Lab-7

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Set Q = {(3,1), (6,1), (0,0), (7,3), (5,5), (5,3), (4,2), (3,4)}

Step1: Find the point P_0 with Min. Y-Co-ordinate

Incase of tie, Find the Leftmost such point

Here (0,0) is the point with Min. Y. Co-ordinate.

$\therefore \boxed{P_0 = (0,0)}$

Step2: Sort other points from Q based on Polar angle in Counter-clockwise order around P_0.

→ For Nao Let's take the points in-order only.

$P_1 = (6,1)$
$P_2 = (3,1)$
$P_3 = (7,3)$
$P_4 = (4,2)$
$P_5 = (5,3)$
$P_6 = (5,5)$
$P_7 = (3,4)$

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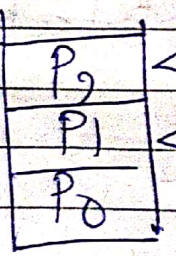
Step 3 Let S be empty stack
push (P_0, S)
push (P_1, S)
push (P_2, S)

Step 4 for $i = 3$ to m
{
while (the angle formed by Points Next-to-Top(S), Top(S), and P_i makes a nonleft turn)
{
Pop(S)
}
push (P_i, S)
}
return S

Initial Stack



$P_1 = (6, 1)$
$P_2 = (3, 1)$
$P_3 = (7, 3)$

☆ $i = 3$ $\quad P_1 - P_2 - P_3$

① direction $(P_1 - P_2 - P_3)$
$= (P_3 - P_1) \times (P_2 - P_1)$

$$= \begin{vmatrix} 7-6 & 3-6 \\ 3-1 & 1-1 \end{vmatrix} = \begin{vmatrix} 1 & -3 \\ 2 & 0 \end{vmatrix} = 6 > 0$$

$\Rightarrow$ $P_1 - P_2 - P_3$ is Right Turn

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$\Rightarrow$ Pop (S)

$\therefore$ Stack S =

P_1 ← Tos
P_0 ← Next_to_Top

(2) $P_0 - P_1 - P_3$

$(P_3 - P_0) \times (P_1 - P_0)$

$= \begin{vmatrix} 7-0 & 6-0 \\ 3-0 & 1-0 \end{vmatrix}$

$= 7 - 18$

$= -14 < 0$

$\Rightarrow$ Left Turn

$\Rightarrow$ Come out of while Loop.

and Push (P_3,S)

$\therefore S =$

P_3 ← Tos
P_1 ← Next to Top
P_0

$P_1 = (6,1)$
$P_3 = (7,3)$
$P_4 = (4,2)$

$\star$ $i = 4$ (Next to Top) (Top) (P_i)

$P_1 - P_3 - P_4$

Direction = $(P_4 - P_1) \times (P_3 - P_1)$

$= \begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} = -5 < 0$

$\Rightarrow$ Left Turn

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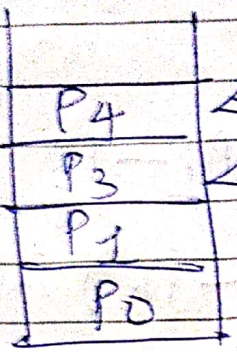
∴ Came out of While Loop

Push(P_4, S)

∴ Stack S



$P_3 = (7,3)$
$P_4 = (4,2)$
$P_5 = (5,3)$

⚠ i=5

$P_3 - P_4 - P_5$

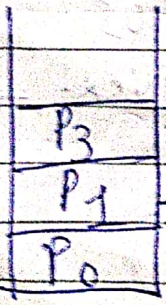
① Direction: $(P_5 - P_3) \times (P_4 - P_3)$

$$= \begin{vmatrix} -2 & -3 \\ 0 & -1 \end{vmatrix}$$

$= 2 > 0$

$\Rightarrow$ Right Turn

∴ Pop Stack S



② Direction: $P_1 - P_3 - P_5$

$(P_5 - P_1) \times (P_3 - P_1)$

$P_1 = (6,1)$
$P_3 = (7,3)$
$P_5 = (5,3)$

$$= \begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} = -4 < 0$$

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⇒ ~~Left Turn~~

⇒ Came out out of while loop

Push (P_5, S)

∴ S = | P_5 | ← Tos
| P_3 | ← Next_To_Tos
| P_1 |
| P_0 |

$P_3 = (2,3)$
$P_5 = (5,3)$
$P_6 = (5,5)$

☆ i = 6

$P_3 - P_5 - P_6$

(1) Direction: $(P_6 - P_3) \times (P_5 - P_3)$

$$= \begin{vmatrix} -2 & -2 \\ 2 & 0 \end{vmatrix}$$

$= 0 - (-4) = 4 > 0$

⇒ Right turn

∴ POP(S)

S = | P_3 | ← Tos
| P_1 | ← Next_To_Top
| P_0 |

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② Direction $P_1 - P_3 - P_6$

$P_1 = (6,1)$
$P_3 = (7,3)$
$P_6 = (5,5)$

$(P_6 - P_1) \times (P_3 - P_1)$

$= \begin{vmatrix} -1 & 1 \\ 4 & 2 \end{vmatrix}$

$= -2 - 4$

$= -6 < 0$

$\Rightarrow$ Left Turn

∴ Came out of while loop and push (P_6, S)

$S =$

P_6 ← TOS
P_3 ← Next_To_Top
P_1
P_0

$P_3 = (7,3)$
$P_6 = (5,5)$
$P_7 = (3,4)$

✡ $i = 7$

$P_3 - P_6 - P_7$

Direction: $(P_7 - P_3) \times (P_6 - P_3)$

$= \begin{vmatrix} -4 & -2 \\ 1 & 2 \end{vmatrix} = -8 + 2$

$= -6 < 0$

$\Rightarrow$ Left Turn $\Rightarrow$ (Came out of while

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Push (S_7, S)

$\therefore$ S =

P_7
P_6
P_3
P_1
P_0

Came out of "for" loop.

