RESEARCH CYCLE - I PROJECT REPORT

Group B4

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Relevant Definitions

Definition 1: Graph

A graph $G = (V, E)$ is an ordered pair of finite sets with $E \subseteq V^2$

Definition 2: Directed Graph

A directed graph is a graph where all the edges are directed from one vertex to another.

Definition 3: u-v walk

A u-v walk is an alternating sequence of vertices starting with u and ending with v

Definition 4: Path

A path is a walk with no repeating vertices except possibly the start and end vertices.

Definition 5: Cycle

A cycle is a closed path which is a sequence of vertices starting and ending at the same vertex.

Definition 6: Tree

Tree is an acyclic graph. (i.e. an undirected graph in which any two vertices are connected by exactly one path.)

Problem 1:

a) Let T = (V, E) be a tree and let u, v ε V be distinct vertices. Then T has exactly one u-v path

Proof **(by contrapositive)**:

(A) : T is a tree
(B) : T has only one u-v path

$Assume\ {\sim}B,$

${\sim}B$ **Case1**:

$Assume\ there\ are\ two\ distinct\ paths\ betwen\ two\ distinct\ vertices\ u\ and\ v\ of\ the\ tree\ T.$
$The\ union\ of\ these\ two\ paths\ contains\ a\ cycle.\ Therefore,\quad T\ is\ not\ a\ tree\ \Rightarrow\ {\sim}\mathrm{A}$

${\sim}B$ **Case2**:

Assume there is no path between U and V. Then a disconnected graph is obtained.

Therefore, T is not a tree $\Rightarrow\ {\sim}\mathrm{A}$

Therefore,

${\sim}B\ \Rightarrow\ {\sim}\mathrm{A},$

By contrapositive $A\ \Rightarrow\ \mathrm{B}$

Problem 2:

b) If T = (V, E) is a graph, then the following are equivalent:

i. T is a tree

ii. For any new edge e, the join T + e has exactly one cycle.

Proof $(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$:

$Assume\ the\ negation\ of\ (ii),$

That is; case (I) For any new edge e the join T + e has more than 1 cycle

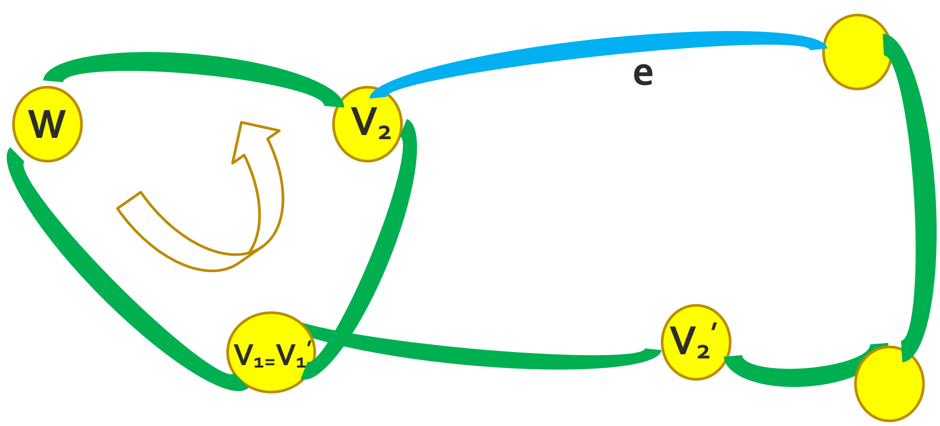
Or case (II) For any new edge e the join T + e has less than 1 i.e. 0 cycles

$Case\ (I)$

$Assume\ that\ there\ are\ two\ cycles\ in\ T + e.$

$$P:\ v_0 = w, v_1, v_2, \dots, v_k = w$$
$$and$$
$$P':\ v'_0 = w, v'_1, v'_2, \dots, v'_l = w$$

