

1. Inserting n elements using

a) Aggregate Method

→ The table doubles in size when it runs out of space.

→ So, if the original size is 1, after insertion it doubles

to size 2, after 2 more insertions it doubles to size 4.

→ In general, after k doublings, the size is 2^k .

Pseudo code:

Initialize table with capacity = 1

for $i = 1$ to n ;

if table is full:

new table = create newtable with size 2

copy elements from old table to new

table = new table.

insert element i into table.

let $k = \log(n+1) - 1$

Total cost = $O(n) + k$

= $O(n \log n)$

Amortized cost per insertion = $O(\log n)$

Runtime per insertion is $O(\log n)$

Total time is $O(n) + \log(n+1)$

b) Accounting Method

- charge 2 unit for each insertion, when the table doubles in size from m to $2m$, credit m units.
- The credit exactly pay for the copy cost of $O(m)$
- Total credit is $m + 2m + 4m + \dots \approx \frac{n}{2} \times m = O(n)$

Pseudo code

Initialize table with capacity = 1

for $i = 1$ to n ;

if table to n ;

new table = create new table with size $2 \times$ current size

copy elements from old table to new table.

table = new table

insert element i into table.

initialize charges = 0

initialize credits = 0

for $i = 1$ to n

charges $++ 2$

if table doubles in size from m to $2 \times m$

credits $++ m$

Total charges = $2 \times n = O(n)$

Total credits = $m + 2m + \dots \approx \frac{n}{2} \times m = O(n)$

Amortized cost per insertion = Total $/ n$

= $O(n) / n$

= $O(1)$

Runtime per insertion = $O(1)$

Total time = $O(n)$