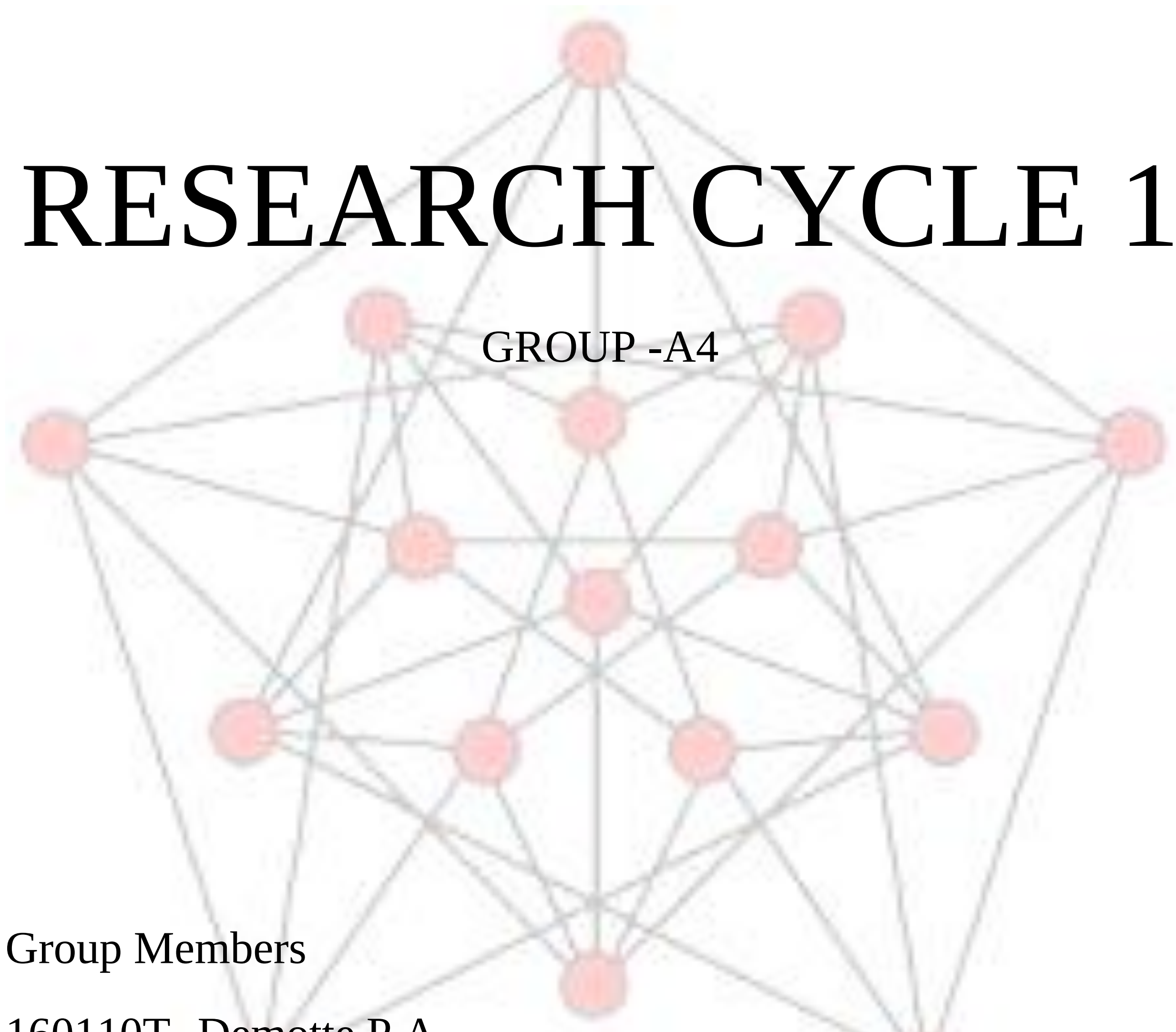
RESEARCH CYCLE 1

GROUP -A4



Group Members

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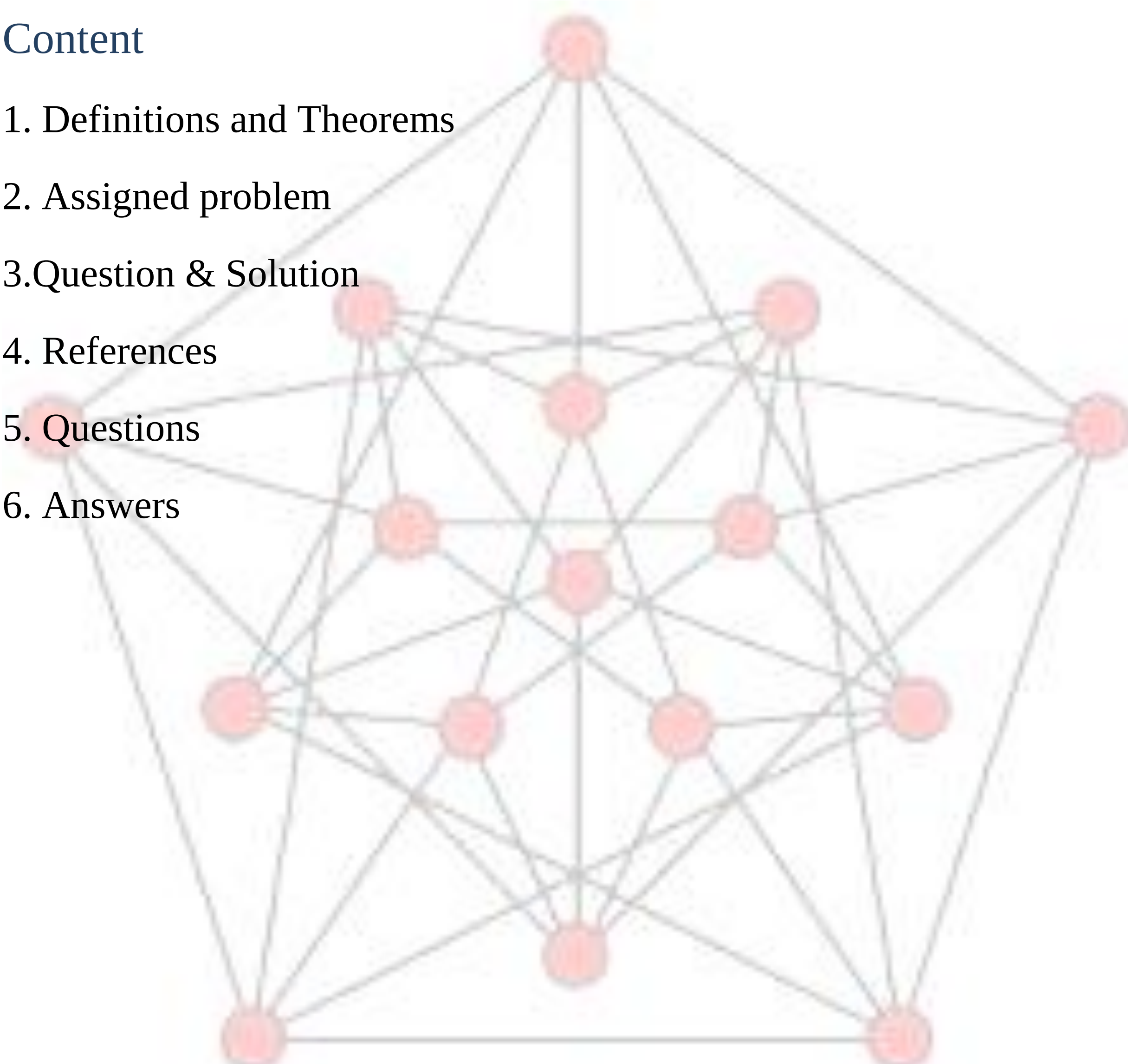
Content



Definitions

Graph

Collection of points and lines connecting some (possibly empty) subset of them.

A graph $G = (V, E)$ is an **ordered pair** of **finite** sets with $E \subseteq V^{(2)}$

Graph vertex (V)

One of the points on which the graph is defined and which may be connected by graph edges

Graph edge

For an undirected graph,

An unordered pair of nodes that specify a line joining these two nodes are said to form an edge

For a directed graph,

A graph with directed edges is called a directed graph.

