

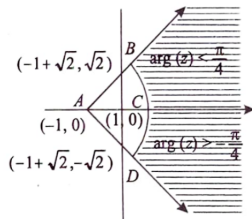
# Assignment-1

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## I. MCQ's WITH SINGLE CORRECT ANSWER

- 1) The locus of  $z$  which lies in the shaded region (excluding the boundaries) is best represented by

(2005S)



- (a)  $z : |z + 1| > 2$  and  $|\arg(z - 1)| < \frac{\pi}{4}$   
 (b)  $z : |z - 1| > 2$  and  $|\arg(z - 1)| < \frac{\pi}{4}$   
 (c)  $z : |z + 1| < 2$  and  $|\arg(z + 1)| < \frac{\pi}{4}$   
 (d)  $z : |z - 1| < 2$  and  $|\arg(z + 1)| < \frac{\pi}{4}$
- 2)  $a, b, c$  are integers, not all simultaneously equal and  $\omega$  is the cube root of unity ( $\omega \neq 1$ ), then the minimum value of  $|a + b\omega + c\omega^2|$  is?

(2005S)

- (a) 0      (b) 1      (c)  $\frac{\sqrt{3}}{2}$       (d)  $\frac{1}{2}$

- 3) Let  $\omega = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ , then the value of  $\det$ ?

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 - \omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} \quad (1)$$

(2002- 2M)

- (a)  $3\omega$       (c)  $3\omega^2$   
 (b)  $3\omega(\omega - 1)$       (d)  $3\omega(1 - \omega)$
- 4) If  $\frac{w - \bar{w}z}{1 - z}$  is purely real where  $w = \alpha + i\beta, \beta \neq 0$  and  $z \neq 1$ . Then the set of values of  $z$  is?

(2006-3M)

- (a)  $z : |z| = 1$       (c)  $z : z \neq 1$   
 (b)  $z : z = \bar{z}$       (d)  $z : |z| = 1, z \neq 1$

- 5) A man walks a distance of 3 units from the origin towards the north-east ( $N 45^\circ E$ ) direction. From there, he walks a distance of 4 units towards the northwest ( $N 45^\circ W$ ) direction to reach a point  $P$ . The position of  $P$  in the Argand Plane is

(2007-3M)

- (a)  $3e^{\frac{i\pi}{4}} + 4i$       (c)  $(4 + 3i)e^{\frac{i\pi}{4}}$   
 (b)  $(3 - 4i)e^{\frac{i\pi}{4}}$       (d)  $(3 + 4i)e^{\frac{i\pi}{4}}$

- 6) If  $|z| = 1$  and  $z \neq \pm 1$ , then all the values of  $\frac{z}{1 - z^2}$  lie on

(2007-3M)

- (a) a line not passing through origin  
 (b)  $|z| = \sqrt{2}$   
 (c) the x-axis  
 (d) the y-axis

- 7) A particle  $P$  starts from the point  $z_0 = 1 + 2i$ , where  $i = \sqrt{-1}$ . It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point  $z_1$ . From  $z_1$ , the particle moves  $\sqrt{2}$  units in the direction of the vector  $\hat{i} + \hat{j}$  and then it moves through an angle  $\frac{\pi}{2}$  in anticlockwise direction on a circle with centre at origin, to reach a point  $z_2$ . The point  $z_2$  is given by

(2008)

- (a)  $6 + 7i$       (c)  $7 + 6i$   
 (b)  $-7 + 6i$       (d)  $-6 + 7i$

- 8) Let  $z = \cos \theta + i \sin \theta$ . Then the value of  $\sum_{m=1}^{15} \operatorname{Im}(z^{2m-1})$  at  $\theta = 2^\circ$  is

(2009)

- (a)  $\frac{1}{\sin 2^\circ}$       (b)  $\frac{1}{3 \sin 2^\circ}$       (c)  $\frac{1}{2 \sin 2^\circ}$       (d)  $\frac{1}{4 \sin 2^\circ}$

- 9) Let  $z = x + iy$  be a complex number where  $x$  and  $y$  are integers. Then the area of the rectangle whose vertices are the roots of the equation:  $z\bar{z}^3 + \bar{z}z^3 = 350$  is

(2009)

- (a) 48      (b) 32      (c) 40      (d) 80

- (a) -1      (c)  $-i$       (e) None  
(b) 0      (d)  $i$

10) Let  $z$  be a complex number such that the imaginary part of  $z$  is non-zero and  $a = z^2 + z + 1$  is real. The  $a$  cannot take the value (2012)

- (a) -1      (b)  $\frac{1}{3}$       (c)  $\frac{1}{2}$       (d)  $\frac{3}{4}$

11) Let complex numbers  $\alpha$  and  $\frac{1}{\alpha}$  lie on circles  $(x-x_0)^2 + (y-y_0)^2 = r^2$  and  $(x-x_0)^2 + (y-y_0)^2 = 4r^2$  respectively. If  $z_0 = x_0 + iy_0$  satisfies the equation  $2|z_0|^2 = r^2 + 2$ , the  $|\alpha| =$  (2013)

- (a)  $\frac{1}{\sqrt{2}}$       (b)  $\frac{1}{2}$       (c)  $\frac{1}{\sqrt{7}}$       (d)  $\frac{1}{3}$

12) Let  $S$  be the set of all complex numbers  $z$  satisfying  $|z - 2 + i| \geq \sqrt{5}$ . If the complex number  $z_0$  is such that  $\frac{1}{|z_0 - 1|}$  is the maximum of the set  $\left\{ \frac{1}{|z - 1|} : z \in S \right\}$ , then the principal argument of  $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$  is (2019)

- (a)  $\frac{\pi}{4}$       (b)  $\frac{3\pi}{4}$       (c)  $\frac{\pi}{2}$       (d)  $-\frac{\pi}{2}$

## II. MCQ'S WITH ONE OR MORE THAN ONE CORRECT

1) If  $z_1 = a + ib$  and  $z_2 = c + id$  are complex numbers such that  $|z_1| = |z_2| = 1$  and  $\text{Re}(z_1 \bar{z}_2) = 0$ , then the pair of complex numbers  $w_1 = a + ic$  and  $w_2 = b + id$  satisfies (1985 - 2 Marks)

- (a)  $|w_1| = 1$       (c)  $\text{Re}(w_1 \bar{w}_2) = 0$   
(b)  $|w_2| = 1$       (d) none of these.

2) Let  $z_1$  and  $z_2$  be complex numbers such that  $z_1 \neq z_2$  and  $|z_1| = |z_2|$ . If  $z_1$  has positive real part and  $z_2$  has negative imaginary part, then  $\frac{z_1 + z_2}{z_1 - z_2}$  may be (1986 - 2 Marks)

- (a) zero      (d) purely imaginary  
(b) real and positive      (e) none of these.  
(c) real and negative

3) If  $z_1$  and  $z_2$  are two non-zero complex numbers such that  $|z_1 + z_2| = |z_1| + |z_2|$ , then  $\arg z_1 - \arg z_2$  is equal to (1987 - 2 Marks)

- (a)  $-\pi$       (c) 0      (e)  $\pi$   
(b)  $-\frac{\pi}{2}$       (d)  $\frac{\pi}{2}$

4) The value of  $\sum_{k=1}^6 \left( \sin \frac{2\pi k}{7} - i \cos \frac{2\pi k}{7} \right)$  is

(1987 - 2 Marks)