1.6.6

AI24BTECH11036 - Yadlapally Shreedhanvi

Question:In each of the following, find the value of k, for which the points are collinear.
a) (7, -2), (5, 1), (3, k)
b) (8, 1), (k, -4), (2, -5)

Solution:

Label	Co-ordinate
Case (a)	
A	$(7, -2)$
B	$(5, 1)$
C	$(3, k)$
Case (b)	
A	$(8, 1)$
B	$(k, -4)$
C	$(2, -5)$

TABLE 0: Co-ordinates

The rank-nullity theorem states that for any linear transformation, (n is the dimension of the space) $\mathbf{M}$,

$$\mathbf{M} = \begin{pmatrix} \mathbf{A_2} - \mathbf{A_1} & \mathbf{A_3} - \mathbf{A_1} & \dots & \mathbf{A_m} - \mathbf{A_1} \end{pmatrix} \tag{0.1}$$

$$\text{rank}\left(\mathbf{M}\right) + \text{nullity}\left(\mathbf{M}\right) = n \tag{0.2}$$

$$\text{rank}\left(\mathbf{M}\right) = 1 \tag{0.3}$$

Consider a set of m points $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3} \dots, \mathbf{A_m}$ in an n-dimensional. These points are said to be collinear if they all lie on a single straight line. For collinearity, the vectors formed by subtracting one point from another must be linearly dependent. All the vectors are scalar multiples of each other so the rank must be 1.
For the points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ to be collinear,

$$\text{rank}\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} = 1 \tag{0.4}$$

In case (a)

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} =$$

$$\begin{pmatrix}-2 & -4\\ 3 & k+2\end{pmatrix} \xleftrightarrow{R_2 \leftarrow 3R_1+2R_2} \begin{pmatrix}-2 & -4\\ 0 & -8+2k\end{pmatrix}$$

Since the rank of the above matrix should be $1, -8+2k = 0$

$$\therefore k = 4.$$

In case (b)

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} =$$

$$\begin{pmatrix}k-8 & -6\\ -5 & -6\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1-R_2} \begin{pmatrix}k-8 & -6\\ k-3 & 0\end{pmatrix}$$

Since the rank of the above matrix should be $1, k-3 = 0$

$$\therefore k = 3.$$

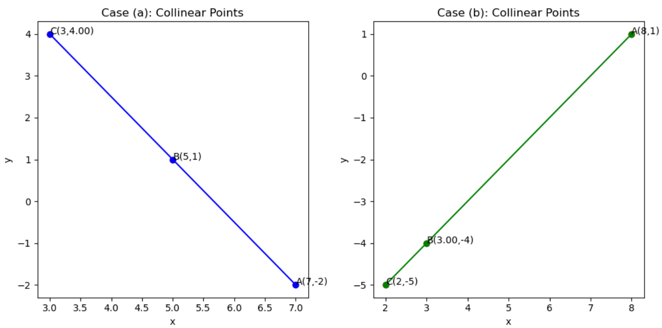

Fig. 0.1: Plots of Lines