CSA-0666

DAA- DESIGN AND ANALYSIS OF ALGORITHM.

ASSIGNMENT.

1) Solve the following recurrance relations.

a) $x(n) = x(n-1) + 5$ for $n>1$, $x(1) = 0$.

$$x(n) = x(n-1) + 5 \longrightarrow (1)$$
$$x(n-1) = x(n-1-1) + 5$$
$$x(n-1) = x(n-2) + 5 \longrightarrow (2)$$
$$x(n-2) = x(n-2-1) + 5$$
$$x(n-2) = x(n-3) + 5 \longrightarrow (3)$$

Sub eq (3) in (2);

$$x(n-1) = x(n-3) + 5 + 5$$
$$x(n-1) = x(n-3) + 10 \rightarrow (4)$$

Sub eq (4) in (1);

$$x(n) = x(n-3) + 10 + 5$$
$$x(n) = x(n-3) + 15 \rightarrow (5)$$

for some k,

$$x(n) = x(n-k) + 5k \rightarrow (6)$$
$$n-k = 1, \quad n-1 = k.$$
$$x(n) = x(1) + 5(n-1)$$
$$= 0 + 5n - 5$$
$$x(n) = 5n - 5.$$

∴ The time complexity $= O(n)$.

b) $x(n) = 3x(n-1)$ for $n>1$, $x(1) = 4$.

$$x(n) = 3x(n-1) \rightarrow (1)$$

$$x(n-1) = 3x(n-1-1) = 3x(n-2)$$

$$= 3x(n-2) \rightarrow (2)$$

$$3(n-2) = 3x(n-3) \rightarrow (3)$$

Sub eq (3) in (2);

$$x(n-1) = 3(3x(n-3))$$

$$x(n-1) = 9x(n-3) \rightarrow (4)$$

Sub eq (4) in (1) :-

$$x(n) = 3[9x(n-3)]$$

$$x(n) = 27x(n-3)$$

At some k;

$$x(n) = 3^k x(n-k) \rightarrow (5)$$

$$n-k = 1$$

$$k = n-1$$

$$x(n) = 3^{n-1} x(1)$$

$$= 3^{n-1} \cdot 4$$

$$= 3^n \cdot 3^{-1} \cdot 4$$

$$= 3^n$$

∴ Time complexity = $O(3^n)$.

c) $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$

(Solve for $n = 2k$)

$$x(n) = x(n/2) + c \rightarrow (1)$$

$$x(n/2) = x(n/4) + c \rightarrow (2)$$

$$x(n/4) = x(n/8) + c \rightarrow (3)$$

Sub (2) in (1) ;

$$x(n) = x(n/4) + c + c$$

$$x(n) = x(n/4) + 2c \rightarrow (4)$$

$$= x(n/2^2) + 2c$$

Sub (3) in (4) :-

$$x(n) = x(n/8) + c + 2c$$

$$x(n) = x(n/2^3) + 3c$$

$$x(n) = x(n/2^k) + kc \rightarrow (n/2^k) = 1$$

$n = 2k$; $x(1) = 1$

$$n = 2^k$$

$$\log n = k \log 2$$

$$x(n) = x(n/n) + kc$$

$$k = \log n$$

$$x(n) = 1 + kc$$

$$n = 2k$$

$$k = n/2$$

$$x(n) = 1 + \log n \cdot c$$

Time complexity = $O(\log n)$

d) $x(n) = x(n/3) + 1$ for $n>1$ $x(1) = 1$

(Solve for $n = 3k$)

$$x(n) = x(n/3) + 1 \rightarrow (1)$$

$$x(n/3) = x(n/9) + 1 \rightarrow (2)$$

$$x(n/9) = x(n/27) + 1 \rightarrow (3)$$

Sub (2) in (1) :-

$$x(n) = x(n/9) + 2 \rightarrow (4)$$

$$= x(n/3^2) + 2$$

Sub (3) in (4) ;