⟶ Return S

S is the Set of points in the Convex-Hull.

$\therefore$ Convex Hull of Q

$= \{P_0, P_1, P_3, P_6, P_7\}$

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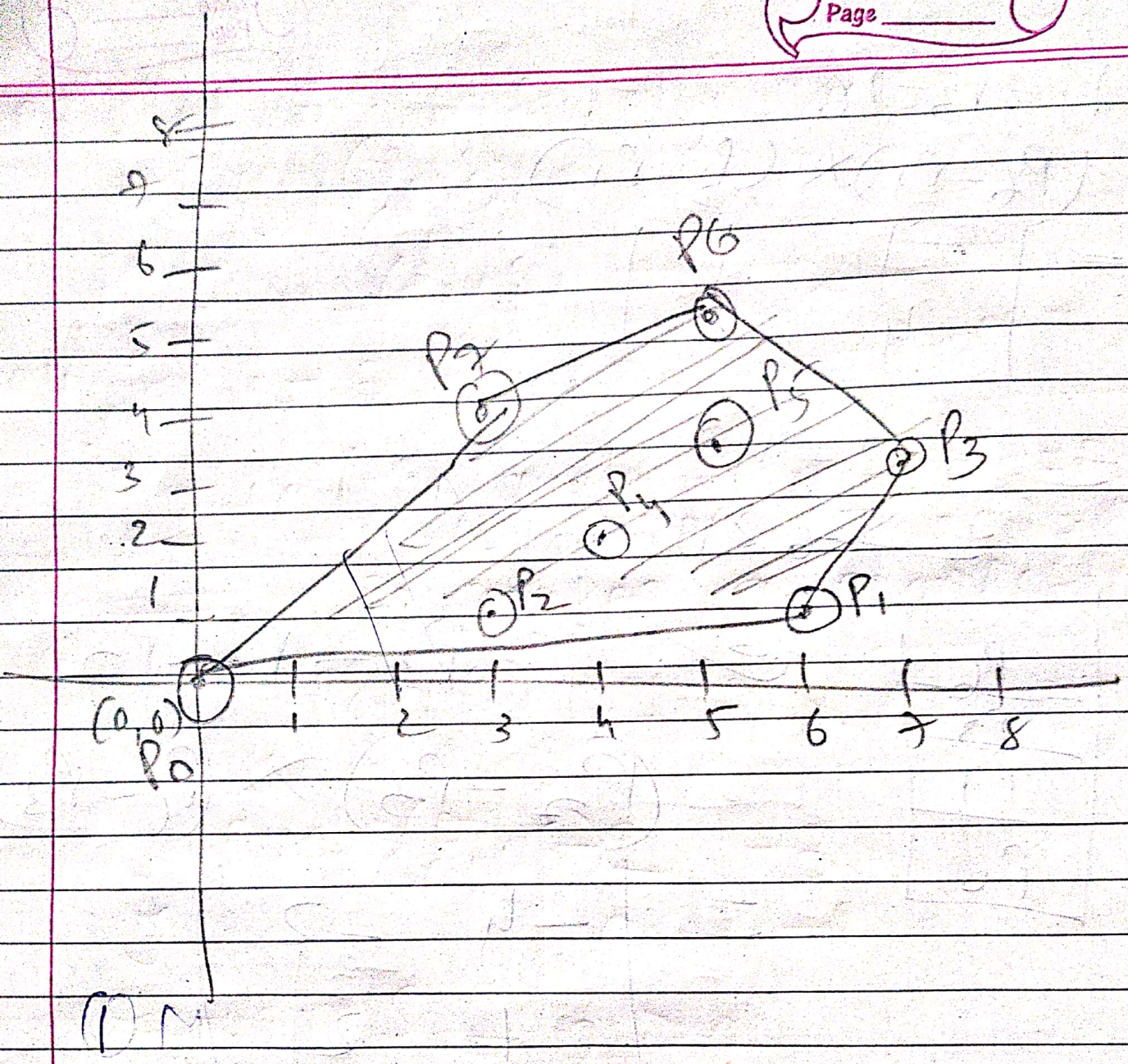

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(2) Sort other points based on polar angle in counter-clockwise order around Po.

(0,0), (3,1) $\tan^{-1}\left(\frac{1}{3}\right) = 18.43$

(6,1) $\tan^{-1}\left(\frac{1}{6}\right) = 9.46$

(7,3) $\tan^{-1}\left(\frac{3}{7}\right) = 23.19$

(5,5) $\tan^{-1}\left(\frac{5}{5}\right) = 45$

(5,13) $\tan^{-1}\left(\frac{3}{5}\right) = 30.96$

(4,2) $\tan^{-1}\left(\frac{1}{2}\right) = 26.56$

(3,4) $\tan^{-1}\left(\frac{4}{3}\right) = 53.13$

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