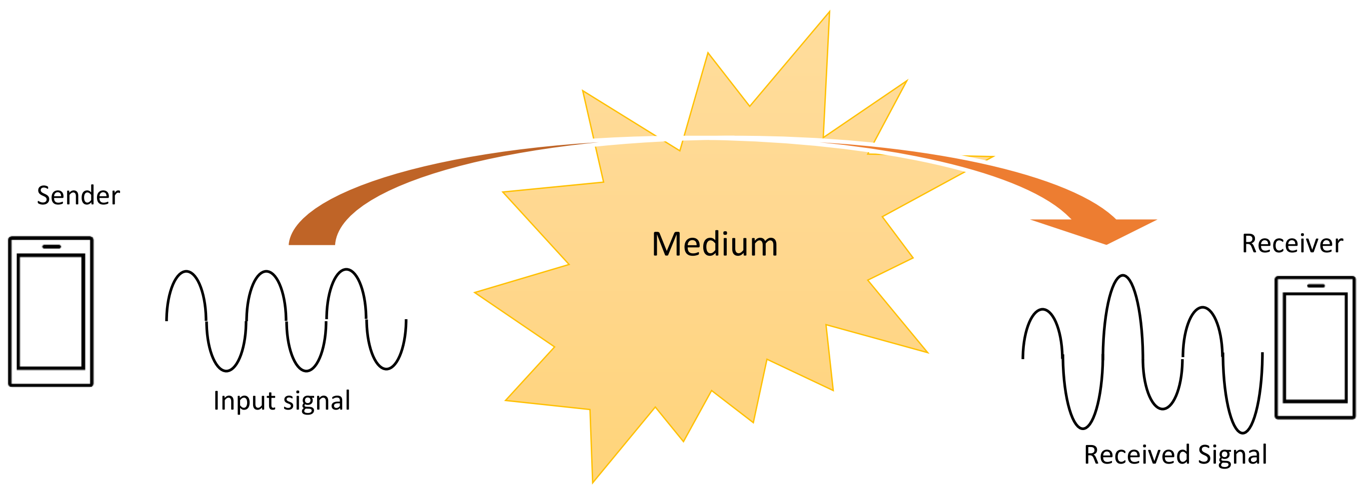
Channel estimation

In all communication systems, data is transferred from source to the destination in form of signals. These signals traverse different medium which can be wired or wireless. Copper wires or fibre cables are two examples of wired medium while air is a wireless medium. These mediums are also called channel. When a signal passes from channel, it is distorted from the noise or from other signals traversing that same medium. This means that when signal is received at its destination, it could have errors. So, in order to remove the noise and distortion effects of channel from the received signal, channel's properties have to be found out. The process of figuring out channel characteristics is called Channel Estimation.



Channel estimation process consists of multiple steps. First a mathematical model is created of the channel. Then a signal which is known by both sender and receiver is transmitted over the channel. When the receiver receives the signal, it is of course distorted and contains noise from the channel, but the receiver also knows the original signal, thus it can compare the original signal and received signal to extract the properties of channel and the noises added to the sent signal in the channel.

To put is in 3 main steps:

1. Mathematical model for channel is created. This model correlates sent and received signal using channel matrix.
2. A signal known by both sender and receiver is sent by sender over the channel.
3. Receiver compares the received signal with original signal and figures out the values in channel matrix.

Note: the signal that is sent and is known by both sender and receiver is usually called reference signal or pilot signal.

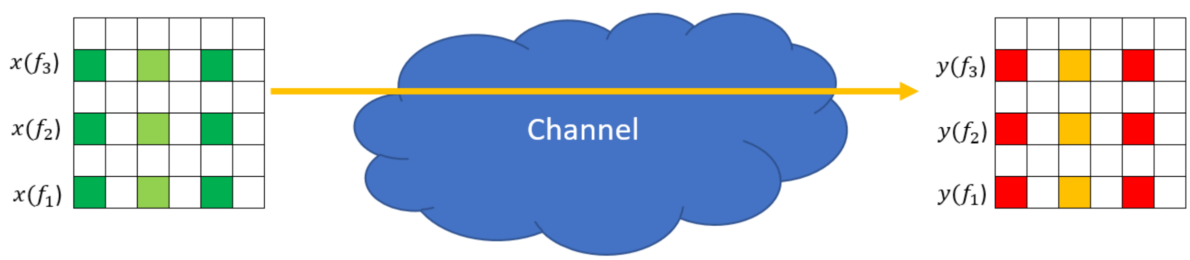
Channel Estimation in Single Input Single Output System

Let's consider a single input single output (SISO) system. As channel can affect different frequency signals differently, so channel estimation have to be done for each frequency channel. Depending on the number of channels it could be complex and resource consuming so often channel estimation is done for fewer channels and estimates of rest of the channels are interpolated from the computed estimates.

We consider 3 frequencies (in our example) from f_1 to f_3 from all the available frequencies. And this will mean that reference signal or pilot signals will be $x(f_1), x(f_2), and\ x(f_3)$. Reference signals are complex signals (having I/Q form of data). For the remaining frequency channels, we can interpolate the channel characteristics via the ones that are calculated.

When these reference signals are received at destination, they contain distortions and noise, and they are represented by $y(f_1), y(f_2), and\ y(f_3)$. Now to represent received signals $y(f)$ in terms of $x(f)$, channel function is required for that specific frequency. It can be given by $h(f)$.

Therefore, the relation of reference signal, received signal, and channel function can be represented by the correlation function.



