Contents

1. The student-level scheduling optimisation problem

Under the hybrid mode, the students will be assigned into groups to take turns attending the classes in person. The formulation of a student-level scheduling optimization problem proposed by Moallemi and Patange (2023) for Columbia Business School can be described as follows.

The required parameters for the problem formulation include the set of students (S), the set of classes in the term (C) and their calendar schedule, the set of students enrolled to each class (A_k, where k is a class in C), the class capacity under the social distancing restriction (c_k, where k is a class in C), the set of hours for every day of the week (T, denoting each hour within T by t), and the number of groups the students would be assigned to for hybrid mode (M). The students are assigned into groups using the integral method, so **the decision variables** (π_{ij}) are binary indicating whether a student i would be assigned to group j during the hybrid mode schedule or not. To assess the effectiveness of the student allocation, the Total Excess (TE), which is the total number of excess students of all classes assigned to the excess room, the Total Deviation (TD), which reflects how the student assignment in all classes differs from the uniform fractional assignment that is used as the benchmark for equally distributed assignment, and the Surplus Simultaneous Excess (SSE), which is the total number of students that exceeds the excess room capacity, are used as the evaluation metrics. In addition, each metric is assigned with weights. TE's weight is normalised to 1 and TD's weight (λ) is 0.25 reflecting the less concern towards the deviation. With **the assumption** that the maximum number of excess students will not exceed the excess room capacity, the SSE's weight (μ) is 0 regardless of being the most important metric in the COVID situation. **The objective function**, therefore, is the minimisation of the linear combination of the three metrics (minimise $\sum_{k\in C}\sum_{j=1}^{M} e_{jk} + \lambda \sum_{k\in C}\sum_{j=1}^{M}\delta_{jk} + \mu s$) subject to the following **constraints:**

- The assignment constraint restricts each student to be assigned to only one of the hybrid mode groups ($\sum_{j=1}^{M} \pi_{ij} = 1, \forall i \epsilon S$).
- The class excess constraint places the minimum value for the excess in a class with the number of students exceeding the social distancing capacity of each class ($\sum_{i\in A_k}\pi_{ij} - c_k \le e_{jk}, 1 \le j \le M, \forall i \epsilon C$).
- The deviation constraint ensures equal distribution of in-person attendance by placing a linearised equation of TD's definition ($-\delta_{jk} \le \sum_{i\epsilon A_k}\pi_{ij} - \frac{|A_k|}{M} \le \delta_{jk}, 1 \le j \le M, \forall i \epsilon C$).
- The surplus constraint (SSE) places the minimum value for the surplus with the number of students exceeding the excess room at time t ($s \ge s_j^t - E, 0 \le t \le T, 1 \le j \le M$).
- The total excess constraint places the minimum value of the total excess with the total number of students exceeding the social distancing capacity from all the classes that are ongoing at same time t. ($s_j^t \ge \sum_{k\epsilon C_t} e_{jk}, 0 \le t \le T, 1 \le j \le M$)

- The binary decision variables constraint ($\pi_{ij} \in \{0,1\}, \forall i \in S, 1 \leq j \leq M$).
- The non-negative constraint for auxiliary variables ($e_{jk}, \delta_{jk}, s \geq 0, 1 \leq j \leq M, \forall k \in C$).

2. Hybrid timetable scheduling for MSBA

To schedule a hybrid timetable for MSBA students, the optimisation problem is formulated regarding the given timetable and the enrolment information. To provide an equal number of in-person attendance to each student, the students are assigned into groups and each group takes turns coming to classes day by day throughout the term.

Assumptions

1. We assume that WBS has a policy during hybrid mode to encourage less travelling by having a student, who was to come to an in-person class on a particular day, also attend all other classes in person on the same day. Therefore, we consider the equal number of in-person attendance based on the number of days and not the number of hours over the term, making a 2-hour Forecasting lecture on week 7 be considered a class to attend on the day, just like the Forecasting lecture in other weeks.
2. The room capacity in the WBS building under hybrid mode is assumed to be restricted to 50% of the full room capacity. This also applies to room 2.006 with a 26-seat capacity, which is selected to be the excess room for this task (University of Warwick, 2016).
3. Assuming that the hybrid mode is for a similar situation as COVID-19, we assign the weights for each evaluation metric as 1, 0.25, and 1 for TE, TD, and SSE respectively, to place importance on providing a safe study environment and minimising disutility for the students as priorities, while concerning less on the equal opportunity of onsite attendance.
4. The class time slots and the groups for each workshop the students were already assigned to cannot be altered, hence they remain as given.

