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ISE 510 – Introduction to Operations Research

Project Report

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Abstract

A study explaining the problems faced in areas of aggregate planning and demand analysis in an installation elevator company which results in huge delivery delays and so having a high impact in customer satisfaction has been identified and solved for maximizing the profit using linear programming. The major concern was that the company was unable to deliver products on due dates which in-turn had a significant effect on customer satisfaction. So, an in-depth analysis has been performed to find the lowest aggregate production cost that in turn gave higher profits. This study includes prediction of customer demand for the next 12-month period. Accuracy of demand predictions is calculated using the mean absolute percent error (MAPE). The predicted demand is the primary input for devising the optimal production plan – a formulated mathematical model solved using Excel solver. A numerical analysis has been performed on changing circumstances in profit earned where different production strategies are introduced in the plan. The results revealed that the maximum profit for the company was **6498611.11 USD** when the reduction in overtime limit for the workforce coupled up with 5% discounting in product price (which led to 10% increase in product demand) is introduced.

Introduction

The primary objective of manufacturing and service industries is to reduce the operational cost to increase their profit margins. Though one might argue that the objectives could include better quality, improved customer service, near to zero wastage etc., our defense is to stick to the main objective of maximizing the profit, which automatically means that the company is having better customer satisfaction, better quality and is staying relevant in the market.

A research paper that discusses about one such instance in an elevator installation company where they find it hard to cater to demand with various challenges involving operational costs such as inventory carrying, stocking out, workforce balancing, material costs has been identified. Owing to the improper planning, the company was spending a lot on production cost and thereby not having a high profit margin. To make use of the resources efficiently with the maximum possible profit margin, this numerical study makes an attempt to propose an optimal aggregate production plan by mathematically formulating and solving a linear programming model.

Right before formulating an efficient model, the demand forecast for the next 12-month period (which acts as the primary component / input for the aggregate production model) is required which is predicted using double exponential smoothing with the help of previous year's demand data.

Secondly, using the demand predicted and the operational costs involved, an efficient mathematical model with the primary objective of maximizing the total profit subject to constraints such as workforce balancing, overtime balancing, inventory balancing and production capacity shall be formulated.

Lastly, the formulated model is solved using Excel Solver and the results are obtained. A numerical analysis has been performed on how the profit margin changes when different strategies such as chase, level, subcontracting, discounting & overtime reduction is introduced in the plan. The results are summarized in a way which makes it convenient for the company to change their plan as per the market requirements.

Problem Description

The collected demand data (previous year) and the predicted forecast (calculation sheet included in appendices) for the next 12-month period is as shown below in Table 1 & Figure 1.

Table 1. Month t vs Predicted Demand

Month	Demand	Forecast
1	13	10
2	8	11
3	9	11
4	12	12
5	10	13
6	17	14
7	11	14
8	19	15
9	12	16
10	13	16
11	17	17
12	20	18

