Assignment-1

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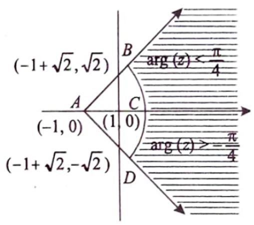
I. MCQ's with Single Correct Answer

1) The locus of z which lies in the shaded region (excluding the boundaries) is best represented by (2005S)



   (a) $z : |z + 1| > 2$ and $|arg(z - 1)| < \frac{\pi}{4}$
   (b) $z : |z - 1| > 2$ and $|arg(z - 1)| < \frac{\pi}{4}$
   (c) $z : |z + 1| < 2$ and $|arg(z + 1)| < \frac{\pi}{2}$
   (d) $z : |z - 1| < 2$ and $|arg(z + 1)| < \frac{\pi}{2}$

2) a, b, c are integers, not all simultaneously equal and ω is the cube root of unity ($\omega \neq 1$), then the minimum value of $|a+b\omega+c\omega^2|$ is?(2005S)
   (a) 0
   (b) 1
   (c) $\frac{\sqrt{3}}{2}$
   (d) $\frac{1}{2}$

3) Let $\omega = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$,then the value of det?

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} \tag{1}$$

(2002- 2M)
   (a) 3ω
   (b) $3\omega(\omega - 1)$
   (c) $3\omega^2$
   (d) $3\omega(1 - \omega)$

4) If $\frac{w-\overline{w}z}{1-z}$ is purely real where $w = \alpha + i\beta, \beta \neq 0$ and $z \neq 1$. Then the set of values of z is? (2006-3M)
   (a) $z : |z| = 1$
   (b) $z : z = \overline{z}$
   (c) $z : z \neq 1$
   (d) $z : |z| = 1, z \neq 1$

5) A man walks a distance of 3 units from the origin towards the north-east (N 45°E) direction. From there, he walks a distance of 4 units towards the northwest (N 45°W) direction to reach a poit P. The position of P in the Argand Plane is (2007-3M)
   (a) $3e^{\frac{i\pi}{4}} + 4i$
   (b) $(3 - 4i)e^{\frac{i\pi}{4}}$
   (c) $(4 + 3i)e^{\frac{i\pi}{4}}$
   (d) $(3 + 4i)e^{\frac{i\pi}{4}}$

6) If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on (2007-3M)
   (a) a line not passing through origin
   (b) $|z| = \sqrt{2}$
   (c) the x-axis
   (d) the y-axis

7) A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach apoint z_1. From z_1, the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by (2008)
   (a) $6 + 7i$
   (b) $-7 + 6i$
   (c) $7 + 6i$
   (d) $-6 + 7i$

8) Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15}$ $\text{Im}(z^{2m-1})$ at $\theta = 2°$ is (2009)
   (a) $\frac{1}{\sin 2°}$
   (b) $\frac{1}{3\sin 2°}$
   (c) $\frac{1}{2\sin 2°}$
   (d) $\frac{1}{4\sin 2°}$

9) Let $z = x+iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation: $z\overline{z}^3 + \overline{z}z^3 = 350$ is (2009)
   (a) 48
   (b) 32
   (c) 40
   (d) 80

10) Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2+z+1$ is real. The a cannot take the value (2012)
   (a) -1
   (b) $\frac{1}{3}$
   (c) $\frac{1}{2}$
   (d) $\frac{3}{4}$
11) Let complex numbers α and $\frac{1}{\alpha}$ lie on circles $(x-x_0)^2+(y-y_0)^2 = r^2$ and $(x-x_0)^2+(y-y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equatio $2|z_0|^2 = r^2 + 2$, the $|\alpha|$= (2013)
   (a) $\frac{1}{\sqrt{2}}$
   (b) $\frac{1}{2}$
   (c) $\frac{1}{\sqrt{7}}$
   (d) $\frac{1}{3}$
12) Let S be the set of all complex numbers z satisfying $|z - 2 + i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\{ \frac{1}{|z-1|} : z \in S \}$, then the principal argument of $\frac{4-z_0-\overline{z_0}}{z_0-\overline{z_0}+2i}$ is (2019)
   (a) $\frac{\pi}{4}$
   (b) $\frac{3\pi}{4}$
   (c) $\frac{\pi}{2}$
   (d) -$\frac{\pi}{2}$

II. MCQ's with One or More than One Correct

1) If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2|$=1 and Re($z_1\overline{z_2}$)=0, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies (1985 - 2 Marks)
   (a) $|w_1| = 1$
   (b) $|w_2| = 1$
   (c) Re($w_1\overline{w_2}$)=0
   (d) none of these.
2) Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$ may be (1986 - 2 Marks)
   (a) zero
   (b) real and positive
   (c) real and negative
   (d) purely imaginary
   (e) none of the these.
3) If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg$z_1$ - argz_2 is equal to (1987 - 2 Marks)
   (a) -π
   (b) -$\frac{\pi}{2}$
   (c) 0
   (d) $\frac{\pi}{2}$
   (e) π
4) The value of $\sum_{k=1}^{6}(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7})$ is (1987 - 2 Marks)
   (a) -1
   (b) 0
   (c) -i
   (d) i
   (e) None