Set -9: Modelling stock price variations as a Bachelier-Wiener process

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CS302, Modeling and Simulation

In this report we model price variations of the stocks of companies listed in NIFTY (National Stock Exchange, India) as a Bachelier-Wiener Process. The data for the modelling cover the period January, 1997 to April, 2019 (22 years), as registered in the NIFTY website. The data have been provided in five data files.

I. THE DAILY GROWTH OF MEAN STOCK PRICES

A. Model

$$\frac{\Delta S}{S} = a\Delta t + b\Delta W \tag{1}$$

Here S be the stock price, Δt be time interval and ΔW expresses Wiener process. Under an idealized volatility-free condition, we set $b = 0$. Also, $a = 0.05\%$ per day.

$$S = S_0 \exp(at) \tag{2}$$

$$\Delta(\ln S) = a\Delta t \tag{3}$$

B. Results

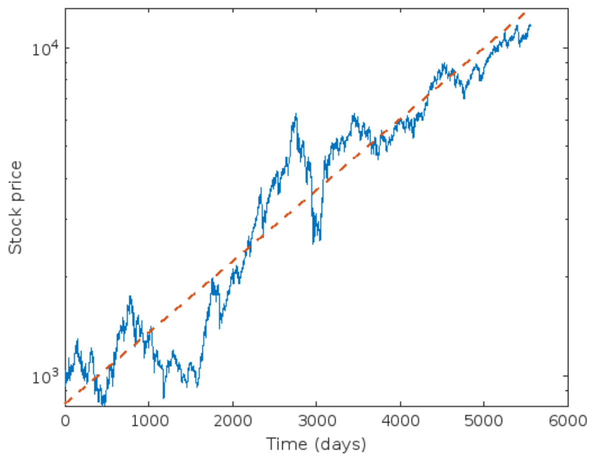

fig. 1. Daily mean growth of the average price of the stock index, NIFTY (NSE, India)

C. Inference

- In this case, t is measured over a period of more than 20 years in days. The least-squares method was used to fit the straight line in this linear-log plot, which shows that S's mean growth is exponential.
- With $b = 0$, the mean relative growth rate of stock values is a

II. GAUSSIAN FLUCTUATIONS IN STOCK PRICES

A. Model

A gaussian function with unity added to it,

$$f(\delta) = 1 + f_0 \exp\left[-\frac{(\delta - \mu)^2}{2\sigma^2}\right] \tag{4}$$

We establish a new variable, δ, which represents the daily percentage change in a stock index's value from the value of the previous day, in order to quantify the fluctuations. $\delta > 0$ indicates positive fluctuations, and $\delta < 0$ indicates negative fluctuations.

Also, here $\mu =< \delta >$ is the mean of the frequency distribution and $\sigma = \sqrt{\langle \delta^2 \rangle - \mu^2}$ is the standard deviation.

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B. Results

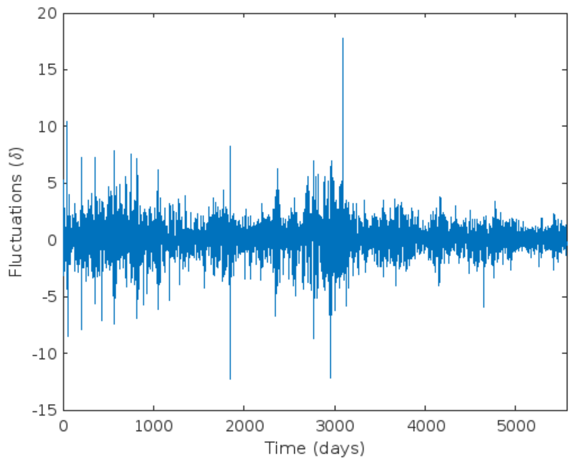

fig. 1. Time series of the daily percentage fluctuation of prices in the stock index, NIFTY (NSE, India)

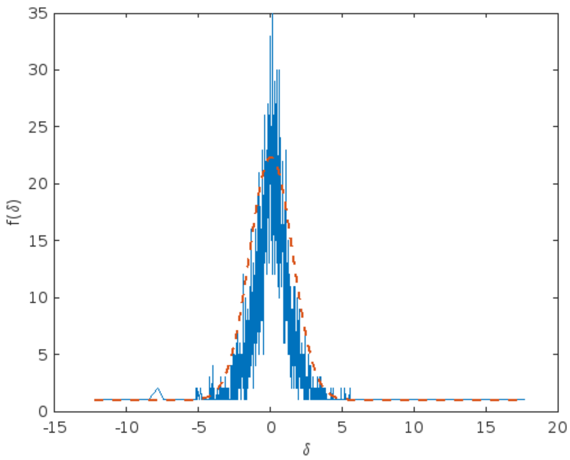

fig. 2. Unnormalized frequency distribution of the daily percentage fluctuation of prices in the stock index, NIFTY (NSE, India)

C. Inference

- The distribution of positive and negative numbers in the daily percentage change of prices is almost equal, with $\delta = 0$. A very small number of variations have huge values at both extremes, while the majority of fluctuations are tiny.