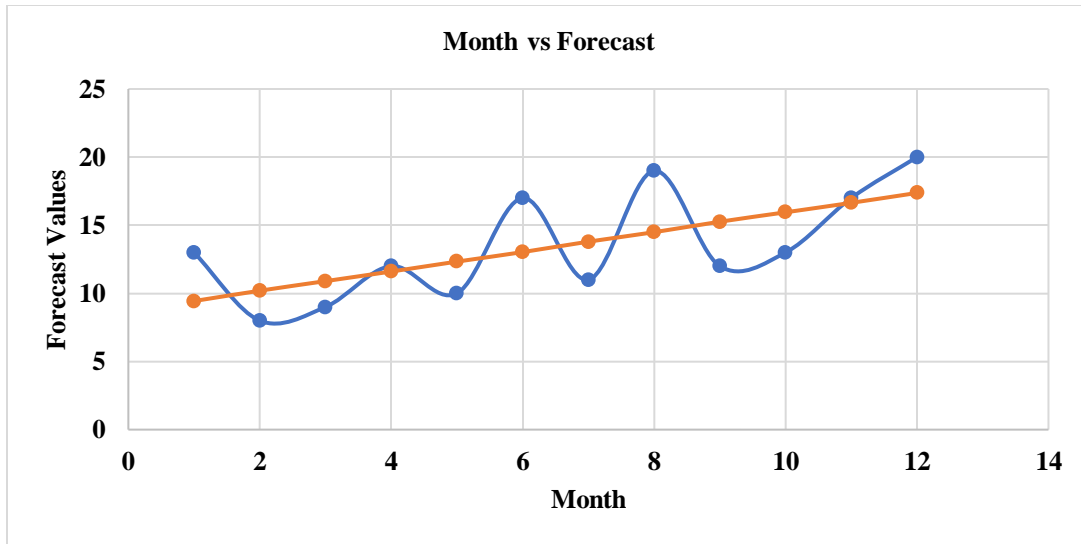


Figure 1. Month t vs Predicted Demand Plot

Necessary data for aggregate production planning consisted of data on production capacity, aggregate units, and expenses such as raw material costs, labor costs, overtime wages, contract hiring, costs from hiring more employees and costs from dismissing employees, etc. is given in the table below.

Table 2. Production Planning Data

Material Cost (in USD)	15000
Production Volume / Worker / Month (Regular time)	5% of overall work / unit
Production Volume / Hour (Overtime)	3% of overall work / unit
Maximum Overtime Working Hours / Worker / Month	30
Inventory Cost (in USD / unit)	1000
Stock-out Cost (in USD / unit)	500
Hiring Cost (in USD / worker)	500
Laying-off Cost (in USD / worker)	1500
Labor Cost (in USD / worker / month)	12000
Overtime Cost (in USD / hour)	100
Subcontracting Cost (in USD / unit)	50000
Initial Inventory Quantity	0
Final Inventory Quantity	0
Stock Out Last Month	0
Starting Workforce	9
Number of Working Days / Month	24
Market Price (Assumption) (in USD)	60000

Linear Programming Model

The aggregate production planning model maximizes the profit margin with the following objective function,

$$\text{Max } Z = \sum_{t=1}^{12} P_c D_t - \sum_{t=1}^{12} R_c W_t - \sum_{t=1}^{12} O_c O_t - \sum_{t=1}^{12} H_c H_t - \sum_{t=1}^{12} L_c L_t - \sum_{t=1}^{12} S_c S_t - \sum_{t=1}^{12} M_c P_t - \sum_{t=1}^{12} C_c C_t - \sum_{t=1}^{12} I_c I_t$$

Subject to constraints,

$$O_t \leq O_{t-\max} * W_t \rightarrow \text{Overtime Balance Constraint}$$

$$P_t = (P_{r-RT} * W_t) + (P_{r-OT} * O_t) \rightarrow \text{Production Capacity Constraint}$$

$$W_t = W_{t-1} + H_t - L_t \rightarrow \text{Workforce Balance Constraint}$$

$$I_{t-1} + P_t + C_t = D_t + S_{t-1} + I_t - S_t \rightarrow \text{Inventory Balance Constraint}$$

$$W_t, O_t, H_t, L_t, I_t, S_t, P_t, C_t \geq 0 \rightarrow \text{Non-negativity Constraint}$$

where,

t – Operational time period

P_c – Market price of the elevator

R_c – Regular time labor cost / month

H_c – Hiring cost

L_c – Layoff cost

S_c – Stockout cost

C_c – Subcontracting cost

I_c – Inventory carrying cost

W – Workforce limit

W_t – Labors in month t

H_t – Hiring in month t

L_t – Layoff in month t

S_t – Stockout in month t

I_t – Inventory in month t

C_t – Subcontracting in month t

P_{r-RT} – Production rate (Regular time)

P_{r-OT} – Production rate (Over time)

P_t – In-house production in month t

D_t – Forecasted Demand in month t

Numerical Study

The following cases has been analyzed in the numerical experimentation (calculation sheet included in appendices) for churning out the recommendation for maximum profit margin possible,

