

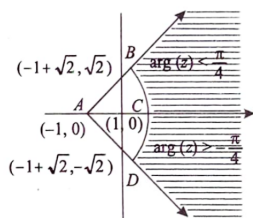
Assignment-1

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I. MCQ's WITH SINGLE CORRECT ANSWER

- 1) The locus of z which lies in the shaded region (excluding the boundaries) is best represented by

(2005S)



- (a) $z : |z + 1| > 2$ and $|\arg(z - 1)| < \frac{\pi}{4}$
 (b) $z : |z - 1| > 2$ and $|\arg(z - 1)| < \frac{\pi}{4}$
 (c) $z : |z + 1| < 2$ and $|\arg(z + 1)| < \frac{\pi}{4}$
 (d) $z : |z - 1| < 2$ and $|\arg(z + 1)| < \frac{\pi}{2}$
- 2) a, b, c are integers, not all simultaneously equal and ω is the cube root of unity ($\omega \neq 1$), then the minimum value of $|a + b\omega + c\omega^2|$ is?

(2005S)

- (a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

- 3) Let $\omega = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, then the value of \det ?

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 - \omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} \quad (1)$$

(2002- 2M)

- (a) 3ω (c) $3\omega^2$
 (b) $3\omega(\omega - 1)$ (d) $3\omega(1 - \omega)$
- 4) If $\frac{w - \bar{w}z}{1 - z}$ is purely real where $w = \alpha + i\beta, \beta \neq 0$ and $z \neq 1$. Then the set of values of z is?

(2006-3M)

- (a) $z : |z| = 1$ (c) $z : z \neq 1$
 (b) $z : z = \bar{z}$ (d) $z : |z| = 1, z \neq 1$

- 5) A man walks a distance of 3 units from the origin towards the north-east ($N 45^\circ E$) direction. From there, he walks a distance of 4 units towards the northwest ($N 45^\circ W$) direction to reach a point P . The position of P in the Argand Plane is

(2007-3M)

- (a) $3e^{\frac{i\pi}{4}} + 4i$ (c) $(4 + 3i)e^{\frac{i\pi}{4}}$
 (b) $(3 - 4i)e^{\frac{i\pi}{4}}$ (d) $(3 + 4i)e^{\frac{i\pi}{4}}$

- 6) If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on

(2007-3M)

- (a) a line not passing through origin
 (b) $|z| = \sqrt{2}$
 (c) the x-axis
 (d) the y-axis

- 7) A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1 . From z_1 , the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2 . The point z_2 is given by

(2008)

- (a) $6 + 7i$ (c) $7 + 6i$
 (b) $-7 + 6i$ (d) $-6 + 7i$

- 8) Let $z = \cos \theta + i \sin \theta$. Then the value of $\sum_{m=1}^{15} \operatorname{Im}(z^{2m-1})$ at $\theta = 2^\circ$ is

(2009)

- (a) $\frac{1}{\sin 2^\circ}$ (b) $\frac{1}{3 \sin 2^\circ}$ (c) $\frac{1}{2 \sin 2^\circ}$ (d) $\frac{1}{4 \sin 2^\circ}$

- 9) Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation: $z\bar{z}^3 + \bar{z}z^3 = 350$ is

(2009)

- (a) 48 (b) 32 (c) 40 (d) 80

- (a) -1 (c) $-i$ (e) None
(b) 0 (d) i

10) Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. The a cannot take the value (2012)

- (a) -1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

11) Let complex numbers α and $\frac{1}{\alpha}$ lie on circles $(x-x_0)^2 + (y-y_0)^2 = r^2$ and $(x-x_0)^2 + (y-y_0)^2 = 4r^2$ respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, the $|\alpha| =$ (2013)

- (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

12) Let S be the set of all complex numbers z satisfying $|z - 2 + i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0 - 1|}$ is the maximum of the set $\left\{ \frac{1}{|z - 1|} : z \in S \right\}$, then the principal argument of $\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}$ is (2019)

- (a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

II. MCQ'S WITH ONE OR MORE THAN ONE CORRECT

1) If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\text{Re}(z_1 \bar{z}_2) = 0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies (1985 - 2 Marks)

- (a) $|w_1| = 1$ (c) $\text{Re}(w_1 \bar{w}_2) = 0$
(b) $|w_2| = 1$ (d) none of these.

2) Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1 + z_2}{z_1 - z_2}$ may be (1986 - 2 Marks)

- (a) zero (d) purely imaginary
(b) real and positive (e) none of these.
(c) real and negative

3) If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to (1987 - 2 Marks)

- (a) $-\pi$ (c) 0 (e) π
(b) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

4) The value of $\sum_{k=1}^6 \left(\sin \frac{2\pi k}{7} - i \cos \frac{2\pi k}{7} \right)$ is

(1987 - 2 Marks)