Matrix Project

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G.Naga Dhanush EE17BTECH11014
B.Gowrishankar Reddy EE17BTECH11009

1 problem

A variable line drawn through the intersection of the lines
(4 3)X = 12
(3 4)X = 12
meets the coordinate axes at A and B, then find the locus of the midpoint of AB.

2 solution

The given linear equations are

$$\begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

$$LetP = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}, Q = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

$$\mathrm{PX} = \mathrm{Q}$$
$$\mathrm{X} = P^{-1}\mathrm{Q}$$
X is point of intersection

$$X = \begin{bmatrix} 1.714 \\ 1.714 \end{bmatrix}$$

Variable line passing through X is

$$\begin{bmatrix} m & -1 \end{bmatrix} X = 1.714(m-1)$$

where m is parameter for variable line
It meets cordinate axes at A and B respectively.

$$A = \begin{bmatrix} a \\ 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

a = $1.714\frac{(m-1)}{m}$,b = 1.714(1-m) The locus of midpoint of A and B is C

$$C = \begin{bmatrix} x \\ y \end{bmatrix}$$

x,y are a/2 and b/2 respectively

x =$\frac{0.8571(m-1)}{m}$, y = 0.8571(1-m)

y/x = m

y = 0.8571(1-$\frac{y}{x}$)

xy = 0.8571(x-y)

Therefore the locus is a hyperbola whose equation is

$$C = \begin{bmatrix} 0.8571(m-1)/m \\ 0.8571(1-m) \end{bmatrix}$$

m is paramter.

FIGURES

The figure of this locus obtained from python code

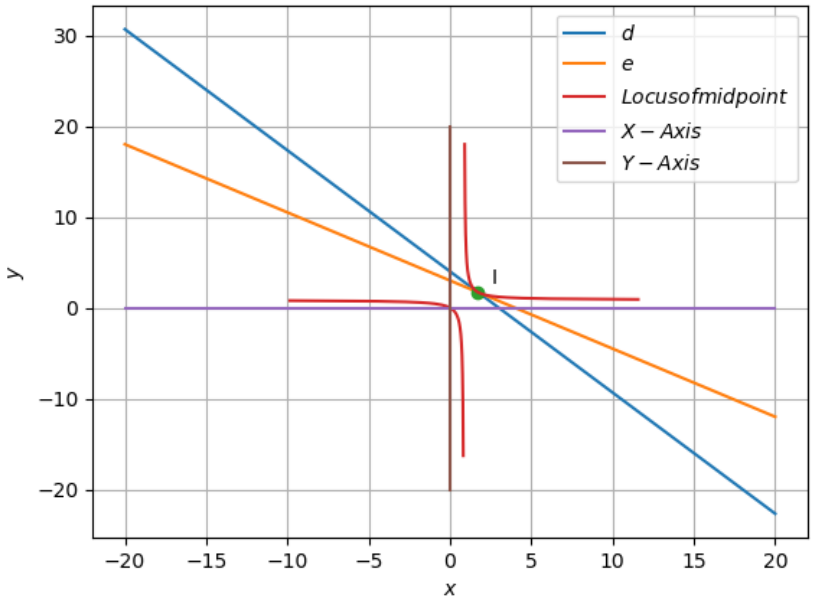

Figure 1: locus diagram

The figure of variable lines passing through X