Data structure The set of 229 MSBA students is stored as an index using their ID numbers. The set of 23 classes, including all the lectures and workshops of 6 main modules, namely, Advanced Analytics: Models and Applications (AAMA), Advanced Data Analysis (ADA), Data Management (DM), Forecasting (F), Financial Analytics (FA), and Supply Chain Analytics (SCA) is also stored as an index by their abbreviations. The data on the attendance of the students to the classes in a week is put into a table filled with the value of 1 if the student ought to attend the class, or 0 otherwise, and stored in the enrolment dataframe. For the timetable, we filled in a 1 or 0 value for each day of the week and each 30-minute slot from 9 am to 6 pm to account for the moment when a particular class is ongoing, both are stored separately in the day and the time dataframes. Additionally, the full capacity of the rooms for all the classes and the excess room are also included in the input file (Appendix 1).

Implementation issues

- **Data structure for decision variables:** As we were exploring 2 groups of students for hybrid mode, we put in 2 sets of 229 MSBA students, one for group 1 and another for group 2, in the data file for decision variables which caused a repetition of the enrolment data and inconsistency in the way we stored the data matrices using Python. Hence, we fixed the issue by keeping only one set of 229 students as the input and generated the combination of students and groups using the function in Python.
- **Data structure for room capacity:** The capacity in the input file was initially stored with the restricted capacity causing inconvenience when we needed to alter the capacity parameters, hence we replaced the input with room capacity in full and placed restrictions for hybrid mode using Python.

3. Problem solving and analysis

After formulating the problem in Python, we used Gurobi Optimizer to solve for the optimal solution with a 50% room capacity and two groups of students. The assigned weights for each evaluation metric are 1, 0.25, and 1 for TE, TD, and SSE respectively. The optimal solution is 2, with 0 TE, 8 TD, and 0 SSE. Group 1 has 114 students and group 2 has 115 students.

Solution analysis

Altering the room capacity: We explored different capacity restrictions while keeping other parameters the same to observe the appropriate number of groups for different capacity restriction policies (Table 1).

Table 1: Solution analysis on different capacity restrictions

Room Capacity Restriction (% full capacity)	Number of Groups	Optimal Solution	TE	TD	SSE	Number of Students			
						Group1	Group2	Group3	Group4
25%	2	583	545	8	36	115	114	NA	NA
	3	62.33	56	25.33	0	76	77	76	NA
	4	8	0	32	0	57	58	57	57
35%	2	155	149	8	4	115	114	NA	NA
	3	6.33	0	25.33	0	76	77	76	NA
	4	8	0	32	0	57	57	58	57
50%	2	2	0	8	0	114	115	NA	NA
	3	6.33	0	25.33	0	76	76	77	NA
	4	8	0	32	0	58	57	57	57

Under 50% of the full capacity, a two-group assignment of students provides the least optimal value. However, when the restriction becomes stricter allowing only 35% and 25% of the full capacity, three and four groups of students, respectively, give better results.

Altering the metric weights: We assign different weights to TE, TD, and SSE with 0.25, 1, and 1 under assumptions that WBS will place importance on providing equal chances of onsite attendance and minimising the surplus excess as the building has limited space available to spare, and concerning less on minimising the disutility for the students (Table 2).

Table 2: Solution analysis on different weights of each evaluation metric

Room Capacity Restriction (% full capacity)	Number of Groups	Optimal Solution	TE	TD	SSE	Number of Students			
						Group1	Group2	Group3	Group4
25%	2	180.25	545	8	36	114	115	NA	NA
	3	39.33	56	25.33	0	76	76	77	NA
	4	32	0	32	0	57	57	57	58
35%	2	49.25	149	8	4	114	115	NA	NA
	3	25.33	0	25.33	0	76	76	77	NA
	4	32	0	32	0	57	57	57	58
50%	2	8	0	8	0	115	114	NA	NA
	3	25.33	0	25.33	0	76	76	77	NA
	4	32	0	32	0	57	57	58	57

Most of the optimal values are higher than those of the original assumptions, however, the number of groups with the least optimal value for each capacity restriction does not change.