Trail

A **trail** is a walk with no repeated edges.

Path

A path is a walk with no repeating vertices, except possibly the start and end vertices.

Cycle

A cycle is a closed path.

Circuit

A circuit is a closed trail (i.e. trail with end points the same vertex) of non zero length.

Euler trail

A trail is called an Euler trail if it includes each edge of the graph exactly once.

Euler circuit

A closed Euler trail is called an Euler circuit.

Euler graph

A graph which has an Euler circuit is called an Euler graph.

Hamiltonian cycle

A Hamiltonian cycle, also called a Hamiltonian circuit, Hamilton cycle, or Hamilton circuit, is a graph cycle (i.e., closed loop) through a graph that visits each node exactly once. A graph possessing a Hamiltonian cycle is said to be a Hamiltonian graph.

Close neighborhood of a graph

The graph close neighborhood of a vertex v in a graph is the set of all the vertices adjacent to including itself.

Open neighborhood of a graph

The graph open neighborhood of a vertex v in a graph is the set of all the vertices adjacent to excluding itself.

Theorem

Define a graph where each vertex corresponds to a person and where two vertices

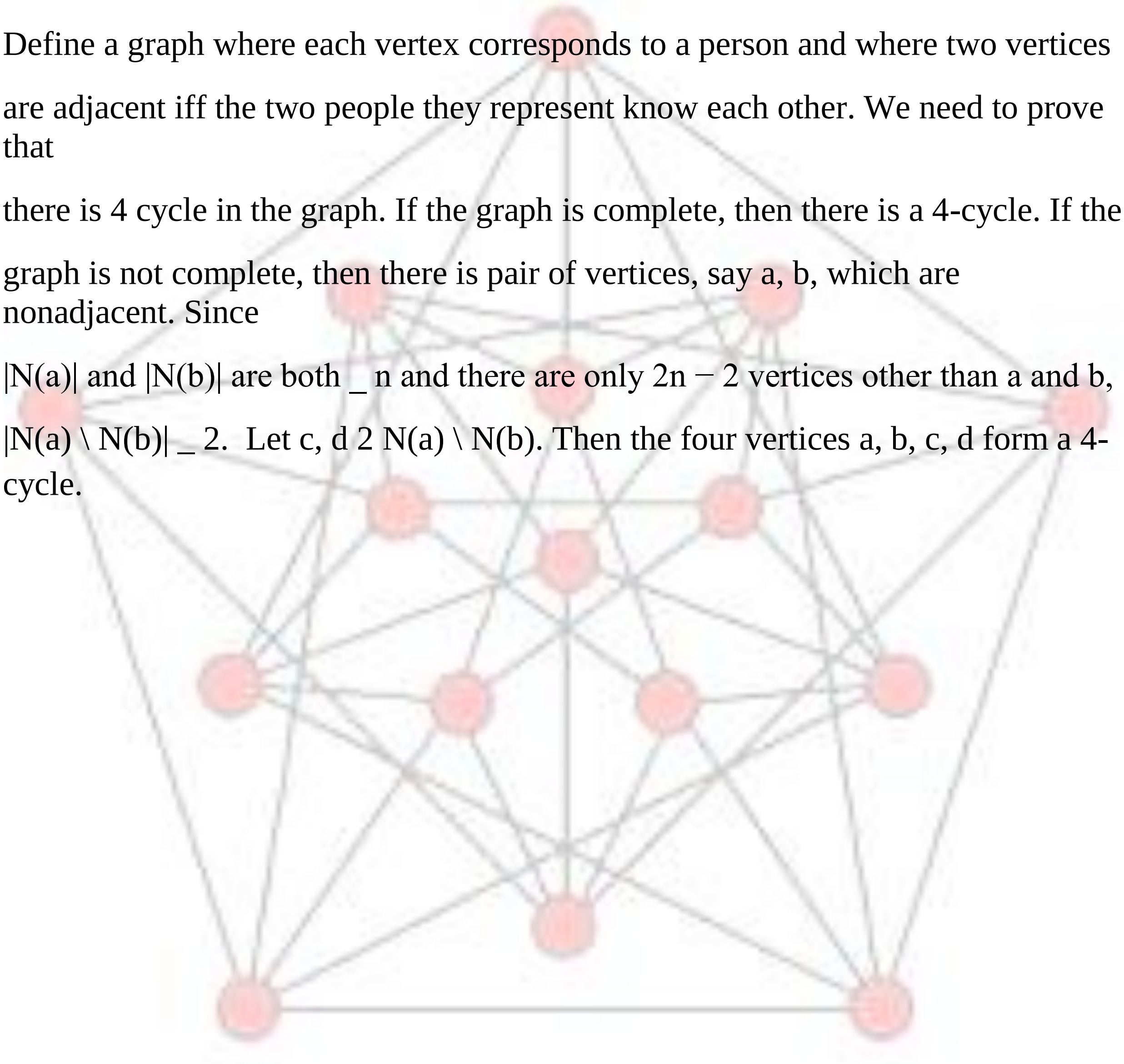
are adjacent iff the two people they represent know each other. We need to prove that

