

MODULE 4 – PERCENTAGE

1. P is eight times as large as Q. By what percent Q is less than P?

- (a) 90% (b) 87.5% (c) 60% (d) $16\frac{2}{3}\%$

Solution:

Let $Q = x$, then $P = 8x$

Percent of Q less than P = $\frac{P-Q}{P} \times 100$

$$\square \frac{8x-x}{8x} \times 100 = 87.5$$

2. A candidate who gets 30% marks fails by 5 marks but another candidate who gets 40% marks gets 5 marks more than the passing marks. Find the maximum marks.

- (a) 50 (b) 100 (c) 150 (d) None

Solution:

Let maximum marks be x .

$$\text{Then, } 30\%x + 5 = 40\%x - 5$$

$$\square x = 100$$

3. Ravi's salary is 50% more than Sunil's salary. Ravi got a raise of 40% on her salary while Sunil got a raise of 30% on her salary. By what percent is Ravi's salary more than Sunil's?

- (a) 61.53% (b) 71.64% (c) 86.47% (d) 56.92%

Solution:

Let Sunil's salary be Rs 100

$$\text{Then Ravi's salary} = 100 \times 50\% + 100 = 150$$

$$\text{Ravi got a raise of 40\%, then } 150 \times 40\% + 150 = 210$$

$$\text{Sunil got a raise of 30\%, then } 100 \times 30\% + 100 = 130$$

Now Ravi's salary is more than Sunil's salary by 80

$$\text{Percent} = \frac{80}{130} \times 100 = 61.53\%$$

4. In XYZ College, 65% of students are less than 20 years of age. The number of students more than 20 years of age is $\frac{2}{3}$ rd of the number of students of 20 years of age which is 42. What is the total number of students in the College?

- (a) 75 (b) 90 (c) 130 (d) 200

Solution:

Let the number of students be x .

Then, Number of students more than 20 years of age = $(100 - 65) \%$ of $x = 35\%$ of x .

$$35\% \text{ of } x = 42 + \frac{2}{3} \text{ of } 42$$

$$\frac{35}{100} x = 70$$

$$x = 200$$

5. A student attempts x number of questions. He answers 15 correctly out of the first 20 questions and of the remaining questions, he answers $\frac{1}{3}$ correctly. If all questions have the same credit and the student gets 50 % marks, then find the value of x ?

- (a) 35 (b) 40 (c) 50 (d) 55

Solution:

1) Student attempts x questions.

2) Out of 20 questions he answers 15 correctly and of $(x - 20)$ questions he answered $\frac{1}{3}$ correctly.

3) The student gets 50 % marks.

Therefore,

$$15 + \frac{1}{3} (x - 20) = 50\% \text{ of } x$$

$$15 + \frac{1}{3} (x - 20) = \frac{1}{2} x$$

$$15 + \frac{1}{3} (x - 20) = \frac{x}{2}$$

$$30 + \frac{2}{3} (x - 20) = x$$

$$90 + 2 (x - 20) = 3x$$

Solving this equation, we get

$$x = 50$$

6. In a medical certificate, by mistake a candidate gave his height as 20% more than normal. In the interview panel, he clarified that his height was 6 feet 6 inches. Find the percentage correction made by the candidate from his stated height to his actual height.

- (a) 16.66% (b) 28.56% (c) 25% (d) 16.66%

Solution:

Actual height = 6 feet 6 inches = 6×12 inches + 6 inches = 78 inches

Height given by mistake = $120\% \times 78$ inches = 93.6 inches

Therefore, Required percentage error = $\frac{(93.6 - 78)}{93.6} \times 100 = 16.66\%$

7. The radius of a sphere is 14 cm. The cost of painting the surface of the sphere is Rs. 25 per square cm. If the radius of the sphere is increased by 20%, then the cost of painting is increased by 20%. What is the percentage increase in the total cost of painting per square cm?

