# Set -6: Battles, war games and strategic conflict

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In this report, we studied Lanchester's Combat Model and predicted the outcome, of two battles: Battle of Iwo Jima and Battle of Trafalgar, based on the no. of troops, combat effectiveness and strategies. Also the report contains Richardson's mathematical model of conflict between nations.

## I. BATTLE OF IWO JIMA

### A. Introduction

The Imperial Japanese Army and the US Army engaged in a bloody fight on the Japanese island of Iwo Jima in 1945.

### B. Model

Applying the Lanchester model of conventional-conventional combat,

$$\dot{J} = -aA \tag{1}$$

$$\dot{A} = -jJ \tag{2}$$

Here the number of Japanese troops and American troops at a time is given by $J(t)$ and $A(t)$ respectively. The combat effectiveness parameters for the American and Japanese forces, respectively, are $a = 0.0106$ and $j = 0.0544$. Also, the initial number of Japanese troops is $J(0) = 18274$ and for American troops is $A(0) = 66454$.

$$K = aA_0^2 - jJ_o^2 \tag{3}$$

$K$ is the constant. If $K > 0$, America wins and if $K < 0$, Japan wins or else battle is tied.

### C. Results

Since $K > 0$, the United States of America wins the battle.

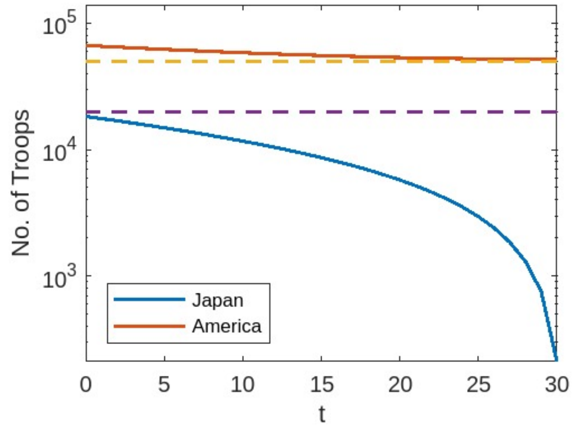


fig. 1. No. of troops at a given time

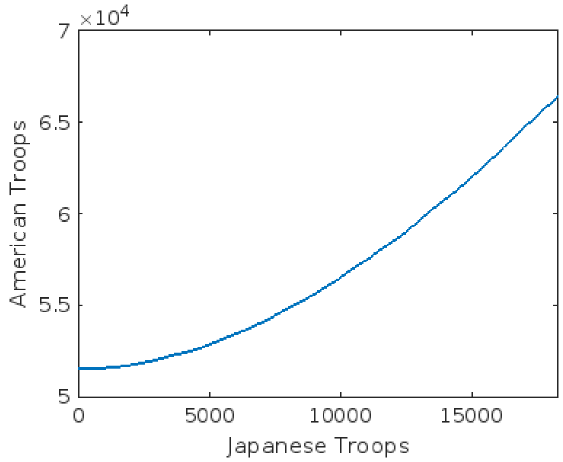


fig. 2. Graph of American troops vs Japanese troops

It took 30 days for American troops to win this battle.

| Result | | |
|---|---|---|
| Country | No. of Surviving troops | No. of Dead troops |
| America | 51580 | 14874 |

### D. Conclusions

- The Lanchester square law fits well and predicts the outcome of the Battle of Iwo Jima correctly.

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## II. BATTLE OF TRAFALGAR

### A. Introduction

Napoleon's French Navy and the British Royal Navy, led by Lord Nelson, engaged in combat at the Battle of Trafalgar in 1805. The British succeeded in this war.

