Graph Theory for Computing
Research Cycle 1
Research Report

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Introduction

The purpose of this report is to elaborate the proof of Konigsberg seven bridge problem.This problem is to cross seven bridges which are situated in a specific manner such that each bridge is crossed only once in a single walk.We have used graph theory concepts to prove this problem mathematically.We have referred various definitions which are related to graph theory in order to prove this problem.Definitions which we have used in order to prove this problem are mainly from ‘Wolfram MathWorld’ site which is maintained by mathematicians all over the world. With the help of these definitions and theorems we were able to find a justifiable solution for the Konigsberg Bridge problem.

Definitions

Connected graph[1]

A graph which is connected in the sense of a topological space, i.e., there is a path from any point to any other point in the graph.

Walk[2]

A walk is a sequence v_0, e_1, v_1, ..., v_k of graph vertices v_i and graph edges e_i such that for $1 \leq i \leq k$, the edge e_i has endpoints v_{i-1} and v_i (West 2000, p. 20). The length of a walk is its number of edges.

A u, v-walk is a walk with first vertex u and last vertex v, where u and v are known as the endpoints. Every u, v-walk contains a u, v-graph path (West 2000, p. 21).

Trail[3]

A trail is a walk v_0, e_1, v_1, ..., v_k with no repeated edge.

Path[4]

A path γ is a continuous mapping $\gamma : [a, b] \mapsto C^0$, where $\gamma(a)$ is the initial point, $\gamma(b)$ is the final point, and C^0 denotes the space of continuous function.The notation for a path parametrized by t is commonly denoted $\sigma(t)$.

Circuit[5]

A closed trail is called a circuit when it is specified in cyclic order but no first vertex is explicitly identified.

Euler Trail[6]

Euler trail is a walk on the graph edges of a graph which uses each graph edge in the original graph exactly once.

Vertex Degree[7]

The degree of a graph vertex of a graph is the number of graph edges which touch .

Theorem[8]

A connected graph has an Euler path iff it has at most two graph vertices of odd degree.

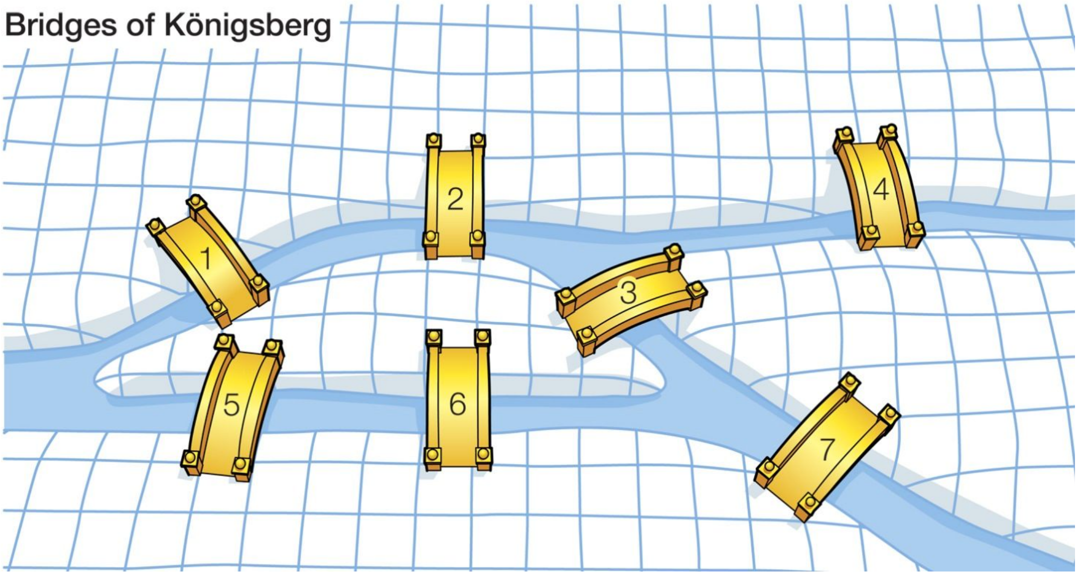
Proof :
Suppose that a graph has an Euler path P. For every vertex v other than the terminal vertices, the path P enters v the same number of times that it leaves v (say 'n' times).
Therefore, there are 2n edges having v as an endpoint.
Therefore, all vertices other than the terminal vertices of P must have an even degree.

