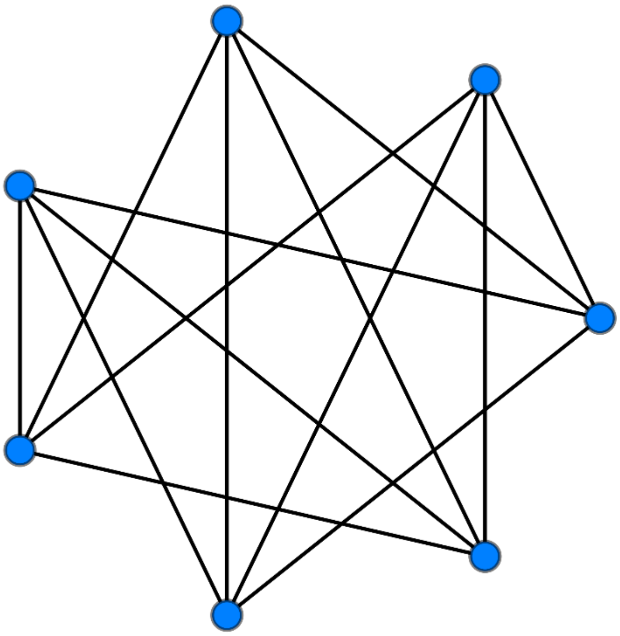

CS 2150: Graph Theory for Computing

Research Cycle I
Group B1

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Is the complement of a regular graph, regular? If yes, prove it! If no, find a counter example!

Question 2

Are there self-complementary graphs of order 2? Of order 3? Of order 4? Of order 5? In each case, if yes, give an example; if no, prove why it is not possible.

Definitions

Graph

A graph is a collection of points and lines connecting some **subset** of them.

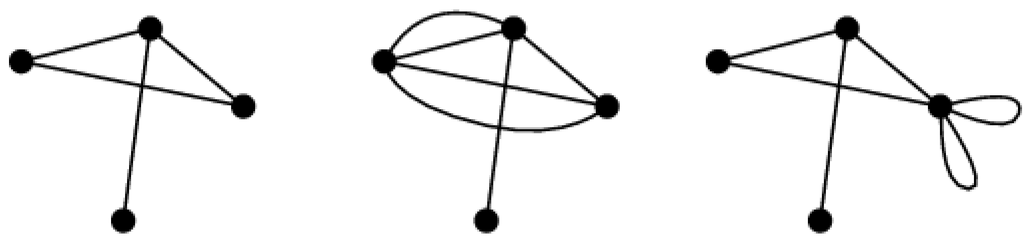
The points of a graph are most commonly known as **graph vertices**, but may also be called "**nodes**" or simply "**points**."

Similarly, the lines connecting the vertices of a graph are most commonly known as **graph edges**, but may also be called "**arcs**" or "**lines**." [1]

Simple Graph

A simple graph, also called a **strict graph**, is an unweighted, undirected graph containing **no graph loops** or **multiple edges**.

A simple graph may be either connected or disconnected.



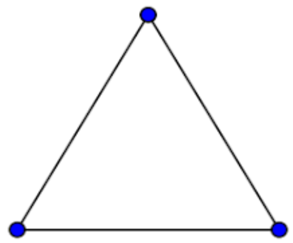
simple graph *nonsimple graph with multiple edges* *nonsimple graph with loops*

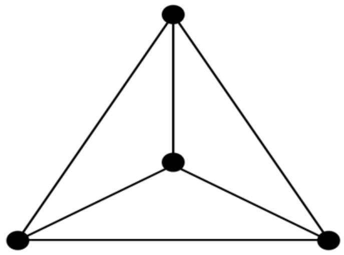
Regular Graph

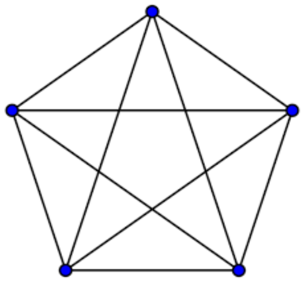
A **graph** is said to be regular of degree **r** if all local degrees are the same number **r**.