Case 1 – Chase Strategy

Condition – Zero Inventory, Stockout & Sub contraction

Additional Constraints - $I_t = 0, S_t = 0, C_t = 0$

Table 3. Decision Table - Chase Strategy

Aggregate Planning Decision Variables									
Month	No. of Workers (W)	Total Decision Making Hours (D)	Hired Workers (H)	Laid off Workers (L)	Inventory (I)	Stock-out Units (S)	Sub-contracted Production (C)	Production (P)	Demand (D)
0	8	-	-	-	8	0	-	-	-
1	8	116.67	0	3	0	0	0	10	10
2	8	108.33	0	0	0	0	0	11	11
3	8	108.33	0	0	0	0	0	11	11
4	7	100.00	1	0	0	0	0	12	12
5	7	101.67	0	0	0	0	0	13	13
6	8	103.33	1	0	0	0	0	14	14
7	8	103.33	0	0	0	0	0	14	14
8	8	220.00	0	0	0	0	0	15	15
9	8	216.67	1	0	0	0	0	16	16
10	8	216.67	0	0	0	0	0	16	16
11	8	238.33	0	0	0	0	0	17	17
12	10	200.00	1	0	0	0	0	18	18
Cost	Labour Cost (Regular)	Labour Cost (Overtime)	Hiring Cost	Layoff Cost	Inventory Cost	Stock-out Cost	Sub-contracting Cost	Production Cost	Total Profit
	1116000	250033.33	2000	4500	0	0	0	2500000	6161066.67

Case 2 – Level Strategy

Assumption – Constant In-house Production (Average Demand), Balanced with Inventory

Additional Constraints - $P_t = 14$ (Average Demand)

Table 4. Decision Table - Level Strategy

Aggregate Planning Decision Variables									
Month	No. of Workers (W)	Total Decision Making Hours (D)	Hired Workers (H)	Laid off Workers (L)	Inventory (I)	Stock-out Units (S)	Sub-contracted Production (C)	Production (P)	Demand (D)
0	8	-	-	-	8	0	-	-	-
1	8	103.33	0	1	4	0	0	14	10
2	8	103.33	0	0	7	0	0	14	11
3	8	103.33	0	0	10	0	0	14	11
4	8	103.33	0	0	12	0	0	14	12
5	8	103.33	0	0	13	0	0	14	13
6	8	103.33	0	0	13	0	0	14	14
7	8	103.33	0	0	13	0	0	14	14
8	8	103.33	0	0	12	0	0	14	15
9	8	103.33	0	0	10	0	0	14	16
10	8	103.33	0	0	6	0	0	14	16
11	8	103.33	0	0	5	0	0	14	17
12	8	103.33	0	0	1	0	0	14	18
Cost	Labour Cost (Regular)	Labour Cost (Overtime)	Hiring Cost	Layoff Cost	Inventory Cost	Stock-out Cost	Sub-contracting Cost	Production Cost	Total Profit
	1152000	220000.00	0	1500	108000	0	0	2520000	6011000.00

Case 3 – Subcontracting Strategy

Assumption – Constant In-house Production (Lowest Demand), Balanced with Sub contraction

Additional Constraints - $P_t = 10$ (Lowest Demand)

Table 5. Decision Table - Subcontracting Strategy

Aggregate Planning Decision Variables									
Month	No. of Workers (W)	Total Decision Making Hours (D)	Hired Workers (H)	Laid off Workers (L)	Inventory (I)	Stock-out Units (S)	Sub-contracted Production (C)	Production (P)	Demand (D)
0	8	-	-	-	8	0	-	-	-
1	8	116.67	0	3	0	0	0	10	10
2	8	116.67	0	0	0	0	1	10	11
3	8	116.67	0	0	0	0	1	10	11
4	8	116.67	0	0	0	0	2	10	12
5	8	116.67	0	0	0	0	3	10	13
6	8	116.67	0	0	0	0	4	10	14
7	8	116.67	0	0	0	0	4	10	14
8	8	116.67	0	0	0	0	5	10	15
9	8	116.67	0	0	0	0	6	10	16
10	8	116.67	0	0	0	0	6	10	16
11	8	116.67	0	0	0	0	7	10	17
12	8	116.67	0	0	0	0	8	10	18
Cost	Labour Cost (Regular)	Labour Cost (Overtime)	Hiring Cost	Layoff Cost	Inventory Cost	Stock-out Cost	Sub-contracting Cost	Production Cost	Total Profit
	864000	140000.00	0	4500	0	0	2300000	1800000	4861000.00

From cases 1, 2 & 3, it is evident that the Chase strategy is the best option possible. Using the same strategy, few other numerical experimentations (with respect to company policies) has been performed to maximize the profit furthermore.

