Assignment-9

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I. Questions with two marks each

1) Let $X_1, \ldots, X_n$ be arandom sample size of $n(\geq 2)$ from $N(\theta, 1)$ distribution where $\theta \in (-\infty, \infty)$. Consider the problem of testing $H_0 : \theta \in [1, 2]$ against $H_1 : \theta < 1$ or $\theta > 2$, based on $X_1, \ldots, X_n$. Which of the following statements is TRUE?
   a) Critical region, of level $\alpha\,(0 < \alpha < 1)$ of uniformly most powerful test for H_0 against H_1 is of the form $\{(x_1, \ldots, x_n) : c_1 \leq \sum_{i=1}^{n} x_i \leq c_2\}$, where c_1 and c_2 are such that the test is of level α
   b) Critical region, of level $\alpha\,(0 < \alpha < 1)$ of uniformly most powerful test for H_0 against H_1 is of the form $\{(x_1, \ldots, x_n) : \sum_{i=1}^{n} x_i > c$ or $\sum_{i=1}^{n} < d\}$, where c and d are such that the test is of level α
   c) At any level $\alpha \in (0, 1)$, uniformly most powerful test for H_0 against H_1 does **NOT** exist
   d) At any level $\alpha \in (0, 1)$, uniformly most powerful test for H_0 against H_1 is less than α
2) In a pure birth process with birth rates $\lambda_n = 2^n, n \geq 0$ let the random variable T denote the time taken for the population size to grow from 0 to 5. If Var(T) denotes the variance of the random variable T, then
$$256 \times \text{Var}(T) = \underline{\qquad}$$
3) Let $\{X_n\}_{n\geq 0}$ be a homogenous Markov chain whose state space is $\{0, 1, 2\}$ and whose one-step transition probability matrix $P = \begin{pmatrix} 0 & 1 & 0 \\ 0.3 & 0 & 0.7 \\ 0 & 1 & 0 \end{pmatrix}$ Then $\lim_{n\to\infty} P(X_{2n} = 2 | X_0 = 2) = \underline{\qquad}$ (coorect upto one decimal place).
4) Let (X, Y) be a random vector such that, for any $y > 0$, the conditional probability density function of X given $Y = y$ is
$$f_{X|Y=y}(x) = ye^{-yx}, x > 0.$$
If the marginal probability density function of Y is
$$g(y) = ye^{-y}, y > 0,$$
then $E(Y|X = 1) = \underline{\qquad}$ (correct up to one decimal place).
5) Let (X, Y) be a random vector with the joint moment generating function
$$M_{X,Y}\{s, t\} = e^{2s^2+t}, \ -\infty < s, t < \infty.$$
Let $\Phi(\cdot)$ denote the distribution function of the standard normal distribution and $p = P(X + 2Y < 1)$. If $\Phi(0) = 0.5, \Phi(0.5) = 0.6915, \Phi(1) = 0.8413$ and $\Phi(1.5) = 0.9332$ then the value of $2p + 1$ (round off to two decimal places) equals $\underline{\qquad}$
6) Consider a homogeneous Markov chain $\{X_n\}_{n\geq 0}$ with state space $\{0, 1, 2, 3\}$ and one-step transition probability matrix
$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Assume that $P(X_0 = 1) = 1$. Let p be the probability that state 0 will be visited before state 3. Then $6 \times p =$ _____

7) Let (X, Y) be a random vector with joint probability mass function

$$f_{X,Y}(x, y) = \begin{cases} {}^{x}C_y \left(\frac{1}{4}\right)^x, & y = 0, 1, 2, \ldots, x;\ x = 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

where ${}^{x}C_y = \frac{x!}{y!(x-y)!}$. Then the variance of Y equals _____.

8) Let X be a discrete random variable with probability mass function $f \in \{f_0, f_1\}$ where The power of

	x=1	x=2	x=3	x=4	x=5
$f_0(x)$	0.10	0.10	0.10	0.10	0.60
$f_1(x)$	0.05	0.06	0.08	0.09	0.72

the most powerful level $\alpha = 0.1$ test for testing $H_0 : X \sim f_0$ against $H_1 : X \sim f_1$ based on X, equals _____ (correct up to two decimal places).

9) Let $\underline{X} = (X_1, X_2, X_3)$ be a random vector following $N_3\left(\underline{0}, \Sigma\right)$ distribution, where $\Sigma = \begin{pmatrix} 1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 1 \end{pmatrix}$. Then the partial correlation coefficient between X_2 and X_3, with fixed X_1, equals _____ (correct up to two decimal places).

10) Let X_1, X_2, X_3 and X_4 be a random sample from a population having probability density function $f_\theta(x) = f(x - \theta), -\infty < x < \infty, \theta \in (-\infty, \infty)$ and $f(-x) = f(x)$, for all $x \in (-\infty, \infty)$. For testing $H_0 : \theta = 0$ against $H_1 : \theta < 0$, let T^+ denote the Wilcoxon Signed-rank statistic. Then under H_0,

$$32 \times P(T^+ \leq 5) = \_\_\_\_\_.$$

11) A simple linear regression model with unknown intercept and unknown slope is fitted to the following data: sing the method of ordinary least squares. Then the predicted value of y corresponding to $x = 5$

x	-2	-1	0	1	2
y	3	5	8	9	10

is _____.

12) Let $D = \{(x, y, z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} : 0 \leq x, y, z \leq 1, x + y + z \leq 1\}$, where $\mathbb{R}$ denotes the set of all real numbers. If

$$I = \iiint_D (x + y)\ dx\, dy\, dz,$$

then $84 \times I =$ _____.

13) Let the random vector (X, Y) have the joint distribution function

$$F(x, y) = \begin{cases} 0, & x < 0 \text{ or } y < 0 \\ 1 - e^{-x}, & x \geq 0, 0 \leq y < 1 \\ \frac{4 - e^{-x}}{1 - e^{-x}}, & x \geq 0, y \geq 1 \end{cases}$$

Let $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$ denote the variances of random variables X and Y, respectively. Then

$$16\,\mathrm{Var}(X) + 32\,\mathrm{Var}(Y) = \_\_\_\_\_.$$