# DIGITAL DESIGN AND COMPUTER ORGANIZATION

## Analysis of Clocked Seq. Circuits

**Team DDCO**
**Department of Computer Science and Engineering**

# Introduction(section 5.5 T1)

- Analysis describes what a given circuit will do under certain operating conditions.

- The **behavior** of a clocked sequential circuit is determined from **the inputs, the outputs, and the state of its flip-flops**.

- The outputs and the next state are both a function of the inputs and the present state.

- The **analysis of a sequential circuit** consists of obtaining **a table or a diagram** for the **time sequence of inputs, outputs, and internal states**.

- It is also possible to write Boolean expressions that describe the behavior of the sequential circuit.

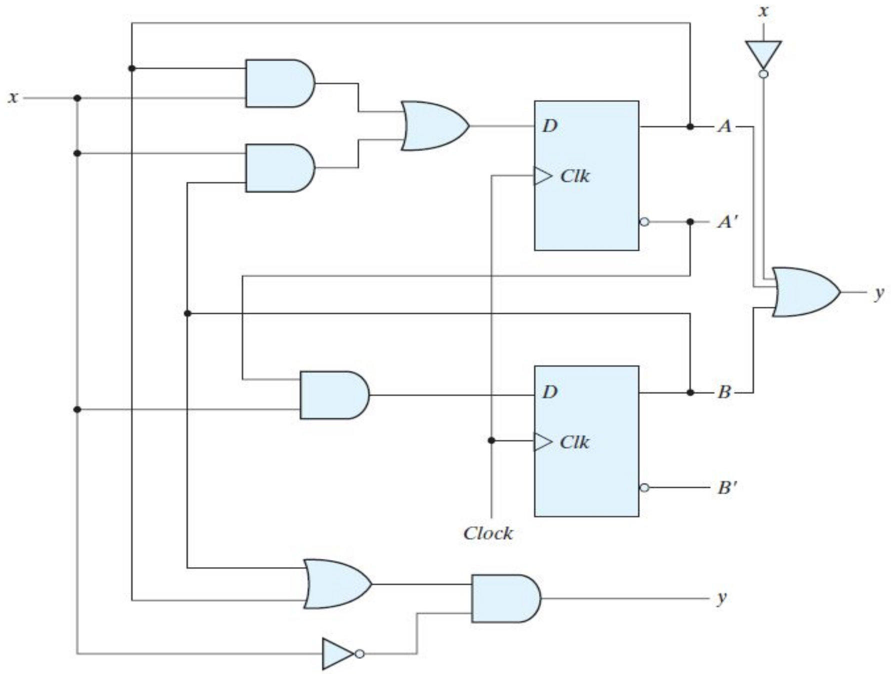
# Introduction

- These expressions must include the necessary time sequence, either directly or indirectly.
- A logic diagram is recognized as a clocked sequential circuit if it includes flip-flops with clock inputs.
- The flip-flops may be of any type, and the logic diagram may or may not include combinational logic gates.
- **Algebraic representation for specifying the next-state condition in terms of the present state and inputs**

# State Equations

- The **behavior** of a clocked sequential circuit can be described algebraically by means of **state equations**.
- A *state equation (also called a transition equation* ) specifies the **next state as a function of the present state and inputs.**
- Consider the sequential circuit shown in the Figure.

# State Equations




Circuit acts as a 0-detector by asserting its output when a 0 is detected in a stream of 1s

Behavior of circuit:

Imagine you're monitoring a signal line that should be continuously 1 (high).

- As long as the signal is 1, the detector stays quiet.
- When a glitch (a single 0) happens, the detector alerts with y = 1.
- But if the line stays 0, it doesn't keep alerting — only the first error is flagged.

