Machine Learning

HOMEWORK - IV

1.) a: For test sample with $x = 4.2$, 1-NN would be $x = 4$.
Hence, it would be classified as 'B'.

b: For test sample with $x = 4.2$, 3-NN would use $x = 3$, $x = 4$ and $x = 5$, having labels {A, B, A} respectively.
The majority here is A, hence it would be classified as 'A'.

c: According to Leave-one-out cross validation of 1-NN,
For each data point in x, if the nearest neighbor has a different label, then x would be misclassified.
Considering all the 18 points, from $x = 0$ to $x = 17$, we errors when,
$x = 3, 4, 5, 8, 9, 12, 13, 14$.
So, 8 out of 18 are errors.

2.)

	X_1 = Acid durability	X_2 = Strength	Y = classification
	7	7	Bad
	7	4	Bad
	3	4	Good
	1	4	Good
Test	3	7	

To find 3-NN we find Euclidean distance between given point & data points

$d_{t_1} = (7-3)^2 + (7-7)^2 = 4^2$ $\quad \sqrt{d_{t_1}} = 4$

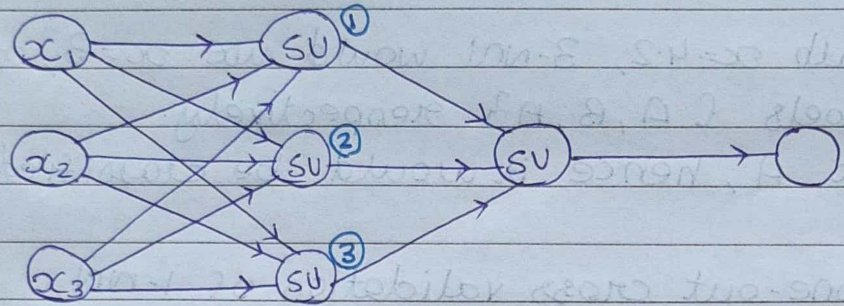
$d_{t_2} = (7-3)^2 + (7-4)^2 = 4^2 + 3^2 = 25$ $\quad \sqrt{d_{t_2}} = 5$

$d_{t_3} = (3-3)^2 + (4-7)^2 = (-3)^2$ $\quad \sqrt{d_{t_3}} = 3$

$d_{t_4} = (1-3)^2 + (4-7)^2 = (-2)^2 + (-3)^2$ $\quad \sqrt{d_{t_4}} = 3.605$

So, the three nearest neighbors are 1, 3 & 4, labelled as Bad, Good and Good respectively. So, the given new tissue will be classified as Good.

3.) To represent $f(x_1, x_2, x_3)$,
we can use 2 hidden layers (one with three signed units, one with ONE signed unit) and an output node.



For each function, if it is True, output = 1 & if False, output = 0.

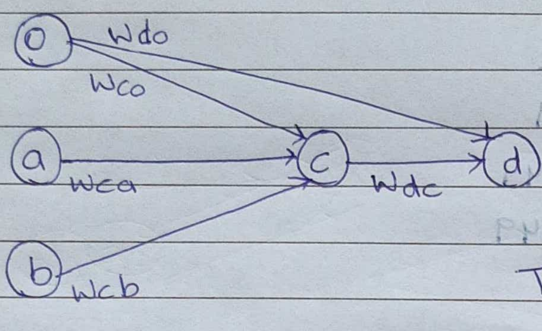
Func at ① ⇒ $\bar{x}_1 \wedge \bar{x}_2 \wedge x_3$, Weights $(w_0, w_1, w_2, w_3) = (-0.5, -1, -1, 1)$
Func at ② ⇒ $\bar{x}_1 \wedge x_2 \wedge \bar{x}_3$, Weights $(w_0, w_1, w_2, w_3) = (-0.5, -1, 1, -1)$
Func at ③ ⇒ $x_1 \wedge \bar{x}_2 \wedge \bar{x}_3$, Weights $(w_0, w_1, w_2, w_3) = (-0.5, 1, -1, -1)$

The input to the second hidden layer is the output of all the hidden nodes in the first hidden layer.
If atleast one of the inputs is a +1, the unit outputs at +1 otherwise it outputs a -1. Thus, it represents an OR node, weights take value $(w_0, w_1, w_2, w_3) = (-0.5, 1, 1, 1)$

The input to the output node is the output of the node in the second hidden layer. It represents the logic function:-
If the input is +1 ⟶ O/P '−5'
−1 ⟶ O/P '10'.

