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It may be noted that Error detection is possible if received code vector is not equal to some other code vector, this shows that the (minimum) transmission errors in the received code vector must be less than minimum distance (d_{min}).

Table shows requirement Error capability of code:-

S.NO	Name of Errors detected/corrected	Distance requirement
01.	Detect upto 's' Errors per word.	$d_{min} \geq s+1$
02	Correct upto 't' Error per word	$d_{min} \geq 2t+1$
03	Correct upto 'ts' and detect $s>t$ Error per word.	$d_{min} \geq t+s+1$

d_{min} = NO Zero Code vector.
↳ Weight of code vector (No of One's)

(8) Code Efficiency:- Expressed as ratio of message bit in a block to the transmitted bit for that particular block by the Encoder:. $\eta = \frac{k}{n}$

(9) Weight of Code:- NON Zero Elements in the transmitted code vector, Eg:- $W(x)$ $x \to$ code vector.

0 → Even
1 → odd } sign

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$X = 01110\ 101$
$W(x) = 5$

ERROR CODING

1) (10) Parity Coding:- { Odd / Even } Parity
2) Vertical Redundancy check
3) longitudinal redundancy check (LRC)
4) Linear Block Codes
5) Hamming codes,
6) Cylic Codes
7) Convolutional Codes.

PARITY Coading:-

Even:- Parity of binary word is known as Even if it contains Even no of One's

Eg:- 0110110110 → 6,1's   b8 → if odd then (1)
b8 → 1 → then Even

odd → 1 } b8 bit.
Even → 0

ODD:- if it contains odd no of ones.

(2) Vertical Redundancy Check:-
ASKEW ASCII

Character	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
k	1	1	0	1	0	0	1	0. Even
0	1	1	1	1	0	0	1	1 odd
	1	0	0	1	0	0	1	1

③ Longitudinal Redundancy check :- (LRC)

In

STX	Inform block	ETB (End Block of Text)	BCC

Start of text character: One charac

Many character

End of Transmitted Block

Block check Charac.

Characte	STX	N	A	I	T	I	S	O	K	ETB		BCC
b_1	0	0	1	1	0	1	1	1	1	1		1
b_2	1	1	0	0	0	0	1	1	1	1		0
b_3	0	1	0	0	1	0	0	1	1	1		0
b_4	0	1	0	1	0	$\boxed{1}$	0	1	0	0		1
b_5	0	0	0	0	1	0	1	0	1	1		1
b_6	0	0	0	0	0	0	0	0	0	0		0
b_7	0	1	1	1	1	1	1	1	0	0		0
b_8	1	0	0	1	1	1	1	0	1	0	0	1

x

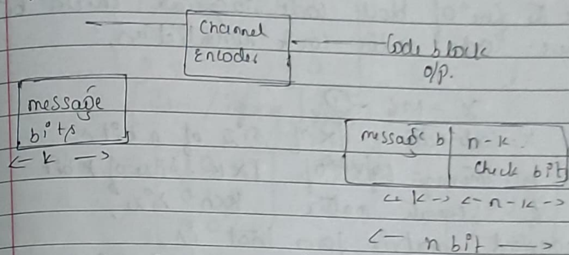
Char parity

row parity

④ Linear Block

For a Block of K mssg bits an (n-k) parity bits are called / Check bits are added. This means that the total bits of o/p Encoder are 'n', such types of codes are known as (n,k) block codes.

Systematic codes



A code is known as linear if the sum of any 2 code vector produce another code vector.

Steps for determination of all code words for a linear block codes

① Let us consider that, the particular code vector consist of $m_1 m_2 \dots m_k$ message bit and $C_1 C_2 \dots C_q$ check bits. Then the code vector may be written as

$m_1 m_2 \dots m_k$

$C_1 C_2 \dots C_q$

i.e.

$$X = (m_1 m_2 \dots m_k \cdot C_1 C_2 \dots C_q)$$

$q = n - k$

② This means that q are the no of redundant bits added by the Encoder.

The above code can be written as

$$X = (M / C)$$

M - k bit message vector

C - 'q' bit check vector

③ Here check bit play the role of error detection and correction (Q) C

④ Here the func of block code is to generate check bit here code vector can be represented as.

$$X = MG \quad -①$$

X : code vector — $1 \times n$ size of n bits

M : message bits / value of — $1 \times k$ size of k bit

G → generator matrix — $k \times n$ size.

① represent matrix form that is.

$$[X]_{1\times n} = [M]_{1\times k}[G]_{k\times n}$$

G matrix depends on linear block code used

(7,4) → (n, k).

$q = (n-k) = 3$ $\quad m = 4$

⑤ Generator is represent as $G = [I_k][P_{k\times q}]_{k\times n}$ (identity; submatrix)

here $I_k = k \times k$ identity matrix

$P = k \times q$ submatrix

NOW check vector may be obtained as.

$$C = M \times P \text{ eq}$$

$C = C_1 C_2 \cdot C_Q$ $\quad P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1Q} \\ \vdots & & & \\ P_{k1} & P_{k2} & \cdots & P_{kQ} \end{bmatrix}_{k \times Q}$

$M = M_1 M_2 \; M_Q$

$0 \oplus 1 = 1$

$1 \oplus 0 = 1$

$0 \oplus 0 = 0$

$1 \oplus 1 = 0$

On solving the matrix Eq^n:

$C_1 = M_1 P_{11} \oplus M_2 P_{21} \oplus M_3 P_{31}$

$C_2 = M_1 P_{12} \oplus M_2 P_{22} \oplus M_3 P_{32}$

$C_3 = M_1 P_{13} \oplus M_2 P_{23} \oplus M_3 P_{33}$

$G = [I_k \;\; P_{k\times q}]_{k\times n}$

$C = MP.$

① Generator matrix for (6,3) matrix block code is shown below obtain all code words for this code.

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

$[I_k \,/\, P_{k\times q}]_{k\times n}.$

$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad P_{k\times q} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$C = MP.$ $\quad n = 6$

$M = 3 = M_1 M_2 M_3$ $\quad k = 3.$

$$\rightarrow [C_1 \; C_2 \; C_3] = [M_1 \; M_2 \; M_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$C_1 = M_2 \oplus M_3$

$C_2 = M_1 \oplus M_3$

$C_3 = M_1 \oplus M_2$

(b) Code vectors of (6,3) linear block codes

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Sl NO	Bits of message vector in one block			C_1	C_2	C_3	Complete code vector
				$M_2 \oplus M_3$	$M_1 \oplus M_3$	$M_1 \oplus M_2$	
	M1	M2	M3				$M_1 M_2 M_3$ $C_1 C_2 C_3$
1	0	0	0	0	0	0	000 000
2	0	0	1	1	1	0	001 110
3	0	1	0	1	0	1	010 101
4	0	1	1	0	1	1	011 011
5	1	0	0	0	1	1	100 011
6	1	0	1	1	0	1	101 101
7	1	1	0	1	1	1	110 111
8	1	1	1	0	0	0	111 000

Hamming Codes:-

Are defined as (n, k) linear block codes, these codes satisfy the following condition.

a. No of check bits $Q \geq 3$

b. Block length $n = 2^{Q} - 1$

c. No of message bits $K = n - Q$

d. Mini Distance $D_{min} = 3$.

e. Code rate $r = \frac{k}{n}$

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Error detection and correction of Hamming Code

$D_{min} = 3$ it can be used to detect double errors and / or correct single error.