# State Equations

- It consists of two D flip-flops A and B, an input x and an output y .
- Since the D input of a flip-flop determines the value of the next state (i.e., the state reached after the clock transition), it is possible to write a set of state equations for the circuit:
    - $A(t + 1) = A(t)x(t) + B(t)x(t)$
    - $B(t + 1) = A'(t)x(t)$

# State Equations

- The Boolean expressions for the state equations can be derived directly from the gates that form the combinational circuit part of the sequential circuit, since the D values of the combinational circuit determine the next state.
- Similarly, the present-state value of the output can be expressed algebraically as
  - $y(t) = [A(t) + B(t)]x'(t)$
- By removing the symbol *(t)* for the present state, we obtain the output Boolean equation:
  - $y = (A + B)x'$

# State Table

- The time sequence of **inputs, outputs, and flip-flop states** can be enumerated in a ***state table*** (sometimes called a ***transition table***).
- The state table for the previous circuit is shown in the table.
- The table consists of 4 sections: **present state, input, next state, and output** .
- The present-state section shows the states of flip-flops A and B at any given time t .
- The input section gives a value of x for each possible present state.
- The next-state section shows the states of the flip-flops one clock cycle later, at time t + 1.
- The output section gives the value of y at time t for each present state and input condition.

# State Table- Type 1

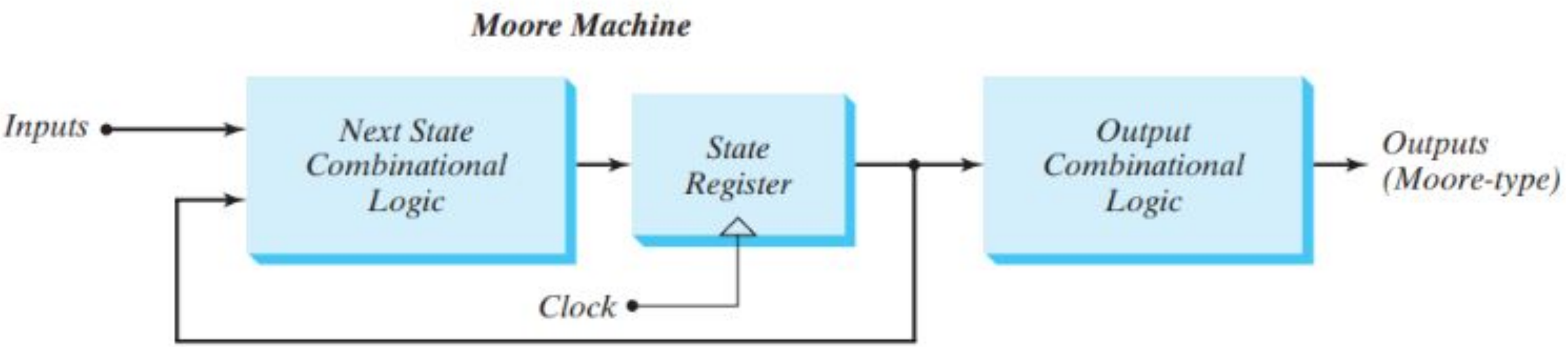
| Present State | | Input | Next State | | Output |
|---|---|---|---|---|---|
| *A* | *B* | *x* | *A* | *B* | *y* |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

$$y = Ax' + Bx'$$

# Moore Models of Finite State Machines




- In the Moore model, the output is a function of only the present state
- Next state is function of present state and input

# State Table

- The state table of a sequential circuit with D -type flip-flops is obtained by the same procedure outlined in the previous example.
- In general, a sequential circuit with m flip-flops and n inputs needs $2^{m+n}$ rows in the state table.
- The binary numbers from 0 through $2^{m+n} - 1$ are listed under the present-state and input columns.
- The next-state section has m columns, one for each flip-flop.
- The binary values for the next state are derived directly from the state equations.
- The output section has as many columns as there are output variables.
- Its binary value is derived from the circuit or from the Boolean function in the same manner as in a truth table.

# State Table

- It is sometimes convenient to express the state table in a slightly different form having only three sections: present state, next state, and output.
- The input conditions are enumerated under the next–state and output sections.
- The new state table is as shown in this second form.
- For each present state, there are two possible next states and outputs, depending on the value of the input.

