Assignment-10

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I. Questions with one mark each

1) Suppose hat the characteristic equation of $M \in \mathbb{C}^{3\times 3}$ is
$$\lambda^3 + \alpha\lambda^2 + \beta\lambda - 1 = 0$$
where $\alpha, \beta \in \mathbb{C}$ with $\alpha + \beta \neq 0$. Which of the following statements is TRUE?
   a) $M(I - \beta M) = M^{-1}(M + \alpha I)$
   b) $M(I + \beta M) = M^{-1}(M - \alpha I)$
   c) $M^{-1}\left(M^{-1} + \beta M\right) = M - \alpha I$
   d) $M^{-1}\left(M^{-1} - \beta M\right) = M + \alpha I$
2) Consider
P: Let $M \in \mathbb{R}^{m\times n}$ with $m > n \geq 2$. If rank$(M) = n$, then the system of linear equations $Mx = 0$ has $x = 0$ as the only solution.
Q: Let $E \in \mathbb{R}^{m\times n}$, $n \geq 2$ be a non-zero matrix such that $E^3 = 0$. Then $I + E^2$ is a singular matrix.
Which of the following statements is TRUE?
   a) Both **P** and **Q** are TRUE
   b) Both **P** and **Q** are FALSE
   c) **P** is TRUE and **Q** is FALSE
   d) **P** is FALSE and **Q** is TRUE
3) Consider the real function of two real variables given by
$$u(x, y) = e^{2x}\{\sin 3x \cos 2y \cosh 3y - \cos 3x \sin 2y \sinh 3y\}$$
Let $v(x, y)$ be the harmonic conjugate of $u(x, y)$ such that $v(0, 0) = 2$. Let $z = x + iy$ and $f(z) = u(x, y) + iv(x, y)$, then the value of $4 + 2if(i\pi)$ is
   a) $e^{3\pi} + e^{-3\pi}$
   b) $e^{3\pi} - e^{-3\pi}$
   c) $-e^{3\pi} + e^{-3\pi}$
   d) $-e^{3\pi} - e^{-3\pi}$
4) The value of the integral
$$\int_C \frac{z^{100}}{z^{101} + 1} dz$$
where C is the circle of radius 2 centred at the origin taken in the anti-clockwise direction is
   a) $-2\pi i$
   b) 2π
   c) 0
   d) $2\pi i$
5) Let X be a real normed linear space. Let $X_0 = \{x \in X : ||x|| = 1\}$. If X_0 contains two distinct points x and y nd the line segment joining them, then, which of the following statements is TRUE?
   a) $||x + y|| = ||x|| + ||y||$ and x, y are linearly independent
   b) $||x + y|| = ||x|| + ||y||$ and x, y are linearly dependent

c) $||x + y||^2 = ||x||^2 + ||y||^2$ and x, y are linearly independent
d) $||x + y|| = 2\,||x||\,||y||$ and x, y are linearly dependent

6) Let $\{e_k : k \in \mathbb{N}\}$ be an orthonormal basis for a Hilbert space H. Define $f_k = e_k + e_{k+1}, k \in \mathbb{N}$ and $g_j = \sum_{n=1}^{j} (-1)^{n+1} e_n$, $j \in \mathbb{N}$. Then $\sum_{k=1}^{\infty} \left|\langle g_j, f_k\rangle\right|^2 =$
a) 0
b) j^2
c) $4j^2$
d) 1

7) Consider $\mathbb{R}^2$ with the usual metric. Let $A = \left\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\right\}$ and $B = \left\{(x, y) \in \mathbb{R}^2 : (x - 2)^2 + y^2 \leq 1\right\}$. Let $M = A \cup B$ and $N = \text{interior}\,(A) \cup \text{interior}\,(B)$. Then, which of the following statements is TRUE?
a) M and N are connected
b) Neither M nor N is connected
c) M is connected and N is not connected
d) M is not connected and N is connected

8) The real sequence generated by the iterative scheme

$$x_n = \frac{x_{n-1}}{2} + \frac{1}{x_{n-1}},\ n \geq 1$$

a) converges to $\sqrt{2}$, for all $x_0 \in \mathbb{R}\ \ \{0\}$
b) converges to $\sqrt{2}$, whenever $x_0 > \sqrt{\frac{2}{3}}$
c) converges to $\sqrt{2}$, whenever $x_0 \in (-1, 1)\ \ \{0\}$
d) diverges for any $x_0 \neq 0$

9) The initial value problem

$$\frac{dy}{dx} = \cos(xy)\,,\ x \in \mathbb{R},\ y(0) = y_0,$$

where y_0 is a real constant, has
a) a unique solution
b) exactly two solutions
c) infinitely many solutions
d) no solution

10) If eigenfunctions corresponding to distinct eigenvalues $\lambda$of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = \lambda y,\ 0 < x < \pi,$$
$$y(0) = y(\pi) = 0$$

are orthogonal with respect to the weight function $w(x)$, then $w(x)$ is
a) e^{-3x}
b) e^{-2x}
c) e^{2x}
d) e^{3x}

11) The steady state solution for the heat equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0,\ 0 < x < 2,\ t > 0,$$

with the initial condition $u(x, 0) = 0, 0 < x < 2$ and the boundary conditions $u(0, t) = 1$ and $u(2, t) = 3,\ t > 0$, at $x = 1$ is
a) 1

b) 2
c) 3
d) 4

12) Consider $(\{0,1\},T_1)$, where T_1 is the subspace topology induced by the Euclidean topology on $\mathbb{R}$, and let T_2 be any topology on $\{0,1\}$. Consider the following statements:
P: If T_1 is a proper subset of T_2, then $(\{0,1\},T_2)$ is not compact.
Q: If T_2 is a proper subset of T_1, then $(\{0,1\},T_2)$ is not Hausdorff.
Then

a) Both **P** and **Q** are TRUE
b) Both **P** and **Q** are FALSE
c) **P** is TRUE and **Q** is FALSE
d) **P** is FALSE and **Q** is TRUE

13) Let $p : (\{0,1\},T_1) \to (\{0,1\},T_2)$ be the quotient map, arising from the characteristic function on $\left\{\frac{1}{2},1\right\}$, where T_1 is the subspace topology induced by the Euclidean topology on $\mathbb{R}$. Which of the following statements is TRUE?

a) p is an open map but not a closed map
b) p is a closed map but not an open map
c) p is a closed map as well as an open map
d) p is neither an open map nor a closed map