$d_{min} \geq s+1$ $\quad d_t \geq 2t+1$

① The parity check matrix of a particular $(7, 4)$ linear block code is expressed as

$$H = \begin{bmatrix} 1110100 \\ 1101010 \\ 1011001 \end{bmatrix}$$

ⓐ obtain the generator matrix G.

ⓑ List all the code vectors

ⓒ what will be the mini distance b/w code vectors

ⓓ How many errors can be detected, corrected.

→ Given that $n = 7$, $k = 4$. and no of check bits are
$Q = n - k = 7 - 4 = 3$.
hence $n = 2^{Q} - 1 = 2^{3} - 1 = 7$

This indicate that given code is hamming code.
parity check matrix is $Q \times n$ size. and is given by the Eq^n.

Also $H = [P^T : I_Q]_{Q \times n}$

→ $P = \begin{bmatrix} 111 \\ 110 \\ 101 \\ 011 \end{bmatrix}$

Further generator matrix is expressed as $G = [I_k : P_{k \times Q}]_{k \times n}$

$\rightarrow$ $G = [I_k : P_{k \times q}]_{k \times n}$

$= [I_4 : P_{4 \times 3}]_{4 \times 7}$

$$G = \left[\begin{array}{cccc:ccc} 1&0&0&0 & 1&1&1 \\ 0&1&0&0 & 1&1&0 \\ 0&0&1&0 & 1&0&1 \\ 0&0&0&1 & 0&1&1 \end{array}\right]_{4 \times 7}$$

$C_p = M_k P$ $\qquad C_3 = M_4 P$

$$[C_1\ C_2\ C_3] = [M_1\ M_2\ M_3\ M_4] \begin{bmatrix} 1&1&1 \\ 1&1&0 \\ 1&0&1 \\ 0&1&1 \end{bmatrix}$$

$C_1 = M_1 \oplus M_2 \oplus M_3$

$C_2 = M_1 \oplus M_2 \oplus M_4$

$C_3 = M_1 \oplus M_3 \oplus M_4$

(b) min distance $= 3$

$d_{min} \geq s+1$ $\qquad s = 2$ Error check

$d_{min} \geq 2t+1$ $\qquad t = 1 \rightarrow$ Corrected

$3 = 2 \times 1 + 1$

SAM

Sl	Message bit $M_1\ M_2\ M_3\ M_4$	C_1 $M_1 M_2 M_3$	C_3 $M_1 M_3 M_4$	C_2 $M_1 M_2 M_4$	
	0 0 0 0	0	0	0	0000000
	0 0 0 1	0	1	1	0001011
	0 0 1 0	1	0	0	0010100
	0 0 1 1	1	0	1	0011101
	0 1 0 0	1	0	1	0100101
	0 1 0 1	1	1	0	0101110
	0 1 1 0	0	1	1	0110011
	0 1 1 1	0	0	0	0111000
	1 0 0 0	1	1	1	1000111
	1 0 0 1	1	0	0	1001100
	1 0 1 0	0	0	1	1010001
	1 0 1 1	0	1	0	1011010
	1 1 0 0	0	1	0	1100010
	1 1 0 1	0	0	1	1101001
	1 1 1 0	1	0	0	1110100
	1 1 1 1	1	1	1	1111111

~~(b) min distance~~

Cyclic code:- These may be described as the sub class of linear block codes. They have the property that cyclic shift of one code word produces another code word.

As an Example, consider an n bit code vector

$X = \{x_{n-1}, x_{n-2}, \ldots x_1, x_0\}$

x_{n-1}, x_{n-2}, x_1 are individual bits of code vector

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'x' i) we shift above code word cyclically then we get another code vector i.e

$$x' = \{x_{n-2}, x_{n-3}, \ldots x_1, x_0, x_{n-1}\}$$

Cyclic code:- A linear code is known as cyclic code if every cyclic shift of the code vector produces some other valid code vector.

① The codevector may be defined with the help of polynomial of degree less than $(n-1)$. Thus we can define codevector in terms of polynomial as :-

$$X(p) = \{x_{n-1}p^{n-1} + x_{n-2}p^{n-2} + \ldots x_1 p + x_0\}$$

where p is an arbitrary variable and power of p represents the position of codeword bits.

2 types - a. systematic cyclic code
b. NON systematic cyclic code.

A generator polynomial of a (7,4) cyclic code is $g(p) = p^3 + p + 1$. obtain all the code vector for the code in non systematic and systematic form

NON sys
=> $n = 7$; $k = 4$; $q = n - k = 3$

message vector $2^k = 2^4 = 16 (M_s)$

Let us consider any message vector as, $M = (M_3 M_2 M_1 M_0)$

$M = (M_3 M_2 M_1 M_0) = \{0101\}$
then the message polynomial will be $M(p) =$

$$M(p) = M_3p^3 + M_2p^2 + M_1p + M_0$$
$$\quad\;\; 0 \quad\;\; 1 \quad\;\; 0 \quad\;\; 1$$

$$M(p) = p^2 + 1$$

given $g(p) = p^3 + p + 1$.

To obtain non systematic code vector

$$X(p) = M_p\, g(p)$$
$$= (p^2+1)(p^3+p+1)$$
$$= p^5 + \underbrace{p^3 + p^2 + p^3}_{\text{take mod}} + p + 1$$
$$= p^6 + p^5 + \big|\; p^5 + 2p^3 + p^2 + p + 1$$
$$= p^5 + p^3(1 \oplus 1) + p^2 + p + 1$$

$$\boxed{X(p) = p^5 + p^2 + p + 1} \quad (0p^6 + 1p^5 + 0p^3 + p^2 + p + 1)$$

Note that the degree of above polynomial is $(n-1) = 7\ (6-1)$

The code vectors corresponding to above polynomial is

$$X = \{x_6\ x_5\ x_4\ x_3\ x_2\ x_1\ x_0\}$$

$X = \{0100111\}$ This is the code vector for message vector (0101)

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This code vector is non systematic cyclic code vector.

(b) Systematic code vector :-

For SCV we will first find check bits. These check bits are given by check bit polynomial $C(p)$. From the derivation $C(p)$ is expressed as

$$C(p) = rem\left[\frac{p^q\,M(p)}{G(p)}\right]$$

Since $Q = 3$, $p^q M(p)$ will be, 0101

$$p^q M(p) = p^3(p^2+1)$$
$$= p^5 + p^3 + p$$
$$= p^5 + 0p^4 + p^3 + 0p^2 + 0p + 0$$

$$G(p) = p^3 + p + 1$$
$$= p^3 + 0p^2 + p + 1$$

$$= rem\left[\frac{p^5 + 0p^4 + p^3 + 0p^2 + 0p + 0}{p^3 + 0p^2 + 0p + 1}\right]$$

$$p^2 + 0\,1\,0$$

$$p^3 + 0p^2 + 0p + 1 \,\big)\, p^5 + 0p^4 + 0p^3 + 0p^2 + 0p + 0$$
$$p^5 + 0p^4 + 0p^3 + 0p^2$$
$$0 \quad 0 \quad 0 \quad 0 \quad 0$$
$$p^2 + 0p + 0 \quad \text{remainder}$$

