# Homework 4

Due Date: April. 23, 2019

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(1) **(10 pts)** Given two strings $s$ and $t$, write a function to determine if $t$ is an anagram of $s$ in at worst case $O(n \log n)$

(2) **(10 pts)** There are **N** students in a class. Some of them are friends, while some are not. Their friendship is transitive in nature. For example, if A is a **direct** friend of B, and B is a *direct* friend of C, then A is an **indirect** friend of C. And we defined a friend circle is a group of students who are direct or indirect friends.

Given a $N*N$ matrix $M$ representing the friend relationship between students in the class. If M[i][j] $= 1$, then the $i^{th}$ and $j^{th}$ students are *direct* friends with each other, otherwise not. And you have to output the number of friend circles among all the students.

So for example:

Input:

$[[1, 1, 0], [1, 1, 0], [0, 0, 1]]$

Output: 2

Explanation:The 0th and 1st students are direct friends, so they are in a friend circle. The 2nd student himself is in a friend circle. So return 2.

Write an algorithm to solve this problem.

(3) **(10 pts)** In a town, there are N people labelled from 1 to N. There is a rumor that one of these people is secretly the town judge.

If the town judge exists, then:

1. The town judge trust nobody.

2. Everybody (except the town judge) trusts the town judge.

3. There is exactly one person that satisfies properties 1 and 2.

You are given trust, an array of pairs trust[i] = [a,b] representing that the person labeled a trusts the person labeled b. If the town judge can be identified, return the label of the town judge, otherwise -1.

Describe *in english* how you would solve this problem.

(4) **(10 pts)** Give the topological sort of the following graph starting at $s$.
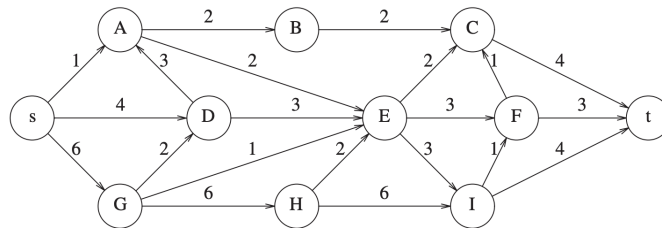
(5) **(10 pts)** Show the table at each step in dijkstra's algorithm for the following graph if s = A
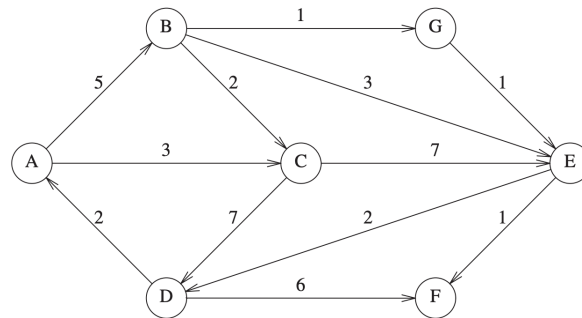


(6) **(10 pts)** Get the maximum flow for the following graph show the residual and flow graph at each step. Then prove the maximum flow is correct by finding the min-cut.
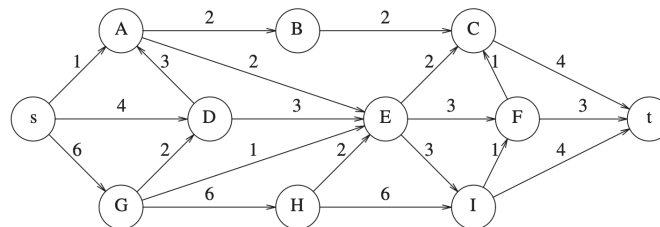


(7) **(10 pts)** Perform quicksort using the median of three method to find the pivot on the following array. [64, 12, 68, 23, 97, 38, 81, 76, 55, 32, 48, 29, 46]. Show each step. If the partition size is $\leq 3$ put the partition in sorted order.

(8) **(10 pts)** Using a set from 0 to 5, perform the following unions using the union-by-height.Show the result of each union. When heights are the same make the second tree a child of the first tree
union(4,5)
union(2,3)
union(0,1)
union(5,3)
union(1,5)

(9) **(10 pts)** Find the minimum spanning tree using prim's algorithm for the following graph starting at vertex $A$. Show each step.
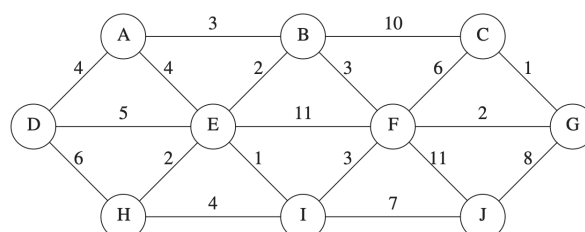
(10) **(10 pts)** Determine if the following graph has a euler circuit.

B 1 G

5 2 3 1

A 3 C 7 E

2 7 2 1

D 6 F