Case 4 – Reduced Overtime with Chase Strategy

Assumption – Maximum Overtime Limit is 25 hours / worker / month

Changes in constraints - $O_t \leq 25W_t$

Table 6. Decision Table - Reduced Overtime

Aggregate Planning Decision Variables									
Month	No. of Workers (W)	Total Overtime Working Hours (O)	Hired Workers (H)	Laid off Workers (L)	Inventory (I)	Stock-out Units (S)	Subcontracted Production (C)	Production (P)	Demand (D)
0	8	-	-	-	0	0	-	-	-
1	8	97.02	0	3	0	0	0	10	10
2	8	131.04	0	0	0	0	0	11	11
3	8	131.04	0	0	0	0	0	11	11
4	7	125.00	1	0	0	0	0	12	12
5	7	109.72	0	0	0	0	0	12	12
6	8	102.70	1	0	0	0	0	14	14
7	8	102.70	0	0	0	0	0	14	14
8	8	187.50	0	0	0	0	0	15	15
9	8	180.56	1	0	0	0	0	16	16
10	8	180.56	0	0	0	0	0	16	16
11	8	215.28	0	0	0	0	0	17	17
12	10	208.33	1	0	0	0	0	18	18
Cost	Labour Cost (Regular)	Labour Cost (Overtime)	Hiring Cost	Layoff Cost	Inventory Cost	Stock-out Cost	Subcontracting Cost	Production Cost	Total Profit
	1110000	192361.11	2000	4500	0	0	0	2500000	6080138.89

Case 5 – 5% Discounting with Chase Strategy

Assumption – 5% Discount in unit price of elevator which in turn increases the demand by 10%

Changes in data – Unit price – 57000 USD, Demand = $1.1D_t$

Table 7. Decision Table - 5% Discounting

Aggregate Planning Decision Variables									
Month	No. of Workers (W)	Total Overtime Working Hours (O)	Hired Workers (H)	Laid off Workers (L)	Inventory (I)	Stock-out Units (S)	Subcontracted Production (C)	Production (P)	Increased Demand (D)
0	8	-	-	-	0	0	-	-	-
1	8	108.33	0	3	0	0	0	11	11
2	7	191.67	1	0	0	0	0	13	13
3	7	191.67	0	0	0	0	0	13	13
4	8	163.33	1	0	0	0	0	14	14
5	8	220.00	0	0	0	0	0	15	15
6	8	216.67	1	0	0	0	0	16	16
7	8	216.67	0	0	0	0	0	16	16
8	9	208.33	0	0	0	0	0	17	17
9	10	200.00	1	0	0	0	0	18	18
10	10	200.00	0	0	0	0	0	18	18
11	10	291.67	0	0	0	0	0	19	19
12	11	283.33	1	0	0	0	0	20	20
Cost	Labour Cost (Regular)	Labour Cost (Overtime)	Hiring Cost	Layoff Cost	Inventory Cost	Stock-out Cost	Subcontracting Cost	Production Cost	Total Profit
	1240000	271886.67	2000	4500	0	0	0	2800000	6483153.33

Case 6 – 10% Discounting with Chase Strategy

Assumption – 10% Discount in unit price of elevator which in turn increases the demand by 20%

Changes in data – Unit price – 54000 USD, Demand = $1.2D_t$

Table 8. Decision Table - 10% Discounting

Aggregate Planning Decision Variables									
Month	No. of Workers (W)	Total Overtime Working Hours (O)	Hired Workers (H)	Laid off Workers (L)	Inventory (I)	Stock-out Units (S)	Subcontracted Production (C)	Production (P)	Increased Demand (D)
0	8	-	-	-	0	0	-	-	-
1	7	160.00	0	2	0	0	0	12	12
2	8	183.33	1	0	0	0	0	14	14
3	8	183.33	0	0	0	0	0	14	14
4	8	225.00	0	0	0	0	0	15	15
5	8	216.67	1	0	0	0	0	16	16
6	8	208.33	0	0	0	0	0	17	17
7	9	208.33	0	0	0	0	0	17	17
8	10	200.00	1	0	0	0	0	18	18
9	11	283.33	1	0	0	0	0	20	20
10	11	283.33	0	0	0	0	0	20	20
11	11	325.00	0	0	0	0	0	21	21
12	12	316.67	1	0	0	0	0	22	22
Cost	Labour Cost (Regular)	Labour Cost (Overtime)	Hiring Cost	Layoff Cost	Inventory Cost	Stock-out Cost	Subcontracting Cost	Production Cost	Total Profit
	1306000	293333.33	2000	3000	0	0	0	3000000	6379166.67

