RESEARCH CYCLE 1

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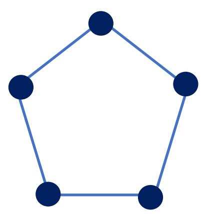
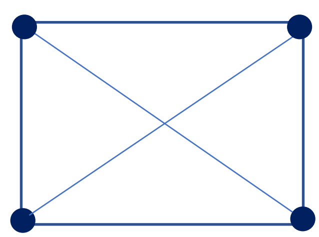
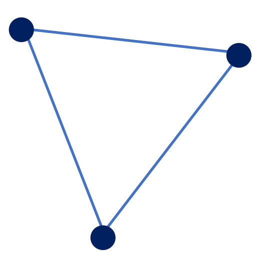
Roles of the team members

	INDEX NUMBER	*NAME*	*ASSIGNED ROLE*
1.	160016K	Bhagya	Group leader
2.	160032F	Uditha	Script writer
3.	160034M	Anushiya	Proof reader
4.	160069A	Thilini	Artistic
5.	160083K	Dulanjaya	Problem solver

Definitions

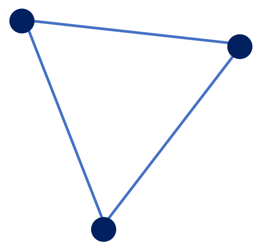
Graph	Graph G = (V,E) where V is the set of vertices and E is the set of edges. If (u,v) € E, then vertex v is adjacent to vertex u (Definition modified from[2])
Vertex degrees	The degree of a vertex of a graph is the number of edges incident to the vertex.[5]
Cycle graph	A cycle or circular graph Cn is define as a graph with n vertices where each vertex is connected to exactly two edges. [5] Eg: C5 
Complete graph	A complete graph Kn is define a a graph with n vertices where each pair of the graph vertices are connected with an edge. [3] Eg: K4 
Triangular graph	A triangular graph is a cycle graph with three vertices (c3). Which is also a complete graph (k3) [4] 
Walk	If u and v are two vertices in a graph G, u-v walk is an alternating sequence of vertices and edges starting with u and ending at v. Consecutive vertices and edges are incident. [2]

Trail	A trail is a walk with no repeated edges. [2]
Closed walk/trail	A walk/trail of length $n \geq 3$ whose start and end vertices are the same is called a closed walk/trail.[2]
Circuit	A circuit is a closed trail (i.e trail with end points the same vertex) of non-zero length.[2]
Euler trail	A trail is called an Euler trail if it includes each edge of the graph exactly once.[2]
Euler circuit	A closed Euler trail is called an Euler circuit.[2]
Recursion	A recursive process is one in which objects are defined in terms of other objects of the same type. Using some sort of repeated method, the entire class of objects can then be built up from a few initial values and a small number of rules.[1]
Graph colouring	Given a graph G, a coloring of G is an assignment of colors to the vertices of G so that if two vertices are adjacent, then they have different colors.[2]

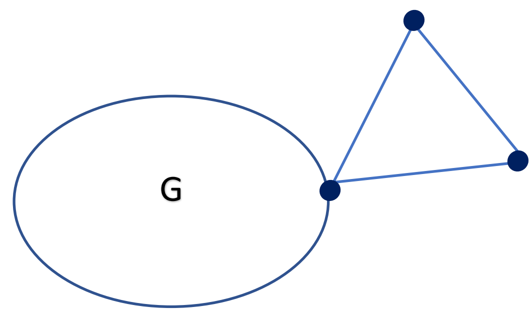
Assigned problem and solution

T-graph can be recursively defined as follows:

- This is a T-Graph



- If G is a T-Graph and v is a vertex of G, then this is also a T-Graph



a) <u>Hypothesis about number of vertices in a T-Graph and proof</u>

In each T graph, Let,

n→ Number of triangles in the T-Graph

E→ Number of edges in the T-Graph

V→ Number of vertices in the T-Graph

Vn→ Number of vertices in the T-Graph with n triangles

Relationship between E and V is given by,

$V = E - (E/3 - 1) = 2E/3+1$

Relationship between E and n is given by,

$E = 3n$, as each triangle adds 3 edges to the T-Graph,

Relationship between n and Vn is given by,

$V_n = 2n+1$

Proof by mathematical induction

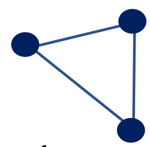
Relationship between V and n is proved by mathematical induction and relationship between E and V can be obtained from that.