# State Table – Type 2

Second Form of the State Table

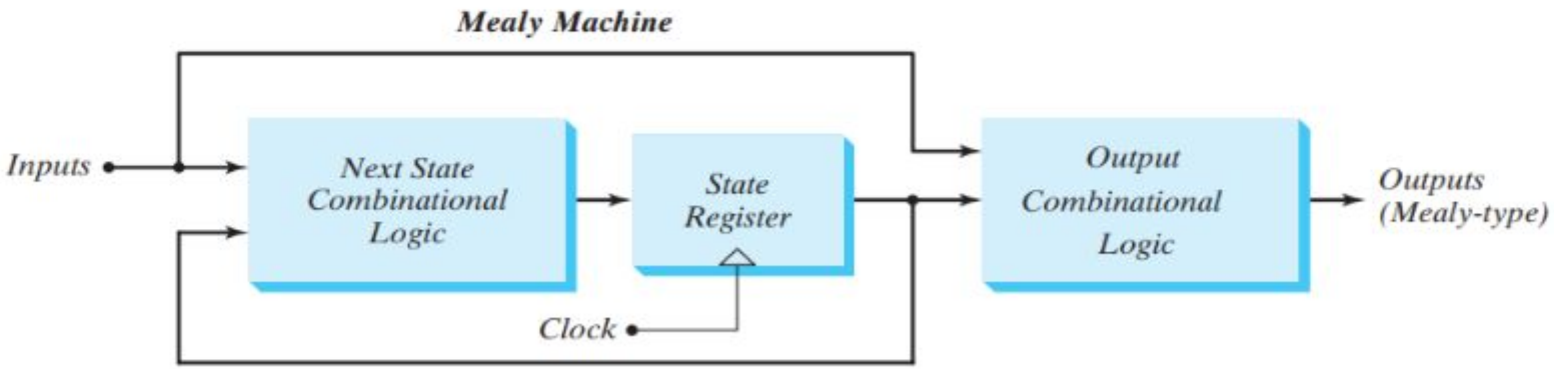
| Present State | | Next State | | | | Output | |
|---|---|---|---|---|---|---|---|
| | | $x = 0$ | | $x = 1$ | | $x = 0$ | $x = 1$ |
| $A$ | $B$ | $A$ | $B$ | $A$ | $B$ | $y$ | $y$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

$$A(t + 1) = Ax + Bx$$

$$B(t + 1) = A'x$$

$$y = Ax' + Bx'$$

# Mealy Models of Finite State Machines



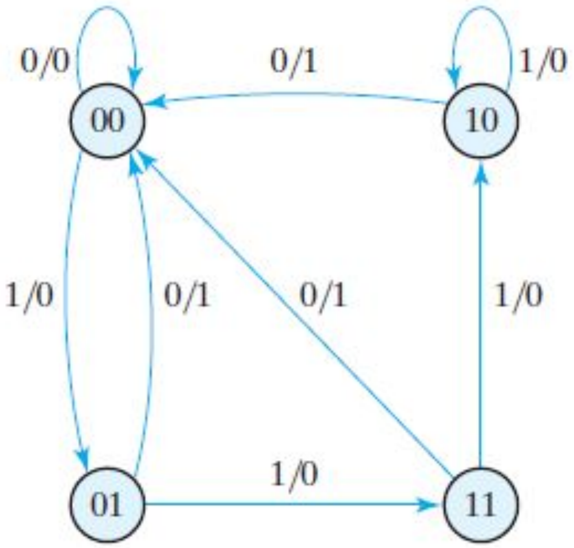
(a)

In the Mealy model, the output is a function of both the present state and the input.
Next state depends on present state and inputs

# State Diagram

Second Form of the State Table

| Present State | | Next State | | | | Output | |
|---|---|---|---|---|---|---|---|
| | | x = 0 | | x = 1 | | x = 0 | x = 1 |
| A | B | A | B | A | B | y | y |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |




Circuit diagram → Equations – State table → State diagram

# State Diagram

- The state diagram provides the same information as the state table and is obtained directly from table. **The information available in a state table can be represented graphically in the form of a state diagram**
- The binary number inside each circle identifies the state of the flip–flops.
- The directed lines are labelled with two binary numbers separated by a slash.
- The input value during the present state is labelled first, and the number after the slash gives the output during the present state with the given input.
- The steps presented in this example are summarized below:

