Assignment-1

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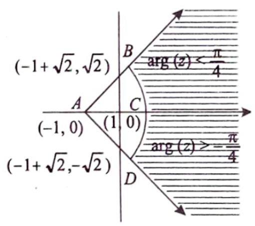
I. MCQ's with Single Correct Answer

1) The locus of z which lies in the shaded region (excluding the boundaries) is best represented by

(2005S)



(a) $z : |z+1| > 2$ and $|arg(z-1)| < \frac{\pi}{4}$
(b) $z : |z-1| > 2$ and $|arg(z-1)| < \frac{\pi}{4}$
(c) $z : |z+1| < 2$ and $|arg(z+1)| < \frac{\pi}{2}$
(d) $z : |z-1| < 2$ and $|arg(z+1)| < \frac{\pi}{2}$

2) a, b, c are integers, not all simultaneously equal and ω is the cube root of unity ($\omega \neq 1$), then the minimum value of $\left|a + b\omega + c\omega^2 l\right|$ is?

(2005S)

(a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

3) Let $\omega = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$,then the value of det?

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} \tag{1}$$

(2002- 2M)

(a) 3ω
(b) $3\omega(\omega - 1)$
(c) $3\omega^2$
(d) $3\omega(1-\omega)$

4) If $\frac{w - \overline{w}z}{1-z}$ is purely real where $w = \alpha + i\beta, \beta \neq 0$ and $z \neq 1$. Then the set of values of z is?

(2006-3M)

(a) $z : |z| = 1$
(b) $z : z = \overline{z}$
(c) $z : z \neq 1$
(d) $z : |z| = 1, z \neq 1$

5) A man walks a distance of 3 units from the origin towards the north-east (N 45°E) direction. From there, he walks a distance of 4 units towards the northwest (N 45°W) direction to reach a poit P. The position of P in the Argand Plane is (2007-3M)

(a) $3e^{\frac{i\pi}{4}} + 4i$
(b) $(3-4i)\, e^{\frac{i\pi}{4}}$
(c) $(4+3i)\, e^{\frac{i\pi}{4}}$
(d) $(3+4i)\, e^{\frac{i\pi}{4}}$

6) If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on (2007-3M)

(a) a line not passing through origin
(b) $|z| = \sqrt{2}$
(c) the x-axis
(d) the y-axis

7) A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach apoint z_1. From z_1, the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by (2008)

(a) $6 + 7i$
(b) -$7 + 6i$
(c) $7 + 6i$
(d) -$6 + 7i$

8) Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is (2009)

(a) $\frac{1}{\sin 2°}$ (b) $\frac{1}{3\sin 2°}$ (c) $\frac{1}{2\sin 2°}$ (d) $\frac{1}{4\sin 2°}$

9) Let $z = x+iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation: $z\overline{z}^3 + \overline{z}z^3 = 350$ is (2009)

(a) 48 (b) 32 (c) 40 (d) 80

10) Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2+z+1$ is real. The a cannot take the value (2012)

(a) -1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

11) Let complex numbers α and $\frac{1}{\overline{\alpha}}$ lie on circles $(x-x_0)^2+(y-y_0)^2 = r^2$ and $(x-x_0)^2+(y-y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equatio $2|z_0|^2 = r^2 + 2$, the $|\alpha|$= (2013)

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

12) Let S be the set of all complex numbers z satisfying $|z-2+i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4-z_0-\overline{z_0}}{z_0-\overline{z_0}+2i}$ is (2019)

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) -$\frac{\pi}{2}$

II. MCQ's with One or More than One Correct

1) If $z_1 = a+ib$ and $z_2 = c+id$ are complex numbers such that $|z_1| = |z_2| = 1$ and Re($z_1\overline{z_2}$)=0, then the pair of complex numbers $w_1 = a+ic$ and $w_2 = b+id$ satisfies (1985 - 2 Marks)

(a) $|w_1| = 1$
(b) $|w_2| = 1$
(c) Re($w_1\overline{w_2}$)=0
(d) none of these.

2) Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$ may be (1986 - 2 Marks)

(a) zero
(b) real and positive
(c) real and negative
(d) purely imaginary
(e) none of the these.

3) If z_1 and z_2 are two non-zero complex numbers such that $|z_1+z_2| = |z_1|+|z_2|$, then argz_1 - argz_2 is equal to (1987 - 2 Marks)

(a) -π
(b) -$\frac{\pi}{2}$
(c) 0
(d) $\frac{\pi}{2}$
(e) π

4) The value of $\sum_{k=1}^{6}(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7})$ is

(1987 - 2 Marks)

(a) -1
(b) 0
(c) -i
(d) i
(e) None