EE2703 : Applied Programming Lab
Assignment 6

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EE20B041

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Assignment

Analyse Linear time invariant system in different scenarios

Q1 : Time response of the spring

The spring satisfies :

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

where f(t) is given as :

$$f(t) = cos(1.5t)e^{-0.5t} * u(t)$$

Solving $equation(1)$ in Laplace domain we get :

$$H(s) = \frac{s + 0.5}{((s + 0.5)^2 + 2.25))(s^2 + 2.25)}$$

Code

The python code for the following problem is :

```
den = np.polymul([1,1,2.5],[1,0,2.25])
H = sp.lti([1,0.5],den)
```

For magnitude and phase responses:

```
w,S,phi = H.bode()

plt.subplot(1,2,1)
plt.semilogx(w,S)
plt.figure(1)
plt.title("Magnitude response")
plt.xlabel("frequency(w)")
plt.ylabel("|H(s)|")
plt.grid()

plt.subplot(1,2,2)
plt.semilogx(w,phi)
plt.title("Phase response")
plt.xlabel("frequency(w)")
plt.ylabel("\u03B8(H(s))")
```

```
plt.grid()
plt.show()
```

For impulse response :

```
t,y = sp.impulse(H,None,np.linspace(0,50,1000))
plt.figure(2)
plt.plot(t,y)
plt.title("Impulse response")
plt.xlabel("time(t)")
plt.ylabel("y")
plt.grid()
```

Results

The magnitude and phase response of the system $H(s)$ is :

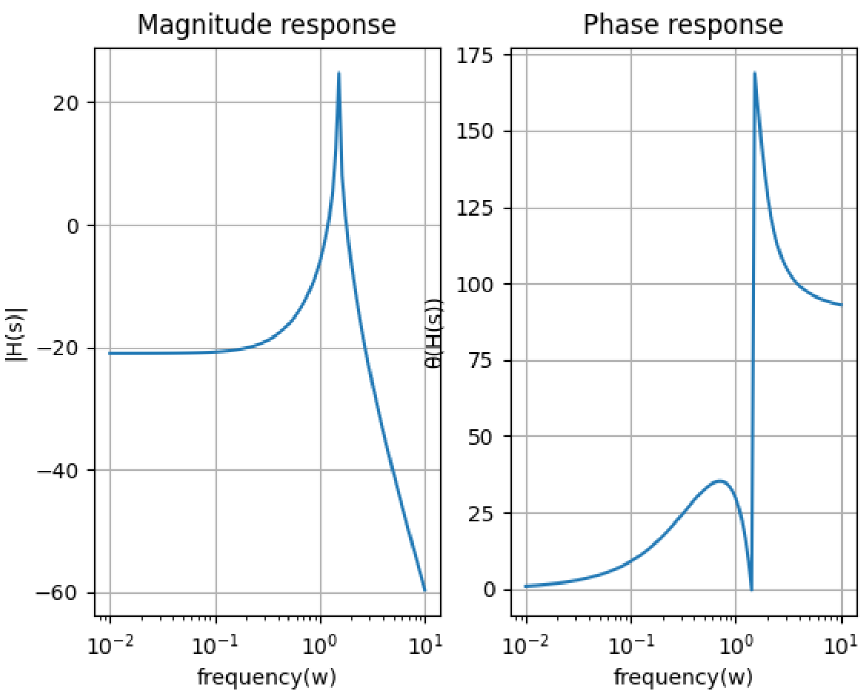

Figure 1: Magnitude and Phase response

The impulse response from 0 to 50sec is :