there is 4 cycle in the graph. If the graph is complete, then there is a 4-cycle. If the

graph is not complete, then there is pair of vertices, say a, b, which are nonadjacent. Since

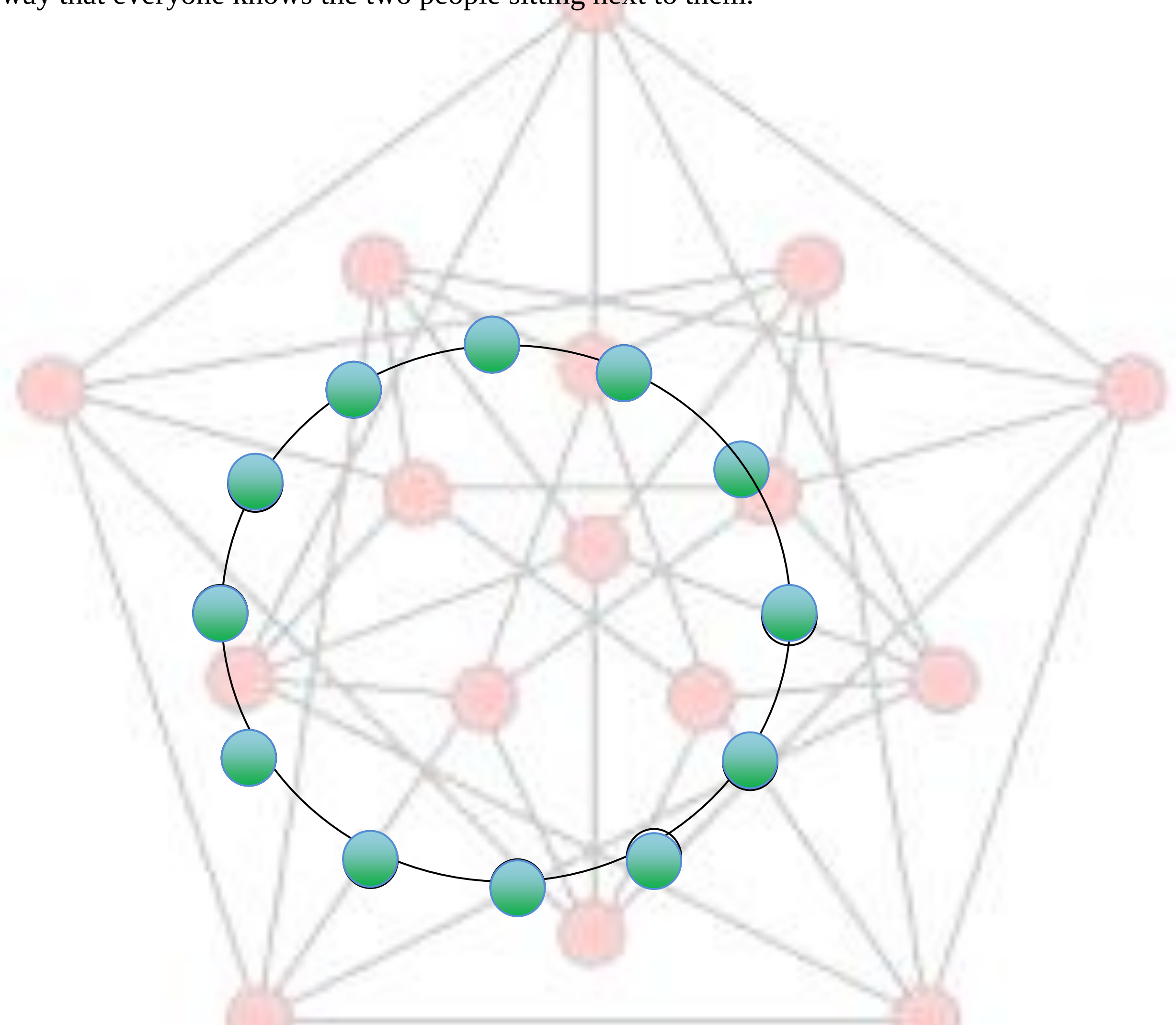
|N(a)| and |N(b)| are both _ n and there are only 2n − 2 vertices other than a and b,