### B. Model

$$\dot{F} = -bB \tag{4}$$

$$\dot{B} = -fF \tag{5}$$

Here the number of French ships and British ships at a time is given by $F(t)$ and $B(t)$ respectively. The combat effectiveness parameters for the French and British Navy, respectively, are $f = 0.05$ and $b = 0.05$. Also, the initial number of French ships is $F(0) = 33$ and for British ships is $B(0) = 27$.

$$K = bB_0^2 - fF_o^2 \tag{6}$$

### C. Results

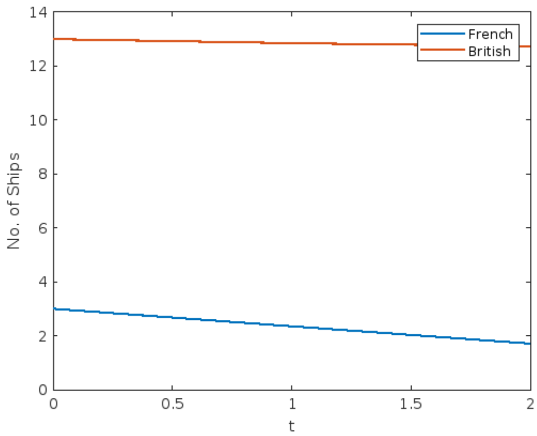


fig. 1. First Stage

**Initial no. of French ships is 3**
**Initial no. of British ships is 13**
**Final no. of French ships is 1.7075**
**Final no. of British ships is 12.7325**

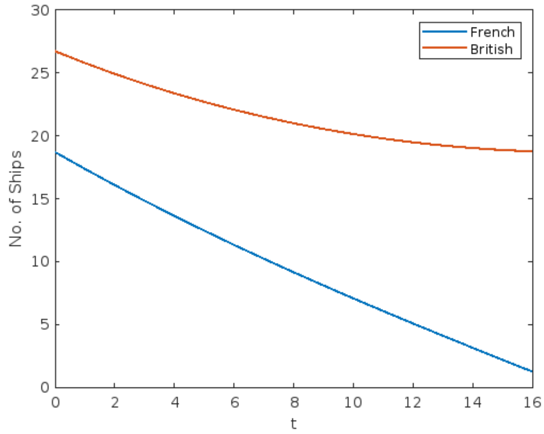


fig. 2. Second Stage

**Initial no. of French ships is 18.7075**
**Initial no. of British ships is 26.7325**
**Final no. of French ships is 1.2409**
**Final no. of British ships is 18.7585**

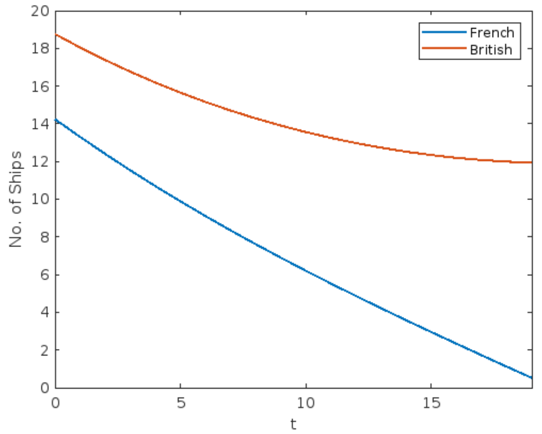


fig. 3. Final Stage

**Initial no. of French ships is 14.2409**
**Initial no. of British ships is 18.7585**
**Final no. of French ships is 0.5182**
**Final no. of British ships is 11.9341**

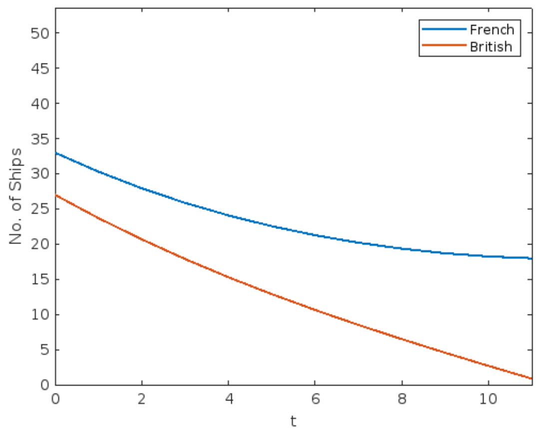


fig. 4. If Lord Nelson had engaged the full French fleet of 33 ships with his 27 ships all at the same time

**Initial no. of French ships is 33**
**Initial no. of British ships is 27**
**Final no. of French ships is 17.9737**
**Final no. of British ships is 0.8550**

### D. Conclusions

- The Lanchester square law fits well and predicts the outcome of the Battle of Trafalgar correctly.
- We can see how dividing the number of combatants and engaging them in different stages of battle helps the British Navy win the battle with lesser number of ships.

