Assignment1

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I. MCQ's with Single Correct Answer

21. The locus of z which lies in the shaded region (excluding the boundaries) is best represented by **(2005S)**

(a) z:$|z+1|$>2 and $|arg$(z+1)$| < \frac{\pi}{4}$

(b) z:$|z-1|$>2 and $|arg$(z-1)$| < \frac{\pi}{4}$

(c) z:$|z+1|$<2 and $|arg$(z+1)$| < \frac{\pi}{2}$

(d) z:$|z-1|$<2 and $|arg$(z-1)$| < \frac{\pi}{2}$

22. a, b, c are integers, not all simultaneously equal and ω is the cube root of unity ($\omega \neq 1$), then the minimum value of $|a + b\omega + c\omega^2|$ is?**(2005S)**
(a) 0
(b) 1

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{1}{2}$

23. Let ω=$\frac{-1}{2} + i\frac{\sqrt{3}}{2}$,then the value of det?
$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ **(2002- 2M)**
(a) 3ω
(b) $3\omega(\omega - 1)$
(c) $3\omega^2$
(d) $3\omega(1 - \omega)$

24. If $\frac{w-\overline{w}z}{1-z}$ is purely real where $w = \alpha + i\beta, \beta \neq 0$ and $z \neq 1$. Then the set of values of z is? **(2006-3M)**
(a) $z : |z| = 1$
(b) $z : z = \overline{z}$
(c) $z : z \neq 1$
(d) $z : |z| = 1, z \neq 1$

25. A man walks a distance of 3 units from the origin towards the north-east (N 45°E) direction. From there, he walks a distance of 4 units towards the northwest (N 45°W) direction to reach a poit P. The position of P in the Argand Plane is **(2007-3M)**
(a) $3e^{\frac{i\pi}{4}} + 4$i

$(b)(3-4\text{i})\text{e}^{\frac{i\pi}{4}}$

$(c)(4+3\text{i})\text{e}^{\frac{i\pi}{4}}$

$(d)(3+4\text{i})\text{e}^{\frac{i\pi}{4}}$

26.If—z—=1andz$\neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on **(2007-3M)**
(a) a line not passing through origin
(b) $|z| = \sqrt{2}$
(c) the x-axis
(d) the y-axis

27. A particle P starts from the point $z_0 = 1 + 2i$, where i=$\sqrt{-1}$. It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach apoint z_1. From z_1, the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by **(2008)**
(a) 6+7i
(b) -7+6i
(c) 7+6i
(d) -6+7i

28. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is **(2009)**
(a) $\frac{1}{\sin 2°}$

(b) $\frac{1}{3\sin 2°}$

(c) $\frac{1}{2\sin 2°}$

(d) $\frac{1}{4\sin 2°}$

29. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation: $z\bar{z}^3 + \bar{z}z^3 = 350$ is **(2009)**
(a) 48
(b) 32
(c) 40
(d) 80

30. Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. The a cannot take the value **(2012)**
(a) -1
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$
(d) $\frac{3}{4}$

31. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on circles $(x-x_0)^2+(y-y_0)^2 = r^2$ and $(x-x_0)^2+(y-y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, the $|\alpha|=$ **(2013)**
(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{7}}$
(d) $\frac{1}{3}$

32. Let S be the set of all complex numbers z satisfying $|z-2+i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\{ \frac{1}{|z-1|} : z \in S \}$, then the principal argument of $\frac{4-z_0-\overline{z_0}}{z_0-\overline{z_0}+2i}$ is **(2019)**
(a) $\frac{\pi}{4}$
(b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{2}$
(d) $-\frac{\pi}{2}$

II. MCQ's with One or More than One Correct

1. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2|=1$ and $\text{Re}(z_1\overline{z_2})=0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies **(1985 - 2 Marks)**
(a) $|w_1|=1$
(b) $|w_2|=1$
(c) $\text{Re}(w_1\overline{w_2})=0$
(d) none of these.

2. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1|=|z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$ may be **(1986 - 2 Marks)**
(a) zero
(b) real and positive
(c) real and negative
(d) purely imaginary
(e) none of the these.

3. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1$ - $\arg z_2$ is equal to **(1987 - 2 Marks)**
(a) $-\pi$
(b) $-\frac{\pi}{2}$
(c) 0
(d) $\frac{\pi}{2}$
(e) π

4. The value of $\sum_{k=1}^{6}(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7})$ is **(1987 - 2 Marks)**
(a) -1
(b) 0
(c) $-i$
(d) i
(e) None