Q4 Exponential distribution:

$$\underbrace{f(x;\lambda)}_{\text{pdf}} = \begin{cases} \lambda e^{-\lambda x} & ; \quad x \geq 0 \\ 0 & ; \quad x < 0 \end{cases}$$

λ = rate parameter
x = random variable
let, $x_1, x_2, \dots, x_n$ be the samples.

Now finding MLE for $x = x_1, x_2, \dots, x_n$
Assuming IID condition:

$$\text{likelihood}, L(\lambda; x_1, \dots, x_n) = P(x_1, x_2, \dots, x_n \mid \lambda)$$

$$= P(x_1 \mid \lambda) \cdot P(x_2 \mid \lambda) \cdots P(x_n \mid \lambda)$$

$$= \lambda e^{-\lambda x_1} \cdot \lambda e^{-\lambda x_2} \cdots \lambda e^{-\lambda x_n} \quad [\text{for } x_1; x_2, \dots, x_n \geq 0]$$

$$= \lambda^n e^{-\lambda(x_1 + x_2 + \dots + x_n)}$$

$$= \lambda^n e^{-\lambda \sum_{j=0}^{n} x_j}$$

$$LL(\lambda; x_1, \dots, x_n) = n \log \lambda - \lambda \sum_{j=0}^{n} x_j$$

$$\underbrace{NLL(\lambda; x_1, \dots, x_n)}_{\text{negative Log likelihood}} = -n\log\lambda + \lambda \sum_{j=0}^{n} x_j$$

$\underline{\lambda > 0}$

Now, $\lambda_{MLE} = \underset{\lambda}{\operatorname{argmin}} \left(NLL(\lambda; x_1, \dots, x_n) \right)$

$\therefore \frac{\partial}{\partial \lambda} NLL(\lambda) = \frac{-n}{\lambda} + \sum_{j=0}^{n} x_j$

for minima, $\frac{\partial}{\partial \lambda} NLL(\lambda) = 0$

$\Rightarrow \frac{-n}{\lambda_{MLE}} + \sum_{j=0}^{n} x_j = 0$

$$\Rightarrow \boxed{\lambda_{MLE} = \frac{n}{\sum_{j=0}^{n} x_j}}$$