MODULE 1 – NUMBER SYSTEM

1. A number when divided by 5 leaves a remainder of 4, when the double (i.e., twice) of that number is divided by 5 the remainder will be:

(a) 0 (b) 1 **(c) 3** (d) Cannot be determined

Solution:

Let the number be N and the quotient be Q
then N = 5Q + 4

If the number is doubled,
then 2N = 2(5Q + 4) = 10 Q + 8
2N = 5 x 2Q + 5 + 3
2N = 5(2Q + 1) + 3

Hence the remainder will be 3.

2. Find the unit's digit of the expression: $78^{5562} \times 56^{256} \times 97^{1250}$.

(a) 4 (b) 5 **(c) 6** (d) 7

Solution:

We can calculate the unit's digit of the exponential expression by looking at the patterns followed by 78, 56 and 97 when they are raised to high powers.
In fact, for the unit's digit of this expression we just need to consider the unit's digit of each part of the product.
A number (like 78) having 8 as the unit's digit yields the following unit's digits based on the powers it is associated with.

78^1 □ 8 78^5 □ 8
78^2 □ 4 78^6 □ 4
78^3 □ 2 78^7 □ 2
78^4 □ 6 78^8 □ 6
8^{4n+1} □ 8
8^{4n+2} □ 4

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Hence 78^{5562} yields 4 as the unit's digit

Similarly, 56^1 □ 6
56^2 □ 6
56^3 □ 6
Hence 56^{256} will yield 6 as the unit's digit

Similarly, 97^1 □ 7
97^2 □ 9
97^3 □ 3
97^4 □ 1
7^{4n+1} □ 7
7^{4n+2} □ 9
Hence, 97^{1250} will yield 9 as the unit digit.

Hence, the required units digit is given by 4 x 6 x 9 □ **6**

3. A number 'A', when divided by 'D', leaves the remainder 18 and if another number 'B' is divided by the same divisor 'D' it leaves the remainder 11. Further, if we divide A + B by 'D' then we obtain the remainder 4. Then the common divisor 'D' is:

(a) 21 (b) 22 (c) 15 **(d) 25**

Solution:
AD □ Remainder 18
BD □ Remainder 11
A+BD □ Remainder 29, but the remainder is 4
Hence, the divisor = (29 – 4) = 25

4. What is the remainder when 3^{256} is divided by 13?

(a) 1 **(b) 3** (c) 9 (d) 5

Solution:
Let us observe the pattern when 3^n is divided by 13.

$\text{Rem}\left(\frac{3^1}{13}\right)=3$, $\text{Rem}\left(\frac{3^2}{13}\right)=9$

$\text{Rem}\left(\frac{3^3}{13}\right)=1$, $\text{Rem}\left(\frac{3^4}{13}\right)=3$

We can observe that the pattern repeats with a cycle of 3.
Now, $256 = 3 \times 85 + 1$

$\therefore \text{Rem}\left(\frac{3^{256}}{13}\right)=\text{Rem}\left(\frac{3^1}{13}\right)=3$

5. A natural number N when successively divided by 4, 5 and 6 leaves remainders of 2, 4 and 5 respectively. What is the sum of the remainders obtained when N is successively divided by 12 and 10?

(a) 19 (b) 20 (c) 10 (d) Cannot be determined

Solution:

Divisors → 4 5 6

Remainders → 2 4 5

Let the last quotient = k (k is a natural number)

Then $N = [(6k + 5)5 + 4]4 + 2 = 120k + 118$

When N is divided by 12 remainder = 10 and quotient = $10k + 9$

When $(10k + 9)$ is divided by 10 the remainder = 9

∴ The sum of the remainders = $10 + 9 = 19$

6. Find the reminder when $3^{2007} + 7^{2007}$ is divided by 8.

(a) 1 **(b) 2** (c) 3 (d) None of these

Solution:

3^2 is divided by 8, rem = 1

$3^{2007} = (32)^{1003} * 3 = (1)^{1003} * 3 = \text{rem} = 3$

7^1 is divided by 8, rem = -1

7^{2007} is divided by 8, rem = $(-1)^{2007} = -1$

Remainder when $3^{2007} + 7^{2007}$ divided by 8 is 3-1= 2.

Thus, the right answer is 2.

7. Sum of 'n' consecutive integers is 900 less than the next 'n' consecutive integers. Find 'n'.

(a) 30 (b) 60 (c) 90 (d) Data insufficient

Solution:

Each integer in the second group is 'n' more than its corresponding integer in the first one.
For example, say the first group of 'n' consecutive integers started from 1 to n.
Then the next group would start from 'n+1' to '2n' where each number is 'n' more than its counterpart.
This means each element in the second group is 'n' more than the corresponding element in the first group.
Thus the difference of sum of the two groups = $n*n = n^2 = 900$
Thus n = 30.

