

# Assignment-2

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## I. SUBJECTIVE PROBLEMS

- 1) Tangent at a point  $P_1$  other than  $(0,0)$  on the curve  $y = x^3$  meets the curve again at  $P_2$ . The tangent at  $P_2$  meets the curve again at  $P_1$ , and so on. Show that the abscissae of  $P_1, P_2, P_3, \dots, P_n$ , form a G.P. Also find the ratio.  $\frac{[area(\Delta P_1 P_2 P_3)]}{[area(\Delta P_2 P_3 P_4)]}$  (1993 - 5 Marks)
- 2) A line through  $A(5,4)$  meets the line  $x + 3y + 2 = 0$ ,  $2x + y + 4 = 0$  and  $x - y - 5 = 0$  at the points  $B$ ,  $C$  and  $D$  respectively. If  $(15/AB)^2 + (10/AC)^2 - (6/AD)^2$ , find the equation of the line. (1993 - 5 Marks)
- 3) A rectangle  $PQRS$  has its side  $PQ$  parallel to the line  $y = mx$  and vertices  $P$ ,  $Q$  and  $S$  on the lines  $y = a$ ,  $x = b$  and  $x = -b$ , respectively. Find the locus of the vertex  $R$ . (1996 - 2 Marks)
- 4) Using co-ordinate geometry, prove that the three altitudes of any triangle are concurrent. (1998 - 8 Marks)
- 5) For points  $P = (x_1, y_1)$  and  $Q = (x_2, y_2)$  of the co-ordinate plane, a new distance  $d(P, Q)$  is defined by  $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$ . Let  $O = (0,0)$  and  $A = (3,2)$ . Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from  $O$  and  $A$  consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. (2000 - 10 Marks)
- 6) Let  $ABC$  and  $PQR$  be any two triangles in the same plane. Assume that the perpendiculars from the points  $A, B, C$  to the sides  $QR, RP, PQ$  respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from  $P, Q, R$  to  $BC, CA, AB$  respectively are also concurrent. (2000 - 10 Marks)
- 7) Let  $a, b, c$  be real numbers with  $a^2 + b^2 + c^2 = 1$ . Show that the equation

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$$

represents a straight line.

(2001 - 6 Marks)

- 8) A straight line  $L$ , through the origin meets the lines  $x + y = 1$  and  $x + y = 3$  at  $P$  and  $Q$  respectively. Through  $P$  and  $Q$  two straight lines  $L_1$ , and  $L_2$  are drawn, parallel to  $2x - y = 5$  and  $3x + y = 5$  respectively. Lines  $L_1$  and  $L_2$  intersect at  $R$ . Show that the locus of  $R$ , as  $L$ , varies, is a straight line. (2002 - 5 Marks)
- 9) A straight line  $L$  with negative slope passes through the point  $(8,2)$  and cuts the positive coordinate axes at points  $P$  and  $Q$ . Find the absolute minimum value of  $OP + OQ$ , as  $L$  varies, where  $O$  is the origin. (2002 - 5 Marks)
- 10) The area of the triangle formed by the intersection of a line parallel to  $x$ -axis and passing through  $P(h,k)$  with the lines  $y = x$  and  $x + y = 2$  is  $4h^2$ , Find the locus of the point  $P$ . (2005 - 2 Marks)

## II. ASSERTION AND REASON TYPE QUESTIONS

- 1) Lines  $L_1 : y - x = 0$  and  $L_2 : 2x + y = 0$  intersect the line  $L_3 : y + 2 = 0$  at  $P$  and  $Q$ , respectively. The bisector of the acute angle between  $L_1$  and  $L_2$  intersects  $L_3$  at  $R$ .  
**STATEMENT-1** : The ratio  $PR : RQ$  equals  $2\sqrt{2} : \sqrt{5}$ .  
**STATEMENT-2** : In any triangle, bisector of an angle divides the triangle into two similar triangles. (2007 - 3 Marks)
  - a) Statement-1 is True, Statement-2 is True  
Statement-2 is not a correct explanation for Statement-1
  - b) Statement-1 is True, Statement-2 is True;  
Statement-2 is NOT a correct explanation for Statement-1
  - c) Statement-1 is True, Statement-2 is False
  - d) Statement-1 is False, Statement-2 is True.

## III. INTEGER VALUE CORRECT TYPE

- 1) For a point  $P$  in the plane, let  $d_1(P)$  and  $d_2(P)$  be the distance of the point  $P$  from the lines  $x - y = 0$  and  $x + y = 0$  respectively. The area of the region  $R$  consisting of all points  $P$  lying in the first quadrant of the plane and satisfying  $2 \leq d_1(P) + d_2(P) \leq 4$ , is

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