EE2703 : Applied Programming Lab
Assignment 4

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EE20B041

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The Assignment

Fourier series expansion of given two functions.

$$f(x) = exp(x)$$
$$g(x) = cos(cos(x))$$

Employed two methods to determine the fourier series coefficients.

- Direct integration
- least squares method

For Direct integration:

$$a_o = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx$$
$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)cos(nx)dx$$
$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)sin(nx)dx$$

For least square method:

$$\begin{bmatrix} 1 & cos(x_1) & sin(x_1) & ... & cos(25x_1) & sin(25x_1) \\ 1 & cos(x_2) & sin(x_2) & ... & cos(25x_2) & sin(25x_2) \\ ... & ... & ... & ... & ... & ... \\ 1 & cos(x_{400}) & sin(x_{400}) & ... & cos(25x_{400}) & sin(x_{400}) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ ... \\ a_{25} \\ b_{25} \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ ... \\ f(x_3) \end{bmatrix}$$

$$Ac = b$$

We will get the coefficients by finding c

Codes

Defining $f(x)$, $g(x)$:

```
def f(x):
    return np.exp(x);
def g(x):
```

```
    return np.cos(np.cos(x));

x = np.linspace(-2*np.pi, 4*np.pi, 200);
```

Ploting the functions over $[-2\pi, 4\pi]$:

```
plt.plot(x, f(x))
plt.title("Plot of f(x)")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.grid()
plt.show()
```

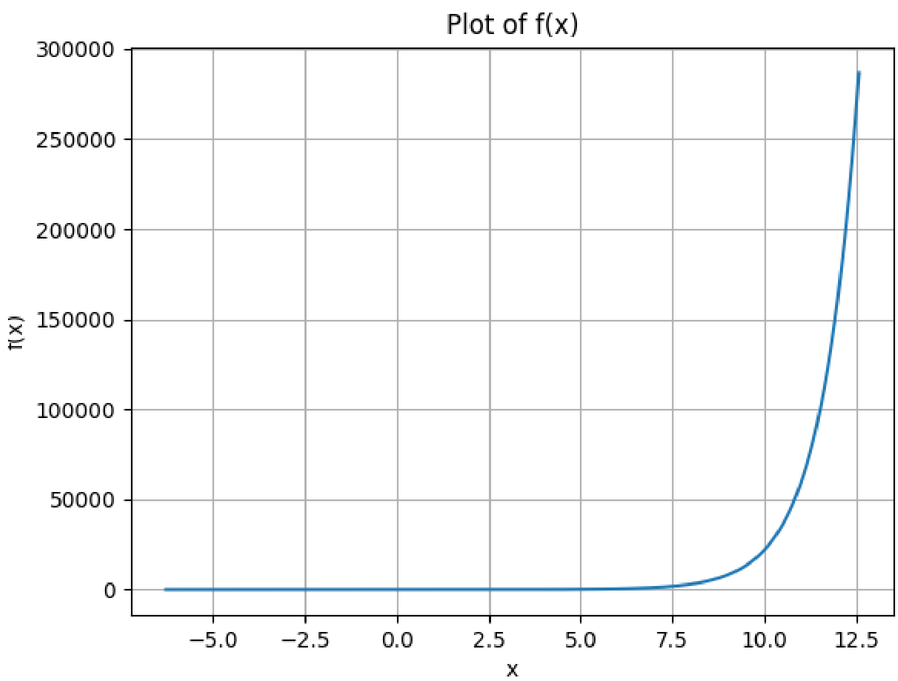

Figure 1: f(x) vs x

```
plt.semilogy(x, f(x))
plt.title("semilog plot of f(x)")
plt.xlabel("x")
plt.ylabel("log(f(x))")
plt.grid()
plt.show()
```

