MODULE 8 – ALLIGATIONS AND MIXTURES

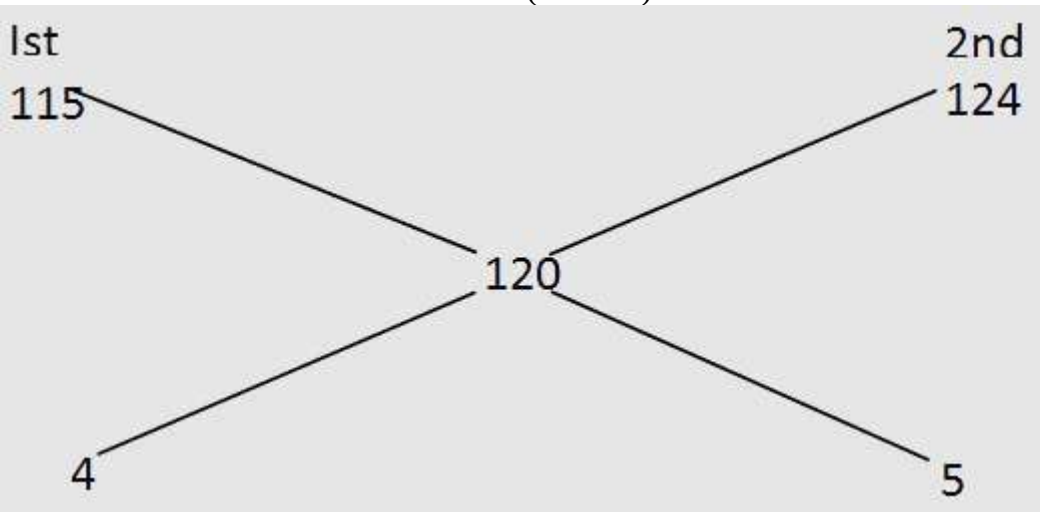
1. A 3:2 milk and water solution is mixed with another 4: 1 milk and water solution. If the volumes are 400 ml and 1,000 ml respectively, then what is the ratio of milk to water in the resultant solution?
(a) 9 : 5 (b) 26 : 9 (c) 5 : 26 (d) 8 : 21

Solution:
Milk in first mixture $M_1 = \frac{3}{5} * 400 = 240$ ml
Water in first mixture $W_1 = 400 – 240 = 160$ ml
Milk in first mixture $M_2 = \frac{4}{5} * 1000 = 840$ ml
Water in second mixture $W_2 = 1000 - 800 = 200$ ml

Hence, total milk = 240 + 800 = 1040 ml
Total water = 160 + 200 = 360 ml
Hence, milk: water = 1040 : 360 = 26 : 9

2. In what ratio must two kinds of sugar at Rs. 1.15 and Rs. 1.24 per kg be mixed so that by selling at Rs. 1.50 per kg, 25% may be gained?
(a) 4:5 (b) 5:4 (c) 1:1 **(**d) 2:3

Solution:
SP = 1.50 Rs. Profit = 25%. CP = 1.50 x (100/125) = 1.20Rs.



3. A thief steals four gallons of liquid soap kept in a train compartment's bathroom from a container that is full of liquid soap. He then fills it with water to avoid detection. Unable to resist the temptation he steals 4 gallons of the mixture again, and fills it with water. When the liquid soap is checked at a station it is found that the ratio of the liquid soap now left in the container to that of the water in it is 36 : 13. What was the initial amount of the liquid soap in the container if it is known that the liquid soap is neither used nor augmented by anybody else during the entire period?
(a)7 gallons (b)14 gallons (c)21 gallons (d)28 gallons

Solution:
Let initial quantity of liquid soap = x
We can solve this by the formula of compound depreciation.

$$x\left(1-\frac{a}{x}\right)^n,$$

$49(1-4/x)^2 = 36$
$(1 - 4/x)=6/7$
$1 - 6/7 = 4/x$
$1/7 = 4/x$
$x = 28$

4. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?
(a) 26 litres (b) 29.16 litres (c) 28 litres (d) 28.2 litres

