Assignment 03 | Advance Algorithms CE-092

Assignment submission for Advance Algorithms subject week 3.

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Task 1:

To Implement fermat's primality testing algorithm.

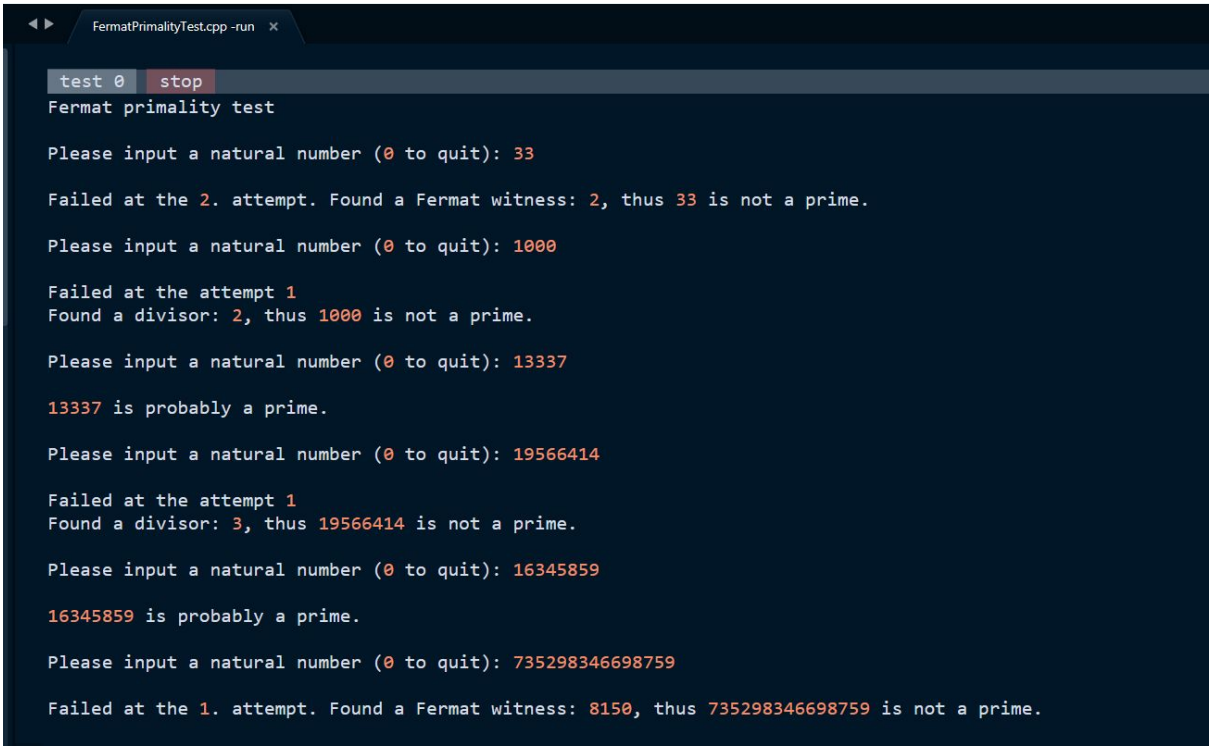
Code:

```
/*
* @Author: nevil
* @Date:   2020-07-24 16:03:04
* @Last Modified by:   nevil
* @Last Modified time: 2020-07-24 16:08:26
*/
#include<bits/stdc++.h>
using namespace std;

void swap(long long &a, long long &b) {
    long long temp = a;
    a = b;
    b = temp;
}

long long gcd(long long a, long long m) {
    if (a > m) swap(a, m);
    long long r;
    for (;;) {
```

```
        r = m % a;
        if (r == 0) return a;
        m = a;
        a = r;
    };
}

long long power(long long a, long long b, long long m)
{
    if (a > m) swap(a, m);
    long long c = 1;
    for (;;) {
        if (b % 2 == 1) c = (c * a) % m;
        b = b / 2;
        if (b == 0) return c;
        a = (a * a) % m;
    };
}

bool fermat(long long m) {
    srand(time(NULL));
    if (m == 1) {
        cout << endl;
        return false;
    };
    for (int i = 0; i < 1000; i++) {
        int cnt = i + 1;
        long long a = rand() % m;
        if (a == 0) a = 1;
        if (gcd(a, m) != 1) {
            cout << "\nFailed at the " << "attempt " <<
cnt << endl;
```

```
            cout << "Found a divisor: " << gcd(a, m) <<
", thus ";
            return false;
        };
        if (power(a, m - 1, m) != 1) {
            cout << "\nFailed at the " << cnt << ".
attempt. ";
            cout << "Found a Fermat witness: " << a <<
", thus ";
            return false;
        };
    };
    cout << endl;
    return true;
}

int main() {

    cout << "Fermat primality test" << endl << endl;
    for (;;) {
        long long n;
        cout << "Please input a natural number (0 to
quit): ";
        cin >> n;
        if (n == 0) return 0;
        bool b = fermat(n);
        if (b) cout << n << " is probably a prime." <<
endl << endl;
        if (!b) cout << n << " is not a prime." << endl
<< endl;
    };
}
```

Output:



Conclusion :

If a given number is prime, then this method always returns true. If a given number is composite (or non-prime), then it may return true or false, but the probability of producing incorrect results for composite is low and can be reduced by doing more iterations.

Complexity :

The fermat's theorem may fail even if we increase the number of iterations (higher k). There exist some composite numbers with the property that for every a < n, gcd(a, n) = 1 and a^n-1 ≡ 1 (mod n). Such numbers are called Carmichael numbers.

Considering the power method takes O(Logn) time. We can clearly say that the complexity of fermat's primality testing algorithm is O(k * Logn).

Application of Fermat's Test :

Fermat's primality test is often used if a rapid method is needed for filtering, for example in the key generation phase of the RSA public key cryptographic algorithm.

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