- (a) 54.27% (b) 20.3% (c) 62.58% (d) 72.8%

Solution:

Total cost of painting = Rate * surface area of sphere

As the radius increases by 20%

Surface area will change by 44% (using formula = $x + y + \frac{xy}{100}$ %)

Total change in cost of painting = $20 + 44 + \frac{20 \times 44}{100}$

□ 72.8%

8. The price of a car is Rs. 8,00,000. It was insured for 90% of its price. The car got completely damaged, and the insurance company paid only 80% of the insured amount. What is the price of the difference between the price of the car and the amount of insurance received?

- (a) 1,28,000 (b) 80,000 (c) 1,60,000 (d) 2,24,000

Solution:

Total value = 100% = 8,00,000

Insurance received (B) = 80% of insured amount

⇒ 80% of 90%

⇒ $(\frac{80 \times 90}{100})\% = 72\%$

Price of the car (A) = 100%

Difference% of A – B = 28%

Difference amount = 28% = x

100% = 8,00,000

By Cross multiplication, $x = \frac{(28 \times 8,00,000)}{100} = \text{Rs. } 2,24,000$

9. The population of New Foundland increases with a uniform rate of 7% per annum, but due to immigration, there is a further increase of population by 1% (however, this 1% increase in population is to be calculated on the population after the 7% increase and not on the previous year's population). What will be the percentage increase in population after 2 years?

- (a) 16.79 (b) 18.81 (c) 18.24 (d) 17.91

Solution:

There is a condition given where a 1 % increase is only after a 7 % increase and not before that. Thus 1 % increase will only be calculated after there is an increase of 7%. This means that for 2 years, we will calculate this increase two times.

For the first year's let's suppose that the population before was 100. Now, with the increase of 7%, the new population after one year will be 107. Now, there is a one percent increase in this population. So, 1% of 107 is 1.07. So, new population will be $107 + 1.07 = 108.07$

Now, for the second year, there is again an increase of 7% in the population. So, the population after two years will be 7% of 108.07 which will be 7.56 ⇒ $108.07 + 7.56 = 115.63$. Increasing 115.63 by 1% we get a new population of 116.791 which is our required answer. So, the increase in population after two will be 16.79 %.

10. In a college election between 2 students, 10% of the votes cast is invalid. The winner gets 70% of the valid votes and defeats the opponent by 1800 votes. How many votes were casted in total?

- (a) 4300 (b) 5000 (c) 5400 (d) 6600

Solution:

Total votes cast = 100%

Invalid votes = 10%

Valid votes = $100 - 10 = 90\%$

Winner gets = 70% of 90%

$\Rightarrow (70 * 90)/100\% = 63\%$

Loser gets = Valid votes – Winner votes

$\Rightarrow 90\% - 63\% = 27\%$

Majority = Votes secured by winner – Votes secured by loser

$\Rightarrow 63\% - 27\% = 36\% = 1800$

Total votes casted = 100% = x

By Cross multiplication, $x = (100 * 1800)/36 = 5000$ votes

Similarly, we can also find out the number of valid votes cast, since we have the value of one percentage.

11. A company has 14 machines of equal efficiency in its factory. The annual manufacturing expenses are Rs. 42, 000 and the establishment charges are Rs. 12,000. The annual output of the company is Rs. 70, 000. The annual output and manufacturing costs are directly proportional to the no. of machines while the shareholders get the 12.5% profit, which is directly proportional to the annual output of the company. If 7.14% machines remained closed throughout the year. Then the percentage decrease in the amount of shareholders is:

- (a) 12% (b) 12.5% (c) 13% (d) 13.5%

Solution:

Manufacturing cost (MC) of 14 machines = Rs. 42000

Output of 14 machines Rs. 70000 Establishment cost (EC) Rs. 12000

Profit Rs. $(70000 - 42000 - 12000) = \text{Rs. } 16000$

Shareholder's profit = 12.5% of Rs. 16000 Rs. 2000

It is given that 7.14% of the machines were non functional which means only 13 machines were functional.

