Set -10: Simulating the time evolution of a diffusive process

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In this report, we model the time evolution of a diffusive process using the Gaussian function and the wave equation. The diffusion equation with beginning conditions is numerically integrated in the second problem, and the outcomes are compared to the analytical solutions found in the first problem. Our findings demonstrate the process' diffusive nature and the precision of numerical integration in modelling it.

I. DIFFUSION EQUATION

A. Model

The point-source solution of the diffusion equation is given by the Gaussian function,

$$\psi(x,t) = (4\pi Dt)^{-1/2} exp\left(\frac{-x^2}{4Dt}\right) \tag{1}$$

Here, take $D = 1$ as a trial value.
Now, consider another Gaussian function, a solution of the wave equation

$$y(x,t) = y_0 \exp\left[-a(x-vt)^2\right] \tag{2}$$

Here, Taking $y_0 = a = v = 1$.

B. Results

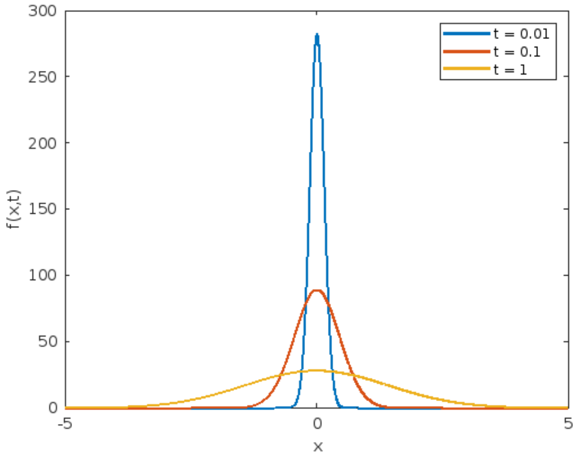

fig. 1. Gaussian function (Diffusion Equation) Plot of Ψ vs. t

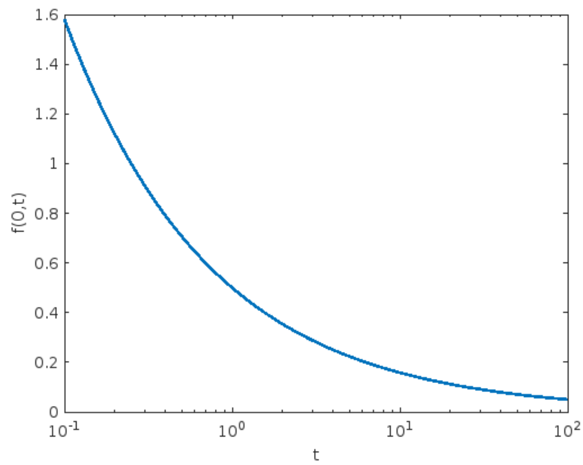

fig. 2. Plot $\Psi(0,t)$ vs. t

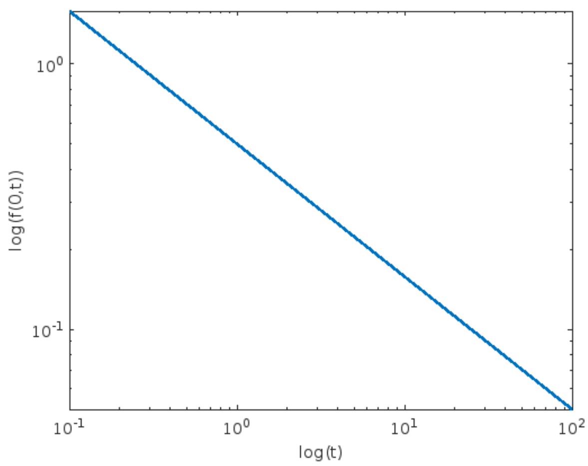

fig. 3. Log-log plot of $\Psi(0,t)$ vs. t

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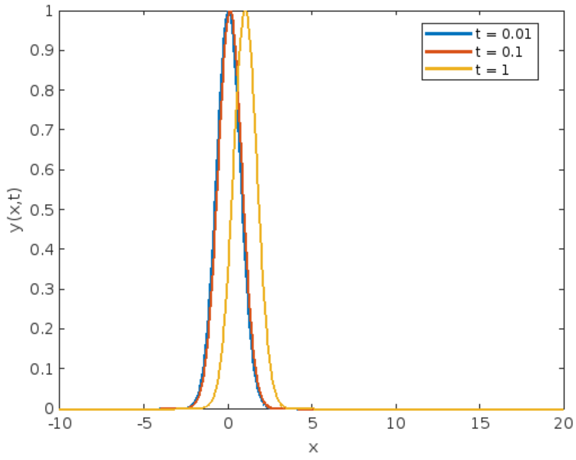

