Set 4 Constrained growth beyond the logistic model

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In this report, we have studied the Gompertz equation for tumor growth and how it behaves for different initial values. Also, with that we have studied the Allee effect, which represents the phenomena where populations develop more quickly when they reach an intermediate size.

I. TUMOUR GROWTH

A. Model

$$\dot{x} = -ax \ln (bx) \tag{1}$$

When we rescale using

$$X = x/b^{-1} \tag{2}$$

$$T = at \tag{3}$$

we get,

$$\frac{d(x/b^{-1})}{dt} = -a(x/b^{-1})ln(bx) \tag{4}$$

$$\frac{d(x/b^{-1})}{d(at)} = -(x/b^{-1})ln(bx) \tag{5}$$

$$\frac{dX}{dT} = -X \ln X \tag{6}$$

and the solution,

$$X = e^{\ln (X_o) e^{-T}} \tag{7}$$

$$x = e^{\ln (x_o/b^{-1}) e^{-at}}/b \tag{8}$$

B. Results

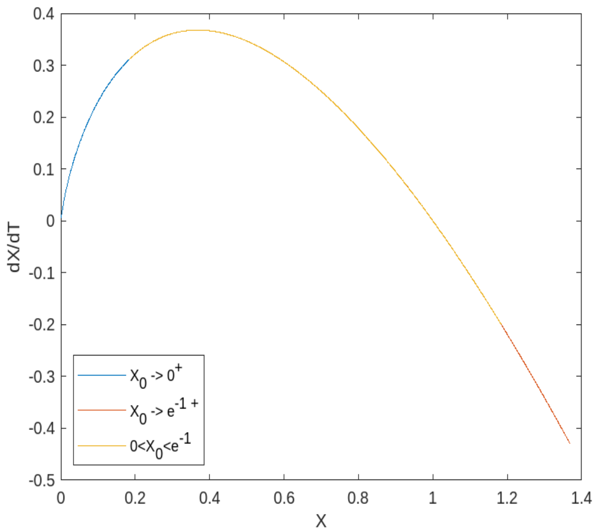

FIG. 1: Phase plot of $X(T)$

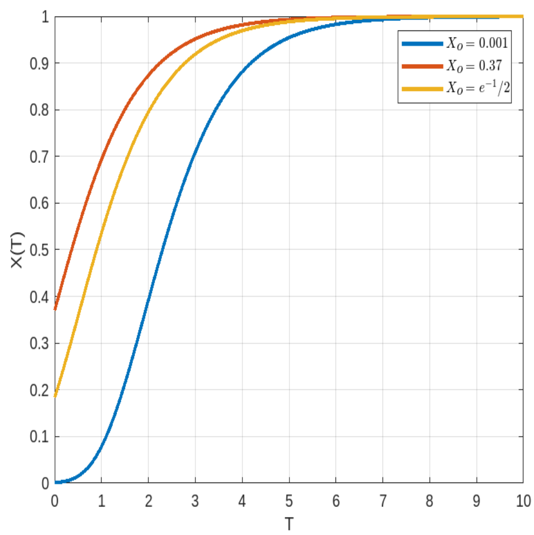

FIG. 2: Plot of $X(T)$ using Euler Method for 3 different values of (X_o)

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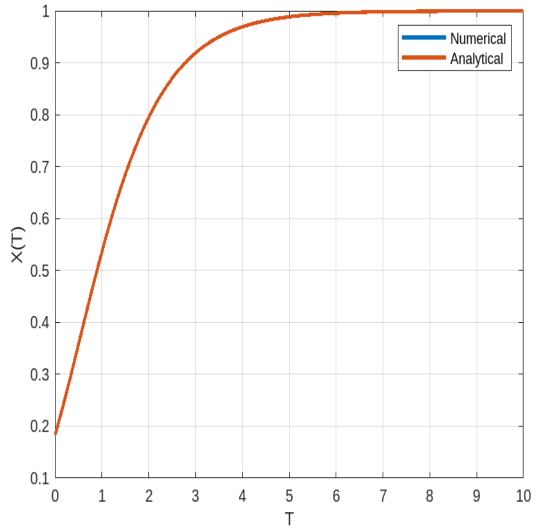

