Assignment-2

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I. Subjective Problems

1) Tangent at a point $\mathbf{P_1}$ other than $(0,0)$ on the curve $y = x^3$ meets the curve again at $\mathbf{P_2}$. The tangent at $\mathbf{P_2}$ meets the curve again at $\mathbf{P_1}$, and so on. Show that the abscissae of $\mathbf{P_1}, \mathbf{P_2}, \mathbf{P_3} \ldots \mathbf{P_n}$, form a G.P. Also find the ratio. $\frac{[area(\Delta P_1P_2P_3)]}{[area(\Delta P_2P_3P_4)]}$
(1993 - 5 Marks)
2) A line through $\mathbf{A}(5,4)$ meets the line $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at the points $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ respectively. If $(15/AB)^2 + (10/AC)^2 - (6/AD)^2$, find the equation of the line.
(1993 - 5 Marks)
3) A rectangle $PQRS$ has it side PQ parallel to the line $y = mx$ and vertices $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{S}$ on the lines $y = a$, $x = b$ and $x = -b$, respectively. Find the locus of the vertex R.
(1996 - 2 Marks)
4) Using co-ordinate geometry, prove that the three altitudes of any triangle are concurrent.
(1998 - 8 Marks)
5) For points $\mathbf{P} = (x_1, y_1)$ and $\mathbf{Q} = (x_2, y_2)$ of the co-ordn=inate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let $\mathbf{O} = (0,0)$ and $\mathbf{A} = (3,2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from $\mathbf{O}$ and $\mathbf{A}$ consists of the union of a line segment of finite length and an infinite ray. Sketch this net in a labelled diagram.
(2000 - 10 Marks)
6) Let ABC and PQR be any two triangles in the same plane. Assume that the perpendiculars from the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the perpendiculars from $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ to BC, CA, AB respectively are also concurrent.
(2000 - 10 Marks)
7) Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation
$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0 \tag{1}$$
represents a straight line.
(2001 - 6 Marks)
8) A straight line L, through the origin meets the lines $x + y = 1$ and $x + y = 3$ at $\mathbf{P}$ and $\mathbf{Q}$ respectively. Through $\mathbf{P}$ and $\mathbf{Q}$ two straight lines L_1, and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Lines L_1 and L_2 intersect at $\mathbf{R}$. Show that the locus of $\mathbf{R}$, as L, varies, is a straight line. (2002 - 5 Marks)
9) A straight line L with negative slope passes through the point $(8,2)$ and cuts the positive coordinate axes at points $\mathbf{P}$ and $\mathbf{Q}$. Find the absolute minimum value of $OP + OQ$, as L varies, where O is the origin.
(2002 - 5 Marks)
10) The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $\mathbf{P}(h, k)$ with the lines $y = x$ and $x + y = 2$ is $4h^2$, Find the locus of the point P.
(2005 - 2 Marks)

II. Assertion and Reason Type Questions

1) Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at **P** and **Q**, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.
STATEMENT-1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.
STATEMENT-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.
(2007 - 3 Marks)
   a) Statement-1 is True, Statement-2 is True Statement-2 is not a correct explanation for Statement-1
   b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   c) Statement-I is True, Statement-2 is False
   d) Statement-1 is False, Statement-2 is True.

III. Integer Value Correct Type

1) For a point **P** in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point **P** from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points **P** lying in the first quadrant of the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$, is
(JEE Adv. 2014)