MODULE 3 – SIMPLIFICATION

1. A gentleman decided to treat a few children in the following manner. He gives half of his total stock of toffees and one extra to the first child, and then the half of the remaining stock along with one extra to the second and continues giving away in this fashion. His total stock exhausts after he takes care of 5 children. How many toffees were there in his stock initially?

(a) 65 (b) 62 (c) 60 (d) 70

Solution:

Let 'n' be the initial number of toffees.
Number of toffees received by 1st child = n/2+1
Remaining toffees = n/2 – 1

Number of toffees received by 2nd child = n/4 - ½ + 1
Remaining toffees = n/4 - 1/2 - 1

Number of toffees received by 3rd child = n/8 - 1/4 - 1/2 + 1
Remaining toffees = n/8 - 1/4 - 1/2 - 1

Number of toffees received by 4th child = n/16 - 1/8 - 1/4 - 1/2 + 1
Remaining toffees = n / 16 - 1/8 - 1/4 - 1/2 - 1

Number of toffees received by 5th child = n/32 - 1/16 - 1/8 - 1/4 - 1/2 + 1
Remaining toffees = n/32 - 1/16 - 1/8 – 1/4 - 1/2 -1

According to problem,
n/32 - 1/16 - 1/8 - 1/4 - 1/2 - 1 = 0
$\Rightarrow$ n/32 = 31/16
$\Rightarrow$ n = 62

2. Aron bought some pencils and sharpeners. Spending the same amount of money as Aron, Aditya bought twice as many pencils and 10 less sharpeners. If the cost of one sharpener is 2 more than the cost of a pencil, then the minimum possible number of pencils bought by Aron and Aditya together is

(a) 30 (b) 27 (c) 33 (d) 36

Solution:

Let the number of pencils and sharpeners bought by Aron are 'p' and 's' respectively.

	No. of pencils	No. of sharpeners
Aron	p	S
Aditya	2p	s -10

Let the cost of one pencil be Rs. 'x'

According to the question,

px + (x + 2) s = 2px + (s – 10) (x + 2)

$\Rightarrow$ px = (x + 2) s - (s - 10)(x + 2)

$\Rightarrow$px = sx +2s - [sx + 2s - 10x - 20]

$\Rightarrow$ px = sx + 2s - sx - 2s + 10x + 20

$\Rightarrow$x = 20p-10

Clearly, the value of 'p' should be more than 10.

The minimum possible number of pencils bought by Aron and Aditya together

= p + 2p = $3p_{mn}$

Now, check options:

1st option is 30 $\Rightarrow$ p = 10 so, not possible.

Similarly, the 2nd option is also not possible.

Both 3rd and 4th options are possible but the minimum value of p, we get from option 3

i.e., 3p = 33

p = 11

3. Students in a college have to choose at least two subjects from chemistry, mathematics and physics. The number of students choosing all three subjects is 18, choosing mathematics as one of their subjects is 23 and choosing physics as one of their subjects is 25. The smallest possible number of students who could choose chemistry as one of their subjects is

(a) 20 (b) 19 (c) 22 (d) 21

Solution:

Now 23 students choose maths as one of their subjects.

This means (MPC) + (MC) + (MP)=23 where MPC denotes students who choose all the three subjects maths, physics and chemistry and so on.

So, MC+ MP = 5. (Math)

Similarly, we have PC + PM = 7. (Physics)

We have to find the smallest number of students choosing chemistry. For that in the first equation let PM = 5 and MC = 0.

In the second equation this PC = 2. Hence, the minimum number of students choosing chemistry will be (18 + 2) = 20. Since 18 students chose all the three subjects.

4. While multiplying three real numbers, Ashok took one of the numbers as 73 instead of 37. As a result, the product went up by 720. Then the minimum possible value of the sum of squares of the other two numbers is

(a) 20 (b) 40 (c) 22 (d) 42

Solution:

Let the other two numbers are x and y

$\therefore$ 73 xy – 37 xy = 720

$\Rightarrow$ xy = 20

$x = \sqrt{20}$ and $y = \sqrt{20}$ will satisfy the above the equation and give the minimum value of $x^2 + y^2$.

