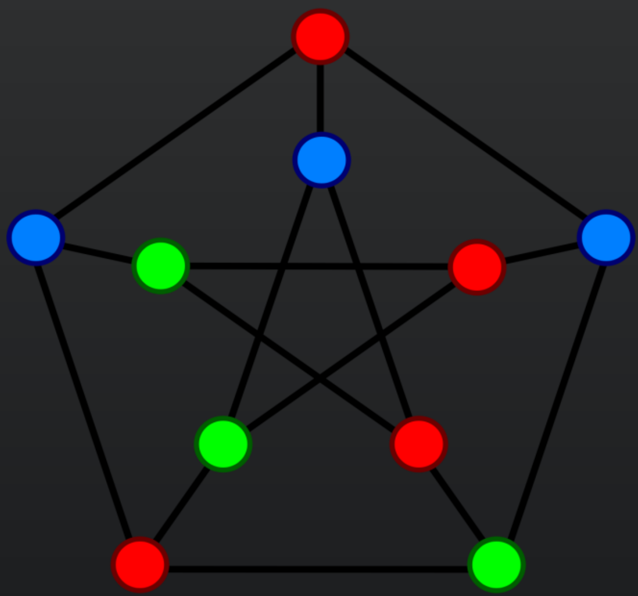
Graph Theory For Computing CS2150

RESEARCH CYCLE I



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RESEARCH CYCLE I

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DEFINITIONS

GRAPH

A graph is a collection of points and lines connecting some (possibly empty) subset of them. [1]

VERTICES

The points of a graph are most commonly known as graph vertices, but may also be called "nodes" or simply "points."[1]

EDGES

The lines connecting the vertices of a graph are most commonly known as graph edges, but may also be called "arcs" or "lines."[1]

DEGREE

The degree (or the local degree) of a vertex of a graph is the number of graph edges which touch the graph vertex. [2]

ORDER OF A GRAPH

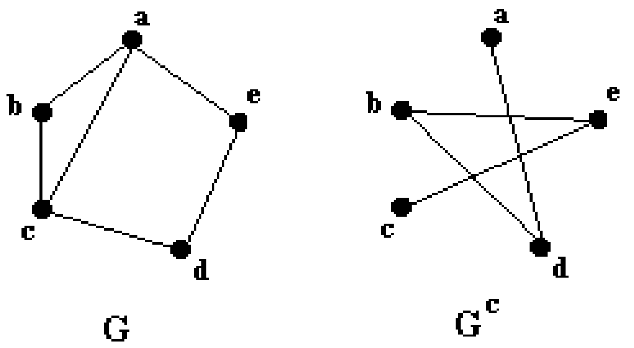
The number of nodes in a graph is called its order.

REGULAR GRAPH

A graph is said to be regular, if all local degrees are the same. [3]

GRAPH COMPLEMENT

The complement of a graph G (sometimes called the edge-complement), is the graph G', with the same vertex set but whose edge set consists of the edges not present in G. [4]

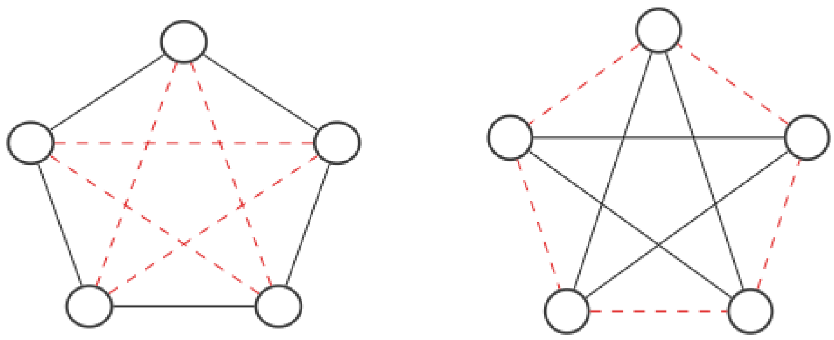


ISOMORPHIC

Two graphs which contain the same number of graph vertices connected in the same way are said to be isomorphic.

SELF-COMPLEMENTARY GRAPH

A self-complementary graph is a graph which is isomorphic to its graph complement. [5]



PROBLEM 2 – PART I

Is the complement of a regular graph, regular? If yes, prove it! If no, find a counterexample.

SOLUTION

YES

Let G be a regular graph with n vertices.

Assume an arbitrary vertex v with degree d.

Since G is regular d is a constant.

In the complement of G (G’), out of the n-1 other vertices v is adjacent only to the (n-1-d) to which v is not adjacent in G.

So the degree of v in G’ is (n-1-d).

(n-1-d) is a constant as n and d, are constant.

So all vertices have degree (n-1-d) in complement of G.

So complement of G is complement.

PROBLEM 2 – PART II

Are there self-complementary graphs of order 3? Of order 4? Of order 5? In each case, if yes, give an example; if no, prove why it is not possible.

SOLUTION

Let's consider a self-complementary graph with n vertices and m edges.

So the total number of edges that can be in G, is (n*(n-1)) / 2.

So the complement of G will have (n*(n-1))/2 – m edges.

Since G is self-complementary,

Number of edges in G = Number of edges in G'

$$m = n*(n-1)/2 - m$$

$$m = (n*(n-1)) / 4$$

m should be a non-negative integer.

n or n-1 is odd.

So n or n-1 should be a multiple of 4.