- The distribution looks to be Gaussian and is centred on a mean value, $\mu = 0.057$, with a standard deviation, $\sigma = 1.495$. When there are significant variations, asymptotic convergence is $f(\delta) \to 1$.

III. THE WIENER PROCESS AND VOLATILITY

A. Model

Variable, W, undergoes a Wiener process, then its change, ΔW, in a discrete interval of time, Δt, is

$$\Delta W = \epsilon\sqrt{\Delta t} \tag{5}$$

Here, ϵ has a standardized normal distribution with a zero mean and a standard deviation of unity.

$$(\Delta S/S) = a\Delta t + b\epsilon\sqrt{\Delta t} \tag{6}$$

$$\Sigma^2 = w\tau \tag{7}$$

Here, Σ^2 is the variance of monthly prices and τ is time.

B. Results

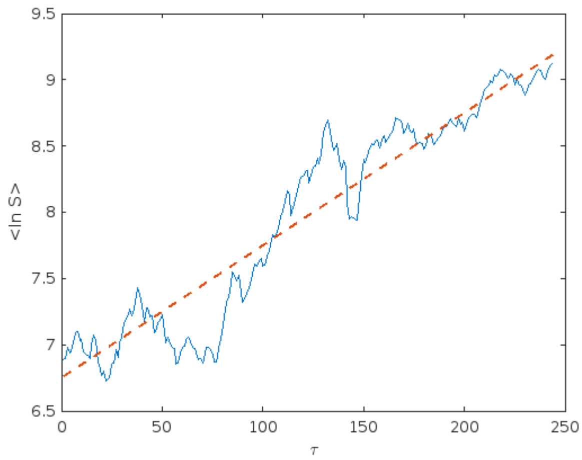

fig. 1. The growth of the monthly average of lnS for NIFTY (NSE, India)

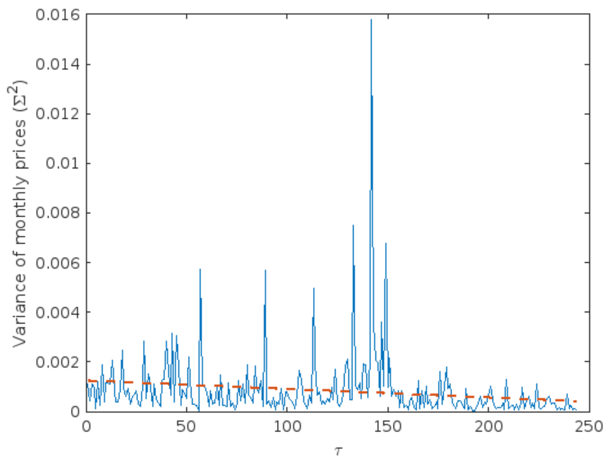

fig. 2. Wiener variance about the monthly average of lnS for NIFTY (NSE, India)

C. Inference

- The period of time, τ, is measured in months and lasts for more than 20 years. The least-squares method is used to fit the straight line that depicts the mean increase. Despite having quite different time scales from the plot in Fig. 1 of Section 1, this map appears statistically close to it.
- With time, the Wiener variance about the monthly average of lnS decreases. Volatility decreases over time when slope $w < 0$ (Here, $w = -3.41$ x 10^{-6} per month).

IV. THE DAILY VOLUME OF TRADE

A. Results

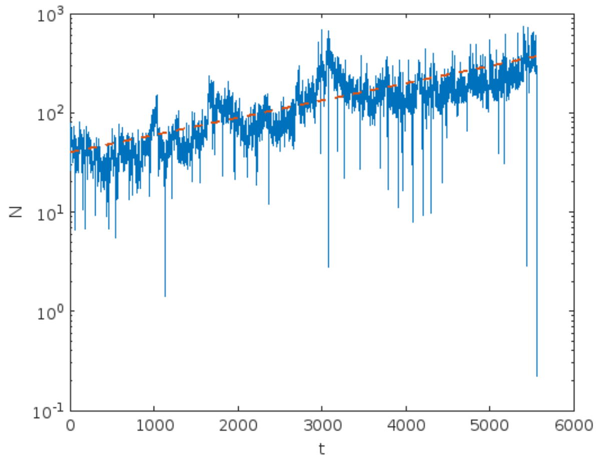

fig. 1. Growth of the daily trade volume of the NIFTY (NSE, India) index

B. Inference

- Number of daily transactions, N scaled by 10^{-6} and time scaled in days. The straight line in this linear-log plot, fitted by the least-squares method, implies an exponential mean growth of N.

V. CONCLUSIONS

- The stock values' daily average rise is exponential. Although there is a nonzero asymptotic convergence, the daily price fluctuations around the mean growth are Gaussian.
- The monthly average stock price growth is statistically close to the daily value growth. The monthly price changes follow a Wiener process, which sees a drop in volatility. The mean growth of the daily volume of trade is exponential.
- Long-term increases in stock values and transaction volume are observed on markets. Markets that are mature have a tendency to resist volatility.

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