

## Lab 5

## Line Segment <sup>Date</sup> <sup>Page</sup> Intersection

Point  $P = (x_1, y_1)$   
 $Q = (x_2, y_2)$

How to check point P is at which side of Q with reference to origin  $O(0,0)$ ?

We need to find following

$$\begin{array}{cc} \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} & = P \times Q \text{ (Notation)} \\ \downarrow & \downarrow \\ \text{Point P} & \text{Point Q} \end{array}$$

$$\det = x_1 y_2 - x_2 y_1$$

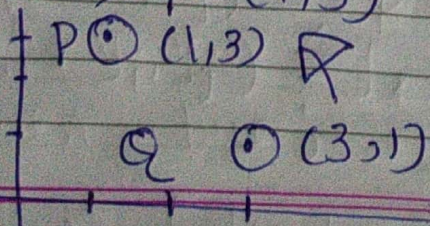
if  $\det > 0 \Rightarrow P$  is clockwise to  $Q$

if  $\det < 0 \Rightarrow P$  is Anticlockwise to  $Q$

E.g.  $P = (1, 3)$   
 $Q = (3, 1)$

$$\det = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 1 - 9 = -8$$

$\det < 0 \Rightarrow P = (1, 3)$  is Anticlockwise to  $Q$





★ Consider  $P(x_1, y_1)$   
 $Q(x_2, y_2)$   
 $R(x_3, y_3)$

Now we need to see that  $P \rightarrow Q \rightarrow R$  makes what type of turn at  $Q$   
 i.e.  $P \rightarrow Q \rightarrow R$  is Left turn or Right turn.

We need to translate  $P$  to Origin

$\therefore P(x_1, y_1)$  will be  $P(0, 0)$

$Q(x_2, y_2)$  will be  $Q(x_2 - x_1, y_2 - y_1)$

$R(x_3, y_3)$  will be  $R(x_3 - x_1, y_3 - y_1)$

Then we need to find

$$\det = \begin{vmatrix} x_3 - x_1 & x_2 - x_1 \\ y_3 - y_1 & y_2 - y_1 \end{vmatrix} \iff \frac{(P_k - P_i) \times (P_j - P_i)}{(P_3 - P_1) \times (P_2 - P_1)}$$

$$= (x_3 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_3 - y_1)$$

If  $\det < 0 \Rightarrow R$  is Anticlockwise to  $Q$

If  $\det > 0 \Rightarrow R$  is clockwise to  $Q$

Direction  $(P_i, P_j, P_k)$

{  
 return  $(P_k - P_i) \times (P_j - P_i)$   
 }



# Segment-Intersect ( $P_1, P_2, P_3, P_4$ )

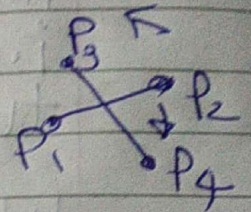
$d_1 = \text{Direction}(P_3, P_4, P_1)$

$d_2 = \text{Direction}(P_3, P_4, P_2)$

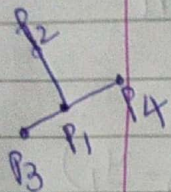
$d_3 = \text{Direction}(P_1, P_2, P_3)$

$d_4 = \text{Direction}(P_1, P_2, P_4)$

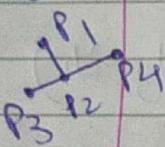
if ( $d_1 + d_2 < 0$  and  $d_3 + d_4 < 0$ )  
     return true;



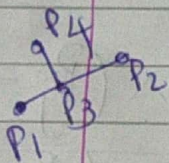
else if  $d_1 == 0$  and onsegment ( $P_3, P_4, P_1$ )  
     return true;



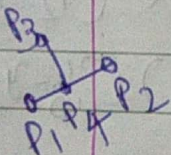
else if  $d_2 == 0$  and onsegment ( $P_3, P_4, P_2$ )  
     return true;



else if  $d_3 == 0$  and onsegment ( $P_1, P_2, P_3$ )  
     return true;



else if  $d_4 == 0$  and onsegment ( $P_1, P_2, P_4$ )  
     return true;



else  
     return false;

}

Onsegment ( $P_i, P_j, P_k$ )

if ( $\min(x_i, x_j) \leq x_k \leq \max(x_i, x_j)$  and  
      $\min(y_i, y_j) \leq y_k \leq \max(y_i, y_j)$ )  
     return true;

else

    return false;

}