<u>CASE I – ORDER 2</u>

NO

n = 2 n-1 = 1

neither n or n-1 is a multiple of 4.

so there cannot be self-complementary graphs of order 2.

CASE II – ORDER 3

NO

n = 3 n-1 = 2

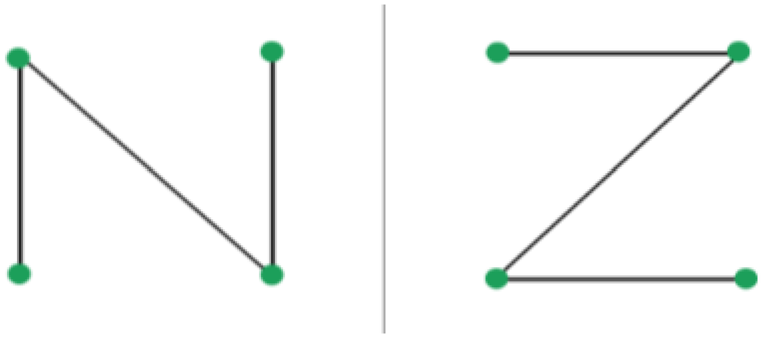
neither n or n-1 is a multiple of 4.

so there cannot be self-complementary graphs of order 2.

CASE III – ORDER 4

YES

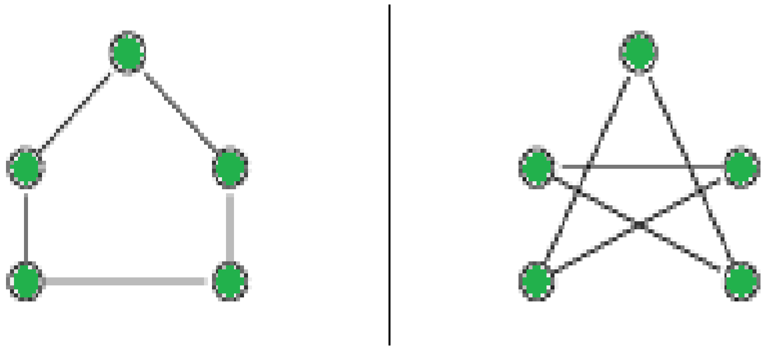
Example: -



CASE IV – ORDER 5

YES

Example: -



ADDITIONAL QUESTIONS & ANSWERS

QUESTIONS

1. Is there a complement for a complete graph?

2. For a graph to be self-complementary, should it be regular?

3. Give an example for graph with 4 edges which is not self- complementary. And explain why?

ANSWERS

1. Yes.

A **complete graph** is a graph in which each pair of graph vertices is connected by an edge.

Let's consider a complete graph, G of order n.

Assume an arbitrary vertex v

So its degree is (n-1).

So degree of v in G' is (n-1) -(n-1) =0

So in G', degree of every vertex is 0. That means G' is edgeless. E= Ø

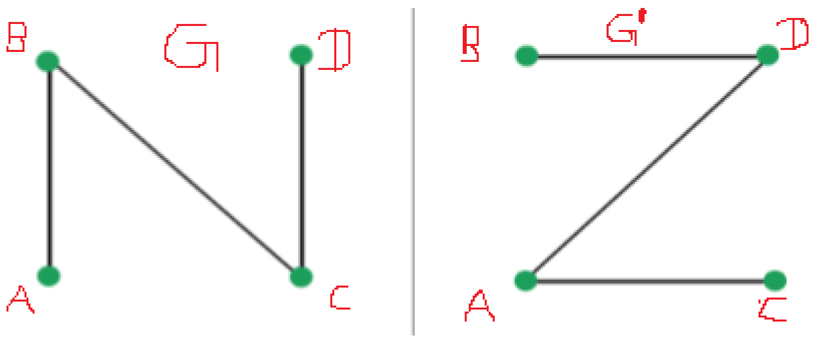
In a graph, set E should be subset of set V*V. Ø is a subset of V*V.

Hence, it is possible to a graph to be edgeless (null graph).

So complement of G exists.

2. No

Let's assume that, it should be regular for a graph to be self-complementary.



This graph G is self-complementary.

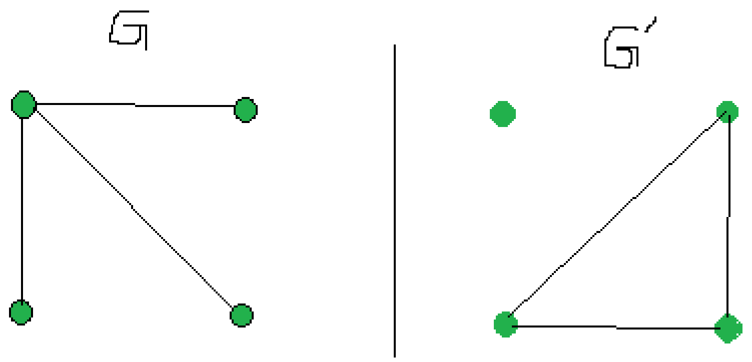
But in G, degree of A is 1 and degree of B is 2.

So G is not regular.

Hence our assumption is wrong.

There can be self-complementary graphs that are not regular.

3.



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If a graph is self-complementary, then it and its complement should have same degree set. But in this scenario, degree set of G is {1,1,1,3} and the degree set of G' is {0,2,2,2}. Those sets are different. So, G is not self-complementary.

REFERENCES

[1] Weisstein, Eric W. (2017-02-14) "Graph." [online] From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/Graph.html

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