Set-3: Modifications of the logistic equations

Dhananjay Vora (2020001417)* and Kashish Shroff (202001425)†
Dhirubhai Ambani Institute of Information & Communication Technology, Gandhinagar, Gujarat 382007, India
CS302, Modeling and Simulation

The paper discusses modifications to the logistic equation and the dynamical equation for agricultural innovations. For the logistic equation with harvesting we plot graph using Euler's method. We compare the numerical solution with the analytical solution for the case without harvesting. For the dynamical equation for agricultural innovations, we rescale the variables and recast the equation in a new form.

I. HARVESTING

A. MODEL

We model the system using the logistic equation,

$$\dot{x} = f(x) = rx[1 - (x/k)] - h \tag{1}$$

Here, x(t) is the population, r=1 is the intrinsic growth rate, k=1000 is the carrying capacity, and h is the harvesting rate.

The integral solution x(t) can be obtained as

$$x = \frac{Nc'[1 - e^{-(Nc+c')t}]}{c' + cNe^{-(Nc+c')t}} \tag{2}$$

B. RESULTS

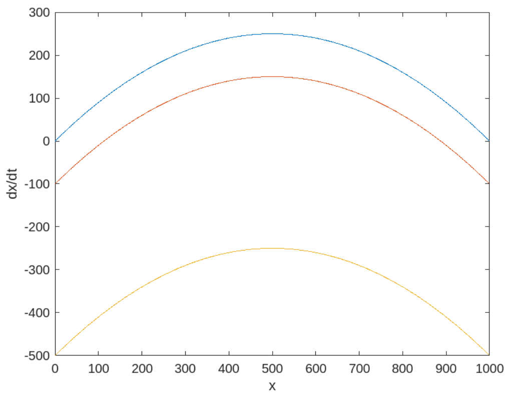

FIG. 1: Plot of $\dot{x}$ vs x for different values of h

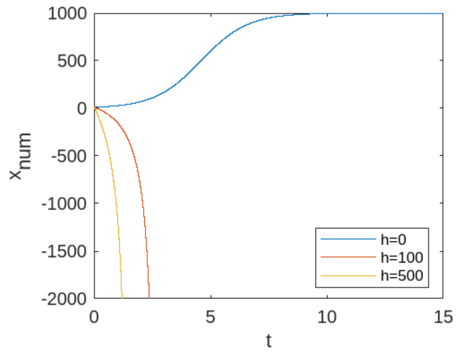

FIG. 2: Plot of x_{num} vs t using Euler's Method

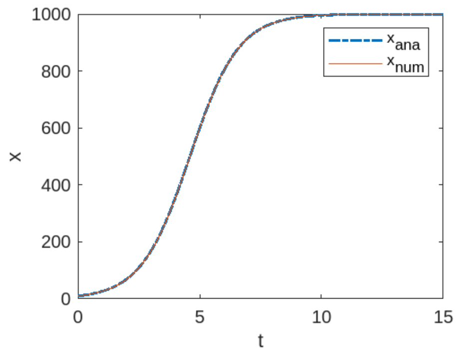

FIG. 3: Analytical Solution Vs Numerical Solution, h=0

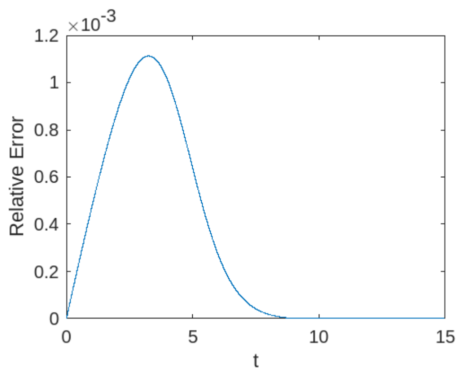

FIG. 4: Relative Error b/w Analytical and Numerical Sol.

*Electronic address: 202001417@daiict.ac.in
†Electronic address: 202001425@daiict.ac.in

Statistical Analysis Of Rel. Error	
Mean	Std
0.301 x 10^{-2}	0.396 x 10^{-2}

II. AGRICULTURAL INNOVATION

A. MODEL

We model the system the logistic equations,

$$\dot{x} = f(x) = cx(N - x) + c'(N - x) \tag{3}$$

with $x(t)$ being the number of farmers who have adopted innovation whereas N-x(t) denotes the number of farmers who have not adopted innovation. Here, N is total number of farmers in the farming community. c' and c are proportional constant due to personal communication and impersonal communication respectively. Rescaling the equation using

$$X = \frac{x}{N} \tag{4}$$

$$t = cNt \tag{5}$$

$$a = \frac{c'}{cN} \tag{6}$$

we get the rescaled equation as,

$$\frac{d(x/N)}{d(t)} = cN[(x/N) + (c'/cN)][1 - (x/N)] \tag{7}$$

$$\frac{d(x/N)}{d(cNt)} = [(x/N) + (c'/cN)][1 - (x/N)] \tag{8}$$

$$\frac{dX}{dT} = (X + a)(1 - X) \tag{9}$$

Rescaling the integral solution $x(t)$ we get

$$x = \frac{Nc'[1 - e^{-(Nc+c')t}]}{c' + cNe^{-(Nc+c')t}} \tag{10}$$

$$(x/N) = \frac{(c'/cN)[1 - e^{-(1+(c'/cN))(cNt)}]}{(c'/cN) + e^{-(1+(c'/cN))(cNt)}} \tag{11}$$

$$X = \frac{a[1 - e^{-(1+a)T}]}{a + e^{-(1+a)T}} \tag{12}$$

B. RESULTS

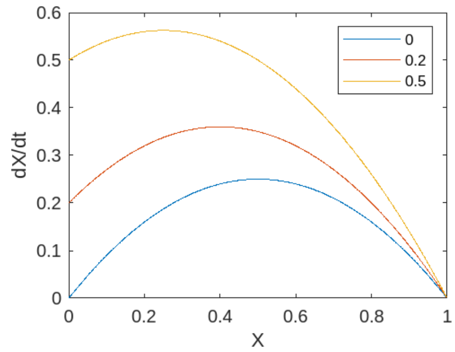

FIG. 1: Plot of $\dot{X}$ vs X for different values of a

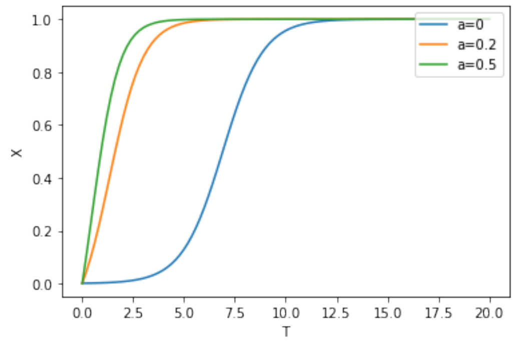

FIG. 2: Plot of X vs t for different values of a

III. CONCLUSIONS

- We can observe that the limiting value obtained by modified logistic equation does not depend on the initial conditions.
- After long time, the convergence is slow as in a power-law.

[1] Ray, A.K. (2010). Modeling Saturation in Industrial Growth. In: Basu, B., Chakravarty, S.R., Chakrabarti, B.K., Gangopadhyay, K. (eds) Econophysics and Economics of Games, Social Choices and Quantitative Techniques. New Economic Windows. Springer, Milano. https://doi.org/10.1007/978-88-470-1501-2$_1$4