So, $x^2 + y^2 = (\sqrt{20})^2 + (\sqrt{20})^2 = 40$

5. A red light flashes three times per minute and a green light flashes five times in 2 minutes at regular intervals. If both lights start flashing at the same time, how many times do they flash together in each hour?

(a) 30 (b) 24 (c) 20 (d) 60

Solution:

A red light flashes three times per minute and a green light flashes five times in 2 min at regular intervals.
So red light flashes after every 1/3 min and green light flashes every 2/5 min.
LCM of both the fractions is 2 min.

Hence they flash together after every 2 min. So in an hour they flash together 30 times.

6. In a call centre at New Delhi, it is observed that it gets a call at an interval of every 10 minutes from California, at every 12 minutes from Texas, at the interval of 20 minutes from Washington DC and after every 25 minutes it gets the call from London. If in the early morning at 5:00 a.m. it has received the calls simultaneously from all the four destinations, then at what time will it receive the calls simultaneously from all the places on the same day?

(a) 10:00 a.m. (b) 3:00 a.m.
(c) 5:00 p.m. (d) both (a) and (b)

Solution:

LCM of 10, 12, 20 and 25 = 300 minutes = 5 hours
The minimum time interval when the call centre receives the calls from all the destinations at the same time Thus 5 + 5 = 10A.M

7. A diamond expert cuts a huge cubical diamond into 960 identical diamond pieces in a minimum number of 'n' cuts. If he wants to maximise the number of identical diamond pieces making same number of n cuts to it, so the maximum number of such diamond pieces are:

(a) 1000 (b) 1331 (c) 1200 (d) none of (a), (b), (c)

Solution:

Since a cube has to be cut from three sides (i.e., along 3 dimensions). Hence the number of cuts will be equal to the sum of the 3 factors of 960, when 960 is expressed as the product of 3 factors.

Again since the number of cuts to be applied are minimum. Hence the sum of these 3 factors of 960 must be minimum. So, in order for the sum of 3 factors of 960 to be minimum we have to have the minimum possible difference between the 3 factors of 960. Thus

960 = 8 * 10 * 12

Hence the minimum possible number of cuts

= (7 + 9 + 11) = 27

Now, if we want to maximise the product of any 3 factors whose sum is constant i.e., 27, then it is possible only when all the 3 factors are equal. Thus a, b, c be the three factors such that:

a + b + c = 27

Then Maximum (1* b* c)= 10 * 10 * 10 = 1000 So, the maximum number of 1000 identical pieces can be formed by applying the same number of cuts.

8. A typist starts to type the serial numbers of candidates in a list, upto 500. Minimum how many times does he need to press the keys of numerals only?
(a) 1389 (b) less than 1000 (c) 1392 (d) can't say

Solution:
For one digit number 1 × 9 = 9 for two digit numbers 2 * 90 = 180 for three digit numbers upto 500 = 3 * 401 = 1203 Thus the required value = 9 + 180 + 1203 = 1392

9. A man sells chocolates which are in the boxes. Only either a full box or half a box of chocolates can be purchased from him. A customer comes and buys half the number of boxes which the seller had plus half a box more. A second customer comes and purchases half the remaining number of boxes plus half a box. After this the seller is left with no chocolate boxes. How many chocolate boxes did the seller have, initially?
(a) 2 (b) 3 (c) 4 (d) 3.5

Solution:

The best way to go through the options

Let there are initially 3 boxes then,

1st customer gets = $\frac{3}{2} + \frac{1}{2} = 2$

Remaining boxes = 3 - 2 = 1

2nd customer = $\frac{1}{2} + \frac{1}{2} = 1$

So, option B is correct.

10. In a soap company a soap is manufactured with 11 parts. For making one soap you will get 1 part as scrap. At the end of the day you have 251 such scraps. From that how many soaps can be manufactured?

a)25 b)20 c)24 d)22

Solution:

Using these 251 scaps, we can make 251/11 = 22 soaps. Leaving 9 scraps unused and producing 22 additional scraps.

