Mean of Binomial Distribution: 10M

expectation of x

$$E(x) = \sum x\, p(x)$$

$$= \sum_{x=0}^{n} x\, nC_x \cdot p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

$n! = 1\times2\times3\ldots(n-1)\times n$

$x! = 1\times2\times3\times\ldots(x-1)\times x$

$$= np \sum_{x=1}^{n} \frac{(n-1)!\, p^{x-1} q^{[(n-1)-(x-1)]}}{(x-1)!\,[(n-1)-(x-1)]}$$

$$= np\,(\underbrace{q+p}_{1})^{n-1}$$

$$= np$$

Variane of Binomial Distribution:

$$\text{Variance}(x) = E(x^2) - (E(x))^2$$

$$n(n-1)p^2 + np - (np)^2$$

$$E(x^2) = \sum x^2\, p(x)$$

$$= \sum (x^2 - x + x)\, p(x)$$

$$= \sum (x^2 - x)\, p(x) + \underbrace{\sum x\, p(x)}_{np}$$

$$= \sum x(x-1)\, nC_x\, p^x q^{n-x}$$

$$= \sum x(x-1) \frac{n!}{x!\,(n-x)!} p^x q^{(n-x)}$$

$$= \sum x(x-1) \frac{1\times2\times\ldots(n-2)\times(n-1)\, n}{1\times2\times\ldots(x-2)\times(x-1)\, x \cdot [n-x]!} p^x q^{(n-x)}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!\, p^{x-2} q^{[(n-2)-(x-2)]}}{(x-2)!\,[(n-2)-(x-2)]!}$$

$$= n(n-1)\,p^2\,(\underbrace{p+q}_{1})^{n-2}$$

$$= n(n-1)\,p^2$$

Varience(x)

$n(n-1)p^2 + np - (np)^2$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$= np - np^2$

$= np[\underbrace{1-p}_{q}]$

$= npq$

(14) The mean and variance of

$B(n,p)$ is 16 & 8. Find i) $P(x =$

Mean $= np = 16 \Rightarrow \frac{1}{2}n = 16 \quad n = 3$

② 10m Find the mean and variance of poisson distribution.

Mean:

$$E(x) = \sum x\, P(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{0}^{\infty} \frac{\cancel{x} \cdot \lambda^x}{1 \times 2 \times \dots (x-1)\cancel{x}}$$

$$= e^{-\lambda}\lambda \sum_{1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= e^{-\lambda} \cdot \lambda \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots\right]$$

$$= e^{-\lambda} \cdot \lambda \cdot e^{\lambda}$$

$$= \lambda$$

Variance:

$$\text{variance}(x) = E(x^2) - (E(x))^2$$

$$\boxed{E(x^n) = \sum x^n P(x)}$$

$$E(x^2) = \sum \frac{x^2 e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum \frac{(x^2 - x + x)\lambda^x}{x!}$$

$$= e^{-\lambda} \sum \frac{(x^2 - x) \cdot \lambda^x}{x!} + \underbrace{e^{-\lambda} \sum \frac{x\lambda^x}{x!}}_{\lambda}$$

$$= e^{-\lambda} \sum \frac{x(x-1) \cdot \lambda^x}{1 \times 2 \times \dots (x-2)(x-1)x} + \lambda$$

$$= \lambda^2 e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= \lambda^2 e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots\right] + \lambda$$

$= \lambda^2 e^{-\lambda} \cdot e^{\lambda} + \lambda$

$= \lambda^2 + \lambda$

$\text{Variance}(x) = E(x^2) - (E(x))^2$

$= \lambda^2 + \lambda - (\lambda)^2$

$= \lambda$

Fitting a Poison Distribution:

Fitting a poisson Distribution for

frequency distribution (x_i, f_i) means a

given distribution is aproximately Poi

find pmf and then finding theoritical

we calculate mean of poisson variable

1. suppose that x as a P.D if P(x

⑧ 10 m
A random variable x has the uniform distribution over the interval $(\underset{\text{lower limit}}{-3}, \overset{\text{upper limit}}{3})$ compute (i) $P(x=2)$ (ii) $P(x<2)$ iii) $P(|x|<2)$ iv) $P(|x-2|<2)$ v) Find k for which $P(x>k) = 1/3$.