A **0-regular** graph is an empty graph. A **1-regular** graph consists of disconnected edges, and a **two-regular graph** consists of one or more (disconnected) cycles.

The first interesting case is **3-regular graphs**, which are called **cubic graphs** [2]





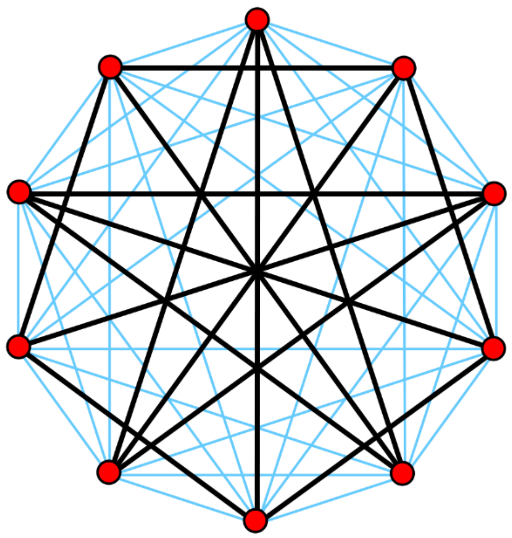


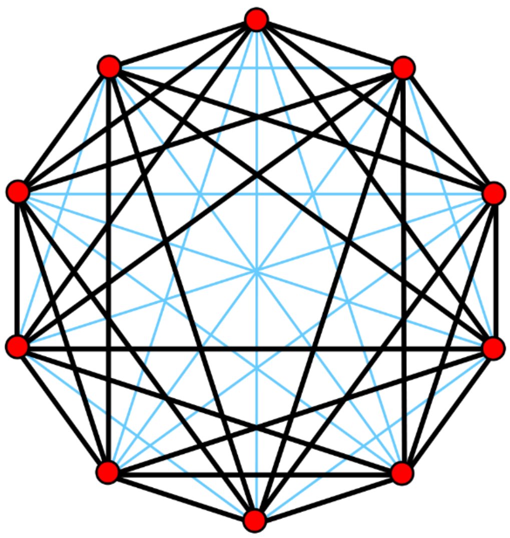
Degree	Name
3	Cubic Graph
4	Quartic Graph
5	Quintic Graph
6	Sextic Graph
7	Septic Graph
8	Optic Graph

Complement of a Graph

The **complement** of a graph G, denoted by G`, is the graph, with the same vertex set but whose edge set consists of the edges not present in the given graph. The graph sum G+G` on a n-node graph G is therefore the Complete Graph $\mathbf{K_n}$ [3]

(The complement is not defined for **multigraphs**. In graphs that allow self- the complement of *G* may be defined by adding a self-loop to every vertex that does not have one in *G*. This operation is, however, different from the one for **simple graphs**, since applying it to a graph with no self-loops would result in a graph with self-loops on all vertices [4

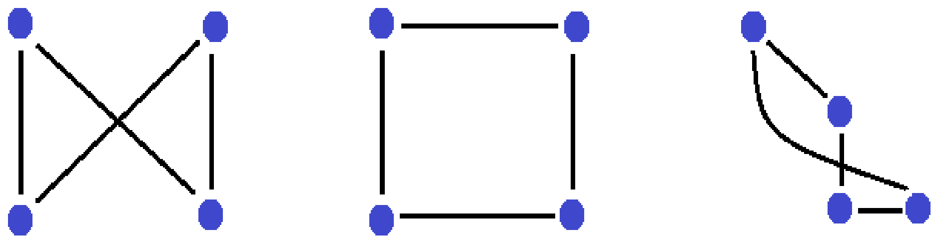




Isomorphism

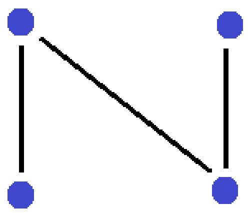
Two graphs which contain the same number of graph vertices connected in the same way are said to be isomorphic.

Formally, two graphs G and H with graph vertices $V_n = \{1, 2, \ldots, n\}$ are said to be isomorphic if there is a permutation p of V_n such that $\{u, v\}$ is in the set of graph edges $E(G)$ iff $\{p(u), p(v)\}$ is in the set of graph edges $E(H)$ [5].

