MODULE 2 – HCF and LCM
AND DECIMAL FRACTIONS

1. Find the smallest number that leaves a remainder of 4 on division by 5, 5 on division by 6, 6 on division by 7, 7 on division by 8, and 8 on division by 9?

(a) 2519 (b) 5039 (c) 1079 (d) 979

Solution:

Note: When a number is divided by 8, a remainder of 7 can be thought of as a reminder of -1. This idea is very useful for a bunch of questions. So, N = 5a - 1 or N + 1 = 5a

N = 6b - 1 or N + 1 = 6b

N = 7c - 1 or N + 1 = 7c

N = 8d - 1 or N + 1 = 8d

N = 9e - 1 or N + 1 = 9e

N + 1 can be expressed as a multiple of (5, 6, 7, 8, 9)

N + 1 = 5a*6b*7c*8d*9e

Or N = (5a*6b*7c*8d*9e) - 1

Smallest value of N will be when we find the smallest common multiple of (5, 6, 7, 8, 9) or LCM of (5, 6, 7, 8, 9)

N = LCM (5, 6, 7, 8, 9) - 1 = 2520 - 1 = 2519.

If we observe the divisor and its respective remainder the difference between them is 1. This means that if the number we are looking for is 1 less than the common multiple of 5, 6, 7, 8, and 9.

The LCM of 5, 6, 7, 8, and 9 is 2520.

Hence the number is 2520-1 = 2519

2. 6 different sweet varieties of count 32, 216, 136, 88, 184, and 120 were ordered for a particular occasion. They need to be packed in such a way that each box has the same variety of sweets and the number of sweets in each box is also the same. What is the minimum number of boxes required to pack?

(a) 129 (b) 64 (c) 48 (d) 97

Solution:

All sweets need to be packed and each box has the same variety.

This implies the number of sweets in each box should be HCF of the different count of sweets

HCF of 32, 216, 136, 88, 184, 120 = 2^3 = 8

Minimum number of boxes = (32 + 216 + 136 + 88 + 184 + 120) / 8 = 97

3. What is the greatest number which when it divides 77, 48, and 34, leaves remainders 2, 3, and 4 respectively?

(a) 15 (b) 14 (c) 25 (d) 30

Solution:
The greatest number would be the HCF of (77 – 2), (48 – 3) and (34 – 4) =HCF (75, 45 and 30), which is 15.
Note: The greatest number that will divide A, B, and C, leaving remainders p, q, and r, respectively, is the HCF of (A-p), (B-q), and (C-r).

4. What is the least number which when divided by 48, 36, and 72 leaves the remainder of 3 in each case?
(a) 154 (b) 147 (c) 125 (d) 130

Solution:
The least number would be LCM of (48, 36, and 72) + 3. LCM = 144. Hence, the required number is 144 + 3 = 147.
Note: The lowest number that is divisible by A, B, and C, leaving the same remainder "r" in each case is LCM of (A, B, and C) + r.

5. Find the greatest number that will divide 65, 81, and 145 leaving the same remainder in each case.
(a) 15 (b) 14 (c) 12 (d) 16

Solution:
Required number = HCF of (81-65), (145-81), and (145-65)
= HCF of 16, 64, and 80 = 16.
Note: The greatest number that will divide p, q, and r leaving the same remainder in each case, then the required number = HCF of the absolute values of (p-q), (q-r), and (r-p).

6. Find the least number which when divided by 6, 7, and 9 leaves the remainder 1, 2, and 4 respectively
(a) 121 (b) 124 (c) 125 (d) 126

Solution:
Here we observe that (6 - 1) = (7 - 2) = (9 - 4) = 5.
Therefore, by applying the formula we get the required number = (LCM of 6,7 and 9) -5 = 126-5=121
Note: If we have to find the least number which when divided by a, b, and c, leaves the same remainder p, q, and r respectively, then if it is observed that (a-p) = (b-q) = (c-r) = k (say), then the required number = (LCM of a, b, and c)-k.

7. The LCM of two numbers is 500 and their HCF is 50. If one of the numbers is 100, the other number is
(a) 250 (b) 400 (c) 500 (d) None

Solution:
LCM * HCF = N_1 * N2
N_1 = 100
Therefore, N_2= 500 * 50 /100 = 250

8. The HCF and LCM of the two numbers are 25 and 500 respectively. If the first number is divided by 2, the quotient is 50. The second number is
(a) 50 (b) 100 (c) 125 (d) 250

Solution:
HCF * LCM = A * B
25 * 500 = A * B
First number, A = 2 * 50 = 100
On substituting the value of A, we can find the value of B
(Since the product of two numbers = HCF * LCM)

25 * 500 = 100 * B
=> B = 125

9. Find the value of 29.94 ÷ 1.45, if the value of 2994 ÷ 14.5 = 172
(a) 17.2 (b) 1.72 (c) 172 (d) 0.172

Solution:
Given that, 2994 ÷ 14.5 = 172
29.94/1.45 can be written as 299.4/14.5
Again, 299.4 /14.5 is written in the form as follows:
= [(2994/14.5)×(1/10)]
Now, substitute 2994 ÷ 14.5 = 172
= 172 × (1/10) = 17.2
Hence, the value of 29.94 ÷ 1.45 is 17.2

10. Simply the value [489.1375 × 0.0483 × 1.956]/[0.0873 × 92.581 × 99.749], and then find the value closest to it.
(a) 0.06 (b) 0.6 (c) 6 (d) 0.006

Solution:
Given expression: [489.1375 × 0.0483 × 1.956]/[0.0873 × 92.581 × 99.749]
Now, write the given values rounded to its nearest value.
Hence, the given value is approximately equal to
[489 × 0.05 × 2]/[0.09 × 93 × 100]= 489/(9 × 93 × 10)
= (163/279) × (1/10)
= 0.58/10 = 0.058, which is approximately equal to 0.06.
Hence, the value closest to the expression [489.1375 × 0.0483 × 1.956]/[0.0873 × 92.581 × 99.749] is 0.06.