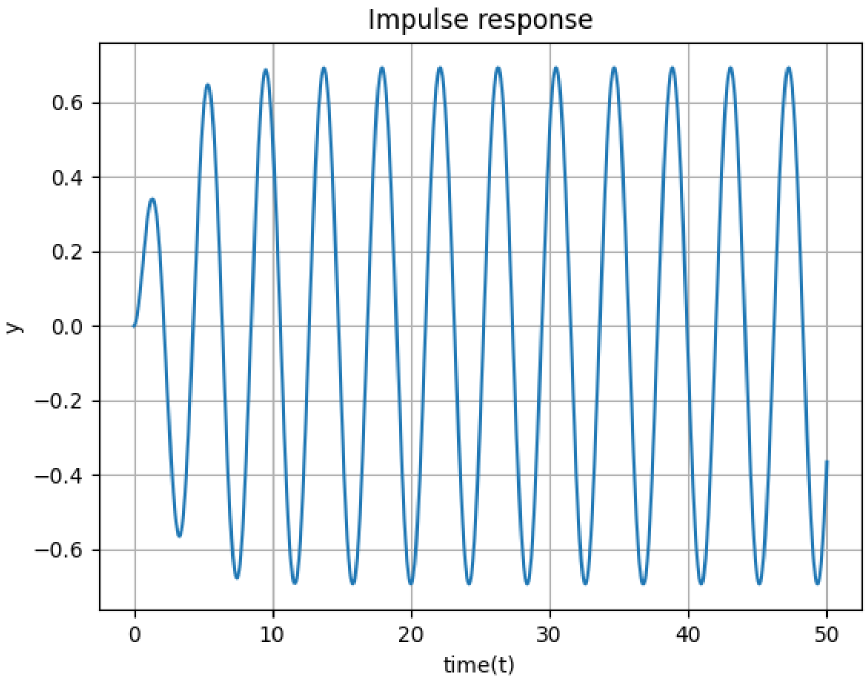

Figure 2: Impulse response of the system

Q2 : Spring response for smaller decay

In this case $f(t)$ is given as :

$$f(t) = cos(1.5t)e^{-0.05t} * u(t)$$

H(s) is given as :

$$H(s) = \frac{s + 0.05}{(s^2 + 0.1s + 2.2525)(s^2 + 2.25)}$$

Code

```
den1 = np.polymul([1,0.1,2.2525], [1,0,2.25])
H1 = sp.lti([1,0.05],den)
```

For magnitude and phase response :

```
w,S,phi = H1.bode()
```

```
plt.figure(3)
plt.subplot(1,2,1)
plt.semilogx(w,S)
plt.title("Magnitude response")
plt.xlabel("frequency(w)")
plt.ylabel("|H(s)|")
plt.grid()

plt.subplot(1,2,2)
plt.semilogx(w,phi)
plt.title("Phase response")
plt.xlabel("frequency(w)")
plt.ylabel("\u03B8(H(s))")
plt.grid()
plt.show()
```

For Impulse response :

```
t,y = sp.impulse(H1,None,np.linspace(0,50,1000))
plt.figure(4)
plt.plot(t,y)
plt.title("Impulse response for smaller decay...")
plt.xlabel("time(t)")
plt.ylabel("y")
plt.grid()
plt.show()
```

Results

Magnitude and phase response:

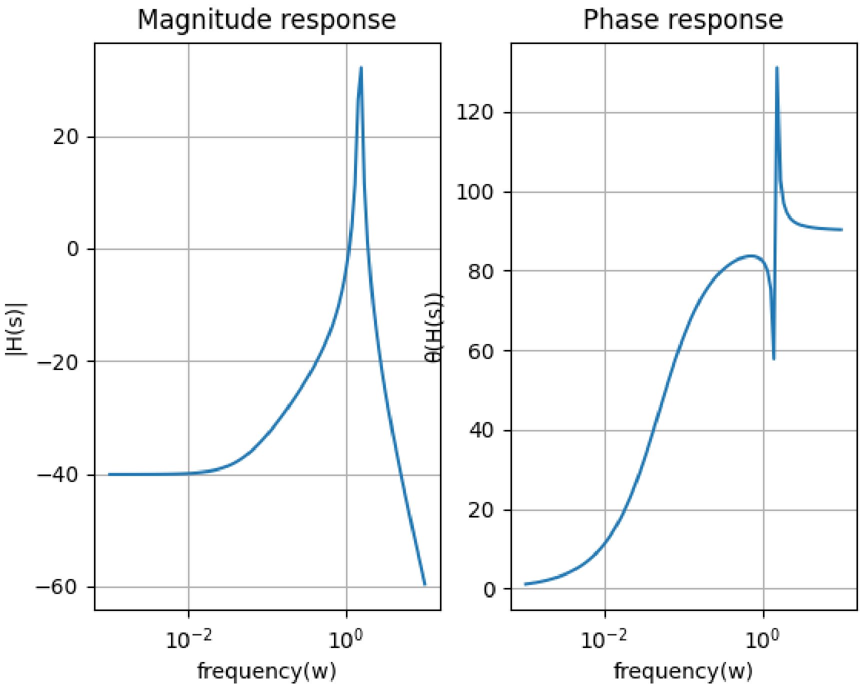

Figure 3: Magnitude and Phase response for smaller decay

Impulse response is :