Hence the remainder polynomial is $p^2 + 0p + 0$ in division of $p^3 M(p)$ by $G(p)$

∴ we have,

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$$\Rightarrow C(p) = rem\left[\frac{p^3\,M(p)}{G(p)}\right]$$

$$C(p) = p^2 + 0p + 0$$

$$0p^3 + p^2 + 0p + 0$$

$$C(p) = C_2 p^2 + C_1 p + C_0$$

$$C_2 p^2 + C_1 p + C_0 = p^2 + 0p + 0,$$

Check bits

$$C = (C_2\, C_1\, C_0) = (1\,0\,0)$$

Code vector

$$X = \{M_3\, M_2\, M_1\, M_0 \;\; C_2\, C_1\, C_0\}$$

$$\boxed{X = \{0101 \,\vdots\, 100\}}$$

27/05/22

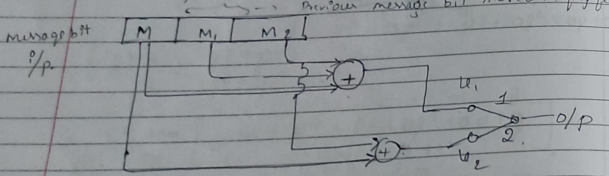
UNIT 02

WAVEFORM CODING.

Convolutional codes:-

A convolutional coding is done by combining the fixed number of input bits. The input bits are stored in fixed length shift register and they are combined with the help of Mod 2 adder.

⊙ This operation is equivalent to binary convolution, hence it is called convolutional coding.
The concept is illustrated in the figure.



Convolutional Encoder with $K=3$, $n=2$

$(M_1 M_2)$ represents the state of shift register.

$V_1 = m \oplus m_1 \oplus m_2$

$V_2 = m \oplus m_2$.

Output V_1V_2 V_1V_2 ....

⊙ Code rate $(r_0) = \frac{k}{n} = \frac{1}{2}$

This type of encoder is called 1/2 encoder.
3 message but i/p is only 1 hence $k=1$ (not)

Operation :-

⊙ Whenever the message bit is shifted to position m, the new value of v_1, v_2 are generated depending upon m, m_1, m_2.

$M_1 M_2$ store the previous 2 message bit, the current bit is present in M, hence we can write

$V_1 = m \oplus m_1 \oplus m_2$ $V_2 = m \oplus m_2$

⊙ The o/p of encoder switch first sample v_1 then v_2
⊙ The shift register can shift the content of the M_2 to M_1

⊙ Next i/p bit is taken and stored in M.
Again $v_1 v_2$ are generated according to this new combination of M, M_1, M_2

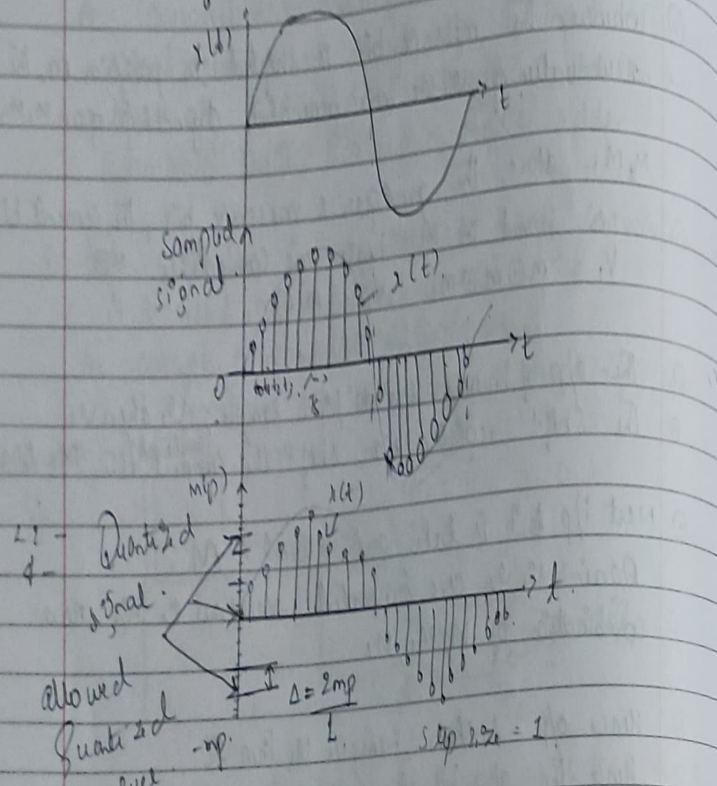
⊙ Hence o/p switch samples v_1 then v_2
Hence the o/p bit stream for successive i/p bits will be
$X = v_1v_2$ v_1v_2 ---- n

⊙ Here it may be noted that for every message bit we have 2 encoded o/p bits v_1 and v_2 are transmitted. In other words for a single message bit the encoded code word is 2 bits.
Therefore the code rate of this channel encoder is.

$r = \frac{k}{n} = \frac{1}{2}$ → o/p of Encoder → n ; i/p → k.

→ Q2:-

Concept of Quantization:



○ As shown, amplitude of signal x(t) lies in range (-mp, mp) which is partitioned into the peak (-mp, mp) which is partitioned to n interval, Each of magnitude / Δp

$\Delta p = \frac{2mp}{L}$ Now Each sample is approximated or rounded to the nearest quantized level as shown in fig.

final

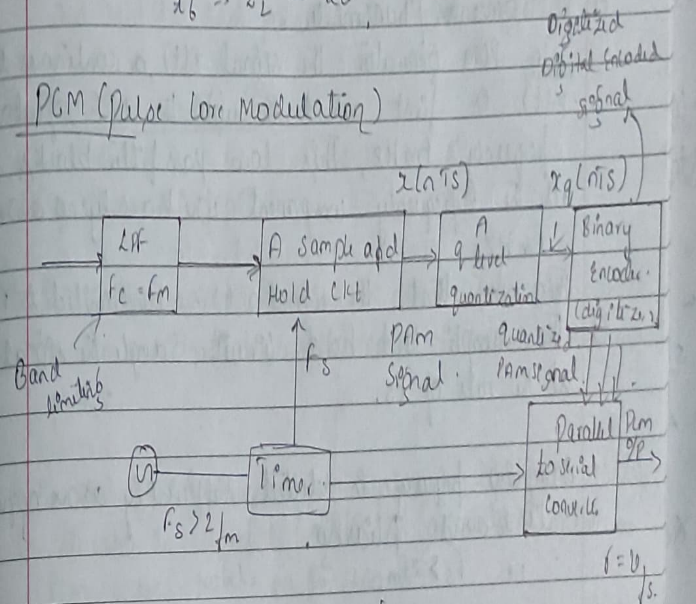
○ Each sample is now approximated to one of the L numbers therefore, the information is digitized.

○ ∴ the quantized is an approximate of original one. we can improve accuracy of quantized signal to any desired degree, simply by increasing the no of levels L.