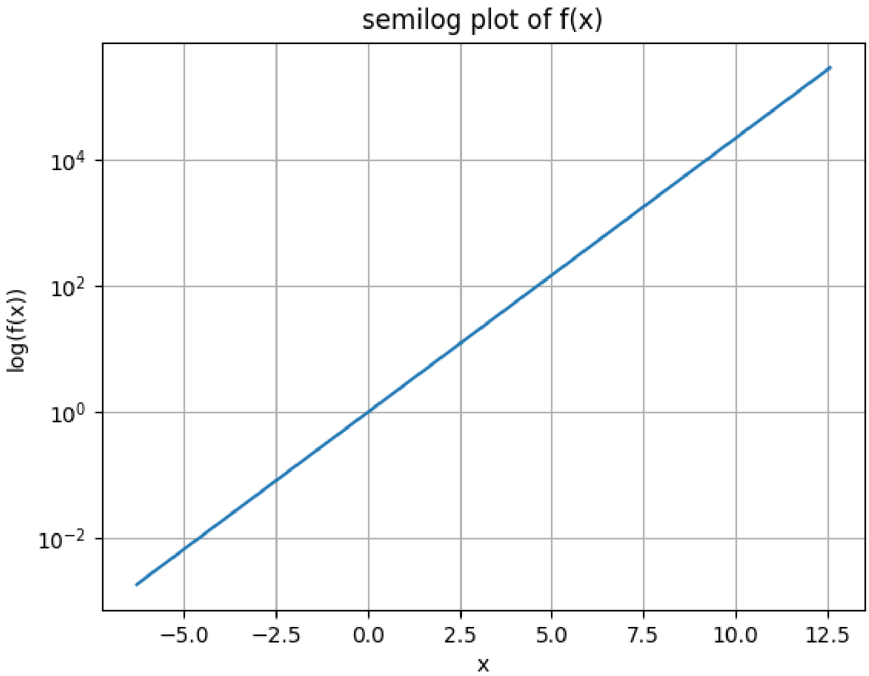

Figure 2: log(f(x)) vs x

```
plt.plot(x, g(x))
plt.title("plot of g(x)")
plt.xlabel("x")
plt.ylabel("g(x)")
plt.show()
```

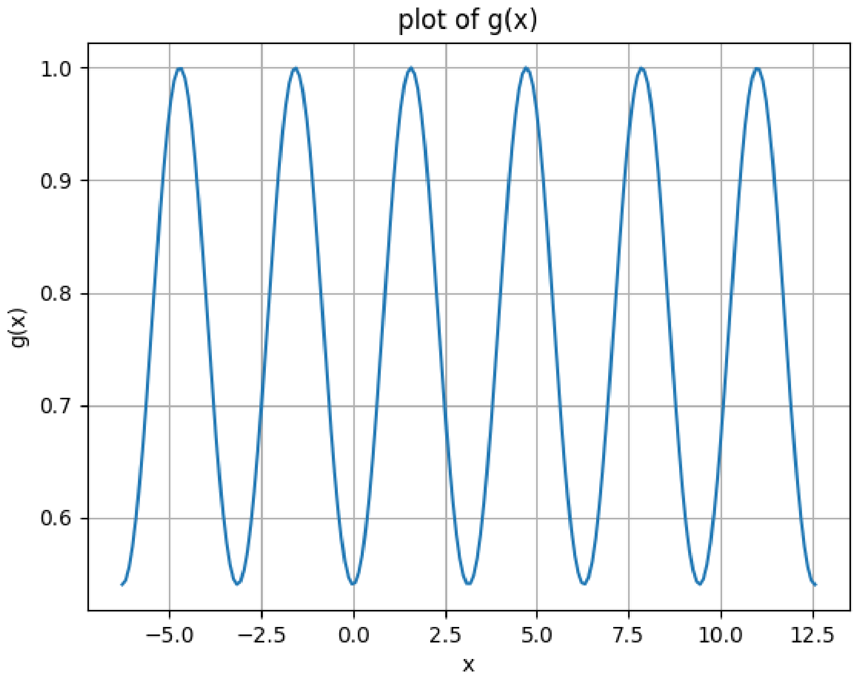

Figure 3: g(x) vs x

The function $f(x) = e^x$ is not periodic.
whereas, The function $g(x) = cos(cos(x))$ is periodic with period π.