FIG. 3: Plot of X_{num} and X_{ana} for intermediate X_0

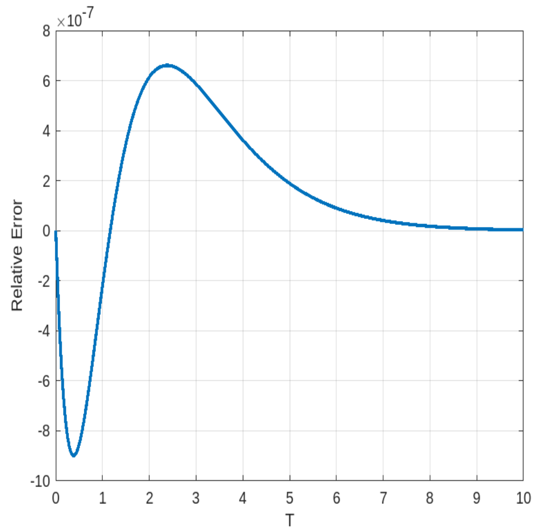

FIG. 4: Plot of Relative Error between Numerical and Analytical Solution of $X(T)$

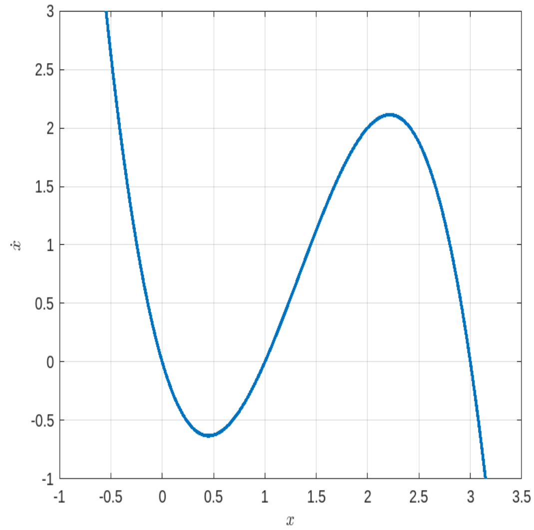

FIG. 5: Phase plot of $x(t)$

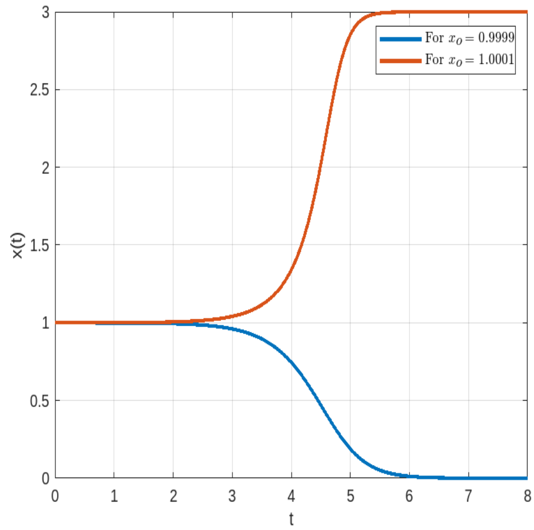

FIG. 6: Plot of $x(t)$ using Euler Method for 2 different values of x_o

II. ALLEE EFFECT

A. Model

$$\dot{x} = x[r - a(x - b)^2] \tag{9}$$

B. Results

As $t \to \infty$	
For $x_o = 0.9999$	Limiting value of $x(t) = 0$
For $x_o = 1.0001$	Limiting value of $x(t) = 3$

III. CONCLUSIONS

- The simplified analysis of tumour growth made possible by the rescaled Gompertz equation revealed that the rate of growth reduces with increasing tumour size. This suggests that early detection and treatment of tumors may be beneficial in controlling their growth.
- The Allee effect can lead to counter-intuitive population dynamics, such as a population declining at low densities and growing at intermediate densities.
- The Allee effect is an important concept in population biology that can have significant implications for conservation efforts and the management of endangered species. The mathematical model used in this task provides a useful tool for predicting and understanding the dynamics of populations exhibiting the Allee effect.

[1] Ray, A.K. (2010). Modeling Saturation in Industrial Growth. In: Basu, B., Chakravarty, S.R., Chakrabarti, B.K., Gangopadhyay, K. (eds) Econophysics and Economics of Games, Social Choices and Quantitative Techniques. New Economic Windows. Springer, Milano. https://doi.org/10.1007/978-88-470-1501-2$_1$4