# 1.6.6

AI24BTECH11036 - Yadlapally Shreedhanvi

**Question**:In each of the following, find the value of k, for which the points are collinear.
a) (7, -2), (5, 1), (3, k)
b) (8, 1), (k, -4), (2, -5)

**Solution:**

| Label | Co-ordinate |
|---|---|
| **Case (a)** | |
| **A** | $(7,-2)$ |
| **B** | $(5,1)$ |
| **C** | $(3,k)$ |
| **Case (b)** | |
| **A** | $(8,1)$ |
| **B** | $(k,-4)$ |
| **C** | $(2,-5)$ |

TABLE 0: Co-ordinates

For the points **A**, **B** and **C** to be collinear,

$$\text{rank}\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} = 1 \tag{0.1}$$

In case (a)

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} =$$
$$\begin{pmatrix}-2 & -4\\ 3 & k+2\end{pmatrix} \xleftrightarrow{R_2 \leftarrow 3R_1+2R_2} \begin{pmatrix}-2 & -4\\ 0 & -8+2k\end{pmatrix}$$

Since the rank of the above matrix should be $1, -8+2k = 0$

$$\therefore k = 4.$$

In case (b)

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} =$$
$$\begin{pmatrix}k-8 & -6\\ -5 & -6\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_1-R_2} \begin{pmatrix}k-8 & -6\\ k-3 & 0\end{pmatrix}$$

Since the rank of the above matrix should be $1, k-3 = 0$

$$\therefore k = 3.$$

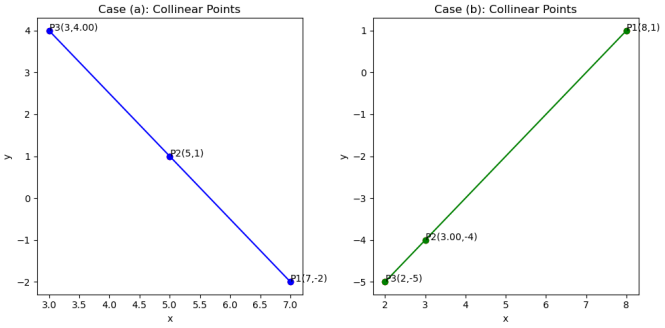


Fig. 0.1: Plots of Lines