With these 22+9 = 31 scraps, another two soaps can be made. Leaving 9 scraps unused and producing additional 2 scraps.

with these 11 scraps, another soap can be made. Hence the total number of soaps that can be made is 22 + 2 + 1 = 25

11. A man spends 2/5th of his salary on house rent, 3/10th of his salary on food and 1/8th of his salary on conveyance. If he has Rs.1400 left with him, find his expenditure on food and conveyance.

(a) Food – Rs. 2400, Conveyance – Rs. 1000
(b) Food – Rs. 2000, Conveyance – Rs. 1600
(c) Food – Rs. 1800, Conveyance – Rs. 1200
(d) Food – Rs. 1000, Conveyance – Rs. 2100

Solution:
Part of salary left = 1 - (2/5 + 3/10 + 1/8)
Let the monthly salary be Rs. x
Then, 7/40 of x = 1400
x = (1400 * 40/7)
= 8000
Expenditure on food = Rs. (3/10*8000) = Rs.2400
Expenditure on conveyance = Rs. (1/8*8000) = Rs.1000

12. In Somnath Temple there are some magical bells which toll 18 times in a day, simultaneously. But every bell tolls at a different interval of time, but not in a fraction of minutes. The maximum number of bells in the temple can be:
(a) 18 (b) 10 (c) 24 (d) 6

Solution:
Since these bells tolls 18 times in 24 hrs
So, the min. time interval when they toll together = 24 18 = 80 minutes
So, the required number of bells = total number of different factors of 80
Now since 80 = 2^4 x 5
Therefore, Total number of factors = (4 + 1) (1 + 1) = 10
Thus, the maximum number of bells = 10

13. When an amount was distributed among 14 boys, each of them got Rs. 80 more than the amount received by each boy when the same amount is distributed equally among 18 boys. What was the amount?
(a) Rs. 5040 (b) Rs. 5820 (c) Rs. 5802 (d) Rs. 3920

Solution:
Let the amount distributed to each individual be x Rs.
When the amount was distributed to 14 boys; amount received by each boy = x / 14
When the amount was distributed to 18 boys; amount received by each boy = x /18
According to question,

$$\frac{x}{14} - \frac{x}{18} = 80$$

$$\Rightarrow \frac{18x - 14x}{18 \times 14} = 80$$

$$\Rightarrow \frac{4x}{18 \times 14} = 80$$

$$\Rightarrow x = \frac{18 \times 14 \times 80}{4}$$

$$\Rightarrow x = 5040$$

14. Three mangoes, four guavas and five watermelons cost Rs.750. Ten watermelons, six mangoes and nine guavas cost Rs.1580. What is the cost of six mangoes, ten watermelons and four guavas?
(a) 1280 (b) 1080 (c) 1180 (d) Cannot be determined

Solution:
3M+4G+5W=750 (1)
6M+9G+10W=1580 (2)

Adding the two equations we get:
9M+13G+15W=2330 □ (3)
Dividing this expression by 3 we get:
3M+4.33G+5W=776.666 □ (4)
(4) – (1) □ 0.33G = 26.666 □ G = 80
Now, if we look at the equation (1) and multiply it by 2, we get: 6M+8G+10W=1500. If we subtract the cost of 4 guavas from this we would get:
6M+4G+10W=1500-320=1180

Alternate solution:

3M+4G+5W=750 □ (1)
6M+9G+10W=1580 □ (2)

On doubling the equation (1) we get
6M+8G+10W=1500 □ (3)

Observing (2) and (3), we get to know that the value of 1 Gauva is 80Rs.