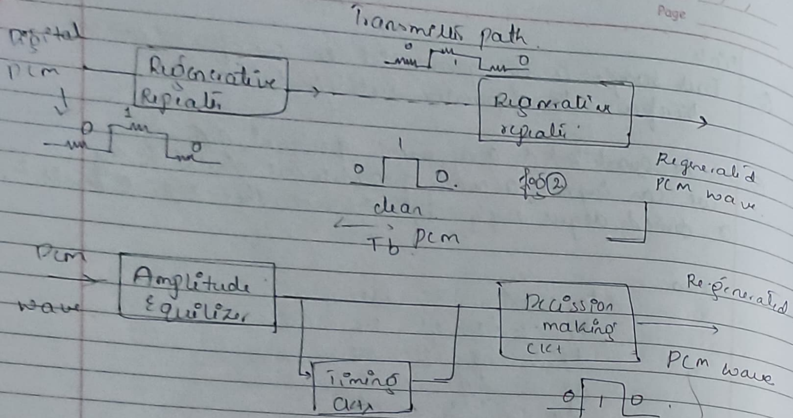
Now

$t_0 \to 4 \to 10$
$t_1 \to L_2 \to 20$
$t_2 \to L_3 \to 30$
$t_3 \to L_4 \to 40$
$t_4 \to L_4 \to 40$
$t_5 \to L_3 \to 30$
$t_6 \to L_2 \to 20$

PCM (Pulse Code Modulation)



Practical pcm generator



⊙ PCM generator / Transmitter :-

In PCM generator the signal $x(t)$, a continuous time $m(t)$ is first passed through the LPF of cutoff frequency f_m Hz. This low pass filter blocks all the frequency component which are lying above f_m Hz.

This means that the signal $x(t)$ is bandlimited to f_m Hz.

⊙ The sample and hold circuit, sample the signal at the rate of f_s.

Sampling frequency is selected sufficiently above Nyquist rate to avoid Aliasing.

i.e $f_s > 2f_m$.

⊙ The o/p of sample and hold ckt is denoted by $x(nT_s)$

This signal is discrete in time and continuous in amplitude.

⊙ A Q level Quantizer compares the i/p $x(nT_s)$ with its fixed digital level and assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or Error.

⊙ This Error is also called as Quantization Error

Now the quantized signal level $x_q(nT_s)$ is given to Binary Encoder:

This Encoder converts i/p signal to v digits ($Q = 2^v$) Binary word. Thus $x_q(nT_s)$ is converted to binary bits.

⊙ This Encoder is also known as Digitizer.

⊙ Also an Oscillator generate clocks and for sample and hold ckt and parallel to serial converter.

PCM transmission Path

As shown in fig ②

⊙ The path b/w PCM transmitter and Receiver over which a PCM signal transmitted / travel is called PCM Transmission path.

⊙ The most imp feature of PCM system is lies in its ability to control the effect of distortion and noise, when the PCM wave travels on the channel.

This is achieved by means of using a chain of regenerative repeaters as shown.

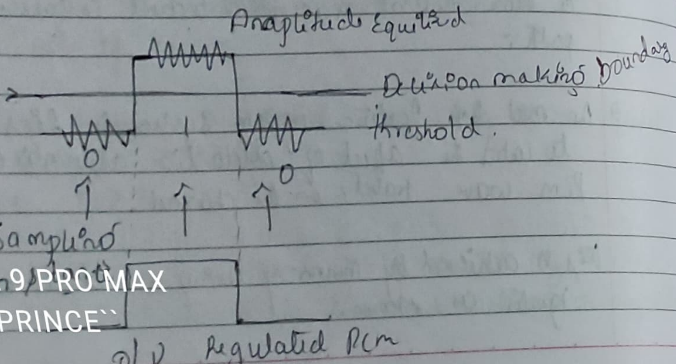
② Such repeaters are spaced close enough to each other on the transmission path.

The regenerative performs basic 3 operations namely
① Equalization.
② Timing.
③ Decision making.

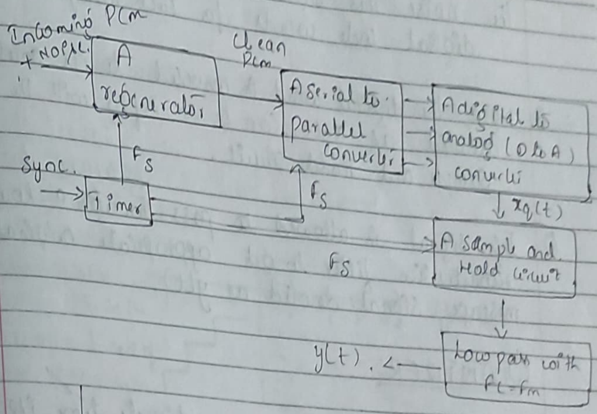
ⓐ As shown in fig ③ Amplitude Equalizer initiates the distorted PCM wave so as to compensate for the effects of Amplitude and phase distortions.

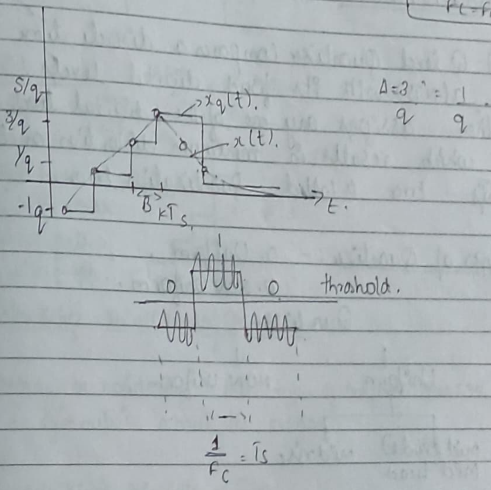
ⓑ Timing ckt produces a periodic pulse train which is derived from the input PCM pulses.

ⓒ This pulse train is then applied to decision making device. The Decision device makes decision about whether the equalized PCM wave has a zero value or 1 value at the instant of sampling. Such a decision is made by comparing equalized PCM with a reference level called threshold as shown in fig.



PCM Reciever:-





① Fig shows the block diagram of PCM reciver and reconstructed filter/signal. The regenerator at the start of the PCM reciver reshapes the pulse and removes

The noise ⊙ This signal is then converted to parallel digital code words for each sample.

⊙ Now, the digital word is converted to its analog value denoted as $x_q(t)$. Can be seen in fig with the help of a sample and hold circuit.

⊙ This signal is allowed to pass through a low pass reconstruction filter to get appropriate original message signal denoted as $y(t)$.

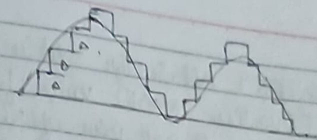
QUANTIZER:-

⊙ A Q level Quantizer compares a discrete time i/p $x(nT_s)$ with its fixed digital levels. Then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or Error. This Error is called Quantization Error.

Types of Quantizer:- a. Uniform.
b. NON Uniform.

Quantizer
- Uniform
  - mid trade / mid tread
  - mid rise
- NON uniform

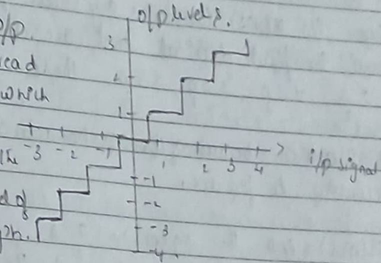
② Uniform Quantizer is type of Quantizer in which step size remains same through out the i/p stage



ⓑ NON :- in which step size varies according to the i/p range

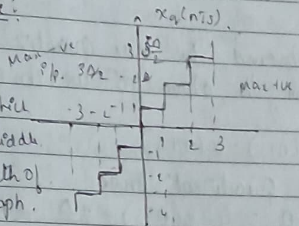
Mid tread:

∴ Fig the i/p and o/p characteristics of mid tread type uniform Quantizer, which is so called because the origin lies in the middle of a tread of staircase like graph.



