INTRO TO AI AND ML

(EE1390)

MATRIX PROJECT

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PROBLEM:31

A variable line drawn through the intersection of lines
$\begin{bmatrix}4 & 3\end{bmatrix}$X=12
$\begin{bmatrix}3 & 4\end{bmatrix}$X = 12
meets the cordinate axes at A and B,then find the locus of the mid point of A and B.

Solution

The given linear equations are

$$\begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

Let $P = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$,

$Q = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$

$PX = Q$

$X = P^{-1}Q$

I is point of intersection

$I = \begin{bmatrix} 1.714 \\ 1.714 \end{bmatrix}$;

Variable lines passing through I is

$\begin{bmatrix} m & -1 \end{bmatrix}$X =1.714(m-1)

where m is paramter

It meets cordinate axes at A and B respectively

A=$\begin{bmatrix} a \\ 0 \end{bmatrix}$ B=$\begin{bmatrix} 0 \\ b \end{bmatrix}$

A=$\begin{bmatrix} 1.714(m-1)/m \\ 0 \end{bmatrix}$ B=$\begin{bmatrix} 0 \\ 1.714(1-m) \end{bmatrix}$

The locus of midpoint of A and B is X

$X = \frac{A+B}{2}$

$X = \begin{bmatrix} 0.8571(m-1)/m \\ 0.8571(1-m) \end{bmatrix}$

EQ(1) :

A random line whose slope is m passing through I is

$\begin{bmatrix} m & -1 \end{bmatrix} \mathrm{X} = 1.714(\mathrm{m}\text{-}1)$

EQ(2) :

Equation of line joining origin and X is

$\begin{bmatrix} m & 1 \end{bmatrix} \mathrm{X} = 0$

By adding both of them we get

Eq(3) :

$\begin{bmatrix} 2m & 0 \end{bmatrix} \mathrm{X} = 1.714(\mathrm{m}\text{-}1)$

By subtracting we get

EQ(4) :

$\begin{bmatrix} 0 & -2 \end{bmatrix} \mathrm{X} = 1.714(\mathrm{m}\text{-}1)$

$\frac{1}{1.714} \begin{bmatrix} 0 & -2 \end{bmatrix} \mathrm{X} + 1 = \mathrm{m}$

Taking transpose on both sides

$\frac{1}{1.714} X^T \begin{bmatrix} 0 \\ -2 \end{bmatrix} + 1 = \mathrm{m}$

Substituting that m value in EQ(3) :

$2m \begin{bmatrix} 1 & 0 \end{bmatrix} X = 1.714(m-1)$

$2 \left(\frac{1}{1.714} X^T \begin{bmatrix} 0 \\ -2 \end{bmatrix} + 1\right) \begin{bmatrix} 1 & 0 \end{bmatrix} X = \begin{bmatrix} 0 & -2 \end{bmatrix} X$

$\left(\frac{2}{1.714} X^T \begin{bmatrix} 0 \\ -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} X\right) + \begin{bmatrix} 2 & 0 \end{bmatrix} X = \begin{bmatrix} 0 & -2 \end{bmatrix} X$

$\frac{2}{1.714} X^T \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix} X = \begin{bmatrix} -2 & -2 \end{bmatrix} X$

$X^T \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix} X = -1.714 \begin{bmatrix} 1 & 1 \end{bmatrix} X$

$X^T \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix} X + 1.714 \begin{bmatrix} 1 & 1 \end{bmatrix} X = 0$

$X^T \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} X - 0.8571 \begin{bmatrix} 1 & 1 \end{bmatrix} X = 0$

$$X^T \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix} X + X^T \begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & 0 \end{bmatrix} X \text{ -0.8571 } \begin{bmatrix} 1 & 1 \end{bmatrix} X = 0$$

$$X^T \begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{1}{2} & 0 \end{bmatrix} X = 0$$

therefore the final equation of locus is

$$X^T \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix} X \text{ -0.8571 } \begin{bmatrix} 1 & 1 \end{bmatrix} X = 0$$

Comparing with general form of conic

$X^T VX + PX + F = 0$

where $V = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ $P = \begin{bmatrix} -0.8571 & -0.8571 \end{bmatrix}$ and F is 0

