1.6.6

AI24BTECH11036 - Yadlapally Shreedhanvi

Question:In each of the following, find the value of k, for which the points are collinear.
a) (7, -2), (5, 1), (3, k)
b) (8, 1), (k, -4), (2, -5)

Solution:

Label	**Co-ordinate**
Case (a)	
A	$(7, -2)$
B	$(5, 1)$
C	$(3, k)$
Case (b)	
A	$(8, 1)$
B	$(k, -4)$
C	$(2, -5)$

TABLE 0: Co-ordinates

The rank-nullity theorem states that for any linear transformation, $\mathbf{M}$,

$$\text{rank}\,(\mathbf{M}) + \text{nullity}\,(\mathbf{M}) = n \tag{0.1}$$

where n is the number of columns, rank is the number of linearly independent columns, nullity is the number of linearly dependent columns. If collinear, the two columns are linearly dependent, the matrix has rank 1, the two columns are linearly dependent, the matrix has rank 1, so nullity is equal to 1,for 2-d vectors.
For the points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ to be collinear,

$$\text{rank}\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} = 1 \tag{0.2}$$

In case (a)

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} =$$

$$\begin{pmatrix}-2 & -4 \\ 3 & k+2\end{pmatrix} \xleftrightarrow{R_2 \leftarrow 3R_1 + 2R_2} \begin{pmatrix}-2 & -4 \\ 0 & -8+2k\end{pmatrix}$$

Since the rank of the above matrix should be $1, -8+2k = 0$

$$\therefore k = 4.$$

In case (b)

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} = \begin{pmatrix} k-8 & -6 \\ -5 & -6 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1 - R_2} \begin{pmatrix} k-8 & -6 \\ k-3 & 0 \end{pmatrix}$$

Since the rank of the above matrix should be 1,$k-3=0$

$$\therefore k = 3.$$

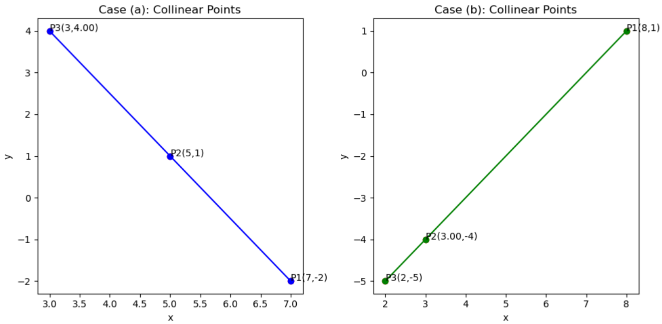

Fig. 0.1: Plots of Lines