Mid rise:

∴ The i/p, o/p characteristics of a uniform mid rise type in which origin lies in the middle of a raising path of staircase like graph.



Working principle of Quantizer:-

→ It may be noted that both types of uniform Quantizers are symmetric about the origin.

→ Working principle of Quo

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Q1 As shown in fig: mid rise type. Let us assume that i/p to the Quantizer $x(nTs)$ varies from -4Δ to 4Δ. This means that peak to peak value of $x(nTs)$ will be b/w -4Δ to 4Δ. (Δ is step size)

Thus i/p $x(nTs)$ can take any value from -4Δ to 4Δ. Now the fixed digital level are available at $\pm\frac{\Delta}{2}$, $\pm\frac{3\Delta}{2}$, $\pm\frac{5\Delta}{2}$, $\pm\frac{7\Delta}{2}$. These levels are available at Quantizer because of its Q characteristic.

According to figure,

$x(nTs) = 4\Delta$ then $x_q(nTs) = \frac{7}{2}\Delta$

if $x(nTs) = -4\Delta$ then $x_q(nTs) = -7/2\,\Delta$

⊙ It may be observed, from fig the max Quantiz^n Error will be $\pm\frac{\Delta}{2}$, from the above fig we conclude that

$\epsilon = x_q(nTs) - x(nTs)$

ϵ - represent Quanti zat^n Error.

Now if $x(nTs) = 0 \rightarrow x_q(nTs) = \pm\frac{\Delta}{2}$

if $x(nTs) = 0$ it will assign any one of nearest Quanti^n level if $\frac{\Delta}{2}$ is assigned then Quant^n Error, $\Delta/2$ if we then $-\frac{\Delta}{2}$.

⊙ Hence the max Quanti z^n Error will be $\pm\frac{\Delta}{2}$

$\epsilon_{max} = \left|\frac{\Delta}{2}\right|$

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Transmission BW of a PCM System

Let us assume that the Quantizer use v no of Binary digits to represent each level. Then the no of level that may be represented by v digits will be $q = 2^v$ here q represents total no of digital levels of a Q level Quantizer.

If $v = 4$ bits, then $q = 2^4 = 16$ levels. Each sample is converted to v Binary bits i.e No of bits/sample is v.

WKT, so No of samples/second = Sampling freque(f_s)

∴ No of bits/second is expressed as No of bits/

So

No of bits/second = (No of bits/sample) × (No of samples/sec)

= (v bit/sample) × (f_s samples/second)

⊙ The No of bits/second is known as signaling rate of PCM.

$r = v \cdot f_s$ bits/sec

$f_s \geq 2f_m$. Since Bandwidth needed for PCM transm^n is given by the Half of signaling rate.

∴ Trans^n BW in PCM is

$$BW \text{ in PCM} = BW \geq \frac{1}{2} r$$

$$= B = \frac{1}{2} v \cdot f_s = \frac{1}{2} v \, 2 f_m$$

$$\boxed{BW \geq v \cdot f_m}$$

$v \cdot f_s \quad v\ 2f_m$

$v \cdot 2\times4 \cdot 2$

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① A TV signal having a BW of 4.2MHz is transmitted using Binary PCM system given that No of Quantization level is 512 determine
(i) Code word length
(ii) Transⁿ Bandwidth
(iii) Find Bit rate
(iv) o/p signal to Quantizⁿ Noise ratio

=> $f_m = 4.2MHz$.
$q = 512$

$q = 2^v \quad 512 = 2^v \; ; \; \boxed{v = 9 bits}$ (Codeword length)

$BW \geq \frac{1}{2} r \quad \geq v f_m$

$BW \geq 37.8MHz$

$r = v \times 2f_m = 9\times 8.4 = 75.6 Mbit/sec$

② The info in analog waveform with max frq $f_m = 3kHz$ is to be transmitted over an (Q) M level where the No of Quantization level is 16 the Quantization distortion is specified not to exceed @ 1% of peak to peak signal
(i) Max No of bits/sample that should be used in this PCM system.
What is (ii) minimum sampling rate and what is resulting bit transmission rate.

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=> $q = 16 = 2^v \quad \boxed{v = 4}$

$r = v \cdot f_s$
$= 4\times 2\times 3k\,Hz = 24k\,bits/sec$

$BW = \frac{1}{2} r = 12kHz$

③ The information in an Analog signal voltage waveform is to be transmitted over a PCM system with an accuracy of ±0.1% (full scale). The analog signal has an voltage of 100Hz and an amplitude of $(-m_p, m_p) = (-10$ to 10 volt$)$.
(i) Find min sampling rate required
(ii) find No of bits in each PCM
(iii) Find mini Bit rate (r)
(iv) Find mini Absolute channel BW required for transmission

=> Here an accuracy is given as ±0.1% => ϵ_{max}

$\epsilon_{max} = \left|\frac{\Delta}{2}\right| = 0.001$

Step size $\Delta = 0.002$

WKT the stepsize, No of Quantizatⁿ level and max value of the signal rate are related as

$\Delta = \frac{2x_{max}}{q} = \frac{2\times10}{q} = 0.002 \qquad x_{max} = m_p, \; |x_{max}| = 10v\,pp.$

$q = \frac{20}{0.002} \quad 10 \qquad \Rightarrow q = 10\times10^3$

Quantization Noise / Error in PCM

A uniform quantiz noise is given by $\varepsilon = x_q(nT_s) - x(nT_s)$
let us assume that i/p $x(nT_s)$ to a linear/uniform quantiz, has continuous amplitude $-x_{max}$ to x_{max}

This means that i/p is 70 and o/p is $\frac{7}{2}\Delta$.
Total amplitude range becomes.

$x_{max} = x_{max} - (-x_{max})$

$x_{max} = 2x_{max}$

∴ This total amplitude range is divided into Q level of quantizer then the step size will be

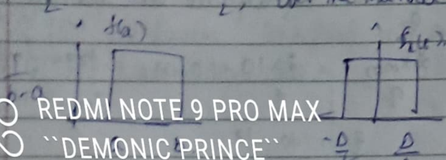
$x_{max} = \frac{2x_{max}}{q}$ $\Delta = \frac{2x_{max}}{q}$

③ Now if the signal $x(t)$ is normalized to minimum and maximum value equal 1. Then we have

$x_{max} = 1; \; -x_{max} = -1; \quad \Delta = \frac{2}{q}$

② Now if the step size 'Δ' is consider sufficiently small then it is may be assumed that quantiz error 'ε' will be uniformly distributed random variable

$-\frac{\Delta}{2} < \varepsilon_{max} < \frac{\Delta}{2}$, over the interval $(-\Delta/2, \Delta/2)$



$= \frac{1}{\frac{\Delta}{2} - \frac{-\Delta}{2}} = \frac{1}{\Delta}$.