In conclusion, the optimal number of groups the students are assigned to for the hybrid scheduling varies in response to the capacity restriction. TE and SSE can be reduced by relaxing the capacity restriction, while TD can be reduced with a lower number of groups.

Recommendations for improvements

1. For practical purposes, the formulation could account for the scheduled day and time separately. Instead of having a set of t for every hour with total weekly hours of 167, having a set of 5 school days of a week, and a set of every 30-minute slot of the school hours in a day, would allow for easier schedule adjustments and input accuracy checks.
2. A dynamic allocation of the students, i.e. a student could be assigned to a different group on a different day, would help maximise the number of in-person attendance. Because some elective modules are not on the same day as the core modules, fixing to a group would reduce the number of attendances for students who are not enrolled in those electives. With dynamic allocation, the students would not be stuck to the day-by-day cycle attendance and could be part of other groups attending the next day's classes.
3. Regarding the day-by-day cycle for groups to attend in-person classes and the fact that we assess equal attendance on the day level, altering the schedule by having the core modules every day of the week would improve equal attendance. For instance, Wednesday and Thursday of the current MSBA student schedule are for elective modules only, which would make the students with those electives come to campus on a greater number of days than those enrolled in the electives that are scheduled on the same day as the core modules.

References

1. Moallemi, C. and Patange, U. (2023). Hybrid Scheduling with Mixed-Integer Programming at Columbia Business School*. INFORMS Journal on Applied Analytics*, Articles in Advance. [online] Available from: https://papers.ssrn.com/sol3/papers.cfm?abstract_id =4252808 (Accessed 16 February 2024).
2. University of Warwick. (2016). *WBS Academic Services.* [online] Available from: https://warwick.ac.uk/fac/soc/wbs/central/academic-services/services/roombookingsand timetabling/roomnumbering/ (Accessed 25 February 2024).

Appendices

Appendix 1: Python input data (Input_data.xlsx attached)

	c	Mon	Tue	Wed	Thu	Fri	t900	t930	t1000	...	t1700	t1730		1	2	...	228	229
SCA_L	120	1	0	0	0	0	1	1	0	...	0	0		0	1	...	0	0
ADA_L	292	1	0	0	0	0	0	0	0	...	0	0		1	1	...	1	1
FA_L	292	1	0	0	0	0	0	0	0	...	0	0		1	0	...	0	0
AAMA_L	292	1	0	0	0	0	0	0	0	...	0	0		1	1	...	1	1
DM_L	292	1	0	0	0	0	0	0	0	...	0	0		1	1	...	1	1
AAMA1	73	0	1	0	0	0	0	0	0	...	0	0		1	0	...	0	0
AAMA2	73	0	1	0	0	0	0	0	0	...	1	1		0	1	...	0	0
AAMA3	73	0	1	0	0	0	1	1	1	...	0	0		0	0	...	1	0
AAMA4	73	0	1	0	0	0	0	0	0	...	0	0		0	0	...	0	1
ADA1	76	0	1	0	0	0	1	1	1	...	0	0		1	0	...	0	0
ADA2	76	0	1	0	0	0	0	0	0	...	0	0		0	1	...	0	0
ADA3	76	0	1	0	0	0	0	0	0	...	0	0		0	0	...	1	0
ADA4	76	0	1	0	0	0	0	0	0	...	1	0		0	0	...	0	1
SCA1	40	0	0	1	0	0	0	0	0	...	0	0		0	0	...	0	0
SCA2	40	0	0	1	0	0	0	0	0	...	0	0		0	1	...	0	0
F_L	120	0	0	0	1	0	1	1	0	...	0	0		0	0	...	1	1
F1	44	0	0	0	1	0	0	0	0	...	0	0		0	0	...	1	0
F2	44	0	0	0	1	0	0	0	0	...	0	0		0	0	...	0	1
FA1	73	0	0	0	0	1	1	1	1	...	0	0		1	0	...	0	0
FA2	73	0	0	0	0	1	0	0	0	...	0	0		0	0	...	0	0
DM1	120	0	0	0	0	1	0	0	1	...	0	0		0	1	...	1	1
DM2	120	0	0	0	0	1	0	0	0	...	0	0		1	0	...	0	0
DM3	120	0	0	0	0	1	0	0	0	...	1	1		0	0	...	0	0
E	26																	