Fourier coefficients using direct integration:

Defining the functions for integration:
for $f(x)$:

```
def u(x,k):
    return f(x)*np.cos(k*x)
def v(x,k):
    return f(x)*np.sin(k*x)
```

for $g(x)$:

```
def w(x,k):
    return g(x)*np.cos(k*x)
def y(x,k):
    return g(x)*np.sin(k*x)
```

Obtaining coefficients for f(x) and g(x):

```
fa = []
fb = []
ga = []
gb = []

for i in range(26):
    fa.append(sp.integrate.quad(u,0,2*np.pi, args=(i))[0])
    ga.append(sp.integrate.quad(w,0,2*np.pi, args=(i))[0])

for i in range(25):
    fb.append(sp.integrate.quad(v,0,2*np.pi, args=(i+1))[0])
    gb.append(sp.integrate.quad(y,0,2*np.pi, args=(i+1))[0])

fa = np.array(fa)/np.pi
fb = np.array(fb)/np.pi

ga = np.array(ga)/np.pi
gb = np.array(gb)/np.pi

coeffd = np.zeros(51)
coefgd = np.zeros(51)
coeffd[0] = fa[0]/2
coefgd[0] = ga[0]/2

j=1
for i in range(1,50,2):
    coeffd[i] = fa[j]
    coeffd[i+1] = fb[j-1]

    coefgd[i] = ga[j]
    coefgd[i+1] = gb[j-1]
    j= j+1
```

The coefficients for $f(x)$ are stored in coeffd and for $g(x)$ in coefgd.

Plotting the obtained coefficients:

for f(x):

```
plt.semilogy(n, abs(coeffd), "o", color="r")
plt.xlabel("n")
```

```
plt.ylabel("Coefficients")
plt.title("semilog plot of coefficients of f(x)")
plt.grid()

plt.loglog(n, abs(coeffd), "o", color="r")
plt.xlabel("n")
plt.ylabel("Coefficients")
plt.title("loglog plot of coefficients of f(x)")
plt.grid()
```