From cases 4, 5 & 6, it is evident that reduced overtime limit & discounting (5% is preferable) has considerable impact in maximizing the profit margin. Lastly, one more numerical study on coupling case 4 & 5 has been performed to check if the profit can be maximized even more.

Case 7 – Reduced Over time + 5% Discounting with Chase Strategy

Assumption – 5% Discount in unit price of elevator which in turn increases the demand by 10%

Changes in data – Unit price – 57000 USD, Demand = $1.1D_t$,

Changes in constraints - $O_t \leq 25W_t$

Table 9. Decision Table - Reduced OT + 5% Discounting

Aggregate Planning Decision Variables										
Month	No. of Workers (W)	Total Overtime Working Hours (O)	Hired Workers (H)	Laid off Workers (L)	Inventory (I)	Stock-out Units (S)	Subcontracted Production (C)	Production (P)	Increased Demand (D)	
0	9	-	-	-	0	0	-	-	-	-
1	6	131.04	0	3	0	0	0	11	11	
2	7	159.72	1	0	0	0	0	13	13	
3	7	159.72	0	0	0	0	0	13	13	
4	8	152.76	1	0	0	0	0	14	14	
5	8	187.50	0	0	0	0	0	15	15	
6	9	180.56	1	0	0	0	0	16	16	
7	9	180.56	0	0	0	0	0	16	16	
8	9	215.28	0	0	0	0	0	17	17	
9	10	208.33	1	0	0	0	0	18	18	
10	10	208.33	0	0	0	0	0	18	18	
11	10	243.06	0	0	0	0	0	19	19	
12	11	236.11	1	0	0	0	0	20	20	
Cost	Labour Cost (Regular)	Labour Cost (Overtime)	Hiring Cost	Layoff Cost	Inventory Cost	Stock-out Cost	Subcontracting Cost	Production Cost	Total Profit	
	1294000	226199.89	2500	4000	0	0	0	2850000	6498611.11	

From case 7, it is evident that the **reduced overtime limit (25 hours / worker / month) coupled up with 5% discounting yields the maximum profit** for the company's model.

Conclusion

With the results obtained, the study proposes an optimal aggregate production plan after examining various operational difficulties using the model linear programming model built with the goal of calculating the maximum possible profit margin in a variety of conditions. It is clear that the organization is capable of increasing productivity and customer satisfaction. The results are summarized in the table below.

Table 10. Results Summary

Case	Strategy	Profit earned	Best option (Maximum Profit)
1	Chase	6161666.67	Reduced Overtime + 5% Discounting
2	Level	6018500.00	
3	Subcontracting	4861500.00	
4	Reduced Overtime	6200138.89	
5	5% Discounting	6453333.33	
6	10% Discounting	6379166.67	
7	Reduced OT + 5% Discounting	6498611.11	

Calculated costs in each case study revealed the state of higher volume of overtime cost spent on workers. As a result, workforce allocation played a vital role in lowering operational costs. With

respect to the analysis performed, chase strategy with overtime limit getting reduced to 25 hours and 5% discounting in unit price paves way for the company to yield maximum possible profit of **6498611.11 USD**. Only business implication in this case can be the fact that the labor market being uncertain for firing and hiring every month (in chase strategy). Otherwise, this strategy could work very well and good for the installation elevator company in this study.

References

[Chanipa Nivasanon., Isaree Srikun., and Pasura Aungkulanon., Aggregate Production Planning: A Case Study of Installation Elevator Company, Proceedings of the 11th Annual International Conference on Industrial Engineering and Operations Management Singapore](#)

Appendices

ISE 501 Project Numericals (Calculation sheet included to this submission)