THEORY

noise, $N(\boldsymbol{k})$, on the MR signal, $S(\boldsymbol{k})$, can be described as an additive, uncorrelated, complex contribution to the pure MR signal

$$S(\boldsymbol{k}) = S_R(\boldsymbol{k}) + N_R(\boldsymbol{k}) + j[S_I(\boldsymbol{k}) + N_I(\boldsymbol{k})],$$

$\boldsymbol{k}$ is the location parameter of the domain in which the signal is acquired, $S_R(\boldsymbol{k})$ and $S_I(\boldsymbol{k})$ are the real and imaginary components of the true (i.e., noiseless) raw signal, respectively, and $N_R(\boldsymbol{k})$ and $N_I(\boldsymbol{k})$ are the real and imaginary noise contributions, respectively.

The primary sources of noise in MR are electronic (i.e., Johnson noise) and dielectric and inductive coupling to the conducting solution inside the body

The noise can generally be considered to be white noise that is both stationary and ergodic.

The Signal in the Image Domain

The MR signal induced in the receiver coil is a continuous complex signal acquired in a frequency domain commonly referred to as *k*-space. To analyze the MRI data statistics in the image domain, the signal needs to be sampled at discrete locations of *k*-space and then reconstructed using a standard reconstruction algorithm, such as the Inverse Discrete Fourier Transform (IDFT).

$$s = \Im^{-1}\{S\} = \Im^{-1}\{S_R + N_R + j(S_I + N_I)\}$$

where $\Im^{-1}\{\}$ represents the IDFT.

Making use of the linearity property of the IDFT,

$$s = \Im^{-1}\{S_R\} + j\Im^{-1}\{S_I\} + \Im^{-1}\{N_R\} + j\Im^{-1}\{N_I\}$$

$$s = A_R + jA_I + n_R + jn_I, \qquad [4]$$

where $A_R + jA_I \equiv \Im^{-1}\{S_R\} + j\Im^{-1}\{S_I\}$ corresponds to the reconstructed complex noiseless MR image and $n = n_R + jn_I \equiv \Im^{-1}\{N_R\} + j\Im^{-1}\{N_I\}$ is the complex noise in the image domain. From Eq. [4] it is clear that the noise in the image domain is still additive.

Magnitude Images

It is a common practice in MR to work with magnitude images rather than the complex ones. The magnitude image, m, is computed as

$$
\begin{aligned}
m &= \sqrt{(A_\mathrm{R} + n_\mathrm{R})^2 + (A_\mathrm{I} + n_\mathrm{I})^2} \\
&= \sqrt{(A\cos\varphi + n_\mathrm{R})^2 + (A\sin\varphi + n_\mathrm{I})^2},
\end{aligned}
$$

where A and φ are the magnitude and phase of the true, noiseless image signal, respectively. Since this is a nonlinear transformation, the distribution of pixel intensities in the resulting image is, in general, not Gaussian; the magnitude operation rectifies low SNR signals causing the noise statistics to change.

PDF for a noisy magnitude MR image

$$
P_\mathrm{m}\left(m|\mathrm{A},\sigma_\mathrm{g}\right) = \frac{m}{\sigma_\mathrm{g}^2}\exp\left[-\frac{(A^2+m^2)}{2\sigma_\mathrm{g}^2}\right] I_0\left(\frac{Am}{\sigma_\mathrm{g}^2}\right) H(m), \qquad [15]
$$

Equation *15* describes the Rician PDF that characterizes the distribution of MR pixel intensities for most standard situations. Importantly, this function does not, in general, describe the distribution of noise in the image—it is the distribution of pixel intensities observed in the presence of noise.