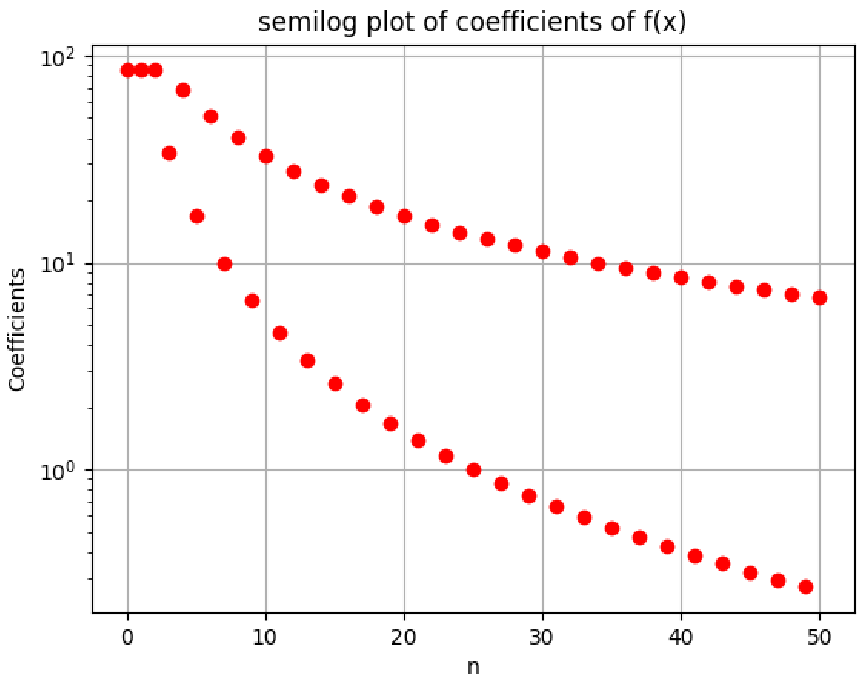

Figure 4: Coefficients of f(x) using direct integration

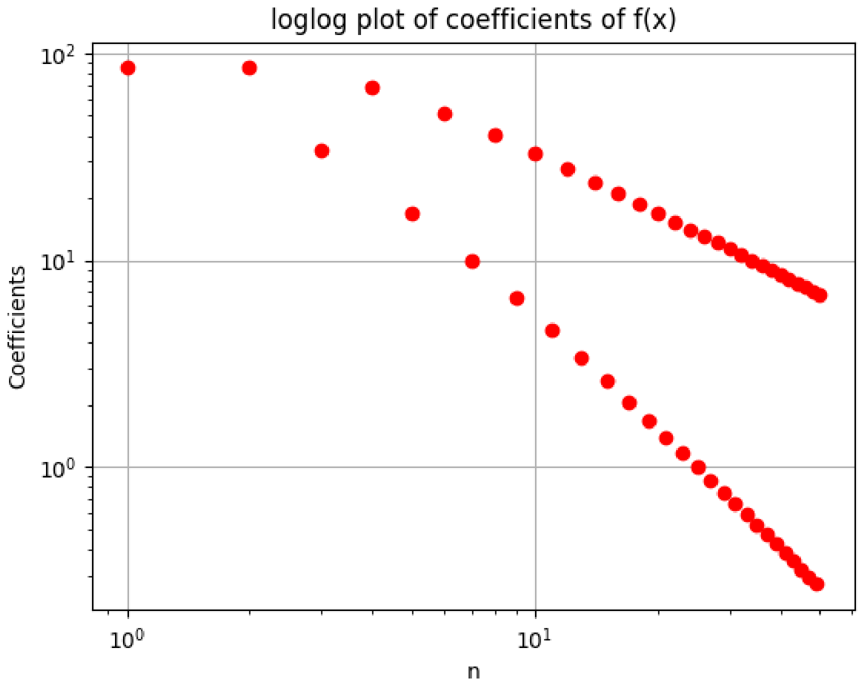

Figure 5: Coefficients of f(x) using direct integration

for g(x):

```
plt.semilogy(n, abs(coefgd), "o", color="r")
plt.xlabel("n")
plt.ylabel("Coefficients")
plt.title("semilog plot of coefficients of g(x)")
plt.grid()

plt.loglog(n, abs(coefgd), "o", color="r")
plt.xlabel("n")
plt.ylabel("Coefficients")
plt.title("loglog plot of coefficients of g(x)")
plt.grid()
```