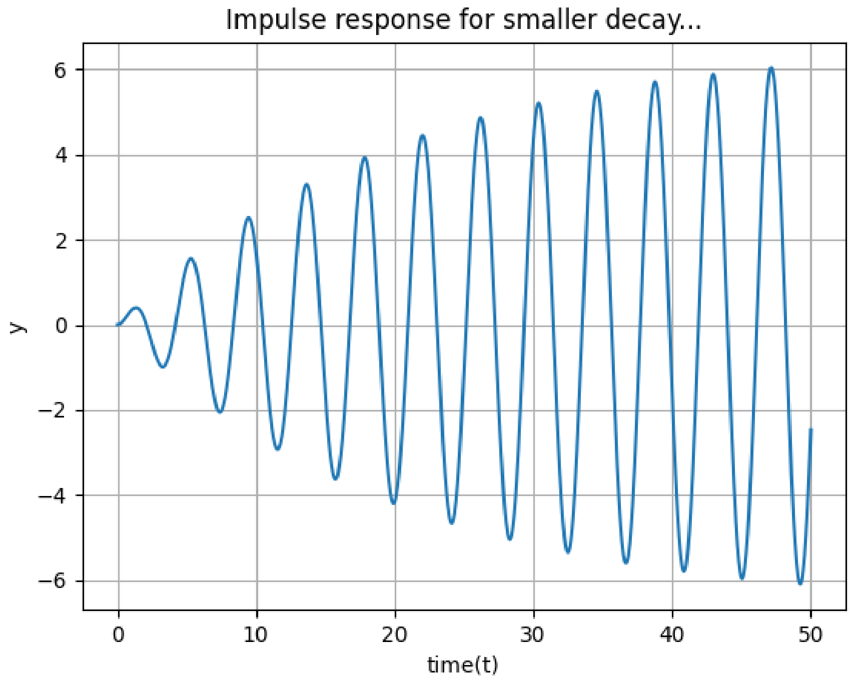

Figure 4: Impulse response for smaller decay

Q3 : Response for different frequencies

Varying the frequency of cosine in $f(t)$ from 1.4 to 1.6 in steps of 0.05.

Code

```
den1 = np.polymul([1,0.1,2.2525], [1,0,2.25])
H1 = sp.lti([1,0.05],den1)
t = np.linspace(0,100,20000)
freq = np.arange(1.4, 1.6, 0.05)

plt.title("Response of the system at different f")
plt.ylabel("y")
plt.xlabel("time(t)")
plt.grid()

for f in freq:
    u = np.cos(f*t)*np.exp(-0.05*t)
```

```
        t,y,_ = sp.lsim(H1,u,t)
        plt.plot(t,y, label= "f = {}".format(f))
    plt.legend()
    plt.show()
```

Results

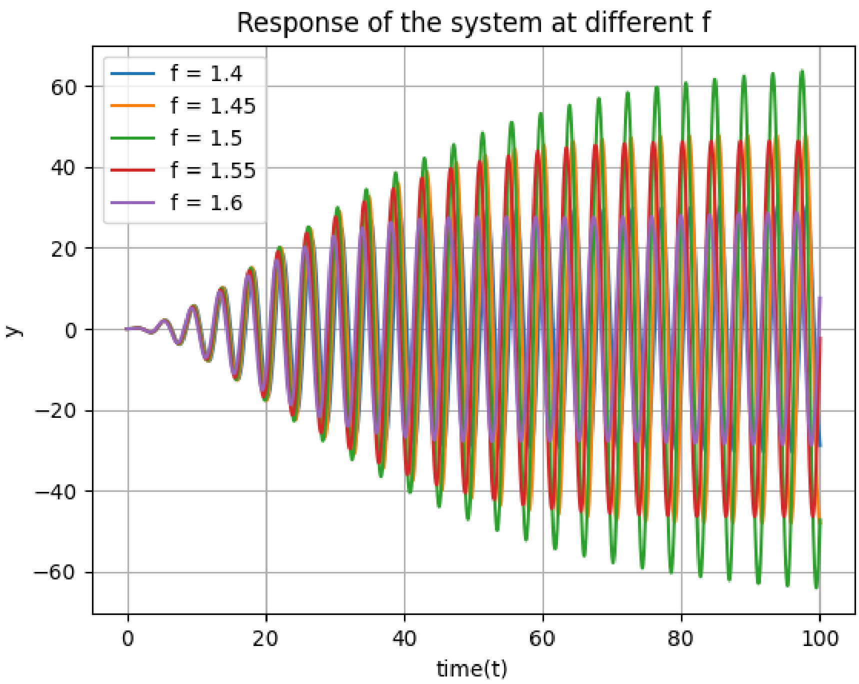

Figure 5: Impulse response for different frequencies

The peak value of the response is observed for frequency f = 1.5, the midpoint of 1.4 and 1.6