Circuit diagram → Equations – State table → State diagram

**The state diagram gives a pictorial view of state transitions and is the form more suitable for human interpretation of the circuit's operation**

# Flip-Flop Input Equations

- The part of the combinational circuit that generates external outputs is described algebraically by a set of Boolean functions **called output equations.**
- The part of the circuit that generates the inputs to flip-flops is described algebraically by a set of Boolean functions called flip-flop **input equations** (or, sometimes, excitation equations).

$$D_A = Ax + Bx$$
$$D_B = A'x$$
$$y = (A + B)x'$$

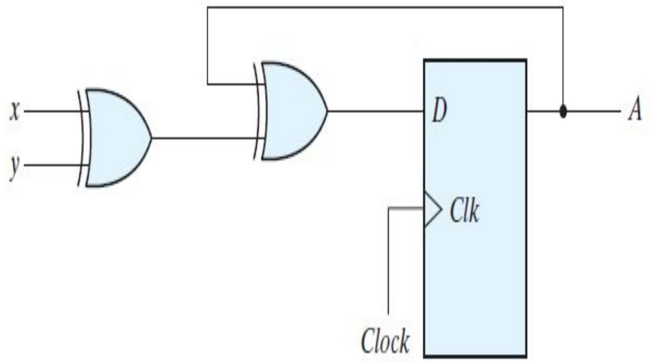
Note that the expression for the input equation for a D flip-flop is identical to the expression for the corresponding state equation. This is because of the characteristic equation that equates the next state to the value of the D input: $Q(t + 1) = D_Q$.

# Analysis with D Flip-Flops

- The procedure for analyzing a clocked sequential circuit with D flip-flops by means of a simple example. The circuit we want to analvze is described by the input equation:

$$D_A = A \oplus x \oplus y$$

- The $D_A$ symbol implies a D flip-flop with output A.
- The x and y variables are the inputs to the circuit.
- **No output equations are given, which implies that the output comes from the output of the flip-flop.**




(a) Circuit diagram

# Analysis with D Flip-Flops

- The state table has one column for the present state of flip–flop $A$, two columns for the two inputs, and one column for the next state of $A$.
- The binary numbers under $Axy$ are listed from 000 through 111 as shown in figure (b).
- The next–state values are obtained from the state equation

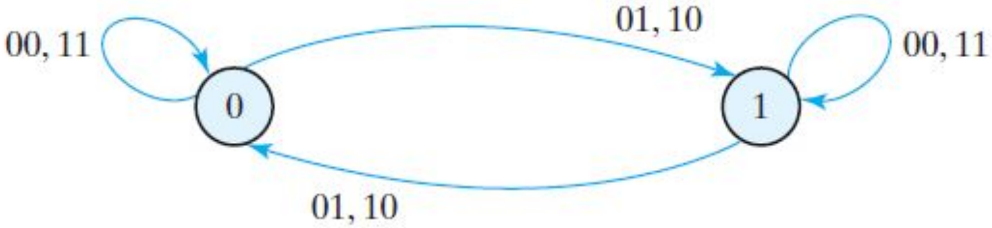
$$A(t+1) = A \oplus x \oplus y$$

- The expression specifies an odd function and is equal to 1 when only one variable is 1 or when all three variables are 1.
- This is indicated in the column for the next state of $A$ .