|N(a) \ N(b)| _ 2. Let c, d 2 N(a) \ N(b). Then the four vertices a, b, c, d form a 4-cycle.



Question & Solution

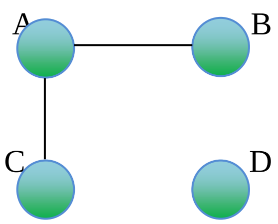
Two CSE grads Mohan and Vathish, both working at Virtusa, have rented a flat together. They have a dinner party where 10 other people from Virtusa are invited. In the group of 12 people, each of them knows at least 6 other people. Prove that they can seat at a round table in such a way that everyone knows the two people sitting next to them.



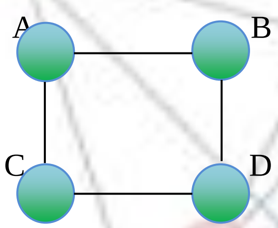
Step 1

For the convenience we have taken a simplifying approach to the solution by considering a group of 4 people where each one know at least 2 other people.

Let’s take them as A,B,C,D. A must know at least 2 others, let’s assume them as B, C.

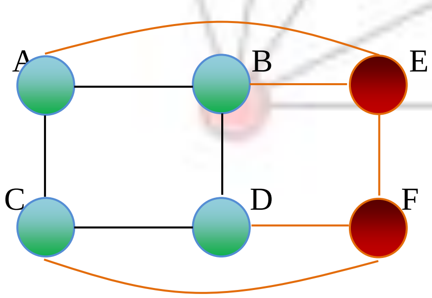


B & C should know at least 2 others, but, as both of them know A already ,they should know one person more. If B knows C, D gets isolated. So the only possible way to fulfill the condition is to consider as C, D and B, D are known to each other. This gives the following graph.



Step 2

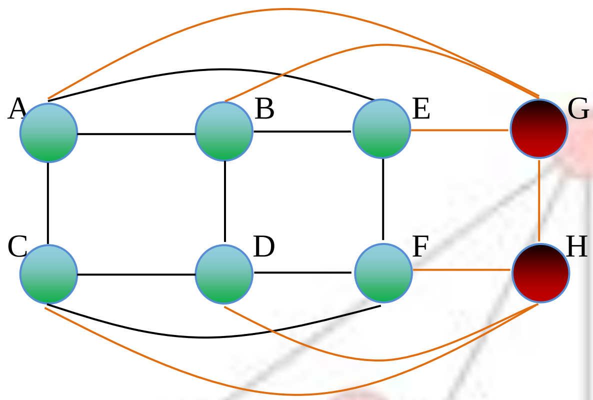
Then we consider the group of 6 people where each one knows 3 others. This case can be developed by adding 2 more people (E, F) to the previous situation.



We can connect E, F as above and it gives a solution for the considered condition.