$$x(n) = x(n/27) + 3 \rightarrow (5)$$

$$= x(n/3^3) + 3$$

$$x(n) = x(n/3^k) + k$$

$n/3^k = 1$

$n = 3^k$

$\log n = \log 3^k$

$\therefore k = \log n.$

$$x(n) = x(n/3^k) + k$$

$$= x(n/n) + k$$

$$= x(1) + k$$

$$= 1 + k$$

$$x(n) = \log n$$

$\therefore$ The time complexity $= O(\log n)$

2) Evaluate following recurrance completely

i)

$$T(n) = T(n/2 + 1), \quad n = 2^k$$

Sub $n = 2^k$

$$T(2^k) = T(2^k/2) + 1 = T(2^{k-1}) + 1$$

now,

$$T(2^{k-1}) = T\left(\frac{2^{k-1}}{2}\right) + 1 = T(2^{k-2}) + 1$$

$$T(2^{k-2}) = T(2^{k-2}/2) + 1 = T(2^{k-3}) + 1$$

$$T(2^1) = T(2^0) + 1$$

$$n = 2^k \Rightarrow k = \log 2^n$$

$$T(2^k) = T(2^{k-1}) + 1 = T(2^{k-2}) + 1 + 1 \ldots$$

Since,

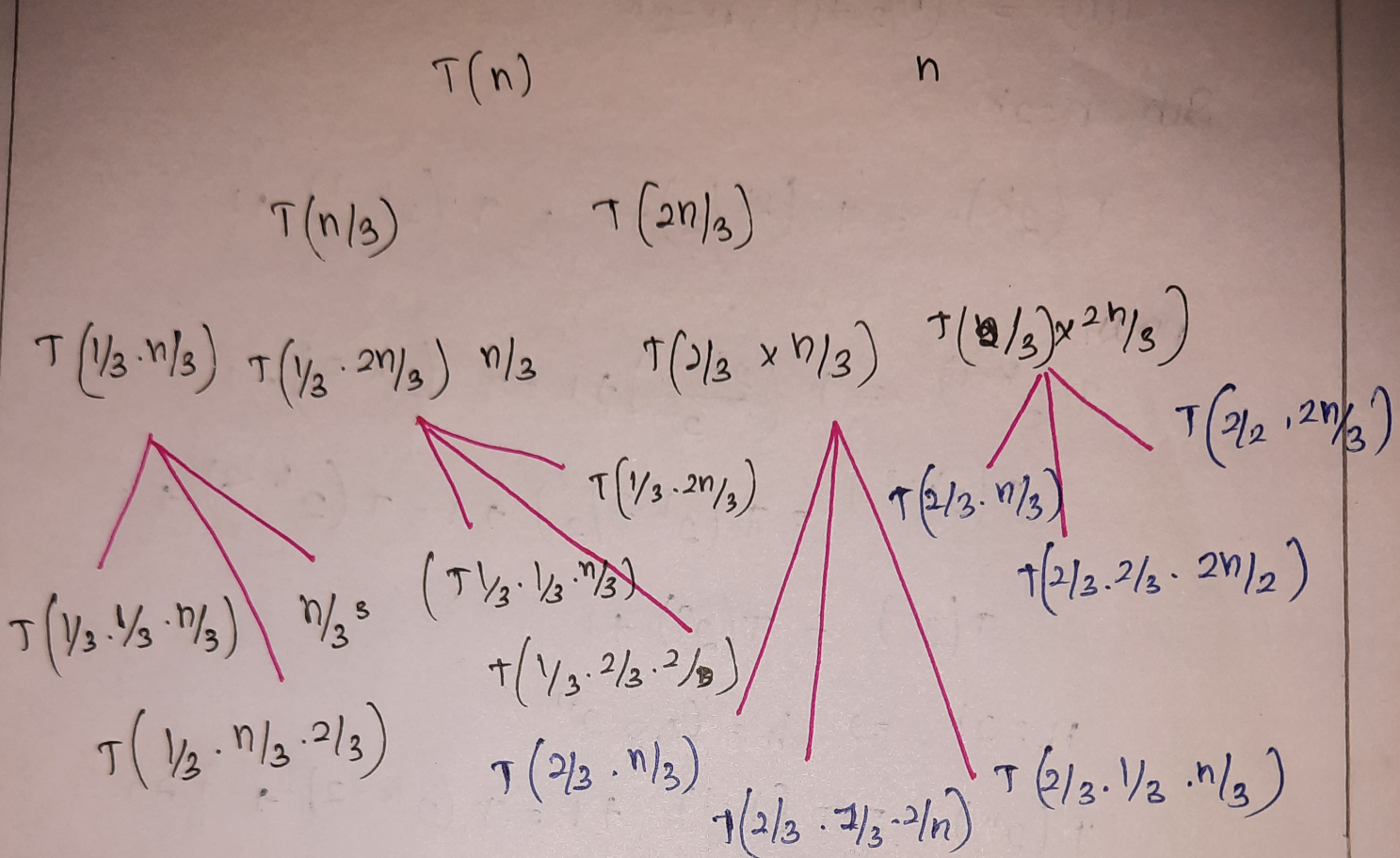
$$2^0 = 1, \quad T(2^0) = T(1)$$

$$T(2^k) = 1 + k$$

$$T(n) = 1 + \log 2^n$$

Time complexity $= O(\log n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn$.



$2^k/3^k \; n = 1$

$n/(3/2)^k = 1$

$n = (3/2)^k$

$\log n = k \log (3/2)$

$k = \log n / \log 3/2$

$k = \log^n 3/2$

$\boxed{k = \log^n 3/2}$

Time complexity $= O(nk)$

$= O(n \log^n 3/2)$

3) Consider following algorithm

```
min1 (A[0...n-1])
if n=1 return A[0]
else temp = min1 (A[0...n-2])
        if temp <= A[n-1] return temp
