A3 Q3

a) $f(y) = \frac{2y}{\theta} e^{-y^2/\theta}$

find MLE

$$L(\theta) = \prod_{i=1}^{n} \frac{2y_i}{\theta} e^{-y_i^2/\theta} \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

$$L(\theta) = \frac{\prod_{i=1}^{n} 2y}{\theta^n} \prod_{i=n}^{n} e^{-y_i^2/\theta} \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

$$L(\theta) = \frac{\prod_{i=1}^{n} 2y}{\theta^n} e^{\sum_{i=1}^{n} \frac{-y_i^2}{\theta}} \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

$$L(\theta) = \frac{\prod_{i=1}^{n} 2y_i}{\theta^n} e^{-\sum_{i=1}^{n} y_i^2/\theta} \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

$$\ell(\theta) = \log\left(\prod_{i=1}^{n} 2y_i\right) - n\log(\theta) - \frac{1}{\theta}\sum_{i=1}^{n} y_i^2 \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

$$\ell'(\theta) = \frac{-n}{\theta} + \frac{1}{\theta^2}\left(\sum_{i=1}^{n} y_i^2\right) = 0 \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

$$0 = -n + \frac{1}{\theta}\left(\sum_{i=1}^{n} y_i^2\right) \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

$$\hat{\theta}_{mle} = \sum_{i=1}^{n} y_i^2 / n \quad [\text{all } y_i \geq 0,\ \theta > 0]$$

Second derivitive test.

$$\ell''(\theta) = \frac{n}{\theta^2} - \frac{2\sum_{i=1}^{n} y_i^2}{\theta^3} < 0 \quad \left[\begin{array}{l} \text{all } y_i \geq 0 \\ \theta > 0 \end{array}\right]$$

Satisfied when $n\theta < 2\sum_{i=1}^{n} y_i^2$

notice that at $\hat{\theta}_{mle}$ it is satisfied when

$$n\hat{\theta} < 2\sum_{i=1}^{n} y_i^2 \iff \frac{n\sum_{i=1}^{n} y_i^2}{n} < 2\sum_{i=1}^{n} y_i^2$$

$$\iff \sum_{i=1}^{n} y_i^2 < 2\sum_{i=1}^{n} y_i^2$$

[Since all $y_i \geq 0$, this is true]

~~Hence~~ hence,

$$\hat{\theta}_{mle} = \sum_{i=1}^{n} y_i^2 / n$$

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b) $\hat{\theta}_{mle} = \sum_{i=1}^{n} y_i^2 / n$

$$= (14.7)^2 + (30.0)^2 + (13.3)^2 + (41.9)^2 + (25.6)^2 + (39.6)^2 + (34.5)^2 + (9.9)^2 + (13.6)^2 + (24.2)^2 + (5.1)^2 + (41.4)^2 + (20.5)^2 + (22.2)^2 / 14$$

$$= 713.145$$

average wind speed of all 14 days $= \sqrt{\theta n}/2$

$= \sqrt{713.145(n)}/2 \quad = 23.7$ km/h

c)

$$W = \frac{2}{\theta}\sum_{i=1}^{n} y_i^2 \sim \chi^2_{2n}$$

for $n=14$, Pivot quantity W, derive confidence interval for 95% for θ.

$$P(c_1 \leq W \leq c_2) = 1-\alpha$$

$$P(\underbrace{\chi^2_{2n,\alpha/2}}_{\text{2.5 Percitile of } \chi^2_{2n}} \leq \underbrace{\frac{2}{\theta}\sum_{i=1}^{n} y_i^2}_{\chi^2_{2n}} \leq \underbrace{\chi^2_{2n,1-\frac{\alpha}{2}}}_{\text{97.5 Percentile of } \chi^2_{2n}}) = 1-\alpha$$

$n=14$, $2n=28$ | $1-\alpha = 0.95$, $\alpha = 0.05$, $\alpha/2 = 0.025$

$$P\left(\frac{2\sum_{i=1}^{n} y_i^2}{\chi^2_{28,0.025}} \geq \theta \geq \frac{2\sum_{i=1}^{n} y_i^2}{\chi^2_{28,0.975}}\right)$$

$$\left(\frac{2\sum_{i=1}^{n} y_i^2}{44.5}, \frac{2\sum_{i=1}^{n} y_i^2}{15.31}\right)$$

We are 95% confident that θ is contained in the general expression $\left(\frac{2\sum_{i=1}^{n} y_i^2}{44.5}, \frac{2\sum_{i=1}^{n} y_i^2}{15.31}\right)$

d) $\sum_{i=1}^{n} y_i^2 = 9984.03$

$$\left(2\frac{\sum_{i=1}^{n} y_i^2}{44.5}, \quad 2\frac{\sum_{i=1}^{n} y_i}{15.31} \right)$$

$= (448.7, 1304.2)$

We are 95 percent confident that the value θ is contained within (448.7, 1304.2).

e) To determine if a wind farm should be situated in windy hill we must compare the effiant mean speed of 16km/h.

From b) we found $\hat{\theta}_{MLE} = 713.145$.

Using $\sqrt{\theta n}/2$ we find that using the data that the point estimated mean wind speed is 23.7 km/h.

From d we found that we are 95% confident that θ is contained within (448.7, 1304.2). Using $\sqrt{\theta n}/2$ we can find 95% confidence interval for mean wind speed.

$\sqrt{\theta n} / z$

lower bound

$\sqrt{n(448.7)} / z$

$= (18.8)$

upper bound.

$\sqrt{n(1304.2)} / z$

$= 32.0$

Thus we are 95 Percent Confident that the mean wind Speed is contained within the interval (18.8 km/h, 32.0 km/h).

Since our Point estimate of mean wind Speed is 23.7 km/h which is greater than 16 and our confidence interval leads us to be 95% confident that the mean wind Speed is in the interval (18.8, 32.0) and all values inside the interval are greater than 16, we can conclude that we can reccomend wind farms be Situated in windy hill.