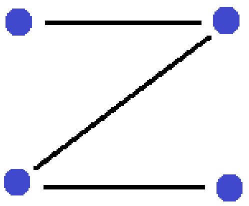


Self-Complementary Graph

A **self-complementary graph** is a graph which is isomorphic to its graph complement. [6]



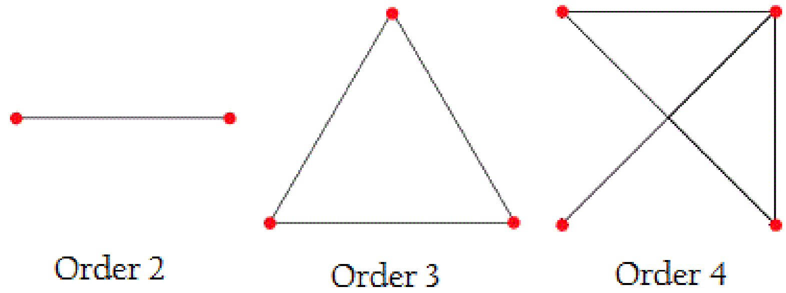
Graph G

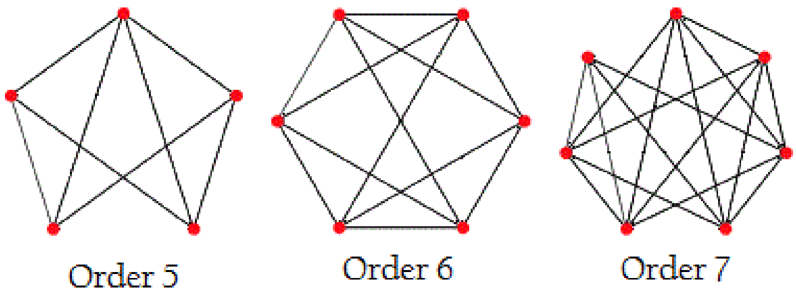


Graph G′

Order of a Graph

The number of nodes in a graph is called its order





Answers for the Questions

Question 1

Is the complement of a regular graph, regular? If yes, prove it! If no, find a counter example!

Proof: - **Yes,**

- Let G be a simple graph with r-regular which has n vertices. **(**Since we are talking about complement of a graph we only take simple graph**)**
- If we take a vertex in G as V, it is connected to r number of other vertices
- The maximum number edges that vertex can have is **(n-1)**
- If we take the complement of graph G, as G′, vertex V will have **[(n-1) - r]**
- Since G is a regular graph this will be the same case for every other vertex.
- In G′ any vertex will have [(n-1)-r] edges.
- Therefore, G′ is a regular graph

Question 2

Are there self-complementary graphs of order 2? Of order 3? Of order 4? Of order 5? In each case, if yes, give an example; if no, prove why it is not possible.

Proof: -

- If a graph is self-complementary the number of edges in the complete graph should be even.
- Let G, be a graph of order n.
- The number of edges in the complete graph = $\frac{n(n-1)}{2}$
- This number should be divisible by 2.

Order 2

Complete number of edges = $\frac{2(2-1)}{2} = 1$

1 is not divisible by 2, ∴ There are no self-complementary graphs of order 2.

Order 3

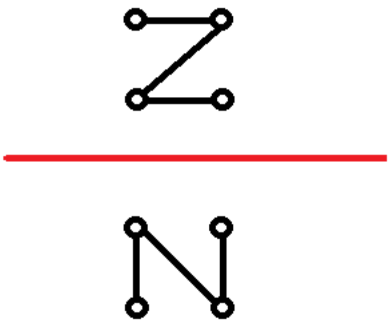
Complete number of edges = $\frac{3(3-1)}{2} = 3$

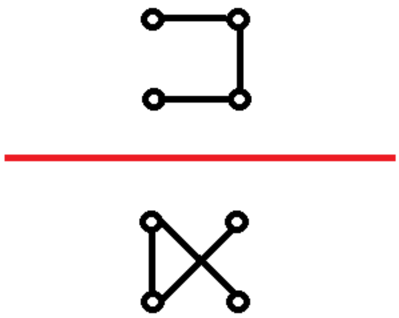
3 is not divisible by 2, ∴ There are no self-complementary graphs of order 3.