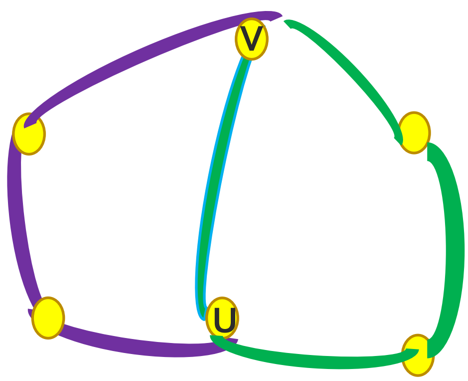
$If\ either\ P\ or\ P'\ does\ not\ contain\ e, say\ P\ does\ not\ contain\ e, then\ P\ is\ a\ cycle\ in\ T.$
$But\ if\ T\ has\ a\ cycle\ (P\ here)\ it\ cannot\ be\ a\ tree\ and\ therefore\ contradicts\ ----(i)$



$We\ may\ now\ suppose\ that\ P\ and\ P'\ both\ contain\ e.$

$Then\ P\ contains\ a\ subpath\ P_0 = P - e\ (which\ is\ not\ closed) that\ is\ the\ same\ as\ P\ except\ it\ lacks\ the\ edge\ from\ u\ to\ v.$

$Likewise, P'\ contains\ a\ subpath\ P'_0 = P' - e\ (which\ is\ not\ closed) that\ is\ the\ same\ as\ P'\ except\ it\ lacks\ the\ edge\ from\ u\ to\ v.$



$But\ as\ proved\ in\ the\ first\ theorem, there\ can\ only\ be\ 1\ \ u - v\ path\ in\ a\ tree$

$Therefore\ T\ is\ not\ a\ tree; contradicts ----(i)$

$Case\ (II)$

$Assume\ that\ there\ are\ no\ cycles\ in\ no\ cycles\ in\ T + e.$

$This\ means\ there\ are\ two\ edges\ u\ and\ v\ connected\ only\ by\ e\ and\ they\ were\ disconnected\ in\ T$

$T\ is\ not\ a\ tree$

Thus, both cases of the negation of (ii) negates (i)

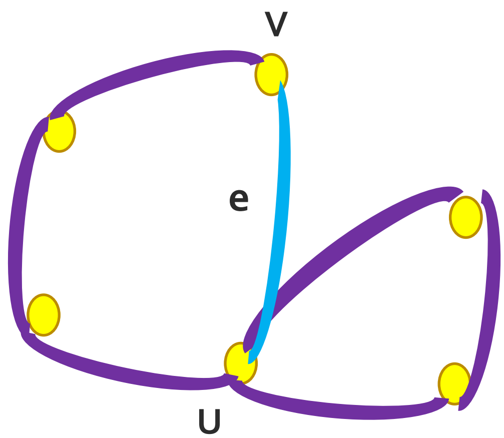
$(\boldsymbol{ii})' \Rightarrow\ (\boldsymbol{i})'$

Therefore, by contrapositive $(\boldsymbol{i}) \Rightarrow\ (\boldsymbol{ii})$ is proved.

Proof $(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$:

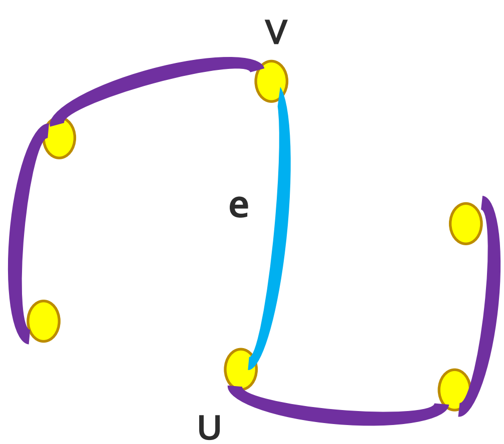
$Assume\ the\ negation\ of\ (i)\ that\ is;$
$T\ is\ either\ (I)\ cyclic\ and\ connected, (II)\ acyclic\ and\ disconnected\ or$
$(III)\ cyclic\ and\ disconnected$