$\therefore (w_0, w_1) \equiv (5/2, -15/2)$

4. Two layer feed forward network



Initial values $(W_{ca}, W_{cb}, W_{co}, W_{dc}, W_{do})$

$(0.1, 0.1, 0.1, 0.1, 0.1)$

$\eta = 0.3, \alpha = 0.9$

Training Set

a	b	d
1	0	1
0	1	0

The network and the sigmoid activation function are as follows :-

$$\sigma(y) = \frac{1}{1+e^{-y}}$$

Training example 1, $a=1$ & $b=0$

$O_c = \sigma((0.1)(1) + (0.1)(0) + (0.1)(1)) = \sigma(0.2) = 0.5498$

~~$O_d = \sigma((0.1)(0.5\ldots) + (0.1)(1)) = \sigma(0.15)$~~

$O_d = \sigma((0.1)(0.5498) + (0.1)(1)) = \sigma(0.15498) = 0.53867$

The error terms for the two neurons, $d=1$

$\partial d = 0.53867 \cdot (1-0.53867) \cdot (1-0.53867) = 0.1146$

$\partial c = 0.5498 \cdot (1-0.5498) \cdot (0.1)(0.1146) = 0.002836$

The correction terms for $a=1, b=0$ and $\eta = 0.3$ are,

$\Delta W_{do} = (0.3)(0.1146)(1) = 0.0342$

$\Delta W_{dc} = (0.3)(0.1146)(0.5498) = 0.0189$

$\Delta W_{co} = (0.3)(0.002836)(1) = 0.000849$

$\Delta W_{ca} = (0.3)(0.002836)(1) = 0.000849$

$\Delta W_{cb} = (0.3)(0.002836)(0) = 0$

The new ~~neighbors~~ weights are,
The new weights are,

$W_{do} = 0.1 + 0.0342 = 0.1342$

$W_{dc} = 0.1 + 0.0189 = 0.1189$

$W_{co} = 0.1 + 0.000849 = 0.100849$

$W_{ca} = 0.1 + 0.000849 = 0.100849$

$W_{cb} = 0.1 + 0 = 0.1$

Training Example 2, $a = 0$ & $b = 1$

$O_c = \sigma(0.100849(0) + (0.1)(1) + (0.100849)(1)) = \sigma(0.200849) = 0.55$

$O_d = \sigma(0.1189(0.55) + (0.1342)(1)) = \sigma(0.1996) = 0.5497$

The error terms for the two neurons, $d = 0$

$\partial d = (0.5497)(1-0.5497)(0-0.5497) = -0.1361$

$\partial_c = 0.55(1-0.55)(0.1189)(-0.1361) = -0.004$

The correction terms for $a = 0, b = 1$ and $\eta = 0.3$ & $\alpha = 0.9$ are

$\Delta W_{do} = (0.3)(-0.1361)(1) + (0.9)(0.0342) = -0.01$

$\Delta W_{dc} = (0.3)(-0.1361)(0.55) + (0.9)(0.0189) = -0.0055$

$\Delta W_{co} = (0.3)(-0.004)(1) + (0.9)(0.000849) = -0.0004$

$\Delta W_{ca} = (0.3)(-0.004)(0) + (0.9)(0.000849) = 0.00086$

$\Delta W_{cb} = (0.3)(-0.004)(1) + (0.9)(0) = -0.0012$

The new weights are,

$W_{do} = 0.1342 - 0.01 = 0.1242$

$W_{dc} = 0.1189 - 0.0055 = 0.1134$

$W_{co} = 0.100849 - 0.0004 = 0.100809$

$W_{ca} = 0.100849 + 0.00086 = 0.1016$

$W_{cb} = 0.1 - 0.0012 = 0.0988$