Step 3

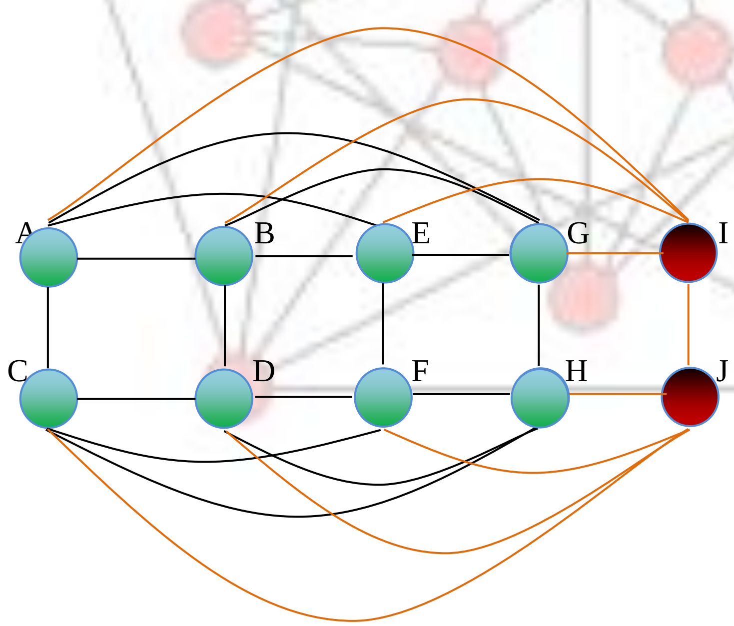
Like this we can add 2 more people (G, H) to the above case and arrange a group of 8 people where each one know at least 4 others.



When newly added nodes are connected we have to connect half of the nodes that the previous condition has to one node and the other half to other node.

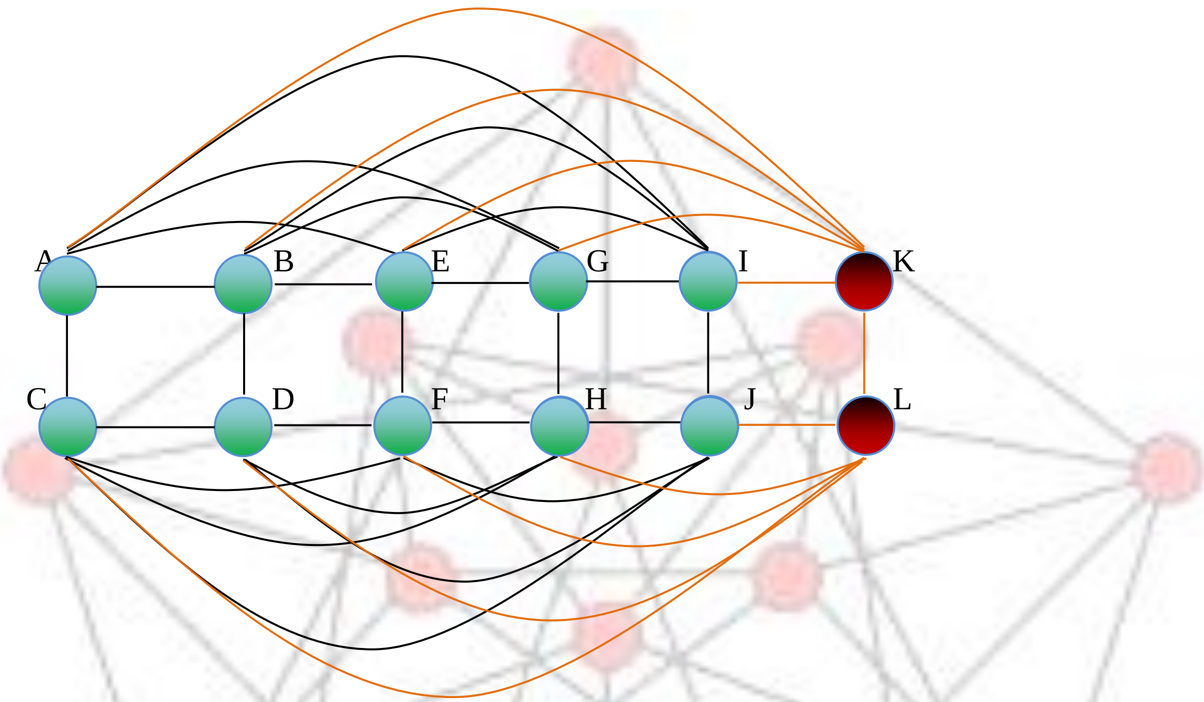
Step 4

Adding 2 more people (I, J) gives a group of 10 people where each one knows at least 5 others.