Probability density funct', ε = for quantiz ε_{max} ε may be defined as.

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon \le \Delta/2 \\ 1/\Delta & -\Delta/2 \le \varepsilon < \Delta/2 \\ 0 & \text{for } \varepsilon > \Delta/2 \end{cases}$$

It may be observed that from fig and eq' of PDF, the Q.E ε has 0 average value further, WKT signal to quantiz' noise ratio of the quantizer is defined as

$\frac{S}{N} = \frac{\text{Signal power}}{\text{Noise power}}$

WKT Noise power $= \frac{V^2_{noise}}{R}$

V^2_{noise} is mean square value of noise voltage. The mean square value is $= \frac{E(\varepsilon^2)}{\varepsilon^2} = V^2_{noise}$

$\therefore \bar{X}^2 = E(\varepsilon^2) = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx$

$E[\varepsilon^2] = \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 f_\varepsilon(\varepsilon)\,d\varepsilon$.

$= \left[\frac{\varepsilon^3}{3}\right]\left[\frac{1}{\Delta}\right] = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right] = \frac{1}{12} = \frac{\Delta^2}{12}$

Mean square noise would be V^2_{noise} = mean square value

$V^2_{noise} = \Delta^2$.

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Signal to Quantization noise ratio for linear quantiz"

1. Signal to noise is given by $\frac{S}{N} = \frac{\text{Normalized signal}}{\Delta^2/12}$ — (1)

2. W.r.t the no of bits (ν) and Quantiz" level are related as $q = 2^{\nu}$.

3. Let us assume that i/p x(nTs) as continuous amplitude in the range → $-x_{max}$ to x_{max}

Total amplitude range is $x_{max} - (-x_{max}) = 2x_{max}$

Now the step size Δ will be $\frac{2x_{max}}{q} = \Delta$.

$q = 2^{\nu}$ $\Delta = \frac{2x_{max}}{2^{\nu}}$

Now in (1) $\frac{S}{N} = \frac{\text{Norm sig}}{\Delta^2/12}$

$= \frac{12 \text{ Norm sig}}{\left(\frac{2x_{max}}{2^{\nu}}\right)^2} = \frac{12 \text{ Norm sig}}{\frac{4x^2_{max}}{2^{2\nu}}}$

Let normalized signal power is denoted as P so S/N is $\frac{3P \times 2^{2\nu}}{x^2_{max}} = \frac{3P}{x^2_{max}} \times 2^{2\nu}$

required sol" for signal to Quantiz" noise ratio for linear Quantiz" in a PCM system

... that S/N power ratio of Quantizer ... with ↑ ν no of bits/sample.
... we assume that i/p x(t) is normalized i.e

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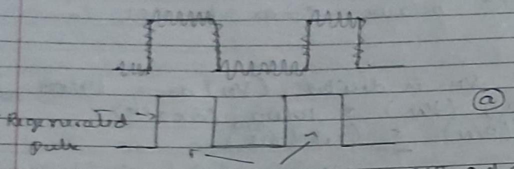
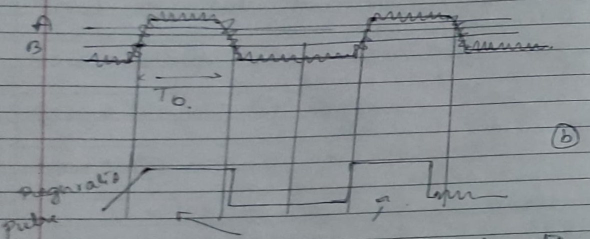
$x_{max} = 1$, then S/N will be $x_{max} = 1 \rightarrow \frac{S}{N} = 3 \times 2^{2\nu} \times P$
also the signal power P is normalized i.e $P \le 1$.
$\frac{S}{N} = 3 \times 2^{2\nu}$ because $x_{max} = 1$, $P \le 1$.

- Expressing S/N in decibel $(S/N)_{dB} = 10\log_{10}(S/N)$
$= 10\log_{10}(S/N) \le 10\log_{10}(3 \times 2^{2\nu})$
$= 10\log_{10}(S/N) \le (4.8 + 6\nu)$

Effect of Noise on PCM System



No change in width and position.



change in width of position.

⊙ In PCM System Noise is unaffected much as compared to PAM, PWM, PPM system.
In PCM system doesn't contain any information in the width or position of pulse hence PCM has much better noise immunity compared to other system.

① Derive an expression for signal to Quantization noise ratio for a PCM system which employ uniform Quantization technique given that input to the PCM system is a sinusoidal signal

(ii) A PCM system uses a uniform Quantizer followed by a ν bit Encoder. Show that RMS signal to Quantiz noise ratio is approximately given as $(1.8 + 6\nu)$ dB

Sol^n:- Let us assume that modulating signal is sinusoidal voltage having amplitude equal to Am/Vm. The power of the signal will be V^2/R Here V is RMS value

$$P = \frac{V^2}{R} \qquad V = \left(\frac{A_m}{\sqrt{2}}\right)^2$$

$$P = \frac{A_m^2}{2R} \quad \text{— ①}$$

In case R = 1 and P is normalised in eq ①.

$$P = \frac{A_m^2}{2} \quad (R = 1)$$

WKT

$$\frac{S}{N} = \frac{3P}{x_{max}^2} \times 2^{2\nu}$$

$$\frac{S}{N} = \frac{3A_m^2}{2x_{max}^2} \times 2^{2\nu}. \qquad x_{max} = A_m$$

$$= \frac{3A_m^2}{2A_m^2} \times 2^{2\nu} = \frac{3}{2} \cdot 2^{2\nu} = (1.5 \times 2^{2\nu})$$

Expressing in dB we get

$$\left[\frac{S}{N}\right]_{dB} = 10\log_{10}\left(\frac{S}{N}\right) = 10\log_{10}(1.5 \times 2^{2\nu})$$

$$= 10\log_{10} 1.5 + 2\nu\, 10\log_{10}(2)$$

$$1.7609 + 2\nu \times 3.010.$$

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$\left(\frac{S}{N}\right)_{dB} \approx 1.8 + 6v.$

NON - Uniform Quantization:

If the Quantizer characteristic is NON linear and the step size is not constant instead if it is variable dependent on amplitude of I/p signal then the Quantization is known as NON uniform Quantization

- In this the step size is reduced with the reduction in signal level. For weak signal (P very $P \ll 1$) the step size is small, therefore the Quantization noise reduces to improve the $(S/N)_q$ ratio for low signals
- The step size is thus varied according to signal level to the signal to noise ratio and adequately high. this is called NUQ.
- This is practically achieved through a process called Companding (Compression + expanding).

The Quant Noise power. $\Delta^2/12$ this shows that in uniform Quant once the step size is fixed the Quantiz Noise power remain constant however the signal power is not constant. it is propotional to the square of the signal amplitude.
Hence the signal power will be small for weak signal therefore $\frac{S}{N_q}$ ratio for low signal is very poor this will effect the Quality of signal

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The Remedy is to use companding

- Companding is a term derived from compression and Expansion.

