Q5. $f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma^2}\right)}$

Let, $x_1, x_2, \ldots, x_n$ be the samples.

Assuming IID condition:

likelihood, $L(\mu,\sigma; x_1, \ldots, x_n) = P(x_1, x_2, \ldots, x_n \mid \mu, \sigma)$

$= P(x_1|\mu,\sigma) \cdot P(x_2|\mu,\sigma) \cdot \ldots \cdot P(x_n|\mu,\sigma)$

$= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2}\left(\frac{(x_1-\mu)^2+(x_2-\mu)^2+\ldots+(x_n-\mu)^2}{\sigma^2}\right)}$

$LL(\mu,\sigma; x_1,\ldots,x_n) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}\sum_{j=1}^{n}(x_j-\mu)^2$

$NLL(\mu,\sigma; x_1,\ldots,x_n) = \frac{n}{2}\log(2\pi) + \frac{n}{2}\log(\sigma^2) + \frac{1}{2\sigma^2}\sum_{j=1}^{n}(x_j-\mu)^2$

$\hookrightarrow \sigma \neq 0$