else
    Return A[n-1]
```

a) what does this algorithm compute?

This algorithm computer minimum element in an array A of size n.

if $i < n$, $A[i]$ is smaller than all elements, then $A[j] = j = i+1$ to $n-1$, then it returns $A[i]$. It also return the leftmost minimal element.

b) Setup a recurrance relation for the algorithm basic operation count and solve it.

$$T(n) = T(n-1) + 1, \text{ when } n > 1$$

$$T(1) = 0 \quad (\text{no comp } n=1)$$

$$T(n) = T(1) + (n-1) * 1$$

$$= 0 + (n-1)$$

$$= n-1$$

$\therefore$ Time Complexity $= O(n)$

$x(n)$: / $T(n/3) + T(2n/3) + cn$.

4) Analyze order of growth.

i) $F(n) = 2n^2 + 5$ and $g(n) = 7n$

use the $\Omega(g(n))$ notation.

$F(n) = 2n^2 + 5$

c. $g(n) = 7n$

$n = 1$

$F(1) = 2(1)^2 + 5 = 7$

$g(1) = 7$

$n = 1, \quad 7 = 7$

$n = 2, \quad 13 = 14$

$n = 3, \quad 23 = 21$

$n \geq 3, \quad F(n) \geq g(n) \cdot c$

$\boxed{F(n) \geq c \cdot g(n)}$

$n = 2$

$F(2) = 2(2)^2 + 5$

$= 8 + 5 = 13$

$g(2) = 7 \times 2 = 14$

$\boxed{n = 3}$

$F(3) = 2(3)^2 + 5$

$= 18 + 5 = 23$

$g(3) = 21.$

$F(n) \geq c \cdot g(n)$ when, n value is $\geq$ to 3

$\therefore F(n) = \Omega(g(n))$

$\Rightarrow$ $F(n)$ is more than $g(n)$ from asymptotically.