Solution:

Suppose a container contains x units of a liquid from which y units are taken out and replaced by water. After n operations, quantity of pure liquid
$= x\left(1-\frac{y}{x}\right)^n$ units.

milk contained by the container now

$$= 40\left(1-\frac{4}{40}\right)^3$$

$$= 40\left(1-\frac{1}{10}\right)^3$$

$$= 40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

$$= \frac{4 \times 9 \times 9 \times 9}{100}$$

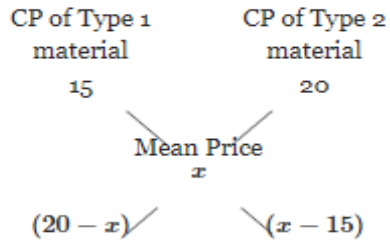
$$= 29.16$$

5. The cost of Type 1 material is Rs. 15 per kg and Type 2 material is Rs.20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then what is the price per kg of the mixed variety of material?
(a) Rs. 19 (b) Rs. 16 (c) Rs. 18 (d) Rs. 17

Solution:

Cost Price(CP) of Type 1 material is Rs. 15 per kg
Cost Price(CP) of Type 2 material is Rs. 20 per kg
Let Cost Price(CP) of resultant mixture be Rs.x per kg

By rule of alligation,



=> Type 1 material : Type 2 material $= (20-x) : (x-15)$

Given that Type 1 material : Type 2 material = 2 : 3
$\Rightarrow (20-x) : (x-15) = 2 : 3$
$\Rightarrow \frac{(20-x)}{(x-15)} = \frac{2}{3}$
$\Rightarrow 3(20-x) = 2(x-15)$
$\Rightarrow 60 - 3x = 2x - 30$
$\Rightarrow 90 = 5x$
$\Rightarrow x = \frac{90}{5} = 18$

=> price per kg of the mixed variety of material = Rs.18

6. A dishonest milkman sells his milk at cost price, but he mixes it with water and thereby gains 25%. What is the percentage of water in the mixture?
(a) 25% (b) 20% (c) 22% (d) 24%

Solution:

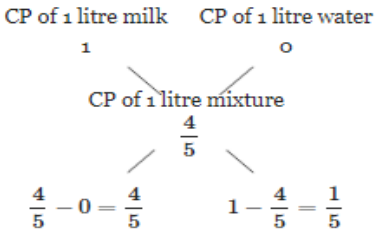
Let CP of 1 litre milk = Rs.1
SP of 1 litre mixture = CP of 1 litre milk = Rs.1
Gain = 25%

Hence CP of 1 litre mixture

$$= \frac{100}{(100 + \text{Gain}\%)} \times \text{SP}$$

$$= \frac{100}{(100 + 25)} \times 1 = \frac{100}{125} = \frac{4}{5}$$

By rule of alligation,



=> Quantity of milk : Quantity of water $= \frac{4}{5} : \frac{1}{5} = 4 : 1$

Hence percentage of water in the mixture $= \frac{1}{5} \times 100 = 20\%$

Alternate Solution:

Profit percentage must be written in terms of fraction which give ¼ in this case.

In this case the numerator is the quantity of water because profit is always for the amount of water added. Denominator is the quantity of the milk.
If we add both numerator and denominator we get the total quantity. Out of the total quantity 5, 1 portion is water. Now, this can be written in percentage as 20%

7. Some amount out of Rs.7000 was lent at 6% per annum and the remaining was lent at 4% per annum. If the total simple interest from both the fractions in 5 years was Rs. 1600, the sum lent at 6% per annum was
(a) Rs. 2000 (b) Rs. 2200 (c) Rs. 2400 (d) Rs. 1800

Solution:

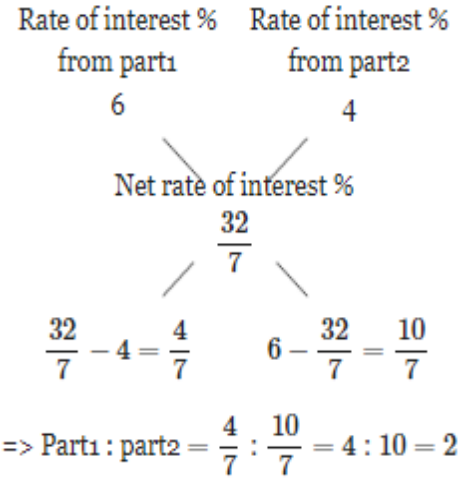
Total simple interest received , I = Rs.1600
Principal , p = 7000
period, n = 5 years
Rate of interest, r = ?

Simple Interest, $I = \frac{pnr}{100}$

$\Rightarrow 1600 = \frac{7000 \times 5 \times r}{100}$

$\Rightarrow r = \frac{1600 \times 100}{7000 \times 5} = \frac{160}{35} = \frac{32}{7}\%$

By rule of alligation,



=> Part1 : part2 $= \frac{4}{7} : \frac{10}{7} = 4 : 10 = 2 : 5$

Given that total amount is Rs.7000. Therefore, the amount lent at 6% per annum (part1 amount)

$= 7000 \times \frac{2}{7} = \text{Rs. } 2000$

8. A chemist mixes two liquids 1 and 2. One litre of liquid 1 weighs 1 kg and one litre of liquid 2 weighs 800 gm. If half litre of the mixture weighs 480 gm, then the percentage of liquid 1 in the mixture, in terms of volume, is

(a) 80 (b) 85 (c) 70 (d) 75

Solution:

In Liquid L1 - 1L = 1000 grams
So, 1000 mL = 1000 grams
1 mL = 1 gram
x mL = x grams
In Liquid L2 - 1L = 800 grams
1000 mL = 800 grams
1mL = 0.8 grams
(500-x) mL = (500-x) x 0.8 grams
Total mass = x + 400 - 0.8x = 480 grams

0.2x = 80 grams
x = 400 grams
Therefore, Liquid 1 has 400 mL and Liquid 2 has 500 - 400 = 100 Ml
Therefore, Percentage of Liquid 1 = (400/500) x 100 = 80%

9. The strength of a salt solution is p% if 100 ml of the solution contains p grams of salt. Each of three vessels A, B, C contains 500 ml of salt solution of strengths 10%, 22%, and 32%, respectively. Now, 100 ml of the solution in vessel A is transferred to vessel B. Then, 100 ml of the solution in vessel B is transferred to vessel C. Finally, 100 ml of the solution in vessel C is transferred to vessel A. The strength, in percentage, of the resulting solution in vessel A is
(a) 15 (b) 12 (c) 13 (d) 14

Solution:
Vessels A, B and C contains salt solution of strengths 10%, 22% and 32% respectively
It is also given that the amount of salt solution = 500 ml
So, Vessels A, B and C contains salt of 50 grams, 110 grams and 160 grams respectively
100 ml of Solution is transferred from A to B:
A would have 400 ml, B would have 600 ml of solution
Amount salt from A which is transferred to B = (Initial salt amount/5) = 10 grams
So, Total salt in B = 110 + 10 = 120 grams (After first transfer)
Total salt in A = 40 grams (After first transfer)
Now, 100 ml from Vessel B is transferred to Vessel C
So similarly, (1/6)th of salt would transfer from B to C
Total Salt in B = 120 - 20 = 100 grams (After second transfer)
Total Salt in C = 160 + 20 = 180 grams (After second transfer)
Now, 100 ml from Vessel C is transferred to Vessel A
So similarly, (1/6)thof salt would transfer from C to A
Total Salt in C = 180 - 30 = 150 grams (After third transfer)
Total Salt in A = 40 + 30 = 70 grams (After third transfer)
So, Vessel A contains 70 grams Salt in 500 ml solution
Strength of Salt Solution in Vessel A = 14%