Q4 : Coupled spring problem

Equation that defines this is :

$$\ddot{x} + (x - y) = 0 \tag{2}$$

$$\ddot{y} + (y - x) = 0 \tag{3}$$

On solving (2) and (3) in Laplace domain :

$$X(s) = \frac{s^2+2}{s^3+3s}$$
$$Y(s) = \frac{2}{s^3+3s}$$

Code

For $X(s)$:

```
X = sp.lti([1,0,2], [1,0,3,0])

t,y = sp.impulse(X, None, np.linspace(0,20,1000))
plt.figure()
plt.plot(t,y)
plt.grid()
plt.title("Impulse response => x(t)")
plt.ylabel("x")
plt.xlabel("time(t)")
plt.show()
```

For $Y(s)$:

```
Y = sp.lti([2],[1,0,3,0])

t,y = sp.impulse(Y, None, np.linspace(0,20,1000))
plt.figure()
plt.plot(t,y)
plt.grid()
plt.title("Impulse response => y(t)")
plt.ylabel("y")
plt.xlabel("time(t)")
plt.show()
```

Results

Impulse response of $X(s)$:

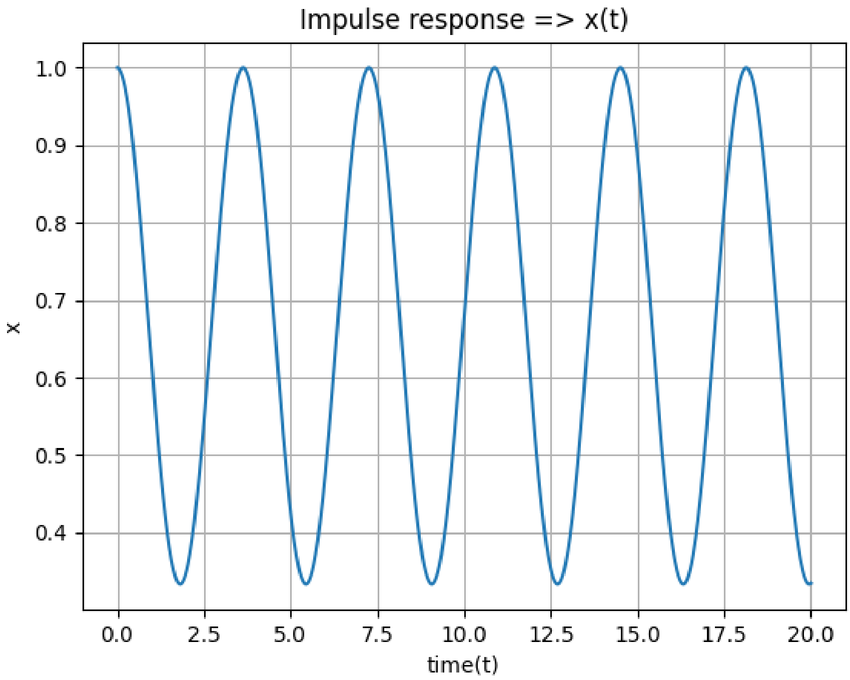

Figure 6: Impulse response for $X(s)$

Impulse response of $Y(s)$:

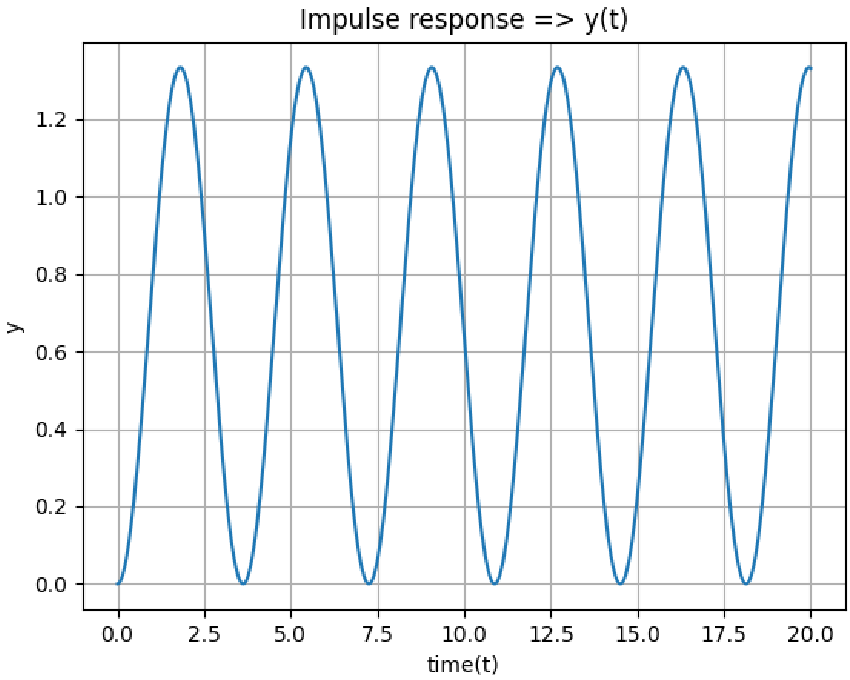

Figure 7: Impulse response for $Y(s)$

Q5 : Two port network

The steady state transfer function of the given two port network is :

$$H(s) = \frac{1}{10^{-12}s^2 + 10^{-4}s + 1}$$

For obtaining the bode plot :

Code

```
H = sp.lti([1], [10**-12, 10**-4, 1])
w,S,phi = H.bode()

plt.figure()
plt.subplot(1,2,1)
plt.semilogx(w,S)
plt.title("Magnitude response")
plt.xlabel("frequency(w)")
```

```
plt.ylabel("|H(s)|")
plt.grid()

plt.subplot(1,2,2)
plt.semilogx(w,phi)
plt.title("Phase response")
plt.xlabel("frequency(w)")
plt.ylabel("\u03B8(H(s))")
plt.grid()
plt.show()
```

0.0.1 Result

The magnitude and phase response of $H(s)$ is :

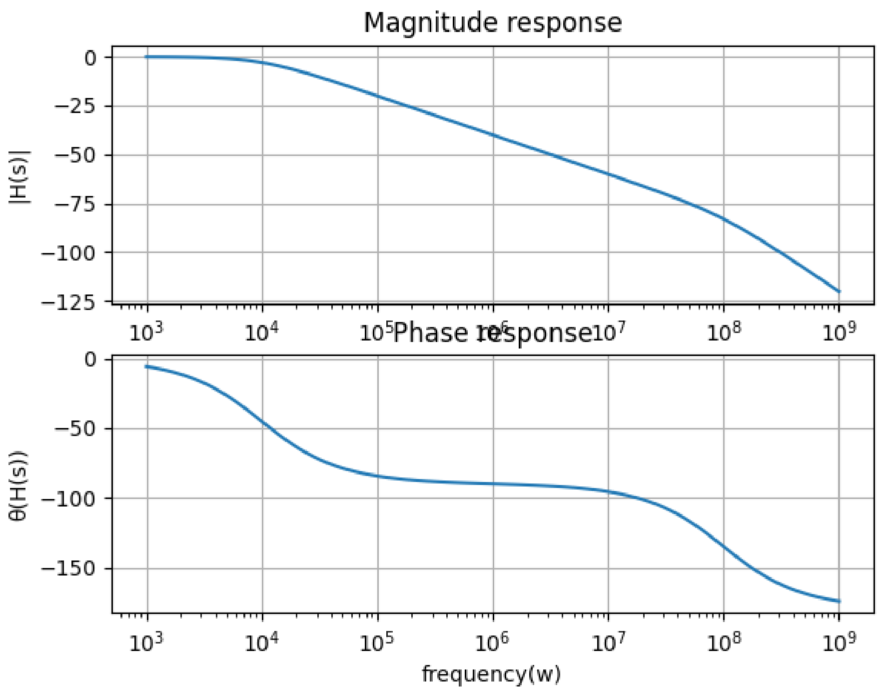

Figure 8: Magnitude and phase response of $H(s)$

Q6 : Low pass filter

The input signal is given as :

$$v_i(t) = cos(10^3 t)u(t) - cos(10^6 t)u(t)$$

Output of the system is given as :

Code

```
H = sp.lti([1], [10**-12, 10**-4, 1])

t = np.linspace(0,30*(10**-6), 1000)
u = np.cos(1000*t) - np.cos((10**6)*t)
t,y,_ = sp.lsim(H, u, t)

plt.figure()
plt.plot(t,y)
plt.grid()
plt.title("Response of the system")
plt.ylabel("y")
plt.xlabel("t")
plt.show()
```