Order 4

Complete number of edges = $\frac{4(4-1)}{2} = 6$

6 is divisible by 2, ∴ There is are self-complementary graphs of order 4.

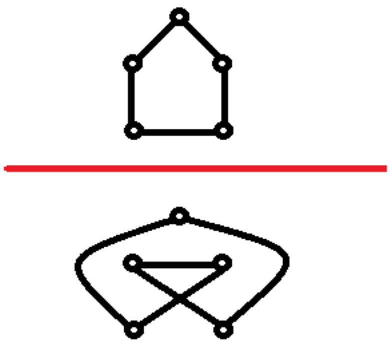




Order 5

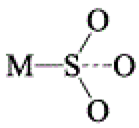
Complete number of edges = $\frac{5(5-1)}{2} = 10$

10 is divisible by 2, ∴ There are self-complementary graphs of order 5.

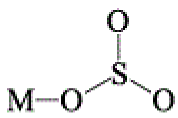


Created Questions

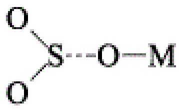
1. A complete graph doesn't have a complement graph.
2. Are there self-complementary graphs that are not regular?
3. Here are some Lewis structures of a compound called double sulfites.



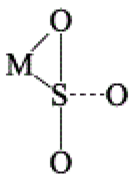
(I) C_{3v}



(II) C_s



(III) C_s



(IV) C_1

Identify the isomorphic structures out of the given 4 states.

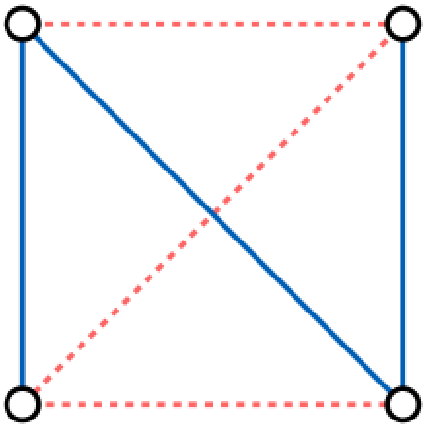
Solutions

Question 1

False, A complete graph is a graph where each pair of graph vertices are connected by an edge. Therefore, the complement of this should be a graph where none of the vertices are connected by an edge, an empty graph. Which is also known as a 0-regular graph.

Question 2

True, for an example take this graph of order 4.



This graph has irregular degree vertices (i.e. not a regular graph). But both the graph and it's complement has the same number of vertices and edges.

If the graph connected in blue lines is rotated 90 degrees clockwise it overlaps with its complementary graph.

∴ This is a self-complementary graph which is not regular.

Question 3

Number 2 and 3 are isomorphic structures. The main idea behind this is to be isomorphic the same vertex need not to take the same place in its complement graph. If there's a permutation such that the vertices match to the complement graph with the same number of degree, then both are isomorphic.

∴ The 2nd and 3rd structures are isomorphic.

References

[1]: Wolfram Math World (Jan 26, 2018). **Graph** [Online]. Available: **http://mathworld.wolfram.com/Graph.html**

[2]: Wolfram Math World (Jan 26, 2018). **Regular Graph** [Online]. Available: **http://mathworld.wolfram.com/RegularGraph.html**

[3]: Wolfram Math World (Jan 26, 2018). **Graph Complement** [Online]. Available: **http://mathworld.wolfram.com/GraphComplement.html**

[4]: Wikipedia (Jan 26, 2018). **Graph Complement** [Online]. Available: **https://en.wikipedia.org/wiki/Complement_graph**

[5]: Wolfram Math World (Jan 26, 2018). **Isomorphic Graphs** [Online]. Available: **http://mathworld.wolfram.com/IsomorphicGraphs.html**

[6]: Wolfram Math World (Jan 26, 2018). **Self- Complementary Graph** [Online]. Available: **http://mathworld.wolfram.com/Self-ComplementaryGraph.html**