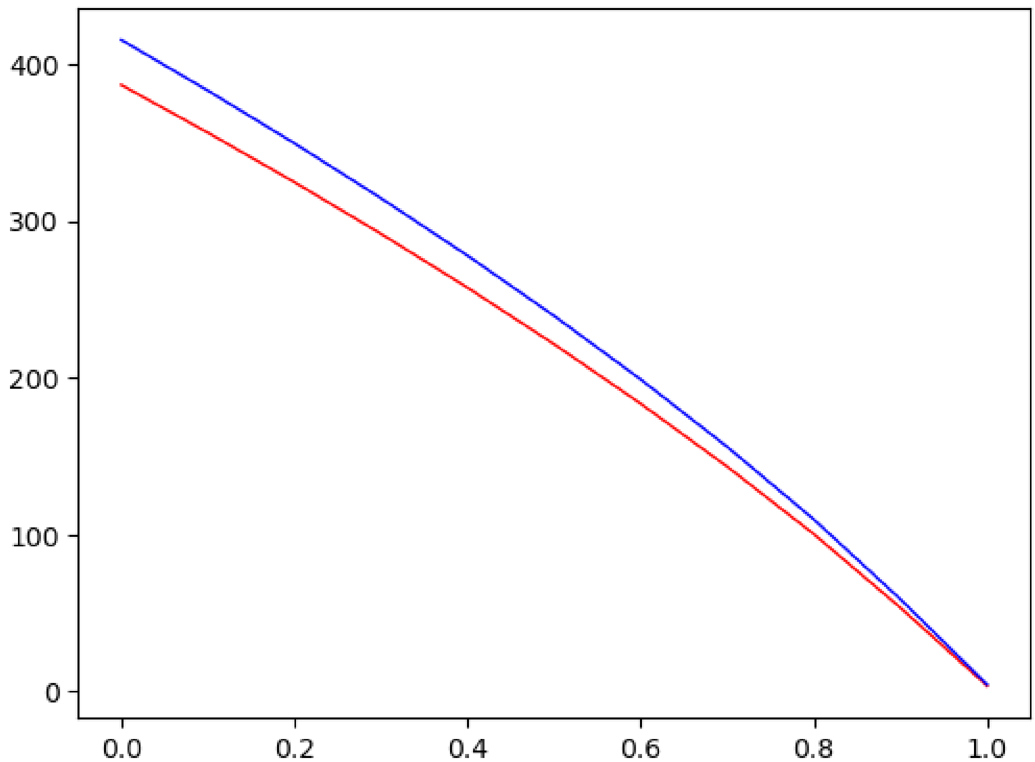
[114]:
```
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(15, 5))

axes[0].plot(np.arange(0.0,1.1,0.1), poly_elastic_net_training_loss, "r-",␣
↪linewidth=1, label="train")
axes[1].plot(np.arange(0.0,1.1,0.1), poly_elastic_net_validation_loss, "b-",␣
↪linewidth=1, label="valid")
plt.show()
```





[115]:
```
plt.plot(np.arange(0.0,1.1,0.1), poly_elastic_net_training_loss, "r-",␣
↪linewidth=1, label="train")
plt.plot(np.arange(0.0,1.1,0.1), poly_elastic_net_validation_loss, "b-",␣
↪linewidth=1, label="valid")
```

[115]: [<matplotlib.lines.Line2D at 0x7fa4072a1d80>]



2.0.2 Task F: Observations:

As we can observe, there is very little diffrence between training loss and testing loss of all the polynomial regression models, when a cross validation method used with 3 parameters 2, Root-Mean Squared Error, and Mean Absolute error. This suggests that the Linear regression model is overfitting but with a very little margin.

2.1 Question G

2.1.1 G - Part 1: Make predictions of the labels on the test data, using the trained model with chosen hyperparameters. Summarize performance using the appropriate evaluation metric.

2.1.2 Model 1: Simple Linear Regression

[116]:
```
reg = LinearRegression().fit(phi_train, t_train)
t_pred = reg.predict(phi_test)
```

[117]:
```
# let's look at the output from k fold
print("Linear Regression: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
↪t_pred)))
```

```
Linear Regression:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 0.000

 Mean Absolute Error(MAE): 0.000
```

2.1.3 Model 2 : Linear Regression with Stochastic Gradient Descent

[118]:
```
sgd_reg = SGDRegressor(max_iter=1000,
                       tol=1e-5,
                       eta0=0.01,
                       n_iter_no_change=100,
                       random_state=42)

sgd_reg.fit(phi_train, t_train)

t_pred = sgd_reg.predict(phi_test)
```

[119]:
```
# let's look at the output from k fold
print("Linear Regression with Stochastic Gradient Descent: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
```

```
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Linear Regression with Stochastic Gradient Descent:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 2.271

 Mean Absolute Error(MAE): 1.639
```