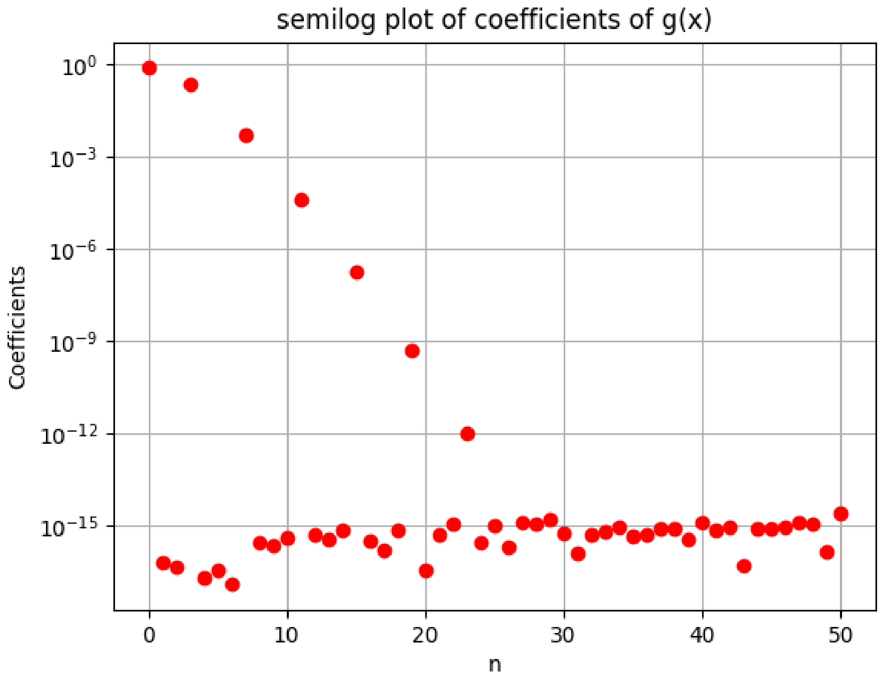

Figure 6: Coefficients of g(x) using direct integration

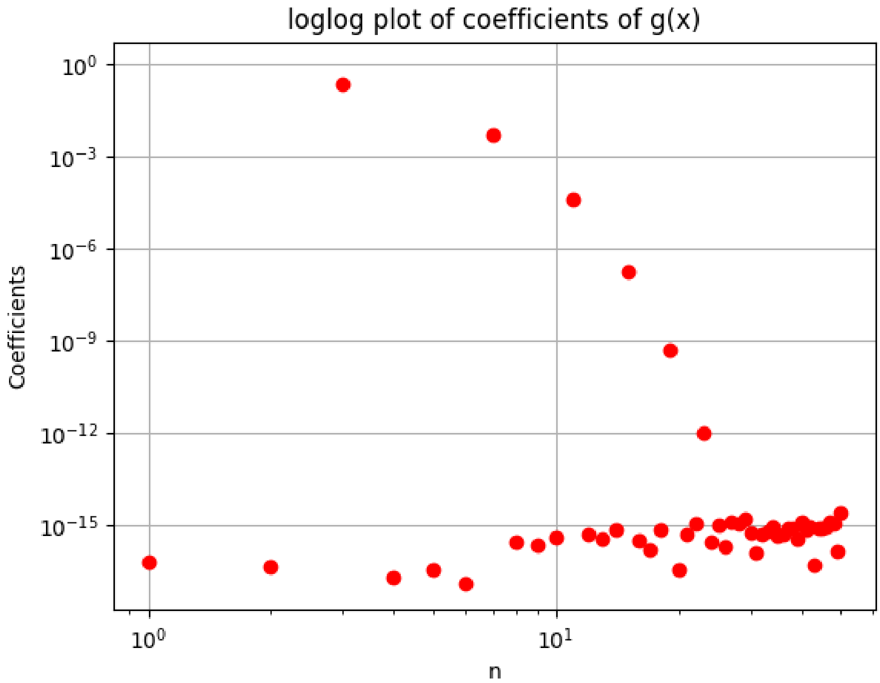

Figure 7: Coefficients of g(x) using direct integration

- The b_n coefficients for $g(x) = cos(cos(x))$ approach to zero as it is an even function which implies the sine signals are negligible compared to cos. hence b_n have low value

- The function $g(x)$ is sinusoidal and the sine and cos signals required to be added to obtain $g(x)$ are less. Therefore, the coefficients of $g(x)$ decrease rapidly towards zero when compared to $f(x)$

- At higher values of n for f(x):

$$log(a_n) = -log(n)$$
$$log(b_n) = -log(n)$$

for g(x):

$$log(a_n) = -log(n)$$

Hence, the plots are linear.

Least square approach for obtaining coefficients :

```
x=np.linspace(0,2*np.pi,401)
x=x[:-1]

bf = f(x)
bg = g(x)
A = np.zeros((400,51))
A[:,0]=1

for k in range(1,26):
    A[:,2*k-1]=np.cos(k*x)
    A[:,2*k]=np.sin(k*x)

coeffl = np.linalg.lstsq(A,bf, rcond=-1)[0]
coefgl = np.linalg.lstsq(A,bg, rcond=-1)[0]
```

Plotting the coefficients:

```
plt.figure(7)
plt.plot(n, coeffl, "o", color="g")
plt.title("Coefficients of f(x) by least square method")
plt.xlabel("n")
plt.ylabel("coefficients")

plt.figure(8)
plt.plot(n, coefgl, "o", color="g")
plt.title("Coefficients of g(x) by least square method")
plt.xlabel("n")
plt.ylabel("coefficients")
plt.show()
```