For (ms) timescale :

```
t = np.linspace(0,10*(10**-3), 100000)
u = np.cos(1000*t) - np.cos((10**6)*t)
t,y,_ = sp.lsim(H, u, t)

plt.figure()
plt.plot(t,y)
plt.grid()
plt.title("Response of the system")
plt.ylabel("y")
plt.xlabel("t")
plt.show()
```

Result

For μ seconds, the response is :

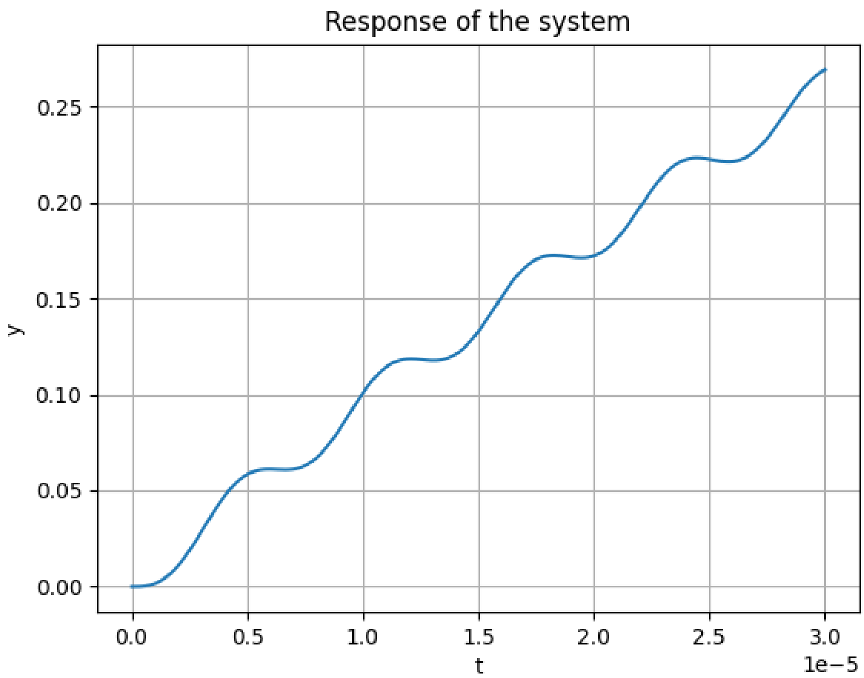

Figure 9: Output signal ($0 < t < 30\mu s$)

For m seconds, the response is :

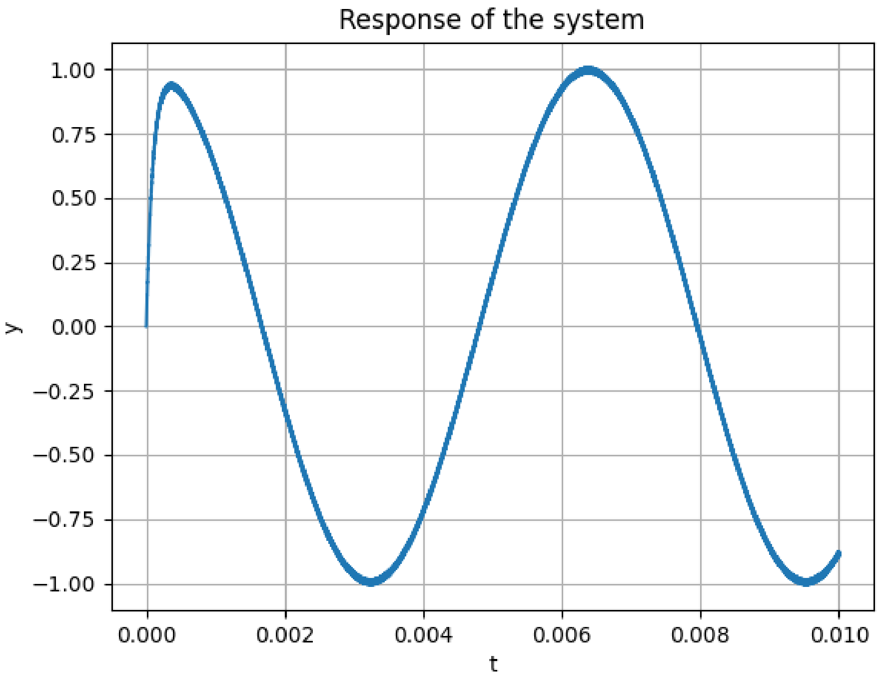

Figure 10: Output signal (0 < t < 10ms)