# Analysis with D Flip-Flops

| Present state | Inputs | | Next state |
|---|---|---|---|
| $A$ | $x$ | $y$ | $A$ |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

(b) State table




(c) State diagram

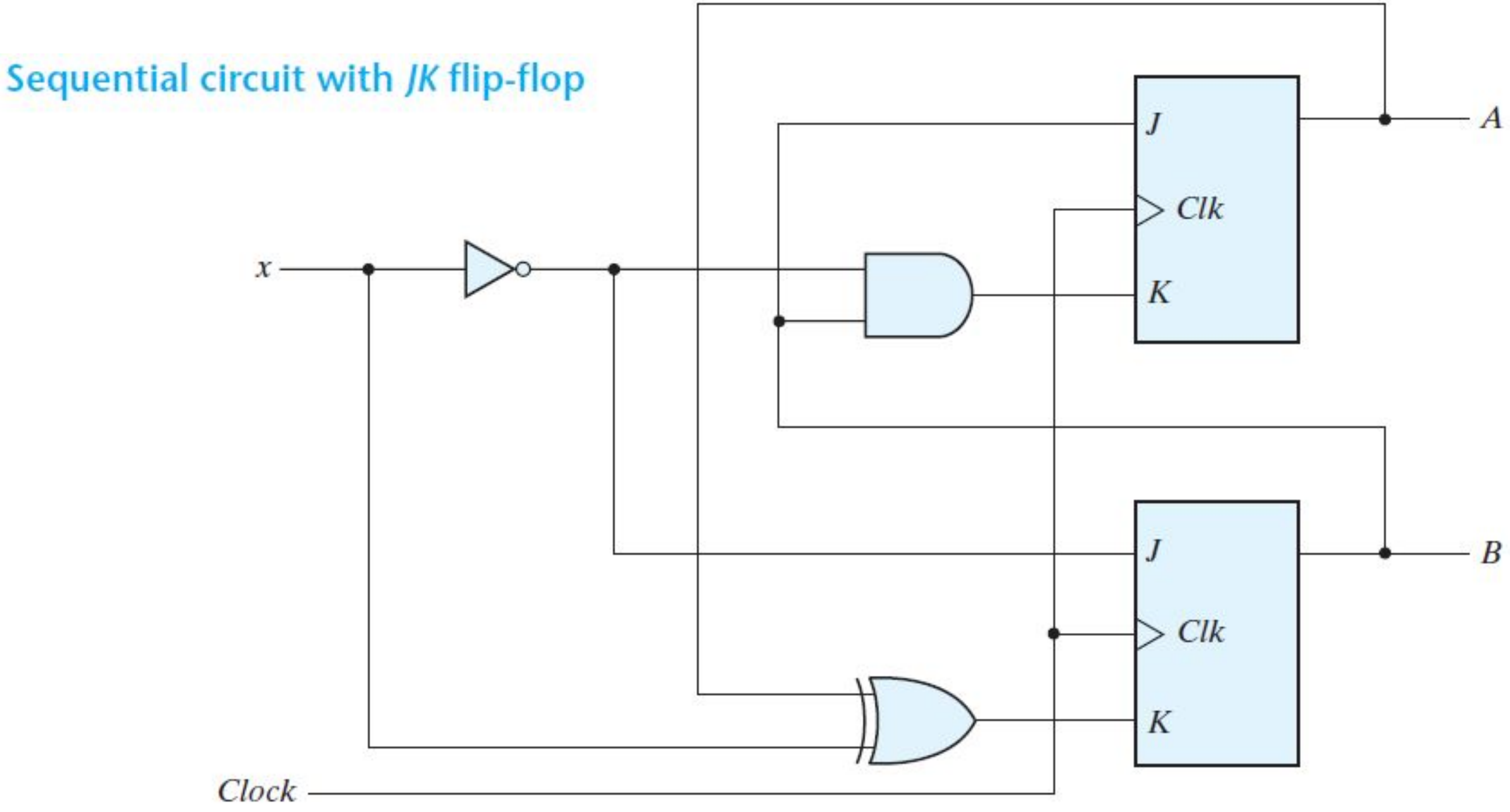
# Analysis with D Flip-Flops

- **The circuit has one flip–flop and two states.**
- The state diagram consists of two circles, one for each state as shown in figure (c).
- **The present state and the output can be either 0 or 1, as indicated by the number inside the circles.(moore model)**
- **A slash on the directed lines is not needed, because there is no output from a combinational circuit**.
- The two inputs can have four possible combinations for each state.
- Two input combinations during each state transition are separated by a comma to simplify the notation.

