Set-2: Compartment modelling of linear systems

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CS302, Modeling and Simulation

The paper presents several examples of compartmental models used in different fields such as concentration of pollution in lake, and concentration of single-dose/course of medicine in bloodstream. It also discusses the mathematical formulation of these compartmental models.

I. LAKE POLLUTION

A. MODEL

We model the system using a compartmental model of a linear function which leads to the logistic equation,

$$\dot{C} = f(C) = a - bC \tag{1}$$

with $C(t)$ being the concentration of pollutant in lake. Fixed parameters a and b are given by

$$a = \frac{FC_{in}}{V} \tag{2}$$

$$b = \frac{F}{V} \tag{3}$$

where $F = 5\text{x}10^8 m^3/day$ is the fixed volumetric inflow rate, $C_{in} = 3$ unit is the constant pollutant concentration of inflow, $V = 10^{12} m^3$ is the fixed volume of the lake and $C(0) = C_0 = 10$ unit.

B. RESULTS

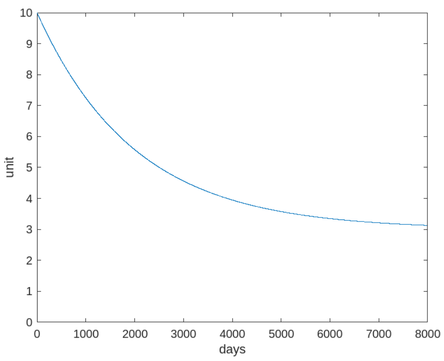

FIG. 1: Concentration of pollutants, $C_{in} = 3$ unit

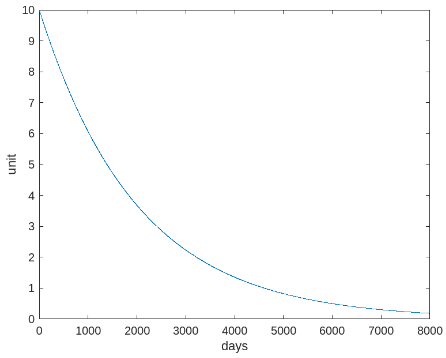

FIG. 2: Concentration of pollutants, $C_{in} = 0$ (fresh water)

C_{in} (in units)	$t_{1/2}$
3	2.5055 x 10^3
0	1.3863 x 10^3

Time taken for $C = 0.5C_0$

C. CONCLUSIONS

- We can observe that concentration of the pollutants will become constant after a long period of time(10000 days)
- The limiting value of concentration of the pollutants is equal to constant pollutant concentration of inflow i.e. C_{in}.

II. SINGLE MEDICINE DOSE

A. MODEL

We model the system using a compartmental model of two linear functions which leads to the logistic equations,

$$\dot{x} = f(x) = -k_1 x \tag{4}$$

$$\dot{y} = f(x, y) = k_1 x - k_2 y \tag{5}$$

where $x(t)$ is the amount of drug in GI tract, $y(t)$ is the amount of drug in blood stream, k_1 and k_2 are fixed parameters and at t=0, $x_0 = 1$ unit and $y_0 = 0$ unit.

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B. RESULTS

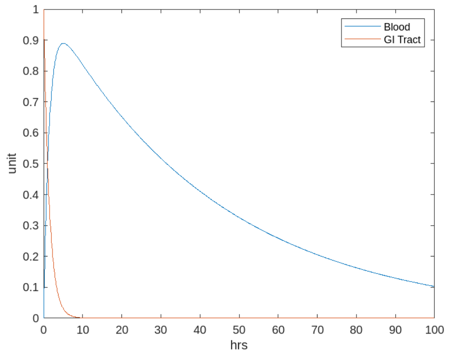

FIG. 1: Amount of drug in GI Tract vs Blood, k1 = 0.6931 & k2 = 0.0231

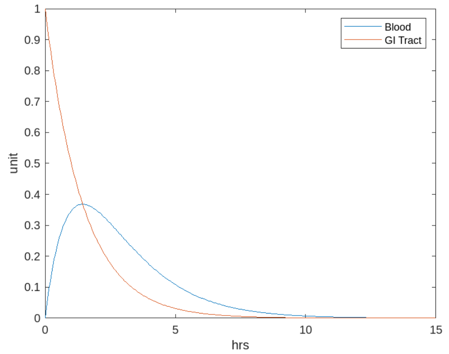

FIG. 2: Amount of drug in GI Tract vs Blood, k1 = k2 = 0.6931

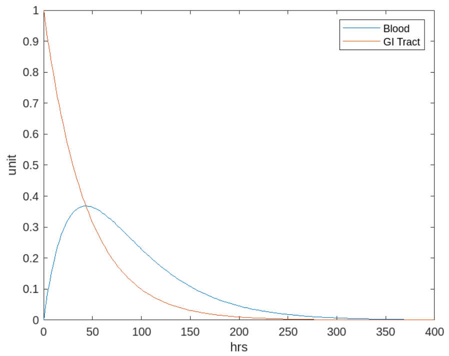

FIG. 3: Amount of drug in GI Tract vs Blood, k1 = k2 = 0.0231

k_1	k_2	t_{peak}	y_{peak}
0.6931	0.0231	5.0766	0.8893
0.6931	0.6931	1.4428	0.3679
0.0231	0.0231	43.2900	0.3679

Peak value of y(t) and its time

C. CONCLUSIONS

- In the single dose drug dosage, there is exponential decay in x(Amount of drug present in GI Tract).
- Whereas, in y(Amount of drug present in Blood Stream) we can observe linear early growth and then exponential decay.