$\frac{1}{b-a}$ $\frac{1}{-3-3}$ $= \frac{1}{6}$

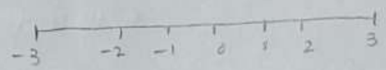
$P(x) = 1/6$

i) $P(x=2)$ is zero

ii) $P(x<2) = \int_{-3}^{2} \frac{1}{6}\, dx$

$= \frac{1}{6}[x]_{-3}^{2}$ $\quad -3-2=5$

$= \frac{5}{6}$



iii) $P(|x|<2) = P(-2<x<2)$

$= \int_{-2}^{2} \frac{1}{6}\, dx$

$= \frac{1}{6}[x]_{-2}^{2}$ $\quad -2-2=4$

$= \frac{4}{6} = 2/3$.

iv) $P(|x-2|<2) = P(\overset{+2}{-2}<(x-2)<2)$

$= P(0<x<4)$

$= \int_{0}^{3} \frac{1}{6}\, dx$

$= \frac{1}{6}[x]_{0}^{3}$

$= \frac{3}{6} = \frac{1}{2}$

v) $P(x>k) = \int_{k}^{3} \frac{1}{6}\, dx = 1/3$

$= \frac{1}{6_2}[x]_{k}^{3} = \frac{1}{3}$

$$\Rightarrow \frac{3-k}{2} = 1$$

$$\Rightarrow 3-k = 2 \quad (\text{or}) \quad k = 1$$

$3-2 = k$

$1 = k$

10m

(1) Find the constant c show that function

$$f(x) = \begin{cases} cx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

is a PDF evaluate $P(1 < x < 2)$

We know

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} + \int_{0}^{3} + \int_{3}^{\infty} = 0 + \int_{0}^{3} cx^2\,dx + 0 = 1$$

ie, $c \int_{0}^{3} x^2\,dx = 1$

$$c\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$c\left[\frac{27}{3} - 0\right] = 1$$

$9c = 1$

$\Rightarrow c = 1/9$

For $x < 0$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \frac{1}{9}\int_{-\infty}^{0} x^2\,dx$$

$$= 0$$

$x \geq 3$

$$F(x) = \frac{1}{9}\int_{3}^{\infty} x^2\,dx$$

$$= \frac{1}{9}\left[\frac{x^3}{3}\right]_3^{\infty} = 1 - \frac{1}{9}\left[\frac{27}{3}\right]$$

$$= 1 - 1 = 0$$

distribution:

$$F(x) = P(X \leq x)$$

$$= \frac{1}{9}\int_0^x x^2\,dx = \frac{1}{9}\left[\frac{x^3}{3}\right]_0^x$$

$$= \frac{x^3}{27}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{27} & 0 \leq x \leq 3 \\ 0 & x \geq 3 \end{cases}$$

$$\frac{d}{dx} F(x) = f(x)$$

$$\frac{d}{dx}\left(\frac{x^3}{27}\right) = \frac{1}{27} 3 \times x^2$$

$$= \frac{1}{9} x^2$$

$$f(x) = \frac{1}{9}x^2, \quad 0 \leq x \leq 3$$

$P(1 < x < 2)$

$$= \frac{1}{9}\int_1^2 x^2\,dx$$

$$= \frac{1}{9}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{27}[8-1]$$

$$= \frac{7}{27}$$

Problem

Urn 1 contains 2 white & 3 black, Urn 2 contains 4 white & 1 black, Urn 3 contains 3 white & 4 black balls. Urn is selected at random and a ball is drawn and it is found to be white. What is the probility Urn 1 was selected.

Given,

Urn 1 - 2w, 3b

Urn 2 - 4w, 1b

Urn 3 - 3w, 4b

E_1, E_2, E_3

$P(E_1) = P(E_2) = P(E_3) = 1/3$

Let A be the event of

$P(A/E_1) = \frac{2C_1}{5C_1} = \frac{2}{5}$

$P(A/E_2) = \frac{4}{5}$

$P(A/E_3) = \frac{3}{7}$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)} = \frac{1/3 \times 2/5}{1/3 \times 2/5 + 1/3 \times 4/5 + 1/3 \times 3/7}$$

$$= \frac{\frac{2}{15}}{\frac{2}{15} + \frac{4}{15} + \frac{\cancel{3}}{\cancel{21}}\ 1/7}$$

$$= \frac{\frac{2}{15}}{\frac{42+15}{105}} = \frac{2}{\cancel{15}} \times \frac{\cancel{105}}{57} = \frac{14}{57}$$

2) Urn contains $\underbrace{5 \text{ white } \& \ 3 \text{ green balls}}_{8}$, and another Urn contains $\underbrace{3 \text{ white } \& \ 7 \text{ green balls}}$. 2 balls are choosen at random from the 1st Urn and put it into the 2nd Urn then a balls is drawn from the 2nd Urn. what is the probability that it is the white ball.