Step 5

Adding 2 more people (K, L) gives a group of 12 people where each one knows at least 6 others.



This graph has a Hamiltonian cycle of 12 people and gives the solution for the problem.

References

http://reference.wolfram.com/language/ref/Graph.html

http://mathworld.wolfram.com/GraphTheory.html

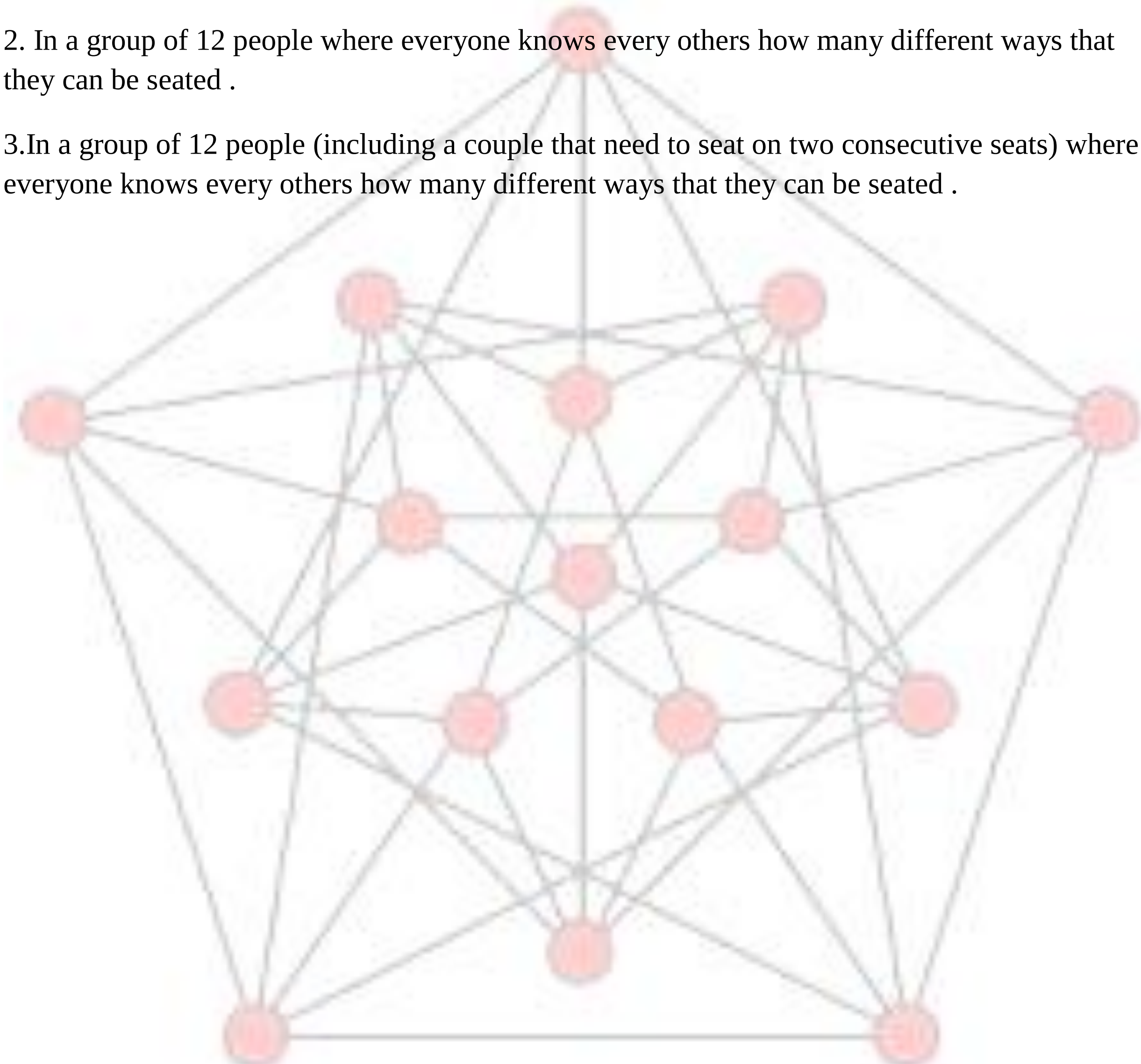
http://mathworld.wolfram.com/GraphVertex.html

Questions

1. In a group of 12 people does each one should know at least 6 others to get seated at a round table in such a way that everyone knows the two people sitting next to them.