If we subtract the values of 4 guavas from (3), we will get the final answer.
So, the value of 6M+4G+10W=1500-320 = 1180

15. Reynolds offers a total of 150 pens to its customers. As per the scheme, one pen will be offered on the purchase of a "Quantitative Aptitude" book. Out of 150 pens, the cost of some pens is Rs. 3 and the cost of the rest of the pens is Rs. 5. At the most, how many customers can avail a pen of Rs. 5 as an offer from the company if the total cost of the pens cannot exceed Rs. 745.
(a) 45 (b) 120 (c) 147 (d) None of these

Solution:
To maximise the number of pens of Rs.5, we have to minimise the number of pens of Rs. 3 and the total cost cannot exceed Rs. 745.
So by hit and trial get the required result. As:

Number of pens of Rs. 3 each	**Number of pens of Rs. 5 each**	**Total cost**
1 x 3 = 3	149 x 5 = 745	748
2 x 3 = 6	148 x 5 = 740	746
3 x 3 = 9	147 x 5 = 735	744

Hence the maximum number of pens of Rs. 5 is 147.

HOME WORK:

1. 4/15 of 5/7 of a number is greater than 4/9 of 2/5 of the same number by 8. What is half of that number?
(a) 275 (b) 315 (c) 240 (d) 475

Solution:

Let the number be x

So from the question, we have

$$\frac{4}{15}.\frac{5}{7}.x - \frac{4}{9}.\frac{2}{5}.x = 8$$

$$\Rightarrow \frac{4x}{21} - \frac{8x}{45} = 8$$

$$\Rightarrow x = \frac{24 \times 7 \times 15}{4}$$

$$\Rightarrow x = 6 \times 7 \times 15$$

$$\Rightarrow x = 630$$

Half of that number is equal to 315.

2. A crate of mangoes contains one bruised mango for every 30 mangoes in the crate. If 3 out of every 4 bruised mango are considered unsaleable, and there are 12 unsaleable mangoes in the crate, how many mangoes are there in the crate?

(a) 480 (b) 500 (c) 440 (d) 520

Solution:

Let x be the total number of mangoes in the crate, Bruised mongo $= \frac{1}{30}x$

unsaleable mangoes $\frac{3}{4}\left(\frac{1}{30}x\right)$

$$\Rightarrow \frac{1}{40}x = 12$$

$$\Rightarrow x = 12 \times 40$$

$$\Rightarrow x = 480$$

3. One third of Arun's marks in Mathematics exceeds a half of his marks in English by 30. If he got 240 marks in the two subjects together, how many marks did he get in English?

(a) 180 (b) 60 (c) 78 (d) 110

Solution:

Let Aruns's marks in Mathematics be x and marks in English be y.

Total marks in both the subject = 240

$\therefore x + y = 240 = 2x + 2y = 480$.....(1)

$$\frac{x}{3} - \frac{y}{2} = 30$$

$= 2x - 3y = 180$ =....(2)

Subtracting equation (1) from equation (2) we get

$$-5y = -300$$

$$y = 60$$

$\therefore$ He scored 60 marks in English

4. $1 \div \frac{1}{1 \div \frac{1}{1 \div \frac{1}{3}}}$ is equal to:

(a) 1/3 (b) 1 (c) 3 (d) 1 (1/3)

Solution:

$$1 \div \frac{1}{1 \div \frac{1}{1 \div \frac{1}{3}}} = 1 \div \frac{1}{1 \div \frac{1}{3}} = 1 \div \frac{1}{3} = 3$$

5. Find the value of *x* in $\sqrt{x + 2\sqrt{x + 2\sqrt{x + 2\sqrt{3x}}}} = x$

(a) 1 (b) 3 (c) 6 (d) 12

Solution:
The value of *x* should be such that the left hand side after completely removing the square root signs should be an integer. For this to happen, first the square root of 3*x* should be an integer. Only 3 and 12 from the options satisfy this requirement. If we try to put *x* as 12, we get the square root of 3*x* as 6. Then the next point at which we need to remove the square root sign would be 12 + 2(6) = 24 whose square root would be an irrational number. This leaves us with only 1 possible value (*x* = 3). Checking for this value of *x* we can see that the expression is satisfied as LHS = RHS.