Assumption: Vn = 2n+1

Base case (n = 1):

V_1 = 2n+1

V_1 = 2*1+1 = 3

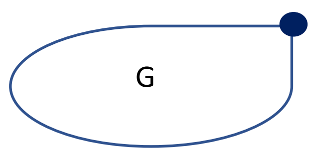


Hence assumption is true for n=1

Inductive hypothesis:

Assume any T graph (G) with n triangles has 2k +1 vertices [n = k (k€Z+)]

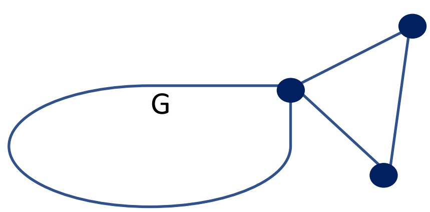
Vk = 2k+1



Inductive step:

Let n = k+1;

When another triangle is joined to an existing T graph, the resultant T-Graph looks as below



Only 2 new vertices are added to the resultant T graph upon adding (k+1)th triangle.

Vk+1 = 2k+1 + 2 = 2(k+1) +1 which is the same as the assumption.

If the assumption is true for n = k, it is true for n = k+1. But the assumption is true for n = 1. Therefore, by the principle of mathematical induction, Vn = 2n+1 is true, for any T-Graph.

b) Hypothesis about the degree of vertex in the T-Graph

1. All vertex have even degree.

2. Maximum possible degree of a vertex in a T-Graph is 2n, where n is the number of triangles in the T-Graph

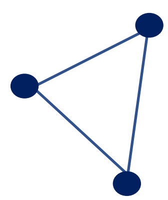
3. Minimum vertex degree in a T graph is 2

1st hypothesis is proven by mathematical induction. 2nd and 3rd hypothesis also can be prove using mathematical induction in a same manner. Hence proofs for 2nd and 3rd is not discussed here

Proof by mathematical induction

Assumption: Every vertex in a T graph has even degree

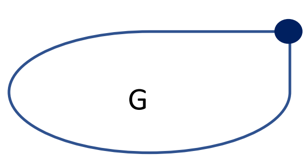
Base case (n=1)



For base case, it can easily be seen that every vertex has a degree of 2 which is even degree. Hence true for n=1.

Inductive hypothesis

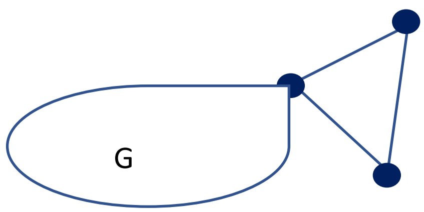
Assume any T graph with n = k triangles has all vertex with even degrees.



Inductive step:

Let n = k+1

When another triangle is joined to existing the T graph (G) both, the T-Graph and the triangle share one node. Hence resultant T-Graph appears as below



If the vertex V_{k+1} is the common vertex after joining triangle, its vertex degree increases by 2

If Vk = 2L

Vk+1 = 2L + 2 = 2(L+1) which is divisible by 2 implies vertex V_{k+1} has even degree.

Hence if n = k true, it is true for n = k+1 also

But the results is true for n = 1. Hence by mathematical induction, vertex degree of every vertex in a T graph is even.

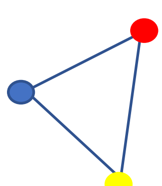
C) Hypothesis about the number of colours required to colour a T-Graph

Three different colours are sufficient colour any T graph.