III. COURSE OF MEDICINE DOSE

A. MODEL

We model the system using a compartmental model of two linear functions which leads to the logistic equations,

$$\dot{x} = f(x) = I - k_1 x \tag{6}$$

$$\dot{y} = f(x, y) = k_1 x - k_2 y \tag{7}$$

where $x(t)$ is the amount of drug in GI tract, $y(t)$ is the amount of drug in blood stream, k_1 and k_2 are fixed parameters, $I = 1$ unit is the ingestion rate and at t=0, $x_0 = 0$ unit and $y_0 = 0$ unit.

B. RESULTS

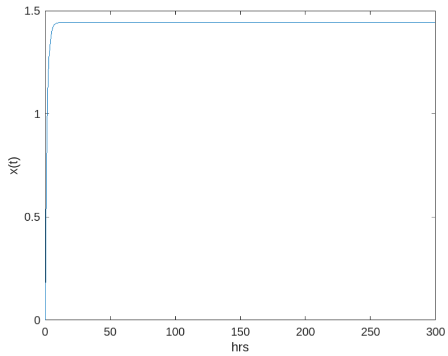

FIG. 1: Amount of drug in GI Tract, k1 = 0.6931 & k2 = 0.0231

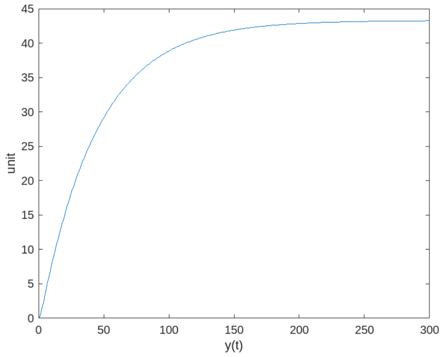

FIG. 2: Amount of drug in Blood,
k1 = 0.6931 & k2 = 0.0231

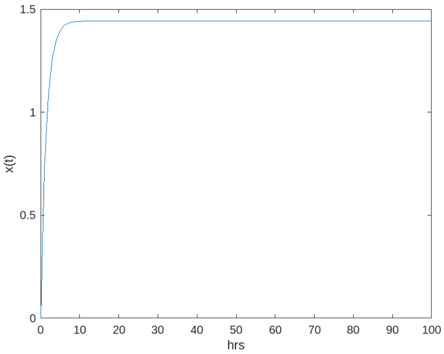

FIG. 3: Amount of drug in GI Tract, k1 = k2 = 0.6931

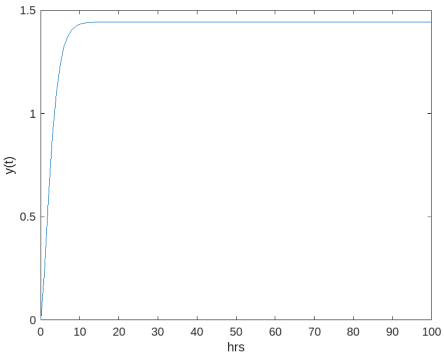

FIG. 4: Amount of drug in Blood,k1 = k2 = 0.6931

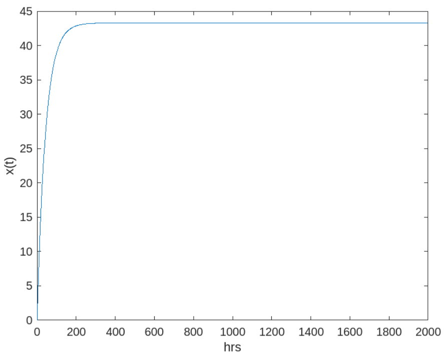

FIG. 5: Amount of drug in GI Tract,k1 = k2 = 0.0231

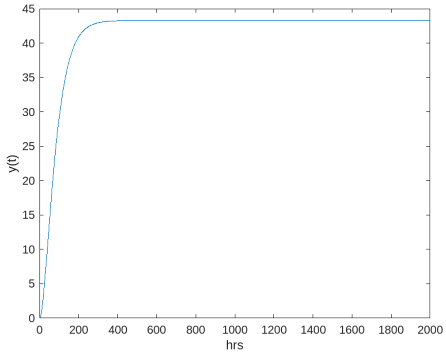

FIG. 6: Amount of drug in Blood,k1 = k2 = 0.0231

k_1	k_2	x_{lim}	y_{lim}
0.6931	0.0231	1.4428	43.2900
0.6931	0.6931	1.4428	1.4428
0.0231	0.0231	43.2900	43.2900

Peak value of y(t) and its time

C. CONCLUSIONS

- When the drug is given periodically in the GI tract, x(Amount of drug present in GI Tract) gets saturated at value I/k_1.
- As is drug is give frequently, we can observe early growth is parabolic in y(Amount of drug in Blood Stream) and after long time y also get saturated at value I/k_2

[1] Ray, A.K. (2010). Modeling Saturation in Industrial Growth. In: Basu, B., Chakravarty, S.R., Chakrabarti, B.K., Gangopadhyay, K. (eds) Econophysics and Economics of Games, Social Choices and Quantitative Techniques. New Economic Windows. Springer, Milano. https://doi.org/10.1007/978-88-470-1501-2$_1$4