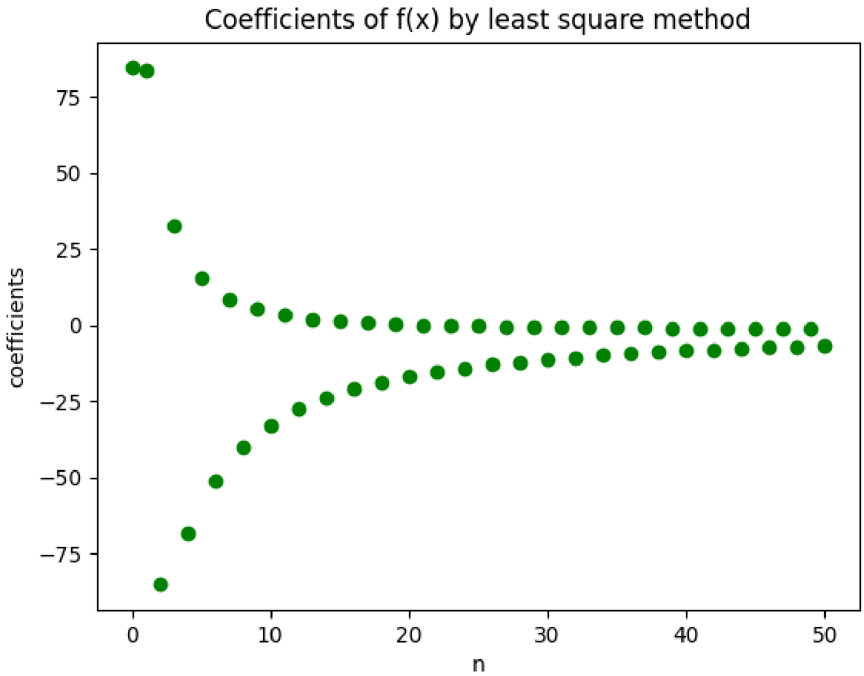

Figure 8: Coefficients of f(x) using least square method

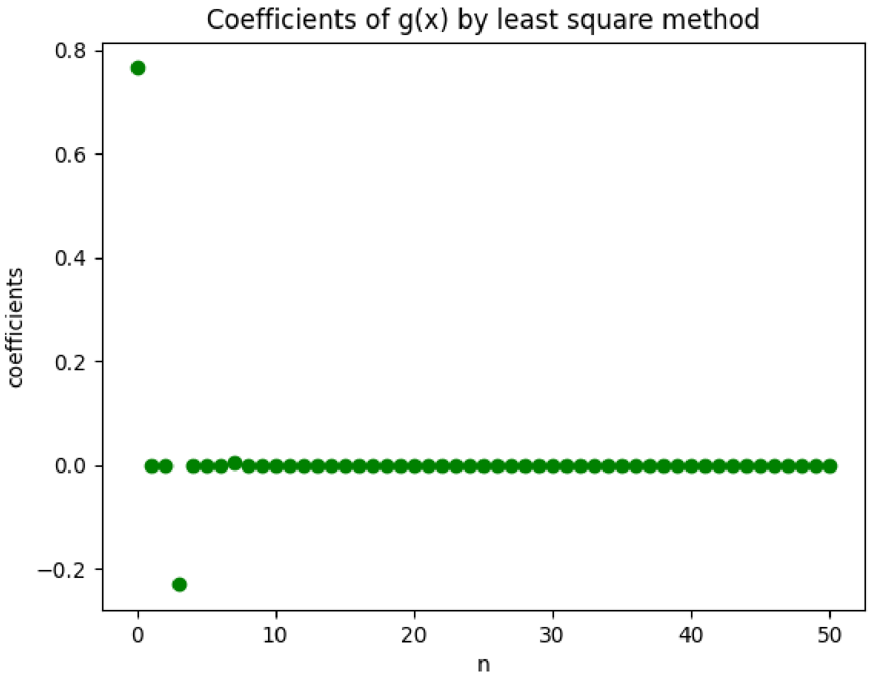

Figure 9: Coefficients of g(x) using least square method

Comparing two methods:

```
devf = abs(abs(coeffl) - abs(coeffd))
devg = abs(abs(coefgl) - abs(coefgd))

maxdevf = devf.max()
maxdevg = devg.max()
```

The maximum deviation for f(x) is 1.33
The maximum deviation for g(x) is $2.53e-15$

Plotting original and estimated fuction:

```
Fls = A.dot(coeffl)
Gls = A.dot(coefgl)

plt.figure(10)
plt.plot(x,Fls, label="least squares")
plt.plot(x,f(x), label="direct integration")
```

```
plt.xlabel("x")
plt.ylabel("f(x)")
plt.legend()

plt.figure(2)
plt.plot(x, Gls, label='least squares')
plt.plot(x, g(x), label='direct integration')
plt.xlabel("x")
plt.ylabel("g(x)")
plt.legend()
plt.show()
```