MC of 13 machines = Rs. $(42000 * 13/14) = \text{Rs. } 39000$ [As it is directly proportional to the number of functional machines]

Output of 13 machines = Rs. $(70000 * 13/14) = \text{Rs. } 65000$ [As it is directly proportional to the number of functional machines]

EC of 13 machines = Rs. 12000 [As it does not depend on the number of functional machines]

Profit Rs. $(65000 - 39000 - 12000) = \text{Rs. } 14000$

Shareholder's profit = 12.5% of Rs. 14000 = Rs. 1750

Reduction in Shareholder's profit = Rs. $(2000 - 1750) = \text{Rs. } 250$

Reduction% = $250/2000 * 100\% = 12.5\%$

12. In a tournament, a team has played 40 matches so far and won 30% of them. If they win 60% of the remaining matches, their overall win percentage will be 50%. Suppose they win 90% of the remaining matches, then the total number of matches won by the team in the tournament will be?

- (a) 86 (b) 84 (c) 78 (d) 80

Solution:

Given that, initially the number of matches the team has played = 40.

The number of matches won by team = 30% of 40 = 12

Let the remaining matches be x .

The number of remaining matches won by team = 60% of $x = 0.6x$

Now, $12 + 0.6x + x = 50$

$$\square 24 + 1.2x = 40 + x$$

$$\square 0.2x = 16$$

$$\square x = 80$$

When the team won 90% of the remaining matches.

Then, the number of remaining matches won by the team = 90% of 80 = 72

Therefore, the total number of matches won by the team in the tournament = $12 + 72 = 84$

13. Hari prepares a budget to visit London. However, he spends 12% of his budget on the first 10% days of his travel when he stays in the city. He knows that he has to spend another 35% of days in city itself, after which he would travel to the countryside. What should be the minimum decrease in spending in the countryside as a percentage of his spending in the city so as to complete his travel on the initial budget itself?

(a) 33.33%

(b) 30.3%

(c) 25%

(d) 32.23%

Solution:

Budget spent on 10% of days = 12%

So, in 1% of days = 12/10

35% remaining days in city = $12/10 \times 35 = 42\%$

Overall budget spent on 45% of days in city = 54%

Days remaining = 55%, Budget remaining = 46%

In 1% of day remaining, he can spend = $46/55$

Therefore, Percentage decrease = $(12/10 - 46/55) \times 100 = 30.3\%$

14. 40% of the employees of a certain company are men and 75% of the men earn more than Rs. 25,000 per year. If 45% of the company's employees earn more than Rs. 25,000 per year, what fraction of the women employed by the company earn Rs. 25,000 or less per year?

(a) 2/11

(b) 1/4

(c) 1/3

(d) 3/4

Solution:

Let the total number of employees in the company be x

Then the number of men and women would be $0.4x$ and $0.6x$ respectively.

75% of men earn more than Rs. 25000 $\Rightarrow 0.75 \times 0.4x = 0.3x$

Total number of employees earning more than Rs. 25000 = $45\% \times x = 0.45x$

Number of women earning more than Rs. 25000 = Total employees earning more than Rs. 25000 – total number of Men earning more than Rs. 25000

$$= 0.45x - 0.30x = 0.15x$$

Number of the women earning Rs. 25000 or less = $0.60x - 0.15x = 0.45x$

Fraction of the women employed by the company who earn Rs. 25000 or less

$$(0.45x/0.60x) = 45/60 = 3/4$$

Alternate Solution:

Let the number of men and women employees in the company be 40 and 60. Now out of 40 men, 75% i.e. 30 earn more than Rs 25000 and 45% of the total employees i.e. 45 employees earn more than Rs 25000.

Hence, there are $45 - 30 = 15$ women who earn more than Rs25000. So, $60 - 15 = 45$ women earn less than Rs 25000.

Hence, the required fraction = $45/60 = 3/4$

15. In an election between two candidates, a person who got 58% of total votes won the election by a majority of 960. Find the total number of votes.

- (a) 6,000 (b) 7,500 (c) 8,000 (d) 9600

Solution:

Assume the population is 100.

→ 58% of 100 = 58 votes got by the winning party.

→ 42% of 100 = 42 votes got by opponents

→ Difference between them = $58\% - 42\% = 16\%$ (majority)

→ Now in the given question, majority = 960 = 16%

→ If 16% = 960 then 100% is 6000.

→ So the total number of votes is 6000.

HOMEWORK:

1. During one year, the population of a town increased by 5% and during the next year, the population decreased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year?