$E_1 = 2w$ $\quad$ 5w, 7g / 3w, 9g

$E_2 = 2g$ $\quad$ 4w, 8g

$E_3 = 1w, 1g$

I: 8, 5w, 3g $\qquad$ II: 3w 7g

the 2 balls are drawn from 1st Urn then E_1 be the event of choosing 2w ball, E_2 be the event of choosing 2 green ball, E_3 be the event of choosing 1w, 1g ball from the 1st Urn to the 2nd after the balls are trasfered from the

Urn 5w, 7g $\quad$ 3w, 9g $\quad$ 4w, 8g

A be event of choosing white ball from Urn

$$P(A/E_1) = \frac{5C_1}{12C_1} = \frac{5}{12} \qquad P(E_1) = \frac{5C_2}{8C_2} \qquad \frac{5\times 4}{1\times 2} \Big/ \frac{8\times 7}{1\times 2} = \frac{10}{28}$$

$$P(A/E_2) = \frac{3C_1}{12} = 3/12 = 1/4 \qquad P(E_2) = \frac{3C_2}{8C_2} \qquad \frac{3\times 2}{1\times 2}$$

$$P(E_3) = \frac{5C_1 \times 3C_1}{8C_2}$$

$$P(A/E_3) = \frac{4C_1}{12C_1} = 4/12 = 1/3$$

$$P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)$$

$$= \frac{\cancel{10}^{5}}{28} \times \frac{5}{\cancel{12}_{6}} + \frac{3}{28} \times \frac{1}{4} + \frac{\cancel{15}^{5}}{28} \times \frac{1}{\cancel{3}_{1}} = \frac{25}{168 \times 2} + \frac{3}{112} + \frac{5}{28}$$

=

In a coin toss in experiment the coin shows Head one die is thrown and the result is recorded but is a coin shows tail 2 dies are thrown and their sum is recorded. What is the probability that recorded number is 2.

when a single die is thrown $P(\text{getting } 2) = \frac{1}{6}$

when a coin is tail $P(\text{getting } 2 \text{ with } 2 \text{ dies}) = \frac{1}{36}$

So, the total probability $P(\text{getting } 2) = P(\text{getting Head}) \cdot P(\text{getting } 2 \text{ with } 1 \text{ dice}) + P(\text{getting tail}) \cdot P(\text{getting } 2 \text{ with } 2 \text{ dice})$

$$= \frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{36}$$

$$= \frac{1}{12} + \frac{1}{72} = \frac{72+12}{864} = \frac{84}{864}$$

$$= \frac{7}{72}$$

A bag contains 5 balls and it is not known how many of them are white 2 balls are drawn at random from their and their noted to be white. What is the chance at all the balls in the bag are all white.

E_1 be event of choosing 2w balls

E_2 be event of choosing 3w balls

E_3 be event of choosing 4w balls

E_4 be event of choosing 5w balls

Let A be event of choosing 2w balls

$P(A/E_1) = \frac{2C_2}{5C_2} \quad \frac{\frac{2\times1}{1\times2}}{\frac{5\times4}{1\times2}} = \frac{1}{10}$

$P(A/E_2) = \frac{3C_2}{5C_2} \quad \frac{\frac{3\times2}{1\times2}}{\frac{5\times4}{1\times2}} = \frac{3}{10}$

$P(A/E_3) = \frac{4C_2}{5C_2} = \frac{\frac{4\times3}{1\times2}}{\frac{5\times4}{1\times2}} = \frac{6}{10}$

$P(A/E_4) = \frac{5C_2}{5C_2} \quad \frac{5\times4}{5\times4} = 1$

since the no. of w balls in the bag is unknown it is equally likely so $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

By Baye's Theorem

$$P(E_4/A) = \frac{P(E_4)\cdot P(A/E_4)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3) + P(E_4)\cdot P(A/E_4)}$$

$$= \frac{\frac{1}{4}\times1}{\frac{1}{4}\times\frac{1}{10} + \frac{1}{4}\times\frac{3}{10} + \frac{1}{4}\times\frac{6}{10} + \frac{1}{4}\times1}$$

$$= \frac{\frac{1}{4}}{\frac{1}{40} + \frac{3}{40} + \frac{6}{40} + \frac{10}{40}} = \frac{1}{4}\times\frac{40}{20}$$

$$= \frac{10}{20} = \frac{1}{2}$$

Random Variable:

Random variable abreviated as R.V is a

A random variable function that assigns a real number $X(s)$ for every element $s\in S$ assign a real number x as $X(s)=x$

Remarks:

Let w be an out come of an expriement then the x of w represents a real number which