2.1.4 Model 3: Linear Regression with Ridge Regularization

[120]:
```
ridge_reg = Ridge(alpha=0.13, solver="cholesky")

ridge_reg.fit(phi_train, t_train)

t_pred = ridge_reg.predict(phi_test)
```

[121]:
```
# let's look at the output from k fold
print("Linear Regression with Ridge Regularization: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Linear Regression with Ridge Regularization:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 2.763

 Mean Absolute Error(MAE): 1.995
```

2.1.5 Model 4 : Linear Regression with Lasso Regularization

[122]:
```
lasso_reg = Lasso(alpha=0.01)

lasso_reg.fit(phi_train, t_train)

t_pred = lasso_reg.predict(phi_test)
```

[123]:
```
# let's look at the output from k fold
print("Linear Regression with Lasso Regularization: ")
```

```
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Linear Regression with Lasso Regularization:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 0.768

 Mean Absolute Error(MAE): 0.584
```

2.1.6 Model 5: Linear Regression with Elastic Net Regularization

[124]:
```
elastic_net = ElasticNet(alpha=0.01, l1_ratio=1.0)

elastic_net.fit(phi_train, t_train)

t_pred = elastic_net.predict(phi_test)
```

[125]:
```
# let's look at the output from k fold
print("Linear Regression with Elastic Net Regularization: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Linear Regression with Elastic Net Regularization:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 0.768

 Mean Absolute Error(MAE): 0.584
```

2.1.7 Model 6: Simple Polynomial Regression

[126]:
```
reg = LinearRegression().fit(poly_phi_train, t_train)

t_pred = reg.predict(poly_phi_test)
```

[127]:
```
# let's look at the output from k fold
print("Simple Polynomial Regression: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
```

```
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Simple Polynomial Regression:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 0.000

 Mean Absolute Error(MAE): 0.000
```

2.1.8 Model 7 : Polynomial Regression with Stochastic Gradient Descent

[128]:
```
sgd_reg = SGDRegressor(max_iter=1000,
                       tol=1e-5,
                       eta0=0.01,
                       n_iter_no_change=100,
                       random_state=42)

sgd_reg.fit(poly_phi_train, t_train)

t_pred = sgd_reg.predict(poly_phi_test)
```

[129]:
```
# let's look at the output from k fold
print("Polynomial Regression with Stochastic Gradient Descent: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Polynomial Regression with Stochastic Gradient Descent:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 43.988

 Mean Absolute Error(MAE): 23.350
```

2.1.9 Model 8: Polynomial Regression with Ridge Regularization

[130]:
```
ridge_reg = Ridge(alpha=0.02, solver="cholesky")

ridge_reg.fit(poly_phi_train, t_train)
```

```
t_pred = ridge_reg.predict(poly_phi_test)
```

[131]:
```
# let's look at the output from k fold
print("Polynomial Regression with Ridge Regularization: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Polynomial Regression with Ridge Regularization:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 1.079

 Mean Absolute Error(MAE): 0.657
```

2.1.10 Model 9 : Polynomial Regression with Lasso Regularization

[132]:
```
lasso_reg = Lasso(alpha=0.01)

lasso_reg.fit(poly_phi_train, t_train)

t_pred = lasso_reg.predict(poly_phi_test)
```

[133]:
```
# let's look at the output from k fold
print("Polynomial Regression with Lasso Regularization: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Polynomial Regression with Lasso Regularization:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 3.074

 Mean Absolute Error(MAE): 1.887
```

2.1.11 Model 10: Polynomial Regression with Elastic Net Regularization

[134]:
```
elastic_net = ElasticNet(alpha=0.01, l1_ratio=1.0)

elastic_net.fit(poly_phi_train, t_train)

t_pred = elastic_net.predict(poly_phi_test)
```

[135]:
```
# let's look at the output from k fold
print("Polynomial Regression with Elastic Net Regularization: ")
print("\n R^2: {:,.3f}".format(r2_score(t_test, t_pred)))
print("\n Root mean Squared Error(RMSE) : {:,.3f}".format(np.
 ↪sqrt(mean_squared_error(t_test, t_pred))))
print("\n Mean Absolute Error(MAE): {:,.3f}".format(mean_absolute_error(t_test,␣
 ↪t_pred)))
```

```
Polynomial Regression with Elastic Net Regularization:

 R^2: 1.000

 Root mean Squared Error(RMSE) : 3.074

 Mean Absolute Error(MAE): 1.887
```

2.1.12 G - Part 2: Discuss the results. Include thoughts about what further can be explored to increase performance.

2.1.13 Conclusion:

```
1. All the models has value of ${R^2}$ equal to 1 hence our dataset is linearly predictable.
2. Out of all 10 models, the linear Regression has the least RMSE and MAE (perfect 0). Thus, fo
```

2.1.14 Improvements :

```
1. Since we have only tried the polynomial linear regression with degree 2, we might consider t
2. Moreover, for the learning rate, we selected range from (0.01, 2) with 0.01 step increase. T
```