Research Cycle 1

Group C1

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Theorems and definitions

- **Binary Triple**

  Binary triple is a number which consists of three binary numbers.

  E.g.

  $\{a, b, c\}, \{0,1,0\}, etc..$

- **Cyclic Order**

  A cyclic order is a way to arrange a set of objects in a circle.

Figure 1 Cyclic path

- **De Bruijn Graph**

  De Bruijn graph is a graph whose nodes are sequences of symbols from some alphabet and whose edges indicate the sequences which might overlap. The below figures show the first $(n-1)-dimentional\ de\ Bruijn\ Graph$ on m symbols and n order, $(m, n-1)$ for $m, n \geq 2$

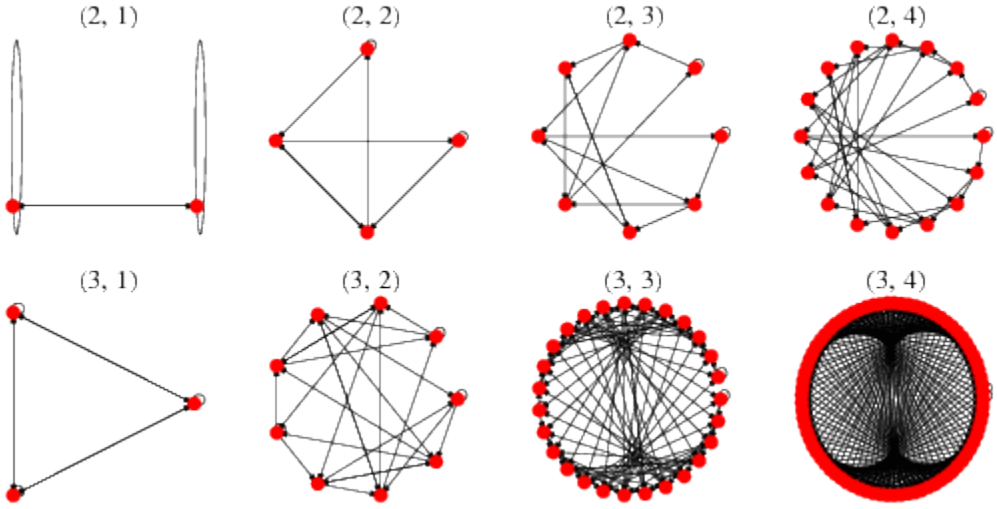


Figure 2 example de Bruijn graph

http://mathworld.wolfram.com/deBruijnGraph.html

http://mathworld.wolfram.com/EulerianCycle.html Eulerian cycle is a trail which starts and ends at the same graph vertex. In other words, it is a graph cycle which uses each graph edge exactly once.

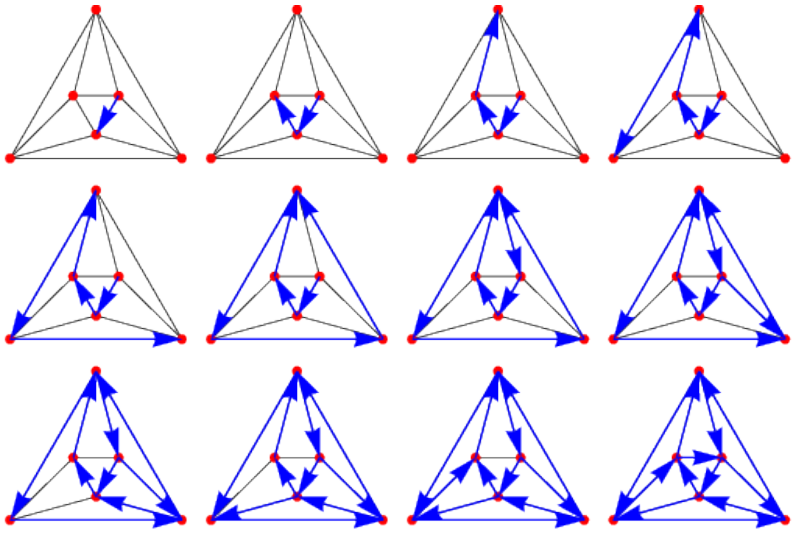


Figure 3 Eulerian cycle on graph

http://mathworld.wolfram.com/EulerianCycle.html

- **De Bruijn Sequence**

The shortest circular sequence of length $\boldsymbol{m^n}$ The shortest circular sequence of length $\boldsymbol{n}$ on the alphabet $\boldsymbol{a}$ size of $\boldsymbol{m}$ m occurs on a contiguous subrange of the sequence described by $\boldsymbol{n}$ such a sequence is denoted by $\boldsymbol{B(m,n)}$