Proof by mathematical induction

Assumption: any T graph can be coloured with 3 different colours

Base case (n = 1):

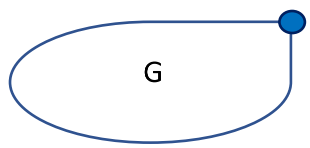


As it is seen in below, base triangle can be coloured using 3 colours.

Hence, the assumption is true for n = 1

Inductive hypothesis

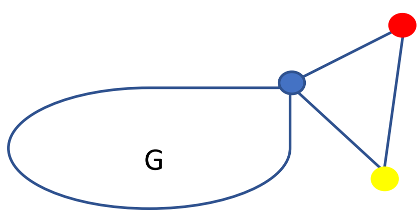
Assume any T graph with k (n = k) triangles can be coloured using 3 colours



Inductive step

Let n = k+1

As seen in below, when another triangle is joined to the T-Graph (G) through a common vertex, the common vertex is coloured using one colour. Hence the other two vertices of the newly added triangle can be colored using two different colours which are previously used to colour the T-Graph. Therefore, 3 colours are sufficient to colour the resultant T-Graph.



Hence if T-Graph with n triangles can be coloured using 3 colours then, T-Graph with n + 1 triangles can also be coloured using 3 colours. As the results is true for n = 1, by the principle of mathematical induction 3 colours are sufficient to colour any T-Graph.

Additional Problems

Problem 01- Application of T-Graphs

A girl bought 5 dark chocolates from a candy shop. The owner of that shop told her, that if she returns 2 chocolate covers of dark chocolate or return a dark chocolate cover while showing a milk chocolate cover, he will give a new milk chocolate. But to get a new milk chocolate, she can't show 2 milk chocolate covers at the same time.

What is the **maximum** number of milk chocolate covers that will be in her hand?

Problem 02 – State whether the statements are true or false

1. T-Graphs are always Euler circuits

2. Are following graphs T-Graphs or not?

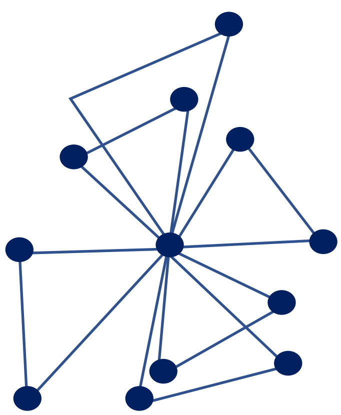


Figure.1

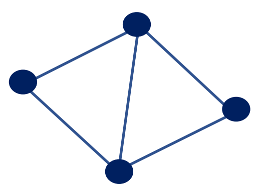


Figure.2

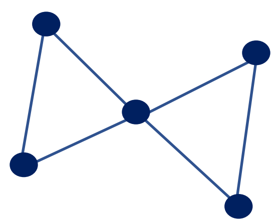


Figure.3

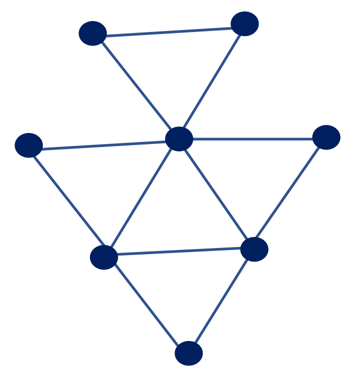


Figure.4

References

[1] Eric W. Weisstein. (2018) Wolfram MathWorld. [Online].
Available:http://mathworld.wolfram.com/Recursion.html

[2] Rapti de Silva(2018) Wk 2 Big Ideas in Graph Theory & Mathematical Thinking[Online].
Available: https://online.mrt.ac.lk/mod/resource/view.php?id=49945

[3] Wolfram Math World (2018). *Complete Graph* [Online].
Available: http://mathworld.wolfram.com/CompleteGraph.html

[4] Wolfram Math World (2018). *Triangle Graph* [Online].
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[5] Wolfram Math World (2018). *Cycle Graph* [Online].
Available: http://mathworld.wolfram.com/CycleGraph.html

[6] (2018, Jan) Wikipedia. [online]:

Available: http://en.wikipedia.org/wiki/Degree_%28graph_theory%29