Which can be written as

$$y(f_1) = h(f_1) \cdot x(f_1)$$

$$y(f_2) = h(f_2) \cdot x(f_2)$$

$$y(f_3) = h(f_3) \cdot x(f_3)$$

From these equations, $x(f)\ and\ y(f)$ are known thus $h(f)$ can be calculated.

$$h(f_1) = y(f_1) \cdot x^H(f_1)$$

$$h(f_2) = y(f_2) \cdot x^H(f_2)$$

$$h(f_3) = y(f_3) \cdot x^H(f_3)$$

Here $x^H(f)$ is the Hermitian of $x(f)$.

Now since, only 3 frequencies had been considered and channel characteristics are estimated for only those frequencies, channel properties for rest of the frequencies can be estimated via interpolation of already known characteristics.

Channel function in these equations represents channel distortion. Noise is also added to the distorted signals, therefore actual equations of the received signal look like following:

$$y(f_1) = h(f_1) \cdot x(f_1) + n(f_1)$$

$$y(f_2) = h(f_2) \cdot x(f_2) + n(f_2)$$

$$y(f_3) = h(f_3) \cdot x(f_3) + n(f_3)$$

Similar to how channel function was estimated, theoretically noise can also be estimated by using averaged channel estimate $\bar{h}(f)$.

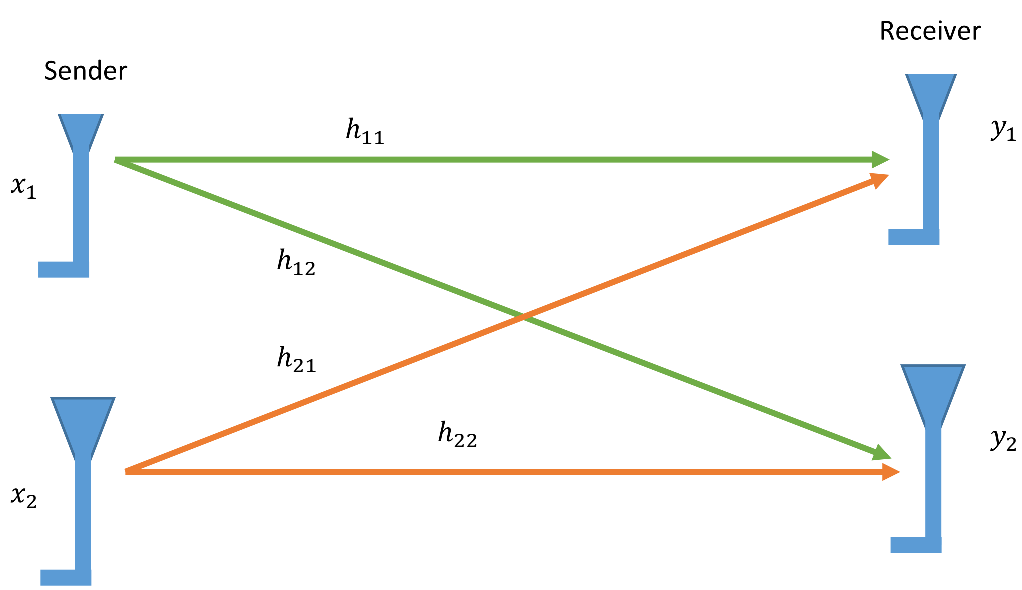
$$n(f_1) = y(f_1) - \bar{h}(f_1) \cdot x(f_1)$$
$$n(f_2) = y(f_2) - \bar{h}(f_2) \cdot x(f_2)$$
$$n(f_3) = y(f_3) - \bar{h}(f_3) \cdot x(f_3)$$

But this provides with absolute values of noise and due to continuously variations in noise in a channel, absolute values of noise are not beneficial in channel estimation. What's beneficial is the estimated function of noise which encompasses and models noise variations too. For that, there are different algorithms and methods. One of them, which is implemented in srsLTE (Open-source LTE implementation on SDR) is to subtract averaged channel estimate from actual channel estimate.

$$n(f_1) = h(f_1) - \bar{h}(f_1)$$
$$n(f_2) = h(f_2) - \bar{h}(f_2)$$
$$n(f_3) = h(f_3) - \bar{h}(f_3)$$

Channel Estimation in Multiple Input Multiple Output (MIMO) system

In a Multiple input multiple output system, there are multiple transmission endpoints and multiple receptions endpoints.



In MIMO system the process of channel estimation remains the same except now there are two signals received from a single source. This means that two paths in the medium were used, one path per signal. Therefore, to compute the final signal $y(f)$ for each frequency, both received signals have to be considered. This results in formation of the matrix of received signals.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Similar to SISO system, Hermitian of input matrix x can be taken to estimate channel matrix h. And noise matrix n can also be calculated in the similar manner via matrix operations.

Resources:

1. https://en.wikipedia.org/wiki/Channel_state_information#Estimation_of_CSI
2. Basics of Channel Estimation: https://www.youtube.com/watch?v=ZsLh01nlRzY
3. Paper: “Different Types of Channel Estimation Techniques Used in MIMO-OFDM for Effective Communication Systems”, by Rakshit Govil, 2018 IJERT.
4. Masters Thesis: “Channel Estimation in mobile wireless systems”, by Idd Pazi Alli, KTH Sweden.
5. Open Source LTE implementation based on SDR : https://www.sharetechnote.com/html/SDR_srsLTE.html
6. Channel Estimation in MATLAB
    a. https://www.mathworks.com/help/lte/ug/channel-estimation.html
    b. https://www.mathworks.com/help/5g/ug/nr-cell-search-and-mib-and-sib1-recovery.html