# Assignment-6

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## I. MCQ - 2 marks

1) Let $u(x,y)$ be the real part of an entire function $f(z) = u(x,y) + iv(x,y)$ for for $z = x + iy \in \mathbb{C}$. If $C$ is the positively oriented boundary of a rectangular region $R$ in $\mathbb{R}^2$, then $\oint_C \left[\frac{\partial u}{\partial y}dx - \frac{\partial u}{\partial x}dy\right] =$

a) 1 b) 0 c) $2\pi$ d) $\pi$

2) Let $\phi : [0,1] \to \mathbb{R}$ be three times continuously differentiable. suppose that the iterates defined by $x_{n+1} = \phi(x_n), n \geq 0$ converge to the fixed point $\xi$ of $\phi$. If the order of convergence is three then

a) $\phi\prime(\xi) = 0,\ \phi\prime\prime(\xi) = 0$
b) $\phi\prime(\xi) \neq 0,\ \phi\prime\prime(\xi) = 0$
c) $\phi\prime(\xi) = 0,\ \phi\prime\prime(\xi) \neq 0$
d) $\phi\prime(\xi) \neq 0,\ \phi\prime\prime(\xi) \neq 0$

3) Let $f : [0,2] \to \mathbb{R}$ be a twice continuously differentiable function. If $\int_0^2 dx = 2f(1)$, then the error in the aproximation is

a) $\frac{f'(\xi)}{12}$ for some $\xi \in (0,2)$
b) $\frac{f'(\xi)}{2}$ for some $\xi \in (0,2)$
c) $\frac{f''(\xi)}{3}$ for some $\xi \in (0,2)$
d) $\frac{f''(\xi)}{6}$ for some $\xi \in (0,2)$

4) For a fixed $t \in \mathbb{R}$, consider the linear programming problem:

$$\begin{aligned} \text{Maximize } z &= 3x + 4y \\ \text{subject to } x + y &\leq 100 \\ x + 3y &\leq t \\ \text{and } x \geq 0, y &\geq 0 \end{aligned}$$

The maximium value of $z$ is 400 for $t =$

a) 50 b) 100 c) 200 d) 300

5) The minimum value of $z = 2x_1 - x_2 + x_3 - 5x_4 + 22x_5$ subject to

$$\begin{aligned} x_1 - 2x_4 + x_5 &= 6 \\ x_2 + x_4 - 4x_5 &= 3 \\ x_3 + 3x_4 + 2x_5 &= 10 \end{aligned}$$

is

a) 28 b) 19 c) 10 d) 9

6) Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

| | | | |
|---|---|---|---|
| 5 | 23 | 14 | 8 |
| 10 | 25 | 1 | 23 |
| 35 | 16 | 15 | 12 |
| 16 | 23 | 11 | 7 |

a) 29 b) 52 c) 26 d) 44

7) Which of the following sequence $\{f_n\}_{n=1}^{\infty}$ of functions does NOT converge uniformly on $[[]\,0,1]$?

a) $f_n(x) = \frac{e^{-x}}{n}$
b) $f_n(x) = (1-x)^n$
c) $f_n(x) = \frac{x^2+nx}{n}$
d) $f_n(x) = \frac{\sin(nx+n)}{n}$

8) Let $E = \left\{(x,y) \in \mathbb{R}^2 : 0 < x < y\right\}$. Then $\iint_E ye^{-(x+y)}dxdy =$

a) $\frac{1}{4}$
b) $\frac{3}{2}$
c) $\frac{4}{3}$
d) $\frac{3}{4}$

9) Let $f_n(x) = \frac{1}{n}\sum_{k=0}^{n} \sqrt{k(n-k)}\binom{n}{k}x^k(1-x)^{n-k}$ for $x \in [0,1]$, $n = 1,2,\ldots$. If $\lim_{n\to\infty} f_n(x) = f(x)$ for $x \in [0,1]$, then the maximum value of $f(x)$ on $[0,1]$ is

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

10) Let $f : (c_{00}, \|\cdot\|_1) \to \mathbb{C}$ be a non-zero continuous linear function. The numberof Hahn-Banach extensions of $f$ to $\left(l^1, \|\cdot\|_1\right)$ is

a) one
b) two
c) three
d) infinite

11) If $I : \left(l^1, \|\cdot\|_2\right) \to \left(l^1, \|\cdot\|_1\right)$ is the identity map, then
a) both $I$ and $I^{-1}$ are continuous
b) $I$ is continuous but $I^{-1}$ is NOT continuous
c) $I^{-1}$ is continuous but $I$ is NOT continuous
d) neither $I$ nor $I^{-1}$ is continuous

12) Consider the topology $\tau = \{G \subseteq \mathbb{R} : \mathbb{R}|G$ is compact in $(\mathbb{R}, \tau_u)\} \cup \{\phi, \mathbb{R}\}$ on $\mathbb{R}$, where $\tau_u$ is the usual topology on $\mathbb{R}$ and $\phi$ is the empty set. Then $(\mathbb{R}, \tau)$ is
a) a connected Hausdorff space
b) connected but NOT Hausdorff
c) Hausdorff but NOT connected
d) neither connected nor Hausdorff