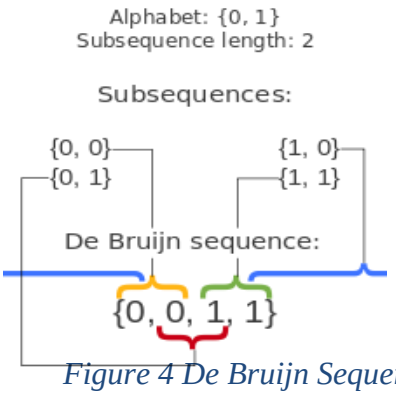


Figure 4 De Bruijn Sequence

"Each De Bruijn graph is Eulerian and Hamiltonian. The Euler cycles and Hamiltonian cycles of these graphs (equivalent to each other via the line graph construction) are De Bruijn sequences[1]"

Source: De Bruijn graph - https://en.wikipedia.org

Assigned problem

Create a sequence of 1s and 0s such that when the digits are arranged in a cycle, each possible binary triple appears exactly once in the cycle?

Solution

- ✓ In this case we must prove existence of $B(2,3)$, alphabet $a = \{0,1\}$.
  1. By definition of de Bruijn graph m=2, n=3.
     Required De Bruijn graph = (2,2)

Length of the sequence	2^3
Dimension of the De Bruijn graph	2-D
Possible combination of De Bruijn graph vertices	(0,0) (0,1) (1,0) (1,1)

The 2-dimensional De Bruijn graph is given below.

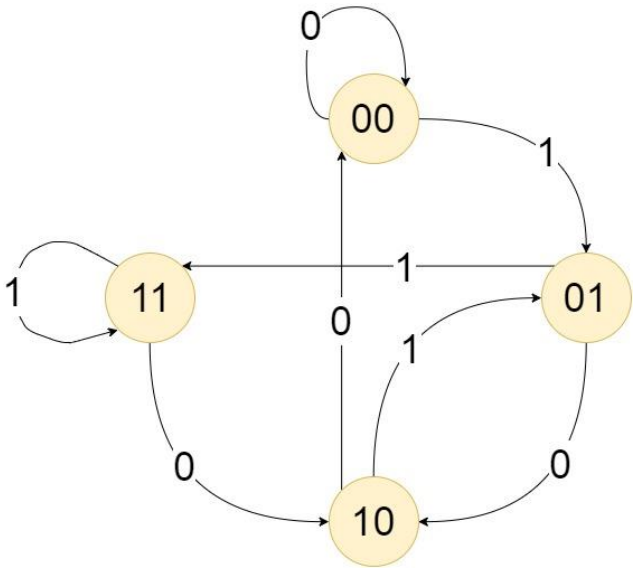


Figure 5 2-D Binary De Bruijn graph

To obtain a De Bruijn sequence, we need to find an *Eulerian cycle* on the above *De Bruijn Graph.*