10. A wholesaler bought walnuts and peanuts, the price of walnuts per kg being thrice that of peanuts per kg. He then sold 8 kg of peanuts at a profit of 10% and 16 kg of walnuts at a profit of 20% to a shopkeeper. However, the shopkeeper lost 5 kg of walnuts and 3 kg of peanuts in transit. He then mixed the remaining nuts and sold the mixture at Rs. 166 per kg, thus making an overall profit of 25%. At what price, in Rs. per kg, did the wholesaler buy the walnuts?
(a) 84 (b) 86 (c) 96 (d) 98

Solution:
Let the wholesaler bought walnuts and peanuts at Rs 3x and Rs x respectively
He sold 8kgs of Peanuts to the shopkeeper at 10% Profit
Cost price of the Peanuts bought by the shopkeeper = Rs. 1.1x per kg
Similarly, he sold 16 kgs of Walnuts to the shopkeeper at 20% Profit
Cost price of the Walnuts bought by the shopkeeper = 1.2 × 3x = Rs. 3.6x per Kg
He lost 5 kgs of Walnuts and 3 Kgs of Peanuts in transit
Remaining = 16 - 5 = 11 kgs of Walnuts & 8-3 = 5 kgs of Peanuts
He mixes them together and sells them at Rs. 166 per kg, making an overall Profit of 25%
Selling Price = 5/4 × Cost Price

Overall Cost Price = Rs.(3.6 × 16x + 1.1 × 8x)
Overall Selling Price = Rs. 16 × 166
Overall Selling Price = (5/4) x Overall Cost Price
(3.6 × 16x + 1.1 × 8x) × (5/4) = 16 × 166
((57.6 × 8.8) x) × 5/4 = 16 × 166
x = Rs. 32
Cost price per Kg of Walnuts bought by the Wholesaler = Rs. 3x = 3 × 32 = >Rs. 96

11. A sample of x litres from a container having a 60 litre mixture of milk and water containing milk and water in the ratio of 2 : 3 is replaced with pure milk so that the container will have milk and water in equal proportions. What is the value of x?
(a) 6 litres (b) 10 litres (c) 30 litres (d) None of these

Solution:
Let us solve this question by back substituting answer choices.

The rigorous method of solving is outlined in the previous question on removing and replacing mixtures.
The mixture of 60 litres has milk and water in the ratio 2 : 3

i.e., 24 litres of milk and 36 litres of water. When x litres of the mixture is removed, 0.4 x litres of milk and 0.6 x litres of water are removed from it.
Take choice (2). According to this choice, x = 10.

So, 10 litres of the mixture is removed.

4 litres of milk and 6 litres of water are removed.
Therefore, there will be 20 litres of milk and 30 litres of water in the container.

Subsequently, when 10 litres of milk is added, the mixture will contain 30 litres of milk and 30 litres of water - i.e. milk and water are in equal proportion.

12. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask originally hold?
(a) 30 litres (b) 26 litres (c) 24 litres (d) 32 litres

Solution:
Let initial quantity of wine = x litre

After a total of 4 operations, quantity of wine
$= x\left(1-\frac{8}{x}\right)^4 = x\left(1-\frac{8}{x}\right)^4$

Given that after a total of 4 operations, the ratio of the quantity of wine left in cask to that of water = 16 : 65

$\Rightarrow \frac{x\left(1-\frac{8}{x}\right)^4}{x} = \frac{16}{81}$
$\Rightarrow \left(1-\frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4$
$\Rightarrow \left(1-\frac{8}{x}\right) = \frac{2}{3}$
$\Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3}$
$\Rightarrow 3x - 24 = 2x$
$\Rightarrow x = 24$

13. Two similar vessels are filled with the mixtures of water and milk in the ratio of 3: 4 and 5:3 respectively. If the mixtures are poured into a third vessel, the respective ratio of water and milk in the third vessel will be
(a) 15:12 (b) 53:59 (c) 20:9 (d) 59:53

Solution:
Let capacity of each vessel be x litres.
Then, amount of water and milk in vessel one = 3/7x and 4/7x
Amount of water and milk in vessel two = 5/8x and 3/8x