8. How many prime numbers exist in $6^7 \times 35^3 \times 11^{10}$?

(a) 30 (b) 29 (c) 27 (d) 31

Solution:

$= (2 \times 3)^7 \times (5 \times 7)^3 \times 11^{10}$

$= 2^7 \times 3^7 \times 5^3 \times 7^3 \times 11^{10}$

Thus, there are (7 + 7 + 3 + 3 + 10) = 30 prime numbers.

9. Find the number of factors of 9321.

(a) 3 (b) 6 **(c) 8** (d) 16

Solution:

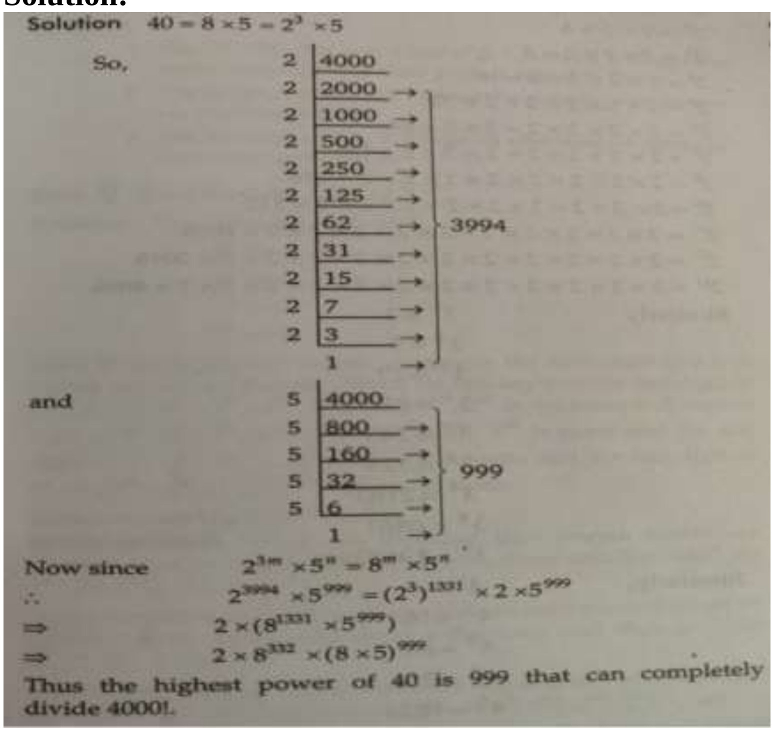
For a natural number N, expressed as the product of prime numbers by prime factorization method, as-

N = $X^a + Y^b + Z^c$ (where X, Y, Z are prime numbers and a, b, c is their respective powers)
The number of factors of N is (a + 1) * (b + 1) *(c + 1)
Here, 9321 can be expressed as-
$9321 = 3^1 * 13^1 * 239^1$
Hence, number of factors of 9321 = (1 + 1) * (1 + 1) * (1 + 1) = 2 * 2 * 2 = **8**

10. What is the rightmost integer of the expression $65776^{759} + 54697^{467}$.
(a) 3 (b) 5 (c) 7 **(d) 9**

Solution:
Last digit for the power of 6 is 6 (always)
Power cycle of 7 is 7, 9, 3, 1.
Now 467/4 gives a remainder of 3
Then the last digit is $7^3 = 3$
Last digit is 6 + 3 = 9

11. Find the highest power of 40 which can completely divide 4000!
(a) 9 (b) 99 **(c) 999** (d) 9999

Solution:

Solution $40 = 8 \times 5 = 2^3 \times 5$

So,

2	4000
2	2000 →
2	1000 →
2	500 →
2	250 →
2	125 →
2	62 →
2	31 →
2	15 →
2	7 →
2	3 →
	1 →

$\} 3994$

and

5	4000
5	800 →
5	160 →
5	32 →
5	6 →
	1 →

$\} 999$

Now since $2^{3m} \times 5^n = 8^m \times 5^n$
∴ $2^{3994} \times 5^{999} = (2^3)^{1331} \times 2 \times 5^{999}$
⇒ $2 \times (8^{1331} \times 5^{999})$
⇒ $2 \times 8^{332} \times (8 \times 5)^{999}$
Thus the highest power of 40 is 999 that can completely divide 4000!.