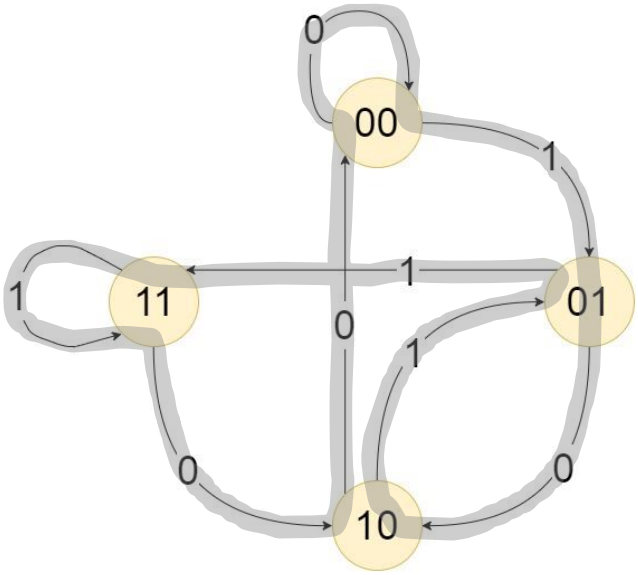
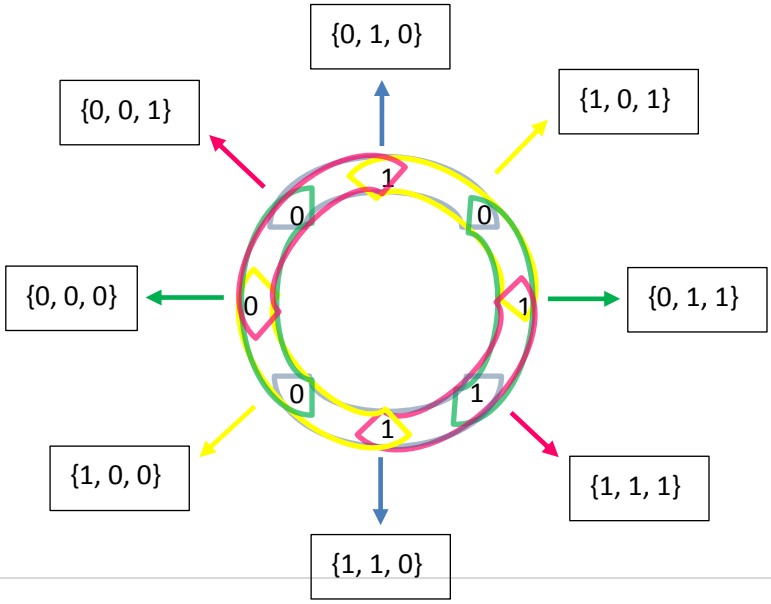


Figure 6 Eulerian cycle on De Bruijn graph

The state changes of the above Eulerian cycle corresponds to a De Bruijn sequence.

Therefore, De Bruijn sequence = {1, 0, 1, 1, 1, 0, 0, 0}

Therefore, by De Bruijn sequence definition, {1, 0, 1, 1, 1, 0, 0, 0}is a sequence of 1s and 0s such that when the digits are arranged in a cycle, each possible binary triple appears exactly once in the cycle.



Related Questions

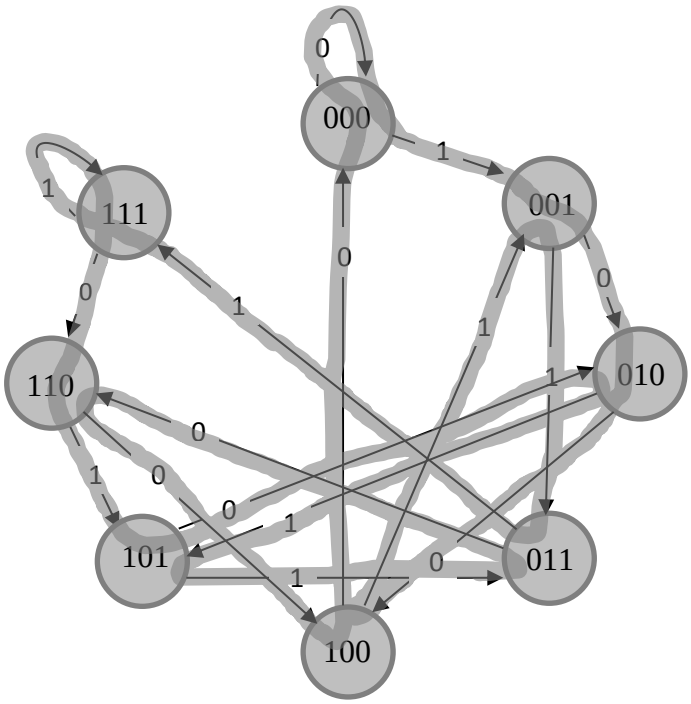
1. Create a sequence of 1s and 0s such that when the digits are arranged in a cycle, each possible binary quadruple appears exactly once in the cycle.

2. Sketch (3,2) De Bruijn graph

3. Find a De Bruijn sequence to the above (3,2) De Bruijn graph

Solutions to Related Questions

1. graph should be 3D
   Possible combinations = 000, 001, 010, 011, 100, 101, 110, 111



De Bruijn sequence = {1, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0}

References

- http://mathworld.wolfram.com/deBruijnGraph.html
- http://mathworld.wolfram.com/EulerianCycle.html
- http://mathworld.wolfram.com/deBruijnSequence.html
- https://en.wikipedia.org/wiki/Cyclic_order
- https://en.wikipedia.org/wiki/De_Bruijn_sequence