Homework V-KEY

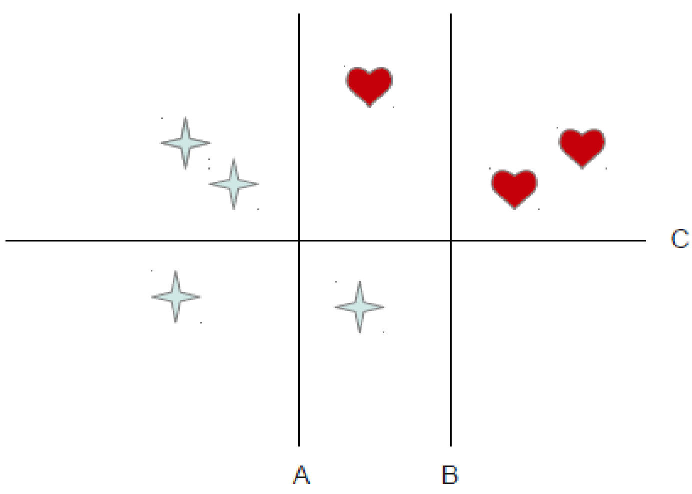
1. Consult the AdaBoost algorithm given in Bishop Chapter 14. Suppose there are two weak leaners h_1 and h_2, and a set of 17 points.
   a) Let say h_1 makes one mistake and h_2 makes four mistakes on the dataset. Which leaner will AdaBoost choose in the first iteration (namely m=1)?
   Will choose h_1.

   b) What is α_1?
   $\epsilon_m = 1/17$. $\alpha_m = \ln\{(1 - 1/17)/1/17\} = \ln(16)$

   c) Calculate the data weighting co-efficient w_2 for the following two cases (1) the points on which chosen leaner made a mistake and (2) the points on which the chosen leaner did not make a mistake. **[10 Points]**

   Case 1: Error made
   $w_2 = 1/17 \times 16 = 16/17$
   Case 2: No Error
   $w_2 = 1/17$.

2. The diagram shows training data for a binary concept where a heart denotes positive examples. Also shown are three decision stumps (A, B and C) each of which consists of a linear decision boundary. Suppose that AdaBoost chooses A as the first stump in an ensemble and it has to decide between B and C as the nest stump. Which will it choose? Explain. What will be the ϵ and α values for the first iteration? **[5 Points]**



It will choose B because the only example mis-classified by A is correctly classified by B (the misclassified examples are assigned higher weight in the next iteration). B also makes another one error C makes 2 errors.

In the first iteration ϵ = 1/7

and $\boldsymbol{\alpha = ln\left\{\frac{1-\epsilon}{\epsilon}\right\} = \ln(6)}$

3. Consider cluster 1D data with a mixture of 2 Guassian using the EM algorithm. You are given the ID data points x = [1 10 20]. Suppose the output of the E step is the following matrix

$$R = \begin{bmatrix} 1 & 0 \\ 0.4 & 0.6 \\ 0 & 1 \end{bmatrix}$$

where entry $r_{i,c}$ is the probability of observation x_i belonging to cluster c (the responsibility of cluster c for data point i). You have to compute the M step. You may state the equations for maximum likelihood estimates of these quantities (which you should know) without proof; you just have to apply the equations to this data set. You may leave your answer in fractional form.

a. Write down the likelihood function you are trying to optimize.
b. After performing M step for the missing weights π_1, π_2, what are the new values?
c. After performing M step for the means μ_1, μ_2, what are the new values?

[10 Points]

a. Likelihood

$$\boldsymbol{p(D|\theta) = \prod_{i=1}^{3}\sum_{k=1}^{2} \pi_k \mathcal{N}(x_i|\mu_k, \sigma_k)}$$

b. Mixing Wights

$$\boldsymbol{\pi_1 = \frac{(1+0.4+0)}{3} = \frac{1.4}{3} = 0.46}$$

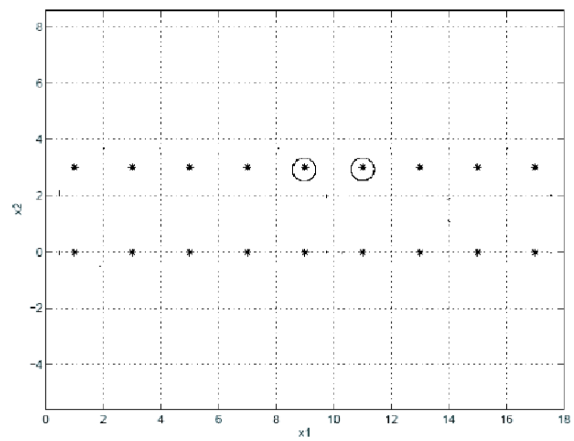
$$\boldsymbol{\pi_2 = \frac{(0+0.6+1)}{3} = \frac{1.6}{3} = 0.53}$$