Total amount of water = $37\ x + 58\ x = 24+3556\ x = 5956\ x$

Total amount of milk= $47\ x + 38\ x = 32+2156\ x = 5356\ x$

Ratio of water and milk = 59/56x / 53/56x = 59/53 = 59:53

14. In 50 L of water and milk mixture; water is 20%. A milkman gives 5 L of this mixture to a customer and then he adds 10 L of pure water in the remaining mixture. The percentage of water in the final mixture is:
(a) 38% (b) 34.54% (c) 20% (d) 46%

Solution:
In 50 L of water and milk mixture; water is
20%. A milkman gives 10 L of this mixture to a customer and then he adds 10 L of pure water in the remaining mixture.

Calculation:
Total mixture is 50 L
20% water
⇒ 50 x 20% = 10 L water

Milk = 50 – 10 = 40 L

Ratio of milk and water
□ 40:10
□ 4:1

5L mixture taken out, It means
□5 × 4/5 = 4L

4 L milk and 1 L water is taken out
Remaining Milk and water
Milk = 40 - 4 = 36
Water = 10 – 1 = 9

Then Add 10 L water, Then new water
□ 9 +10 = 19L

Percentage of water in the mixture
□ (19/55) * 100 = 34.54%

15. A milkman brings 100 litres of pure milk from a dairy farmer and he sells 10 litres of it to the first customer, then he refills his vessel by adding 10 litres of water. After this, he proceeds to the next house and sells 10 litres of it to the second customer and then he refills his vessel again by adding 10 litres of water. Thus, every time he sells 10 litres of milk - pure or impure - he keeps on replacing it with 10 litres of pure water.
Maximum how many customers can get at least 50% milk in the mixture that they purchase from this milkman?
(a) 5 (b) 6 (c) 7 (d) None of these

Solution:
After the first customer, who gets absolutely pure milk, every next customer gets less pure milk than the previous customer. The purity of milk for the nth customer who receives only impure milk can be calculated as follows.
$50 >= 100 * (1 - 10/100)^n$

The highest value of n that satisfies the above relation is 6. Therefore, we have 6 customers who get milk, which is less than 100% pure but more than 50% pure. However, there is one customer who gets 100% pure milk. So, there are actually 7 customers who get more than 50% pure milk.

Hence, choice (c) is the correct one.

Alternatively
We can do it manually, if we are really good at calculation.

The First customer gets 100% pure milk.
The Second customer gets 90% pure milk.
The Third customer gets 81% pure milk.
The Fourth customer gets 72.9% pure milk.
The Fifth customer gets 65.61% pure milk.
The Sixth customer gets 59.05% pure milk.
The Seventh customer gets 53.14% pure milk.
The eighth customer gets 47.83% pure milk.
Thus, we see that a maximum of 7 customers get at least 50% pure milk.

HOMEWORK:

1. In a solution of 35 litres, the respective ratio of milk and water is 4:1. If 14 litres of water is added to the solution, then the ratio of milk and water in the resulting solution will be:
(a) 3:1 (b) 4:3 (c) 3:4 (d) 4:3

Solution:
Initial quantity of milk in the solution = 4/5 * 35 = 28 litres
Initial quantity of water in solution = 35 – 28 = 7 litres
Hence, required ratio = 28 : (7+14) = 28 : 21 = 4 : 3

2. 3 litres of water is added to 11 litres of a solution containing 42% of alcohol in the water. The percentage of alcohol in the new mixture is?
(a) 25% (b) 20% (c) 30% (d) 33%

Solution:

We have a 11 litre solution containing 42% of alcohol in the water.

=> Quantity of alcohol in the solution $= \frac{11 \times 42}{100}$

Now 3 litre of water is added to the solution.