12. How many ways can 1146600 be written as the product of two factors?
(a) 100 (b) 108 (c) 216 (d) 273

Solution:
$1146600 = 2^3 * 3^2 * 5^2 * 7^2 * 13$
Thus the total number of factors of
1146600 = (3+1) (2+1) (2+1) (2+1) (1+1)

= 4 * 3 * 3 * 3 * 2
= 216
So the number of ways in which 1146600 can be expressed as a product of two factors = 216/2 = 108

13. The sum of all four digit numbers which are divisible by 7 is
(a) 7071071 (b) 7^7 (c) 7107073 (d) 10019996

Solution:
The first four-digit number which is divisible by 7 is 1001 and the last four digit number is 9996. Thus the total numbers between 1001 and 9996 are (9996-1001)/ 7 = 89957 + 1=1286
The sum of n terms of an arithmetic progression when first, last and tota number of terms are known is,
Sn = n/2(First term + Last term)
Sn =1286/2 (9996+10012) = 7071071
[Since all these numbers are in A.P. with common difference 7]

14. The number of zeros at the end of 100! is
(a) 36 (b) 18 **(c) 24** (d) 10

Solution:
In terms of prime factors 100! can be written as $2^a.3^b.5^c.7^d$....
Now we use the formula to determine the factorial number 100! and that is given by

$$E_2(100!) = \frac{100}{2} + \frac{100}{2^2} + \frac{100}{2^3} + \frac{100}{2^4} + \frac{100}{2^5} + \frac{100}{2^6} = 50 + 25 + 12 + 6 + 3 + 1 = 97$$

And

$$E_5(100!) = \frac{100}{5} + \frac{100}{5^2} = 20 + 4 = 24$$

Thus $(100!) = 2^{97}.3^b.5^{24}.7^d.... = 2^{73}.3^b.(2 \times 5)^{24}.7^d... = 2^{73}.3^b.(10)^{24}.7^d..$
So the number of zeros at the end of 100! are 24.

<u>Alternate solution</u>: By default in any factorial value that is bigger than 5!, the number of 2s will be more than that of the 5s. Since trailing zeros(A sequence of zeros in the decimal representation of a number, after which there are no digits follow) are just the number of 5x2 combinations, we need to go with the number whichever is lesser in count.

So, 100/5 = 20
since 20 is still bigger than the 5, again divide it by 5, until the dividend becomes lesser than that of the 5

20/5 = 4; Hence the number of 5's will be 24. Therefore the number of zeros at the end of 100! will be 24.

15. The sum of all the factors of 45000 which are exactly the multiples of 10 is:
(a) 152295 **(b) 141960** (c) 600 (d) None of these

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Solution:
Since 45000 = 10 * 4500
So, the sum of all the factors of 45000 which are the multiples of 10 will be the same as the sum of all the factors of 4500 multiplied by 10.
$4500 = 2^2 * 3^2 * 5^3$

Now, Therefore the sum of all the factors of
4500 = ((2 ^ 3 - 1) *(3 ^ 3 - 1)(5 ^ 4 - 1))2-1(3-1)(5-1)
= 7 * 26 * 6241 * 2 * 4 = 14196

So, the sum of all the factors of 45000 which are the multiples of 10 will be equal to 14196x10= 141960

HOMEWORK:

1. Total number of digits in the product of $4^{1111} * 5^{2222}$ is:
(a) 3333 (b) 2223 (c) 2222 (d) Can't be determined

Solution:
$4^{1111} * 5^{2222} = 2^{2222} * 5^{2222} = 1 * 10^{2222}$
Hence there will be 2223 digits in the product.

2. If p = N + 5 when N is the product of any three consecutive positive integers. Then :
(a) p is prime (a) p is odd (c) p is divisible by 6 (d) either of (b), (c)

Solution:
Since N is always even. Thus p is always odd
[As even + odd = odd]

3. What is the least number which must be multiplied to 5400 to get a perfect square?
(a) 2 (b) 3 (c) 6 (d) 10

Solution:
$5400 = 2^3 \times 3^3 \times 5^2$
To get a perfect square, there must be an even number of powers of each prime factor.
So the least such perfect square= $2^3 \times 3^3 \times 5^2 \times 2 \times 3 = 32400$
Thus we have to multiply it by 6 in order to get a perfect square.

4. What is the remainder when the square of the smallest five-digit prime number is divided by 24?
(a) 1 (b) 2 (c) 3 (d) None of these

Solution:

Smallest five digit prime number will be of the form = $6k \pm 1$.

And its square is = $(6k \pm 1)^2 = 36k^2 \pm 12k + 1 = 12k(3k \pm 1) + 1$ Now $k(3k \pm 1)$ is always even whether k is even or odd.Hence first term is divisible by 24 and remainder of the number when it is divided by 24 is 1.

5. How many factors of 1080 are perfect squares?
(a) 4 (b) 6 (c) 8 (d) 5

Solution:
$1080 = 2^3 * 3^3 * 5$. For any perfect square, all the powers of the primes have to be even numbers. So, if the factor is of the form $2^a * 3^b * 5^c$.
The values 'a' can take are 0 and 2, b can take are 0 and 2, and c can take the value 0.
In total, there are 4 possibilities. 1, 4, 9, and 36
Hence the answer is 4.
1.