## III. RICHARDSONS MATHEMATICAL MODEL OF CONFLICT BETWEEN NATIONS

### A. Introduction

The strategic conflict between two nations is captured here.

### B. Model

$$\dot{x} = ky + g - \alpha x \tag{7}$$

$$\dot{y} = lx + h - \beta y \tag{8}$$

Here, the two countries' capacity to wage conflict is represented by the variables $x$ and $y$, with fixed parameters $k$, $l$, $h$, $g$, $\alpha$, and $\beta(> 0)$.

### C. Results

#### *1. Mutual disarmament without grievance*

For $k = 2$, $l = 1$, $h = g = 0$, $\alpha = 4$ ,$\beta = 3$, $x_o = 100$ and $y_o = 50$

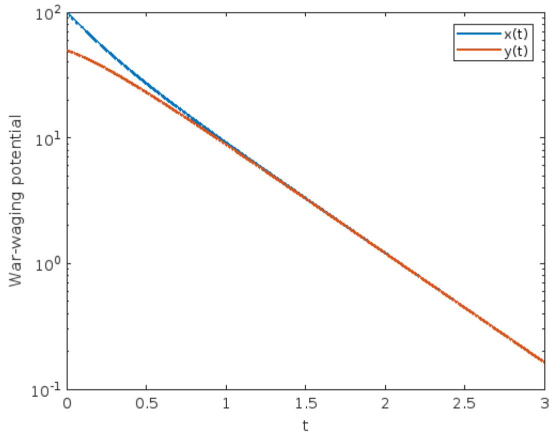


fig. 1. Plot of Mutual disarmament without grievance

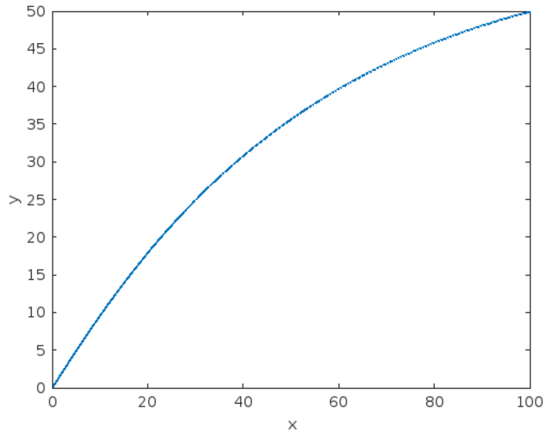


fig. 2. Plot y versus x

#### *2. Mutual disarmament with grievance*

For $k = 2$, $l = 1$, $h = 6$, $g = 5$, $\alpha = 4$, $\beta = 3$ and $x_o = y_o = 0$

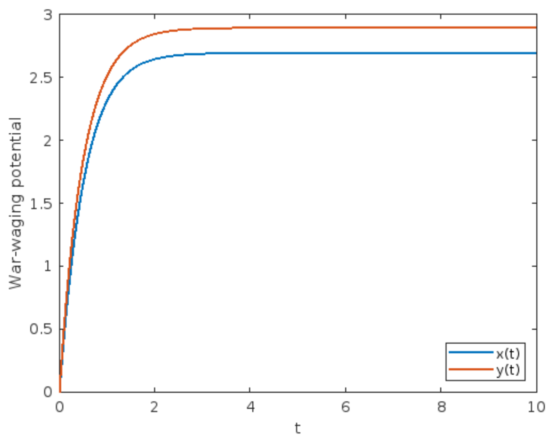


fig. 3. Plot of Mutual disarmament with grievance

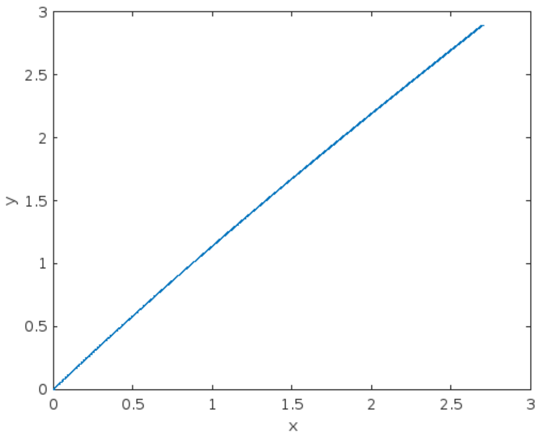


fig. 4. Plot y versus x

### 3. *Unilateral disarmament*

For $k = 2$, $l = 1$, $h = 6$, $g = 5$, $\alpha = 4$, $\beta = 3$, $x_o = 1$ and $y_o = 0$