- (a) 10000 (b) 11000 (c) 12000 (d) 15000

Solution:

Let the population of a town is 100

Then 5% increased population after first year = $100 + 5 = 105$

So, 5% decreased population after second year = $105 - (5/100 \times 105)$

$\square 105 - 5.25 = 99.75$

But, the population after second year is 9975

Then, the population at the beginning of the year is $10099.75 \times 9975 = 10000$

2. The height of a triangle has increased by 40%. What will be the maximum % increase in the length of the base so that the increase in the area is restricted to a maximum of 60%?

- (a) 50% (b) 20% (c) 14.28% (d) 25%

Solution:

Let the height be h and base be b .

Height increases 40%, So new height = $h + 40\%$ of $h = 7/5 \times h$

Area of triangle = $(1/2 \times h \times b)$

Area increases by 60%. so, new area, = $(1/2 \times h \times b) + 60\%$ of $(1/2 \times h \times b)$

$= 8/5 \times 1/2 \times b \times h$

Let new base be x , then

New area, $= 1/2 \times 7/5 \times x = 8/5 \times 1/2 \times b \times h$

which will result into $x = 8/7 \times b$

So, base will increase by $1/7$ th of $b = 14.28\%$.

3. Rakesh is working in the Life Insurance Corporation of India (LIC). He was hired on the basis of commission and he got the bonus only on the first year's commission. He got the policies of 2 lakh having a maturity period of 10 years. His commission in the first, second, third, fourth and for the

rest of the years is 20%, 16%, 12%, 10% and 4% respectively. The bonus is 25% of the commission. If the annual premium is ₹20,000 then what is his total commission if the completion of the maturity of all the policies is mandatory :

- (a) Rs. 17400 (b) Rs. 23600 (c) Rs. 15000 (d) Rs. 15500

Solution:

1st year – 20% commission = $0.2 * 20000 = 4000$
 1st year – 25% bonus commission = $0.25 * 4000 = 1000$
 2nd year - 16% commission = $0.16 * 20000 = 3200$
 3rd year - 12% commission = $0.12 * 20000 = 2400$
 4th year - 10% commission = $0.10 * 20000 = 2000$
 5-10th year - 4% commission = $6 * 0.04 * 20000 = 4800$

Total commission = $4000 + 1000 + 3200 + 2400 + 2000 + 4800 = 17400$

4. If an equal number of people are born on each day, Find the approximate percentage of the people whose birthday will fall on 29th February if we are to consider people born in the 20th century (1901 – 2000) and assuming no deaths.

- (a) 0.374 (b) 0.5732 (c) 0.0664 (d) None of these

Solution:

In 20th century, 1901–2000, or in 100 years total people born will be = $365 * 100 * 1 + 25 * 1$
 And people born on 29th February = $25 * 1$ (25 days date will be 29 Feb in 100 years)
 Hence percentage will be = $\frac{25 * 1}{365 * 100 * 1 + 25 * 1} * 100 = 0.0684$

5. In a local election, 2400 people were to vote for Party A or Party B. Party A was bound to win the election. However, on Election Day, 33% of the voters of Party A were kidnapped. Party B was also able to influence the remaining Party A voters and thus double the strength of its voters. In this way, Party A lost by a majority which was half of that by which it would have won had the elections been fair. How many people finally voted for Party A and Party B?

- (a) 600(A), 1200(B) (b) 300(A), 600(B)
 (c) 450(A), 900(B) (b) 600(A), 900(B)

Solution:

Easiest way to solve this question is through options

Option a) If 600 voted for A and 1200 voted for B then

Before influencing, A = 600 + 600 = 1200, B = 1200 – 600 = 600 (As half of B's vote were through influencing)

Before kidnapping, A = 1200 + $\frac{1}{3}$ of 1200 = 1800 (As $\frac{1}{3}$ of A's voters were kidnapped which is half of the $\frac{2}{3}$ that remained)

Thus, total voter initially = 1800 (A) + 600 (B) = 2400

Thus option a) fits the scenario given in the question completely.

600 would have voted for Party A, 1200 would have voted for Party B.

Hence, the answer is 600(A), 1200(B)