Lab-9 Solve Sum of Subset using 0/1 Knapsack

0/1 Knapsack using D.P.

```
0/1 Knapsack(P, W, m)
{ /* Here let P[1...n], W[1..n] be
     arrays of Profits and
     weights respectively
     n is the number of items */
  /* Let T be 2-D Array of following
  Dimension T[0..n, 0...m] */

  for i = 0 to n
  {   T[i][0] <- 0;
  }
  for j = 1 to m
  {   T[0][j] <- 0;
  }
  for i = 1 to n
  {   for j = 1 to m
      { if j < W[i]
            T[i][j] <- T[i-1][j];
        else
            T[i][j] <- Max{T[i-1][j],
                       P[i] + T[i-1][j-W[i]]};
      }
  }
```

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return T[n, m];
}

→ Now Sum of subset problem has two inputs

{ Set S
Sum m } Instance of Sum of Subset

⇓

→ To solve sum of Subset using 0/1 knapsack

→ We need to create instance of 0/1 knapsack from instance of sum of subset.

{ Profit P = S
Weight W = S
Capacity m = Sum m } Instance of 0/1 knapsack equivalen to Sum of Subset

⇓

→ Now solve 0/1 Knapsack using Dynamic Programming

→ If profit given by 0/1 knapsack i.e. profit value at last row, last column is same as capacity then there exist atleast 1 subset which sums to m i.e. capacity.

→ If profit returned is less than m then there doesn't exist any subset which sums to m.

If $T[n,m] == m$

Yes → There exist at least 1 subset which sums to m → **Find the subset using Table T.**

No → There is no subset which sums to m but there is subset such that its sum is less than m.

We assume that we give proper input for sum of subset i.e.

$$\text{Sum } m \leq \text{Sum of all the elements of set } S$$

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★ Sum of Subset Instance

Set $S = \{10, 20, 30, 40\}$

$m = 50$

⇓

Is there any subset of S which sums to 50?

⇓

We want to Solve it Using 0/1 Knapsack.

$P = \{10, 20, 30, 40\} = S$

$W = \{10, 20, 30, 40\} = S$

Capacity of Knapsack $= 50 = m$

⇓

Now solve it using 0/1 knapsack Dynamic Programming algorithm.

⇓

Create a Table. Solve 0/1 knapsack. Then using that try to get the solution of Sum Of Subset problem.

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Profit $P = \{1,2,3,4\}$
$W = \{1,2,3,4\}$
Capacity $m=5$

→ Capacity

↓ Item

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	1	1	1	1
2	0	1	2	3	3	3
3	0	1	2	3	4	5
4	0	1	2	3	4	5

$5 == 5(m)$
⇒ There exist at least 1 subset

$$T[i,j] = Max\{T[i-1,j], P[i] + T[i-1, j-W[i]]\}$$

Initialize $T[0,:] = 0$ i.e. 0^{th} row to 0.
$T[:,0] = 0$ i.e. 0^{th} column to 0.

If $j < W[i]$
$T[i,j] = T[i-1,j]$
Else
$T[i,j] = Max\{T[i-1,j], P[i] + T[i-1, j-W[i]]\}$

∴ Here $T[4,5] = 5 =$ Desired sum
∴ It has exact sol^n.
5 $\{1,0,0,1\}$ and $\{0,1,1,0\}$ are two $sol^n s$

Ex 2

$S = \{1, 4, 5\}$

$m = 8$

$\Downarrow$

$P = \{1, 4, 5\}$

$W = \{1, 4, 5\}$

$m = 8$

Table	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	1	1
2	0	1	1	1	4	5	5	5	5
3	0	1	1	1	4	5	6	6	6

$T[1,1] = Max\{T[0,1], T[0,0]+1\}$

$= Max\{0, 0+1\} = 1$ No Subs exists

$T[1,2] = Max\{T[0,2], T[0,2-1]+$

$= 1$

$T[2,2] = Max\{T[1,2], \quad + T[1, -ve$

$= 1$

$T[3,8] = Max\{T[2,8], 5+T[2,3]\}$

$= 6$

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