Input → Compression → Uniform Quantizer → Expander → o/p

- in practice it is difficult to implement the NON uniform Quantiz because it is not known in advance about the changes in the signal level
therefore a particular method is used the weak signal are amplified and strong signal are attenuated before applying them to a uniform Quantizer
- At receiver Exactly opposite is followed which is called expansion. The compression of the signal at the transmitter and Expansion at receiver is combined to be called. Companding.

(1) A PCM System uses a Uniform Quantizer followed by 7 Bit binary Encoder. the Bit-rate of the system is 50×10^6 bits/sec. what is the maximum message signal band width for which the system operates satisfactorily calculate o/p signal to quantizat noise ratio when a full load sinusoidal modulating wave of frequency 1 MHZ is applied to the input.

WKT, given $r = 5\times10^6$ $v = 7$

$r = vf_s$

$f_s = \frac{5\times10^6}{7} = 714.28\times10^3$

$f_s = 2f_m$

F

$f_s = 3.57\,MHz$

$\left(\frac{S}{N}\right)_q = 1.76 + 6\nu$

$= 1.76 + 6(2)$

$= 13.76\,dB$

(1) A signal having a band width of 3.5kHz is sample Quantized and coded by a PCM system. The coded o/p signal is then transmitted over a "baseband" channel of supporting a transmission rate of 50kbits/sec. Determine the max signal to noise ratio that can be obtained by the system. The input signal has peak to peak value of 4V and rms value of 0.2V.

Sol) $\frac{S}{N} = \frac{3P}{x^2_{max}} \cdot 2^{2\nu}$

$r = 50k$ bits

$f_s = 2 \times 3.5k$

$= 7\,kHz$

$P = \frac{V^2}{R} = \frac{(0.2)^2}{1} = 0.04\,W$

$r = \nu \cdot f_s$

$\nu = \frac{50k}{7k}$ $\nu = 7.14$ bits

$x_{max} = 4$ take $\nu = 8$

$(x_{max} = 2)$

Now $\frac{S}{N} = \frac{3(0.04)}{4} \times 2^{2 \times 7.14}$

$= 1973.3722$ dB

$= 1966.08$

$= 10\log_{10}(1966.08)$

$= 33\,dB$

(2) The bandwidth of i/p signal to the PCM is restricted to 4kHz. The i/p signal varies in amplitude from -3.8V to 3.8V has the average power of 30mW. The required signal to noise ratio is given 20dB. The PCM modulator produces binary o/p assuming uniform Quantiz.

(i) Find the no of bits required for sample

(ii) o/p of 30 such PCM coders are multiplexed what would be the minimum required transmission bandwidth for the multiplexed signal.

→ $B = 4\,kHz$ $\frac{20}{10} = 10^2$

$x_{max} = 3.8$

$P = 30\,mWatt$

$\frac{S}{N} = 20\,dB$

$= 100$

$\frac{S}{N} = \frac{3P}{x^2_{max}} 2^{2\nu}$ $r = \nu \cdot f_s$

$100 = \frac{3(30 \times 10^{-3})}{(3.8)^2} \times 2^{2\nu}$

$16044.44 = 2^{2\nu}$

$\log(16044.44) = 2\nu \cdot \log(2)$

$2\nu = 13.96$

$\nu = 6.98 \approx 7$ bits //

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BW = 16 fm
BW = 30 × 10 × fm. (30 PCM channel

30 × 7 × 4k
BW
= 840 KHZ

r = 840 × 2
= 1680 k bits/sec.

Compressor Companding:-

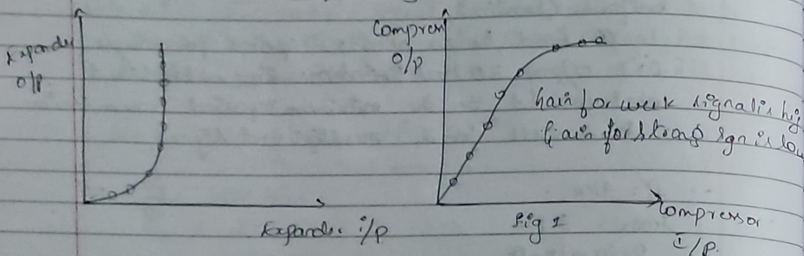
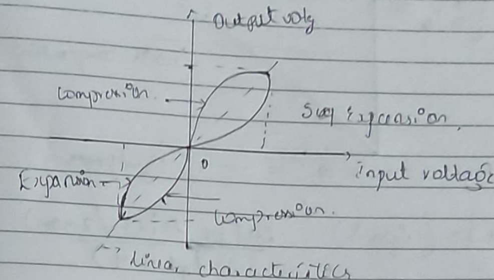


Fig 1

As shown in fig 1 the compressor provides higher gain to the weak signal and smaller gain to the strong input signal.
Thus weak signals are artificial boosted to improve the (S/N)q ratio.

Expander Chara

As shown, this characteristic is exactly the inverse of compressor characte, this ensures that artificial boosted signal by the compressor are brought back to their original amplitude at the receiver end.



Companding curve for PCM system.

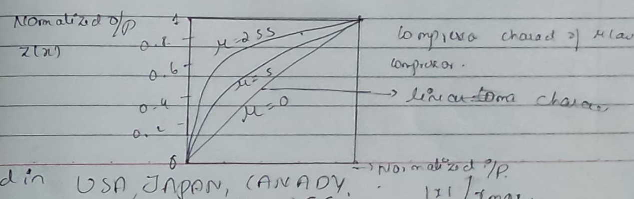
Different types of compressor characteristics

1. μ-law companding 2. A law companding.

Ideally we need a linear compressor characteristic for small amplitude of the input signal and a logarithmic characteristics.

In practice this is achived by using following method

1. μ-law companding :- m.



Used in USA, JAPAN, CANADA.
μ = 255

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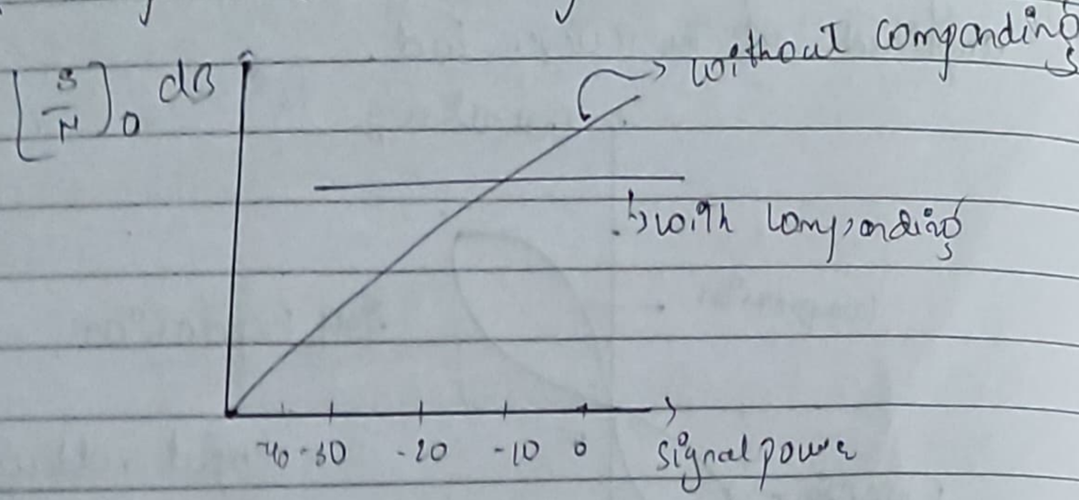
(3) Used in Speech and music PCM system



$$z(x) = (\operatorname{sgn} x) \frac{\ln(1 + \mu |x| / x_{max})}{\ln(1 + \mu)}$$

where $0 \leq |x| / x_{max} \leq 1$.