11. 11.98 × 11.98 + 11.98 × m + 0.02 × 0.02 should be a perfect square for "m" equal to
(a) 0.04 (b) 0.4 (c) 4 (d) 0.004
Solution:
Given expression: (11.98 × 11.98 + 11.98 × m + 0.02 × 0.02)
11.98 × 11.98 + 11.98 × m + 0.02 × 0.02 = $(11.98)^2 + (0.02)^2$ + 11.98 × m.
For the expression to be a perfect square, we should have,
11.98 × m = 2 × 11.98 × 0.02
By observation, m should be equal to 0.02 × 2 = 0.04
m = 0.04
Thus, 11.98 × 11.98 + 11.98 × m + 0.02 × 0.02 should be a perfect square for "m" equal to 0.04.

12. Find the unknown value in the given equation: 3889 + 12.952 –? = 3854.002
(a) 479.5 (b) 47.95 (c) 4.795 (d) 4795

Solution:
Let the unknown value be a.
Thus, 3889 + 12.952 – a = 3854.002.
Rearranging the above equation, we can write
a = (3889 + 12.952) – 3854.002
a = 3901.952 – 3854.002
a = 47.95.
Thus, the unknown value is 47.95.

13. Evaluate:

i) 8.71 x 1.2
ii) 3.7496 x 1.3
iii) 0.6 x 0.06 x 0.006 x 60

Solution:

i) 871 x 12 = 10452. Sum of decimal places of given numbers = (2+1) = 3
Hence, 8.71 x 1.2 = 10.452
ii) 37496 x 13 = 487448. Sum of decimal places of given numbers = (4+1) = 5
Hence, 3.7496 x 1.3 = 4.87448
iii) 6 x 6 x 6 x 60 = 12960. Sum of decimal places of given numbers = (1+2+3) = 6
Hence, 0.6 x 0.06 x 0.006 x 60 = 0.012960 = 0.01296

14. Evaluate:
i) 0.72 / 9
ii) 0.0216 / 18
iii) 4.2096 / 16

Solution:

i) 72 / 9 = 8. The dividend contains 2 places of decimal. Hence, 0.72 / 9 = 0.08
ii) 216 / 18 = 12. The dividend contains 4 places of decimal. Hence, 0.0216 / 18 = 0.0012
iii) 42096 / 16 = 2631. The dividend contains 4 places of decimal. Hence, 4.2096 / 16 = 0.2631

15. Evaluate (2.392-1.612)(2.39-1.61)

(a) 2 (b) 4 (c) 3 (d) 5

Solution:

Use following formula to solve it quickly

$$\frac{a^2 - b^2}{a - b} = \frac{(a+b)(a-b)}{(a-b)} = (a+b)$$

$= (2.39+1.61) = 4$

HOMEWORK:

1. Three numbers are in the ratio 2: 3: 4 and their HCF is 12. The LCM of the numbers is

(a) 144 (b) 192 (c) 96 (d) 72

Solution:

Let the number be 2x, 3x, and 4x respectively HCF = x = 12
Numbers are:
2*12 = 24
3*12 = 36
4*12 = 48
LCM of 24, 36, 48 = 2 x 2 x 2 x 3 x 3 x 2 = 144

2. The sum of the HCF and LCM of the two numbers is 680 and the LCM is 84 times the HCF. If one of the numbers is 56, the other is

(a) 84 (b) 12 (c) 8 (d) 96

Solution:

Let HCF be h and LCM be l
Then l=84h and l+h = 680
84h + h = 680
Therefore, h = 8
Therefore, l = 680 – 8 = 672
Therefore, another number = $672*856$ = 96
3. The LCM of the two numbers is 4 times their HCF. The sum of LCM and HCF is 125. If one of the numbers is 100, then the other number is
(a) 5 (b) 25 (c) 100 (d) 125
Solution:
Let LCM be L and HCF be H. Then L = 4H
Therefore, H + 4H = 125
5H = 125
H = 5
Therefore, L = 4 * 25 = 100
Therefore, Second number = $L*H \text{First number} = 100*25100$ = 25

4. Arrange the fractions 5/8, 7/12, 13/16, 16/29 and 3/4 in ascending order.

Solution:
Converting each of the given fractions into decimal form, we get:
5/8 = 0.625, 7/12 = 0.5833, 13/16 = 0.8125, 16/29 = 0.5517, and 3/4 = 0.75
Now, 0.5517 < 0.5833 < 0.625 < 0.75 < 0.8125
Thus, 16/29 < 7/12 < 5/8 < 3/4 < 13/16

5. Convert the following into vulgar fractions:
i) 0.25
ii) 4.004
iii) 0.0056

Solution:
i) 0.25 = 25/100 = ¼
ii) 4.004 = 4004/1000 = 1001/250
iii) 0.0056 = 56/10000 = 7/1250