fig. 4. Gaussian function (Wave Equation) Plot of Ψ vs. t

C. Conclusions

- As time (t) increases, the Gaussian function $\Psi(x,t) = (4\pi Dt)^{-1/2}$ spreads out, resulting in a wider distribution with lower amplitude.
- The Gaussian function $y(x,t) = y_0 e^{[-a(x-vt)^2]}$ with $y_0 = a = v = 1$ represents a traveling wave pulse that propagates through a medium. As time (t) increases, the traveling wave pulse shifts in the x-direction, simulating the propagation of a wave. The width of the pulse remains constant, but its position changes with time.
- The diffusion Gaussian function represents a spreading process, where the distribution gradually spreads out over time. On the other hand, the traveling wave pulse Gaussian function represents a propagating process, where the distribution shifts in the x-direction with time while maintaining its shape.

II. GENERAL DIFFERENTIAL EQUATION OF DIFFUSION

A. Model

$$\psi(i,n+1) = \psi(i,n) + \frac{D(\Delta t)}{(\Delta x)^2} \times [\psi(i+1,n) + \psi(i-1,n) - 2\psi(i,n)] \tag{3}$$

Here, with the initial conditions $\psi(0,0) = 1000$ and $\psi(x \neq 0, 0) = 0$.
Also taking $D = 1$, $\Delta t = 0.0001$, $\Delta x \geq \sqrt{2D\Delta t}$ and the same range of x as above.

B. Results

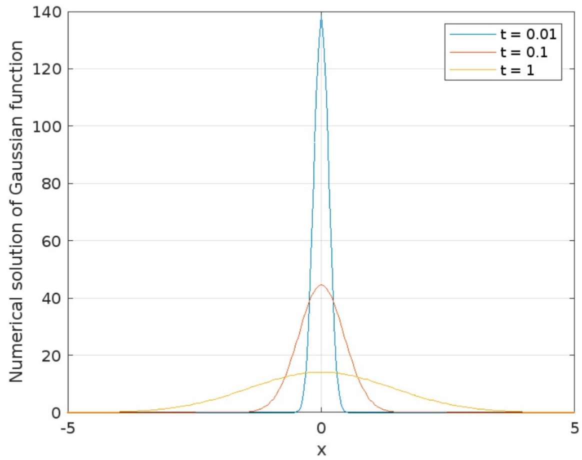

fig. 1. Plot of Ψ vs. t

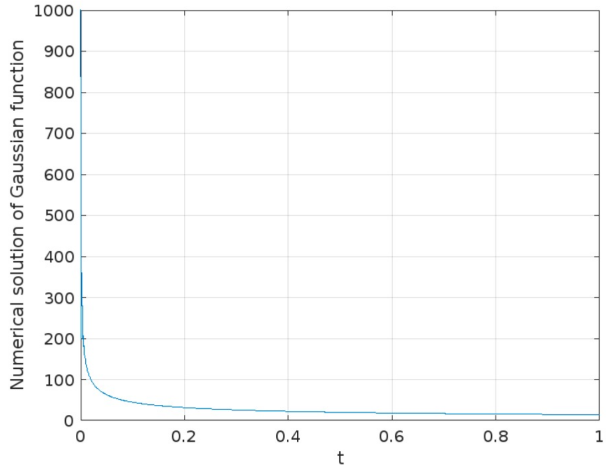

fig. 2. Plot $\Psi(0,t)$ vs. t

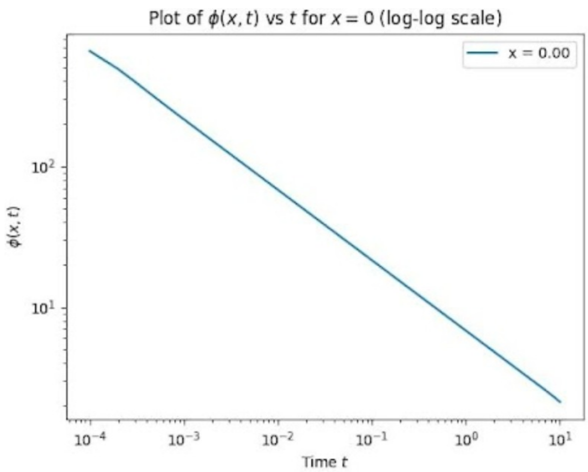

fig. 3. Log-log plot of $\Psi(0,t)$ vs. t

C. Conclusions

- The accuracy of the numerical solution depends on the choice of the time step Δt and the spatial step Δx, which are subject to the stability condition $\Delta x \geq \sqrt{2D\Delta t}$.
- Comparing the numerical solution with the analytical solution obtained in the previous question shows that they are in good agreement, thus validating the accuracy of the numerical method.
- The plots of ψ versus x and $(0, t)$ versus t obtained from the numerical solution exhibit similar behavior as those obtained from the analytical solution.

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