FIGURES

The figure of locus diagram

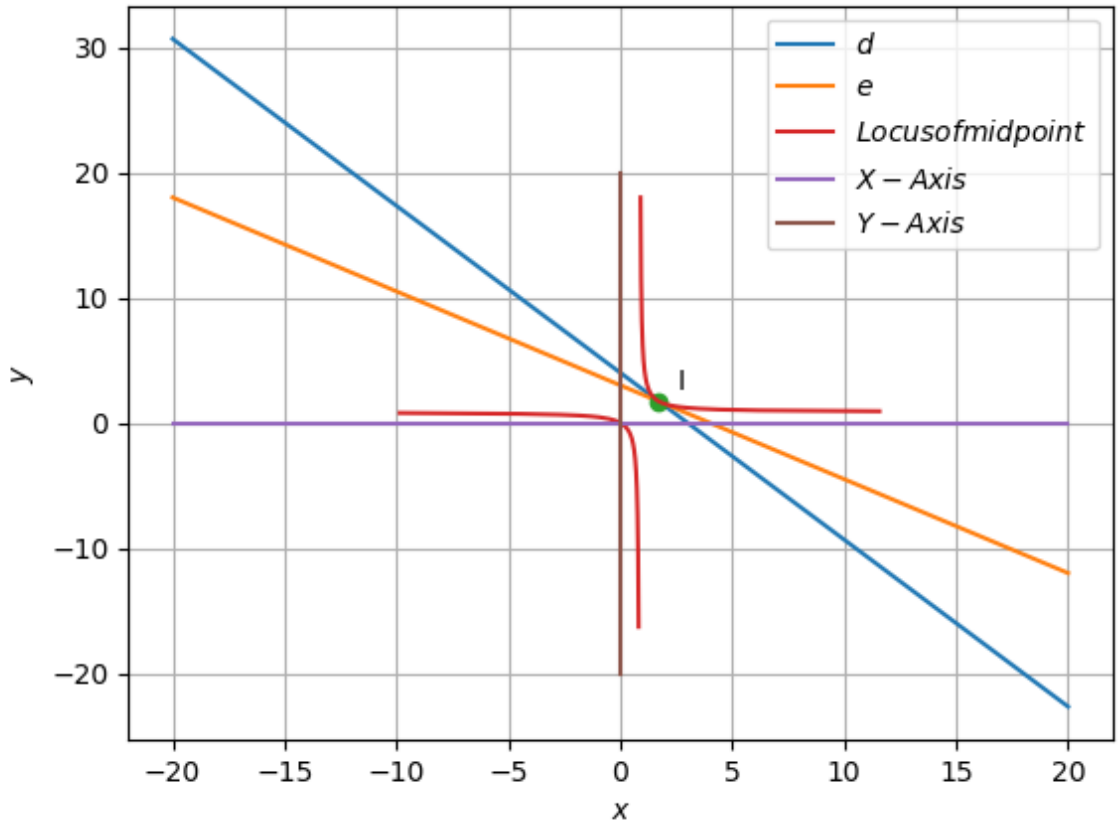

Figure: locus diagram

The figure of variable lines

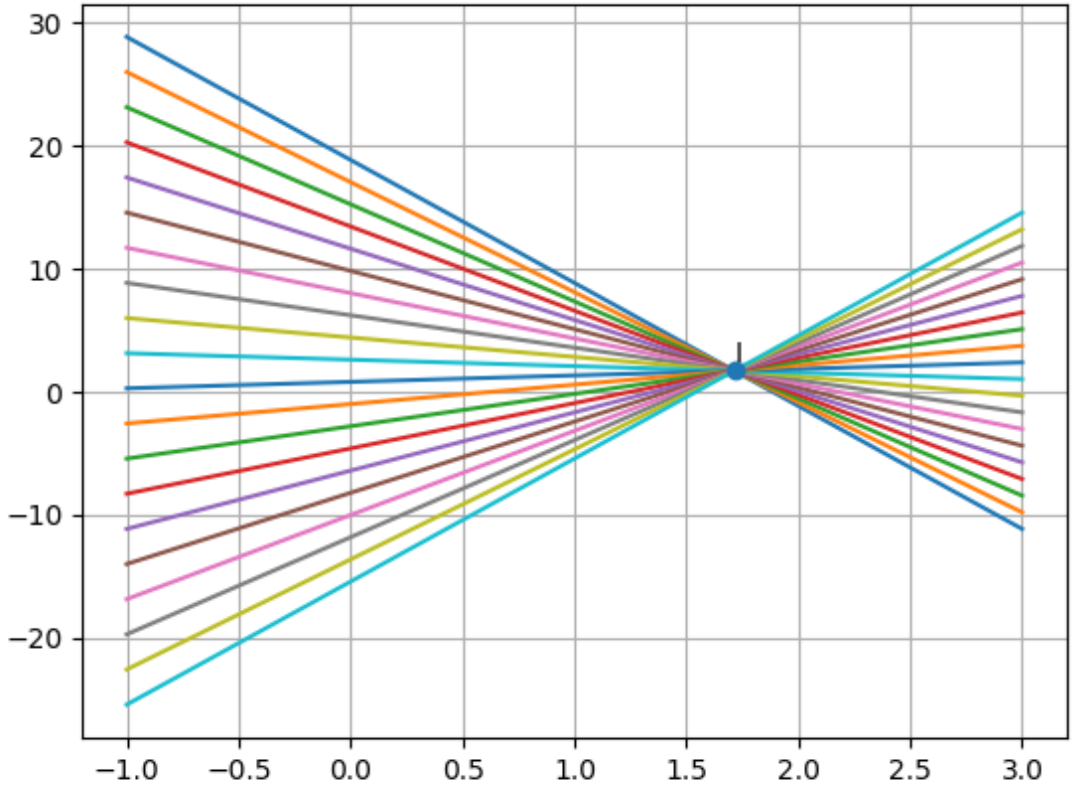

Figure: variable lines

The figure of a random line among variable lines and line joining origin and X