# Analysis of JK Flip-Flop

- When a flip–flop other than the *D* type is used, such as *JK* or *T*, it is necessary to refer to the corresponding characteristic table or characteristic equation to obtain the next–state values.
- The procedure is illustrated first by using the characteristic table and again by using the characteristic equation.
- The next–state values of a sequential circuit that uses *JK* – or *T* –type flip–flops can be derived as follows:
    a. Determine the flip–flop input equations in terms of the present state and input variables.
    b. List the binary values of each input equation.
    c. Use the corresponding flip–flop characteristic table to determine the next–state values in the state table.

# Analysis of JK Flip-Flop



# Analysis of JK Flip-Flop

- As an example, consider the sequential circuit with two *JK* flip–flops *A* and *B* and one input *x*.
- The circuit has no outputs; therefore, the state table does not need an output column.
- The circuit can be specified by the flip–flop input equations

$$J_A = B \quad K_A = Bx'$$

$$J_B = x' \quad K_B = A'x + Ax' = A \oplus x$$

# Analysis of JK Flip-Flop

PES UNIVERSITY

**State Table for Sequential Circuit with JK Flip-Flops**

| Present State | | Input | Next State | | Flip-Flop Inputs | | | |
|---|---|---|---|---|---|---|---|---|
| $A$ | $B$ | $x$ | $A$ | $B$ | $J_A$ | $K_A$ | $J_B$ | $K_B$ |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

# Analysis of JK Flip-Flop

The next-state values can also be obtained by evaluating the state equations from the characteristic equation. This is done by using the following procedure:

1. Determine the flip-flop input equations in terms of the present state and input variables.
2. Substitute the input equations into the flip-flop characteristic equation to obtain the state equations.
3. Use the corresponding state equations to determine the next-state values in the state table.

# Analysis of JK Flip-Flop

The input equations for the two $JK$ flip-flops of Fig. 5.18 were listed a couple of paragraphs ago. The characteristic equations for the flip-flops are obtained by substituting $A$ or $B$ for the name of the flip-flop, instead of $Q$:

$$A(t+1) = JA' + K'A$$
$$B(t+1) = JB' + K'B$$

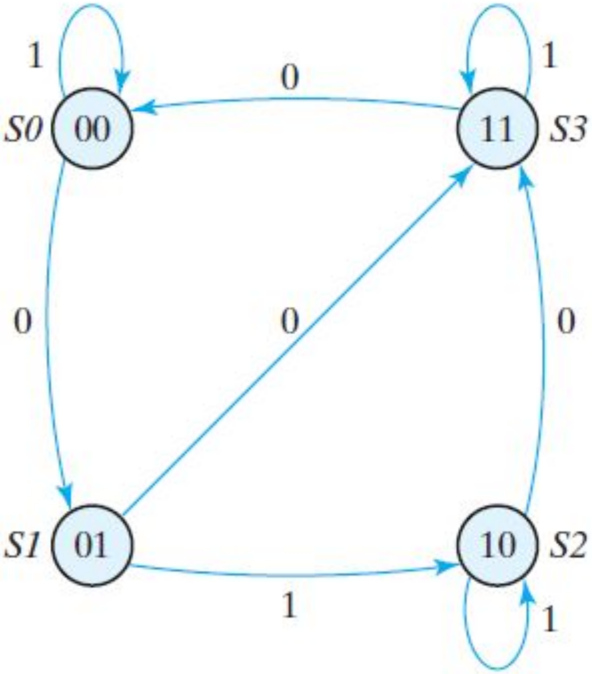
Substituting the values of $J_A$ and $K_A$ from the input equations, we obtain the state equation for $A$:

$$A(t+1) = BA' + (Bx')'A = A'B + AB' + Ax$$

The state equation provides the bit values for the column headed "Next State" for $A$ in the state table. Similarly, the state equation for flip-flop $B$ can be derived from the characteristic equation by substituting the values of $J_B$ and $K_B$:

$$B(t+1) = x'B' + (A \oplus x)'B = B'x' + ABx + A'Bx'$$

# Analysis of JK Flip-Flop



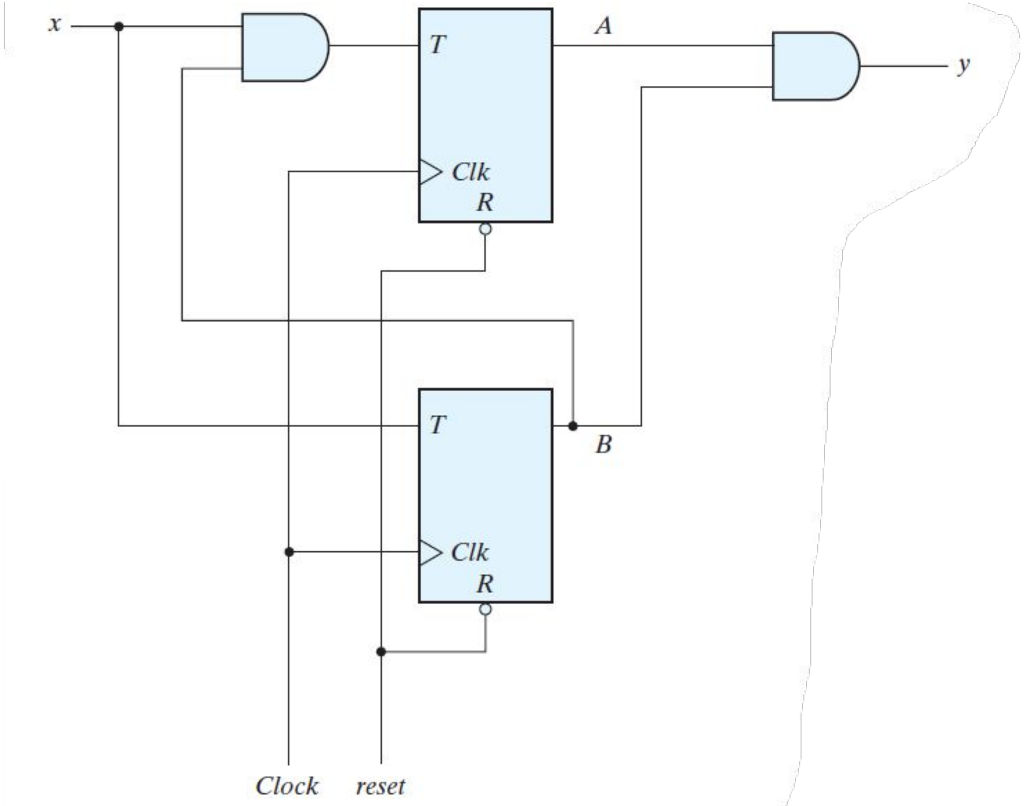

State diagram of the circuit

# Analysis with T Flip-Flops

- The analysis of a sequential circuit with T flip-flops follows the same procedure.
- The next-state values in the state table can be obtained by using either using the characteristic table or characteristic  equation.

$$Q(t+1) = T \oplus Q = T'Q + TQ'$$

# Analysis with T Flip-Flops




(a) Circuit diagram

# Analysis with T Flip-Flops

It has two flip-flops A and B, one input x, and one output y and can be described algebraically by two input equations and an output equation:

$$T_A = Bx$$
$$T_B = x$$
$$y = AB$$

# Analysis with T Flip-Flops

*State Table for Sequential Circuit with T Flip-Flops*

| Present State | | Input | Next State | | Output |
|---|---|---|---|---|---|
| *A* | *B* | *x* | *A* | *B* | *y* |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 |