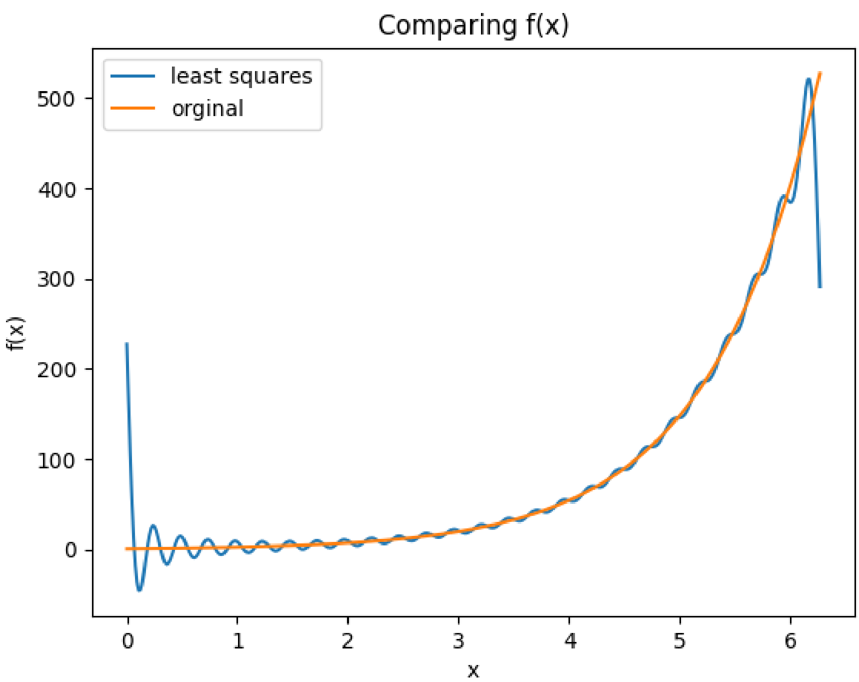

Figure 10: f(x) - Estimated and Original

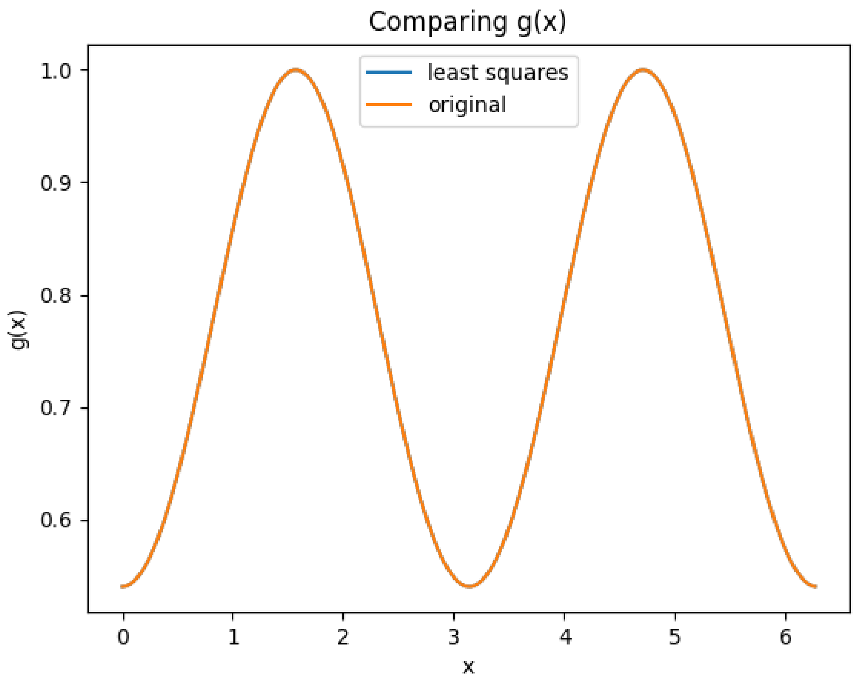

Figure 11: g(x) - Estimated and Original

- $f(x) = exp(x)$ is not a periodic function. Hence, there is a deviation when we use fourier series estimate.
- On the other hand, $g(x) = cos(cos(x))$ is a periodic function. Hence on fourier series estimate the deviation is very less.

Conclusion

Fourier series expansion of $f(x) = exp(x)$ and $g(x) = cos(cos(x))$ are calculated and the deviation from the original function was observed.