c. Means

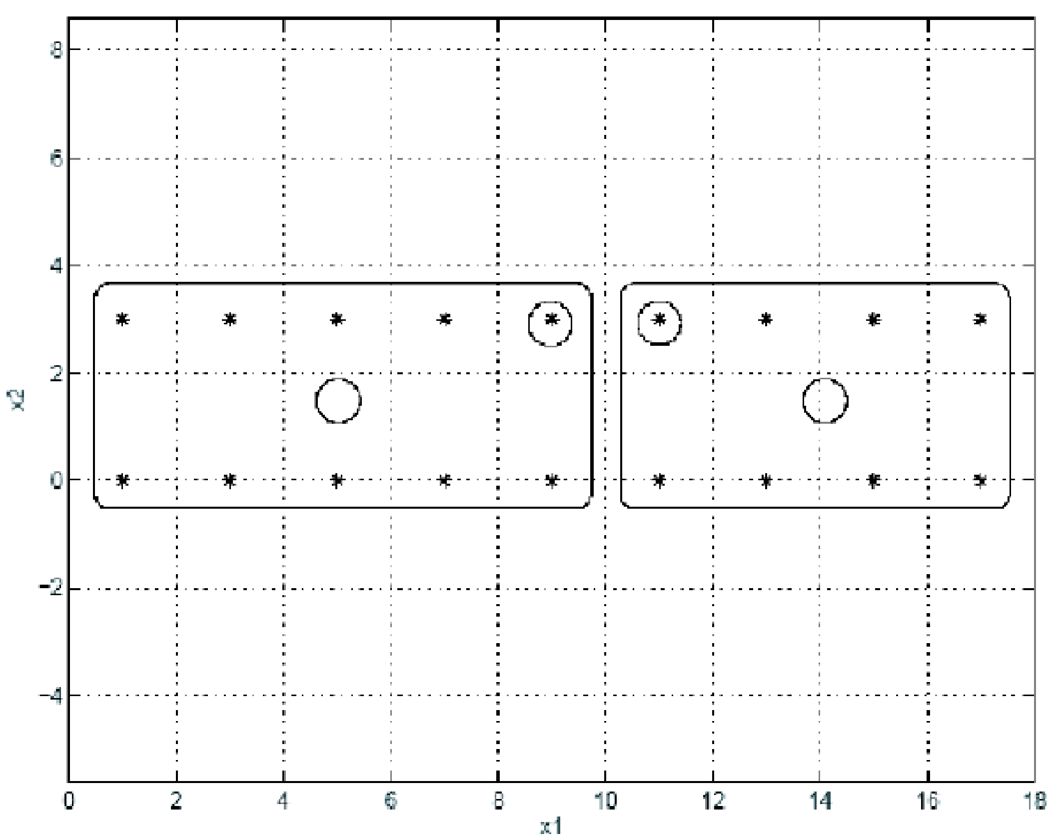
$$\boldsymbol{\mu_1 = \frac{(1)1 + O.4(10)}{1.4} = \frac{5}{1.4} = 3.57}$$

$$\boldsymbol{\mu_2 = \frac{O.6(10) + 1(20)}{1.6} = \frac{26}{1.6} = 16.25}$$

4. In the following figure some data points are shown which lie on integer grid. Suppose we apply the K-means algorithm to this data, using K =2 and with the centers initialized at the two circled data points. Draw the final clusters obtained after K-means converges. **[5 Points]**



Solution: K-means algorithm converges in 2 steps. The following figure shows the final means and clusters.



5. Consider training a two-input perceptron. Give an upper bound on the number of training examples sufficient to assure with 90% confidence that the learned perceptron will have true error of at most 5%? **[5 Points]**

$$m \geq \frac{1}{\epsilon}(4\log_2(2/\delta) + 8VC(H)\log_2(13/\epsilon))$$

that makes

$$\boldsymbol{m \ \geq \frac{1}{0.05}\left(4 * \ log_2\left(\frac{2}{0.1}\right) + 8 * 3 * \ log_2\left(\frac{13}{0.05}\right)\right)}$$

$\boldsymbol{m \ \geq \ 20 * \ (4 * \ 4.32193 + 8 * 3 * \ 8.0224\ )}$ **= 4196.496**

$$\boldsymbol{m \ \geq \ 4197}$$

6. The VC dimension is always less than size of the hypothesis space. True/False? **[5 Points]**

True VC dimension of a hypothesis is at most the log of the hypothesis space.

7. Computational Learning Theory **[10 Points]**

(a) Consider the class C of concepts of the form: $(a \leq x_1 \leq b) \wedge (c \leq \ x_2 \leq d)$. Note that each concept in this class corresponds to a rectangle in 2-dimensions. Let a, b be integers in the range [0, 199] and c, d be integers in the range [0, 99]. Give an upper bound on the number of training examples sufficient to assure that for any target concept c ∈ C, any consistent learner using H = C will, with probability 0.99, output a hypothesis with error at most 0.05.

Since, the learner is consistent, we will use the formula
m >= (1/ε) * (ln (1/d) + ln |H|) to get a bound on m

where, d = 0.01 and ε = 0.05

|H| is the number of rectangles = [(n1 * (n1 – 1))/2] * [(n2 * (n2 – 1))/2] where n1 is 200 and n2 is 100.

|H| = (200*199*100*99) / 4 = 98505000

Therefore, m >= (1/0.05) * (ln(1/0.01) + ln (98505000))

m>=460.216

Number of training examples sufficient to satisfy the required conditions is 461.

(b) Consider the class C of concepts of the form: $(a \leq x_1 \leq b) \wedge (c \leq x_2 \leq d) \wedge (e \leq x_3 \leq f)$. Note that each concept in this class corresponds to a hyper-rectangle in 3-d. Now suppose that a, b, c, d, e, f take on real values instead of integers. Give an upper bound on the number of training examples sufficient to assure that for any target concept c ϵ C, a learner will, with probability 0.95, output a hypothesis with error at most 0.01.

We will use the formula
m >= (1/ε) * [4 * $\log_2$(2/d) + 8*VC(H)*$\log_2$(13/ε)]

where, d = 0.05, ε = 0.01

VC(H) = 2 * Number of dimensions = 2 * 3 = 6

m>= (1/0.01) * [4 * $\log_2$(2/0.05) + 8*6*$\log_2$(13/0.01)]

m>= (1/0.01) * [4 * $\log_2$(40) + 48*$\log_2$(1300)]

m>=100*(4*5.3219 + 48*10.3443)

m>=51781.4

Number of training examples sufficient to satisfy the required conditions is 51782.