Rician noise in MRI

Magnetic Resonance Imaging (MRI) is an indispensable tool in the diagnosis of brain diseases due to painlessness and safety. Nevertheless, Rician noise is inevitably injected during the image acquisition process, which leads to poor observation and interferes with the treatment.

Previous research has established that the noise in MRI is governed by the Rician distribution, in which both real and imaginary parts are corrupted by Gaussian noise with equal variance .

Rician Noise in MRI

The raw image generated by magnetic resonance equipment is K-space, including the real channel $P_r$and the imaginary channelP_i. Both channels are governed by Gaussian noise with equal variance σ^2 and a mean value of zero (Zhu et al., 2009), which can be given by

$$\begin{cases} P_r = R\cos\alpha + \Re \\ P_i = R\sin\alpha + \Im \end{cases} \quad (1)$$

where R is the amplitude and α is the phase of raw signal. In addition, $\Re$ and $\Im$ denote the independent Gaussian noise which is injected into the real and the imaginary channel, respectively. An inverse discrete Fourier transform (Briggs and Henson, 1995) and the modular operation are exploited to reconstruct the MR

images, which satisfies the human visual sense. The modular operation can be expressed as follows:

$$\begin{aligned} D &= \sqrt{P_r^2 + P_i^2} \\ &= \sqrt{(R\cos\alpha + \Re)^2 + (R\sin\alpha + \Im)^2} \end{aligned} \tag{2}$$

After the non-linear transformation, the noise distribution is converted from Gaussian to Rician (He and Greenshields, 2008). The probability distribution function (PDF) of Rician noise can be estimated as

$$p(D|R,\sigma) = \frac{D}{\sigma^2} e^{-\frac{D^2+R^2}{2\sigma^2}} I_0\left(\frac{RD}{\sigma^2}\right) \tag{3}$$

where I_0 stands for the zeroth-order modified Bessel function (Sijbers and den Dekker, 2004) when the discrete grid is utilized to define MRI. From the PDF, it can be inferred that Rician noise is associated with images. For images with different signal-to-noise ratios (SNR), the distributions of Rician noise are disparate. If the value of SNR is relatively high, Rician distribution degenerates into a Gaussian distribution. Conversely, it tends to the Rayleigh distribution in low SNR. Therefore, compared with Gaussian noise, Rician noise is more complicated.