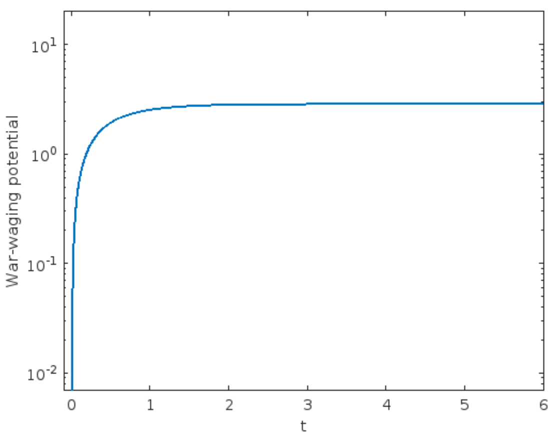


fig. 5. Plot of Unilateral disarmament

### 4. *Arms race*

For $k = 2$, $l = 1$, $h = g = \alpha = \beta = 0$, $x_o = 1000$ and $y_o = 700$

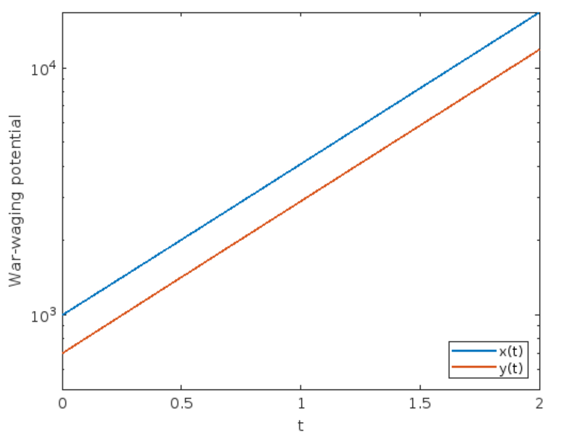


fig. 6. Plot of Arms race

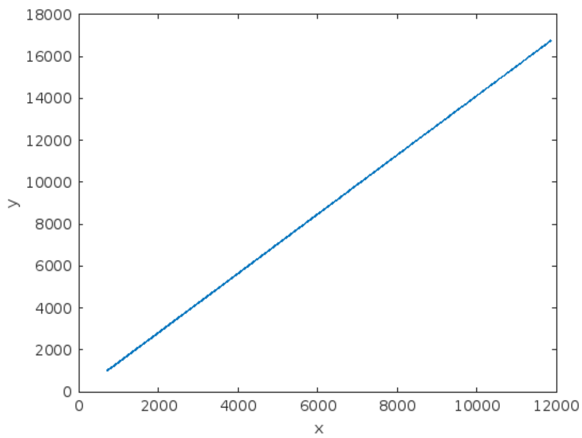


fig. 7. Plot y versus x

## D. Conclusions

- The nation with zero armament will grow to have more armaments than before as for Unilateral disarmament.

[1] Nair, R., Nagarjuna, G. and Ray, A.K., 2009. Finite-size effects in the dependency networks of free and open-source software. arXiv preprint arXiv:0901.4904.

[2] Mullick, A. and Ray, A.K., 2016, October. Nonlinear dynamics of bimodality in vehicular traffic. In Journal of Physics: Conference Series (Vol. 755, No. 1, p. 012009). IOP Publishing.

[3] Ray, A.K. (2010). Modeling Saturation in Industrial Growth. In: Basu, B., Chakravarty, S.R., Chakrabarti, B.K., Gangopadhyay, K. (eds) Econophysics and Economics of Games, Social Choices and Quantitative Techniques. New Economic Windows. Springer, Milano. https://doi.org/10.1007/978-88-470-1501-2$_1$4