$Case(I)\ Suppose\ T\ is\ cyclic\ and\ connected,$



$Let\ e\ be\ the\ new\ edge\ connecting\ two\ vertices\ u\ and\ v. But\ as\ u\ and\ v\ were\ connected\ before,$
$This\ additional\ edge\ would\ create\ a\ new\ cycle. But\ also\ as\ T\ is\ cyclic\ (i.e.\ has\ atleast\ one$
$cycle)\ this\ means\ now\ T + e\ has\ atleast\ 2\ cycles. And\ therefore\ contradicts\ - - - (ii)$

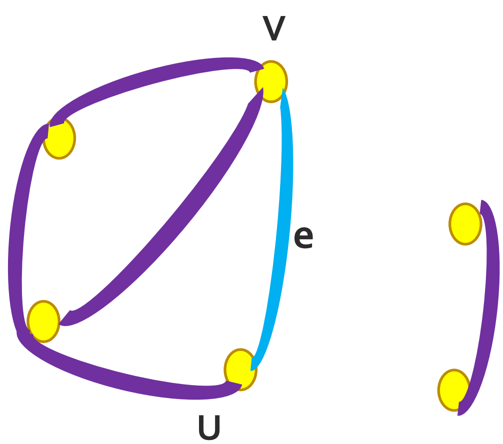
$Case(II)\ Suppose\ T\ is\ acyclic\ and\ disconnected,$



$Let\ u\ be\ a\ vertex\ in\ one\ component, T_1\ say, of\ T\ and\ v\ a\ vertex\ in\ another\ component,$
$T_2\ say, of\ T.$

$Adding\ the\ edge\ e = uv\ does\ not\ create\ a\ cycle\ (if\ it\ did\ then\ T_1\ and\ T_2\ would\ not\ be\ disjoint)\ Contradicts\ ---\ (ii)$

$Case(III)\ Suppose\ T\ is\ cyclic\ and\ disconnected,$



$Take\ two\ vertices\ u\ and\ v\ from\ the\ same\ component\ T_1\ (which\ is\ connected) and\ connect\ with\ new\ edge\ e. This\ additional\ edge\ would\ create\ a\ new\ cycle. But\ also\ as\ T\ is\ cyclic\ (i.e. has\ atleast\ one\ cycle)$
$this\ means\ now\ T + e\ has\ atleast\ 2\ cycles. And\ therefore\ contradicts\ ---\ (ii)$

$All\ three\ cases\ of\ the\ negation\ of\ \ (i)\ negates\ (ii)$

$(\boldsymbol{i})' \Rightarrow\ (\boldsymbol{ii})'$

$Therefore\ by\ contrapositive, (\boldsymbol{ii}) \Rightarrow\ (\boldsymbol{i})$

Designed Problems

1. Consider undirected simple graph G with 100 nodes. The maximum number of edges to be included in G so that the graph is not connected is?

a) 4950
b) 4900
c) 4851
d) 9800

2. Degree of a vertex is defined as edges connected to the particular vertex. At least two vertices have same degree for a simple, connected, undirected graph having more than 2 vertices.

a) True b) False

3. Simple graph having 24 vertices and maximum number of edges (every vertex is connected to every other vertices) cannot be an Euler graph.

a) True b) False

Answers:

1) **C**
2) **True**
3) **False**

References:

[1] Wolfram Math World (Feb 20, 2016). *Path* [Online]. Available: **http://mathworld.wolfram.com/Path.html**
[2] Wolfram Math World (Feb 20, 2016). *Tree* [Online]. Available: **http://mathworld.wolfram.com/Tree.html**
[3] Wolfram Math World (Feb 20, 2016). *Connected Graph* [Online]. Available: **http://mathworld.wolfram.com/ConnectedGraph.html**
[4] Theorem: Available: **http://compalg.inf.elte.hu/~tony/Oktatas/TDK/FINAL/Chap%204.PDF**