Now let's we consider the degree of terminal vertices.
case 1
If Euler path P starts and ends at the same vertex x then number of times(n) enters to x and number of times leaves(n) from x is equal.Thus, degree of vertex x is 2s.
Hence,The degree of the vertex x must be even.
case 2
If Euler path P strats at vertex x and ends at y.
Suppose the Euler path P starts at vertex x and ends at y. Then number of times leaves from x vertex is one more than
number of times enters to it , and number of times leaves y one fewer time than it enters.
Therefore, the degree of the terminal vertices must be odd.

therefore , a connected graph has an euler path iff it has most two vertices of odd degree .
and if all vertices have even degree we can come up with an euler circuit.

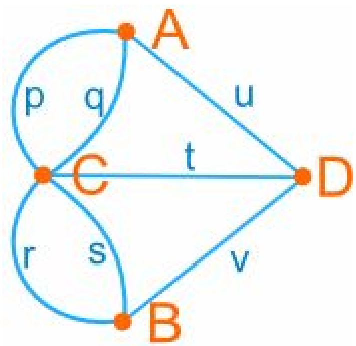
Assigned Problem

Solve the following problem using graph theoretic concepts. The Pregel River flowed through the town of Konigsberg, (present day Kaliningrad in Russia.) Two islands protruded from the river. On either side of the mainland, two bridges joined one side of the mainland with one island and a third bridge joined the same side of the mainland with the other island. A bridge connected the two islands. In total, seven bridges thus connected the two islands with both sides of the mainland. Is it possible to cross each bridge exactly once during a single walk?



Solution

By considering the lands as vertices and bridges as edges, we can represent the above scenario in terms of a graph as shown below.



According to graph theory, crossing each bridge exactly once during a single walk means we should find an Eulerian path from the above shown graph.

By the definition of the Eulerian Path it is stated that in a graph there can be only zero or exactly two odd degree vertices.So in this case, there are 4 odd degree vertices.Therefore there is no any Eulerian Path in the above graph. So it is impossible to make a single move which crosses all the bridges exactly at once.

Questions

1.Let G be a connected graph that has an Euler circuit. Prove that if G is bipartite then it has an even number of edges ?

2. G is a simple graph with order 10 and size 20 , should G be a connected graph?

3.Let G be a connected graph that has an Euler circuit. If G has an even number of vertices then is it not necessary to have even number of edges?

Questions and Solutions

1.Let G be a connected graph that has an Euler circuit. Prove that if G is bipartite then it has an even number of edges ?
•The number of edges in a bipartite graph is equal to the sum of the degrees in one part. If there is an Euler tour then all degrees are even, and the sum of even numbers is also even.

2. G is a simple graph with order 10 and size 20 , should G be a connected graph?
•False
•Simple graphs haven't loops. So order 10 graph can have maximum of 1+2+..+9=45 edges. Here only have 20 edges. So we can create graph with isolated vertices.

3.Let G be a connected graph that has an Euler circuit.If G has an even number of vertices then is it not necessary to have even number of edges?
•True
•Take a cycle of length 3 and a cycle of length 4, joined at a single vertex. It has 6 vertices and 7 edges, and it has an Euler circuit.

References

http://mathworld.wolfram.com/ConnectedGraph.html[1]
http://mathworld.wolfram.com/Walk.html[2]
http://mathworld.wolfram.com/Trail.html[3]
http://mathworld.wolfram.com/Path.html [4]
http://mathworld.wolfram.com/Circuit.html[5]
http://mathworld.wolfram.com/EulerianPath.html[6]
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