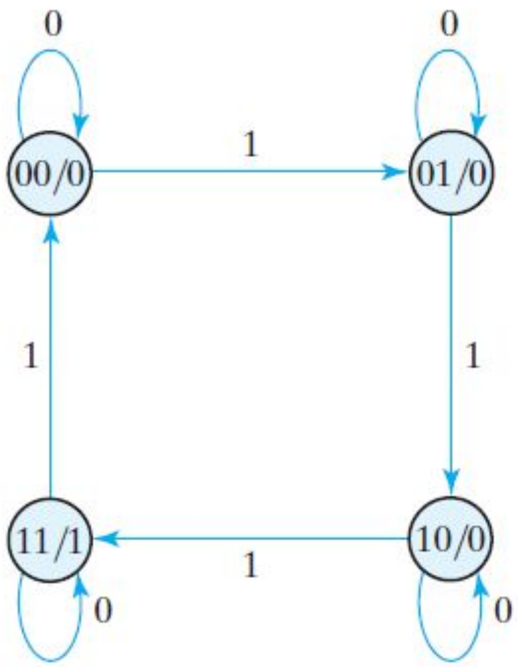
# Analysis with T Flip-Flops

The values for y are obtained from the output equation. The values for the next state can be derived from the state equations by substituting TA and TB in the characteristic equations, yielding

$$A(t+1) = (Bx)'A + (Bx)A' = AB' + Ax' + A'Bx$$

$$B(t+1) = x \oplus B$$

# Analysis with T Flip-Flops




(b) State diagram

As long as input x is equal to 1, the circuit behaves as a binary counter with a sequence of states 00, 01, 10, 11, and back to 00.

When x = 0, the circuit remains in the same state. Output y is equal to 1 when the present state is 11.

Here, the output depends on the present state only and is independent of the input.

The two values inside each circle and separated by a slash are for the present state and output.

# Applications

**Elevator Controller (Multi-floor Building)**

- **Description:**
The elevator logic determines movement between floors, door operation, and priority handling.
- **How it relates to your concept:**
    - States: Floor positions + Door open/close status.
    - Inputs: Floor request buttons, sensors.
    - Flip-flops: Store current floor/door state;
    - **state equations** decide next movement.

**Traffic Light Controller**

- **Description:**
A four-way traffic signal is controlled by a **finite state machine (FSM)** implemented using flip-flops.
- **How it relates to your concept:**
    - **States:** Red → Green → Yellow → Red.
    - **Flip-flops:** Store the current light state.
    - **State table & equations:** Determine which light turns on next based on current state and timer pulse.

# Applications

**Digital Lock System**
**Description:**
Unlocks only when a correct sequence of digits/buttons is entered.
**How it relates to your concept:**
**States:** Each entered digit represents a state transition.
**Inputs:** Keypad inputs.
**Flip-flops:** Store entered sequence progress.
**State equations:** Verify input sequence and transition to “Unlock” state if correct.

# MCQ

1. The present state of a D flip-flop is 1. If the input equation is $D=A\oplus x\oplus y$ where A is the current flip-flop output, $x=1$ and $y=0$, what will be the next state after the clock edge?

   **A)** 0

   **B)** 1

   **C)** Cannot be determined

   **D)** Remains same

2. A sequential circuit uses two D flip-flops A and B, and one input x. The next state equations are:

   $A(t+1)=A\cdot x+B\cdot x$

   $B(t+1)=A`\cdot x$

   If present state is $A=1,B=0,x=1$, what is the next state?

   **A)** (1,0)

   **B)** (1,1)

   **C)** (0,1)

   **D)** (0,0)

# MCQ

1. The present state of a D flip-flop is 1. If the input equation is $D=A\oplus x\oplus yD$ where A is the current flip-flop output, x=1, and y=0, what will be the next state after the clock edge?

   **A)** 0

   **B)** 1

   **C)** Cannot be determined

   **D)** Remains same

   **Answer: A** — $D=1\oplus1\oplus0$, so next state is 0.

2. A sequential circuit uses two D flip-flops A and B, and one input x. The next state equations are:

   $A(t+1)=A\cdot x+B\cdot x$

   $B(t+1)=A`\cdot x$

   If present state is A=1,B=0,x=1, what is the next state?

   **A)** (1,0)

   **B)** (1,1)

   **C)** (0,1)

   **D)** (0,0)

   **Answer: A** — Direct substitution into the state equations.

# THANK YOU

Team DDCO

Department of Computer Science and Engineering