=> Total quantity of the new solution = 11 + 3 = 14

$$\text{Percentage of alcohol in the new solution} = \frac{\frac{11 \times 42}{100}}{14} \times 100$$

$$= \frac{11 \times 3}{100} = 33\%$$

3. In a 729 litres mixture of milk and water, the ratio of milk to water is 7: 2. To get a new mixture containing milk and water in the ratio 7:3, the amount of water to be added is

(a) 81 litres (b) 71 litres (c) 56 litres (d) 50 litres

Solution:

Let the amount of water to be added be x litres.

Quantity of water in 1st mixture

□ 27+2 * 729 = 162 litres

Quantity of milk in 1st mixture

□ 77+2 * 729 = 567 litres

Therefore, 162+x 567 = 37

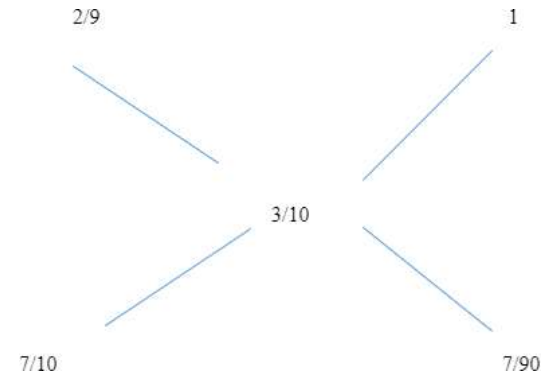
□ 1134 + 7x = 1701

□ 7x = 1701 – 1134

x = 567/7 = 81 litres

Alternative method:

Using Alligation



□ 9:1

Hence, water to be added = 729* 1/9 = 81 litres

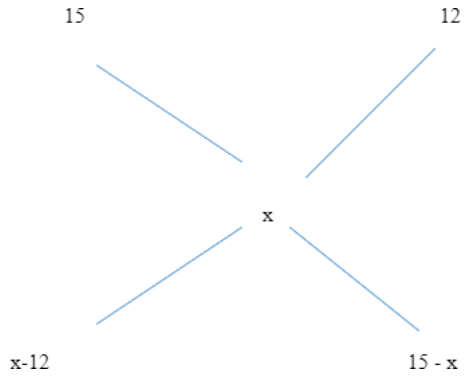
4. Mohan bought 12 kg mango at the rate of Rs.15 per kg and 18 kg mango at the rate of Rs. 12 per kg. Now both varieties are mixed and sold for Rs. 16.50 per kg. What was his total percentage gain?

(a) 20% (b) 25% (c) 162% (d) 33 1/3%

Solution:

Let mean price of mixture be Rs. x per kg

Then



□ x-1215-x = 1218

□ x = 13.2

Therefore, Total Percentage gain = 16.5-13.213.2 * 100

□ 3.313.2 * 100 = 25%

5. Some amount out of 6,000 was lent at 9% per annum and the remaining was lent at 5% per annum. If total simple interest from both amounts in 3 years was Rs. 1,350, the sum lent on 9% per annum was

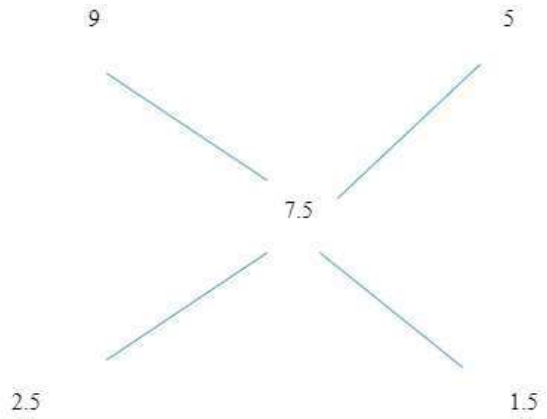
(a) Rs. 3,000 (b) Rs. 2,250 (c) Rs. 3,750 (d) Rs. 4,500

Solution:

Effective interest rate per annum

R = 1350*1006000*3 = 7.5%

Using Alligation on rate percentage



Ratio of amount lent on 9% and 5% per annum = 5 : 3

Hence, amount on 9% per annum = $\frac{5}{8} * 6000$ = Rs. 3,750