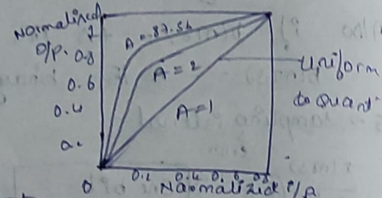
$z(x) \rightarrow$ o/p of compressor.

$x \rightarrow$ i/p to the compressor.

$x / x_{max} \rightarrow$ normalised value of input i.e $|x|$ w.r.t. max value X_{max}.

→ In the A law companding compound characteristic is piecewise made up of a linear segment for a low level i/p and a logarithmic i/p for high level i/p

fig shows A law compressor characteristics for different value of A. A = 1 corresponds to linear characteristic which corresponds to uniform Quantization



Application of PCM:-

1. it is used in Telephone systems
2. In the space communication, space craft transmits signal to Earth. here the transmitted power is very low (10, 15w) and the distance are huge (few million km). still due to high noise immunity only PCM system can be use in such application.

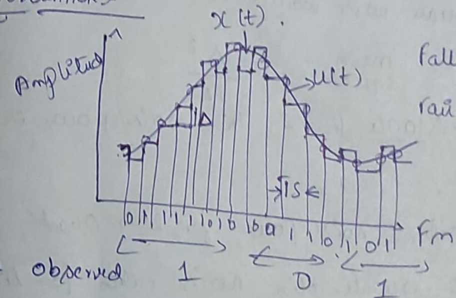
Advantage

1. Very high noise immunity
2. Due to digital nature of the signal, repeaters can be placed b/w transmitter and receiver.
3. repeaters actually regenerate the received pcm signal. this is not possible in analog system.
4. Repeaters further reduce the effect of noise.
5. it is possible to store the pcm system can use various coding techniques so that only the desired person can decode the received signal due to its digital nature.

Dis-Advantages:

1. the encoding, Decoding, Quantizing circuitry of pcm is complex
2. PCM requires large bandwidth as compared to other system

DELTA MODULATION:-



We have observed in pcm that it transmits all the bits which are used to code a sample. Hence signaling rate and transmission channel Bandwidth are Quite large in PCM

1. To overcome this problem Δ modulation is used
2. Δ modulation transmits only 1 bits/sample. Here the present sample value is compared with the previous sample value and this result whether the amplitude is ↑ or ↓ is transmitted

• i/p signal $x(t)$ is approximated to step signal by the Δ modulator. The step size is kept fixed. difference b/w i/p signal $x(t)$ and staircase approximated signal is confined to 2 level $+\Delta$ and $-\Delta$.

• If the difference is +ve the approximated signal is increased by one step (Δ) if the difference is −ve, the approximated signal is reduced by Δ.

• When the step is reduced 0 is transmitted and if the step is i[ncreased] 1 is transmitted. Hence for each sample, only one binary bit is transmitted.

Fig shows the analog signal $x(t)$ and its staircase approximated signal $u(t)$ by the delta modulator.

• Thus the principle of Δ mod^n can be explained with the help ① The error b/w sampled value $x(t)$ and last approximated sample is given as

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

error at present sample ↓ sampled signal of $x(t)$ — last sample approximation of staircase waveform

If we assume $u(nT_s)$ as the present sample approximation staircase o/p. Then $u[(n-1)T_s] = \hat{x}(nT_s)$

let us define a quantity $b(nT_s)$ in such a way that $b(nT_s) = \Delta\, \mathrm{sgn}\,[e(nT_s)]$

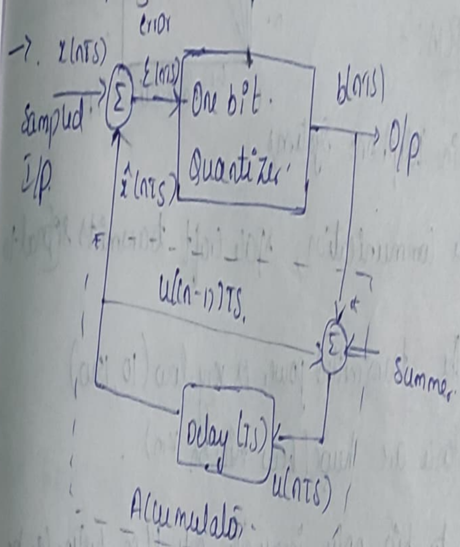
• This means that depending on sign of $e(nT_s)$, the sign of step size Δ is decided. In other words we can write

$$b(nT_s) = \begin{cases} +\Delta & \text{if } x(nT_s) \geq \hat{x}(nT_s) \\ -\Delta & \text{if } x(nT_s) < \hat{x}(nT_s) \end{cases}$$

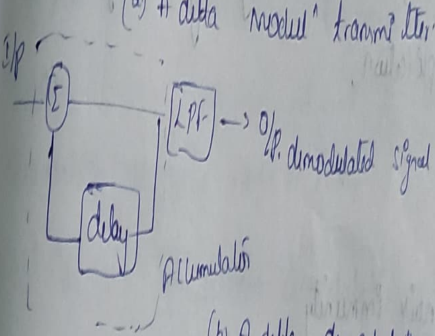
Also if $b(nT_s) = +\Delta$ then binary 1 is transmitted.

$b(nT_s) = -\Delta$ then binary 0 is transmitted.

T_s → sampling interval.

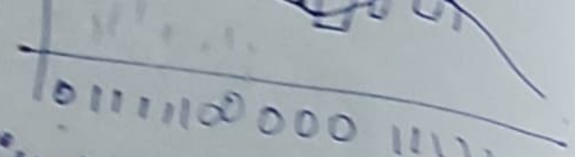


(a) A delta modul^n transmitter



(b) A delta demodulation receiver.

$+\Delta \rightarrow$ one step if $x(nTs) > \hat{x}$

binary sequ. 

0111110000 11111

raise $\rightarrow$ 111

fall edge $\rightarrow$ 00.

① Fig shows the Δ modulated signal. The summer in the accumulator adds $\pm\Delta$ (adds Quantizer o/p) with previous sample approximation.

0 1 1 0 0 1

② This gives present sample approximation i.e $u(nTs) =$

$$u(nTs) = u(nTs - Ts) + (\pm\Delta)$$

$$u(nTs) = u(nTs \; [u((n-1)Ts)] + b(nTs)$$

Previous ou.

③ The previous sample approx $u((n-1)Ts)$ is restored by delaying one sample period Ts. The sampled signal i/p $x(nTs)$ and staircase approximated signal $\hat{x}(nTs)$ are subtracted to get an error signal $E(nTs)$.

④ Then depending on sign of $E(nTs)$. One bit Quantizer generate an o/p of $+\Delta$ or $-\Delta$

⑤ If the step size is $+\Delta$ then binary one is transmitted and if it is $-\Delta$ then binary zero is transmitted.