2. In a group of 12 people where everyone knows every others how many different ways that they can be seated .

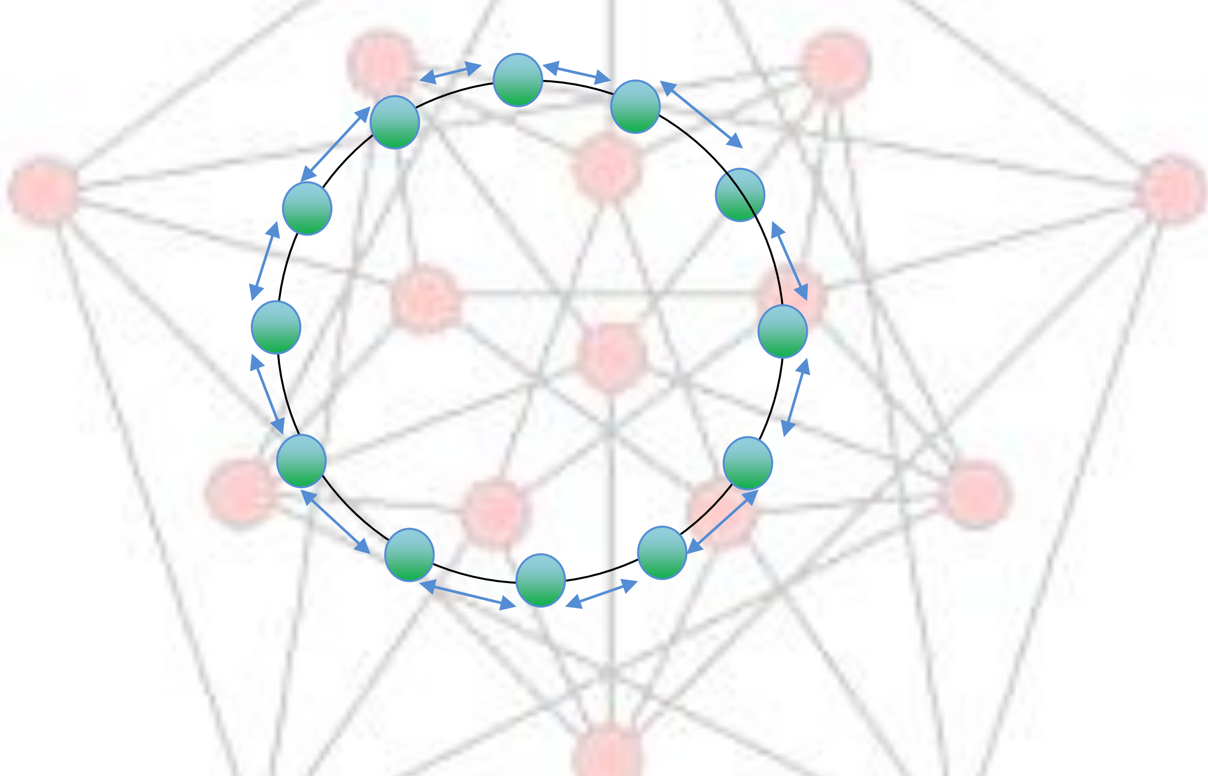
3.In a group of 12 people (including a couple that need to seat on two consecutive seats) where everyone knows every others how many different ways that they can be seated .



Answers

1. No

In a group of 12 people where each ne knows 6 others, definitely they can be seated in a way that everyone knows the two people sitting next to them. But it's not needed each one to know at least 6 others as , if in the group where each one knows exactly 2 others there can be a way that we can arrange them in the needed way. This doesn't say that in every time it is possible, but the question asks each one should know at least 6 others. So the answer is No.



2. There are 12! different ways.

So the answer is 479001600

3. In this problem the couple needs to seat on two consecutive seats . So we can consider them as one person out of 11 people.

So there is 11! ways that they can be seated.

But the positions of the couple can be swapped. So the number of ways is equal to 2 * 11!

So the answer is 79833600.