MATHEMATICS FOR COMPUTER SCIENCE ENGINEERS

Unit 2: Confidence Intervals

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Topics to be covered

- Confidence Intervals - Introduction

Textbook 1: Chapter 5, section 5.1, 5.2

Confidence Intervals: Introduction

Assume that a large number of independent measurements, using the same procedure, are made on the diameter of a piston.

The **sample mean** of the measurements **is 14.0 cm**
The **standard deviation** of the sample mean is **0.1 cm.**

The value 14.0 comes from a **normal distribution**, because it is the average of a large number of measurements.
Now the **true diameter** of the piston **will certainly not be exactly equal to the sample mean of 14.0 cm**.
We can use its standard deviation to determine how close it is likely to be to the true diameter.

Textbook1: Chapter 5, Page 322

Confidence Intervals: Introduction

For example, it is very **unlikely that the sample mean will differ from the true diameter by more than three standard deviations.** Therefore we have a **high level of confidence that the true diameter is in the interval (13.7, 14.3).**

On the other hand, it is **likely for the sample mean to differ from the true value by more than one standard deviation.** Therefore we have a **lower level of confidence that the true diameter is in the the interval (13.9, 14.1).**

The intervals (13.7, 14.3) and (13.9, 14.1) are **confidence intervals** for the true diameter of the piston.

Real Life Examples of Confidence Intervals

- A 2008 Gallup survey found that TV ownership may be good for wellbeing. The results from the poll stated that the confidence level was 95% +/-3, which means that if Gallup repeated the poll over and over, using the same techniques, 95% of the time the results would fall within the published results. The 95% is the confidence level and the +/-3 is called a margin of error.

Real Life Examples of Confidence Intervals

- The U.S. Census Bureau routinely uses confidence levels of 90% in their surveys. One survey of the number of people in poverty in 1995 stated a confidence level of 90% for the statistics "The number of people in poverty in the United States is 35,534,124 to 37,315,094." That means if the Census Bureau repeated the survey using the same techniques, 90 percent of the time the results would fall between 35,534,124 and 37,315,094 people in poverty. The stated figure (35,534,124 to 37,315,094) is the confidence interval.

Real Life Examples of Confidence Intervals

- **Example:** A recent article on Rasmussen Reports states that "38% of Likely U.S. Voters now say their health insurance coverage has changed because of Obama care".

- "The margin of sampling error is +/- 3 percentage points with a **95% level of confidence.**"

- What a 95% confidence level is saying is that **if the poll or survey were repeated over and over again, the results would match the results from the actual population 95% of the time.**

What about “+/- 3 percentage points”?

- The **width of the confidence interval** tells us more about **how certain (or uncertain) we are about the true figure in the population.**

- This width is stated as a plus or minus (in this case,+/- 3).

- In this case, you would expect the actual results to be between (35 , 41) or (38-3 , 38+3) percent, 95% of the time.

Factors that Affect Confidence Intervals (CI)

- **Population size**: Affects the CI when you are working with small groups.
- **Sample Size:** the smaller your sample, the less likely it is you can be confident the results reflect the true population parameter.
- **Percentage:** Extreme answers come with better accuracy.
- *For example*, if 99% of voters are for a particular party, the chances of error are small.
- However, if 49.9% of voters are “for” and 50.1 percent are “against” then the chances of error are bigger.

0% and 100% Confidence Level

- A **0% confidence level** means you have **no faith at all that if you repeated the survey that you would get the same results.**
- A **100% confidence level** means there is **no doubt at all that if you repeated the survey you would get the same results**
- In reality, you would never publish the results from a survey where you had no confidence at all that your statistics were accurate.
- A 100% confidence level doesn’t exist in statistics, unless you surveyed an entire population 100 percent sure that your survey wasn’t open to some kind of error or bias.

What is a Confidence Interval?

- **A confidence interval is how much uncertainty there is with any particular statistic.**
- Confidence intervals are often used with a margin of error.
- It tells you how confident you can be that the results from a poll or survey reflect what you would expect to find if it were possible to survey the entire population.
- Confidence intervals are intrinsically connected to confidence levels.

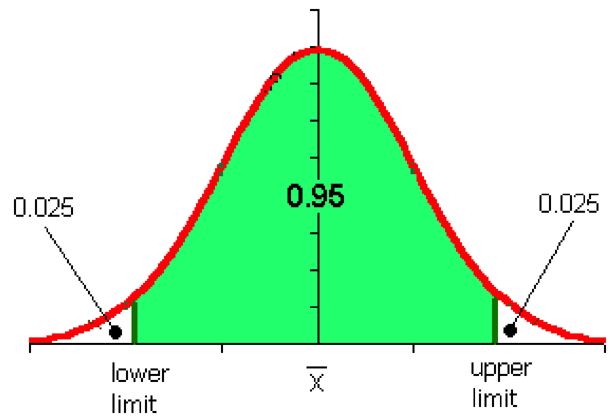
Confidence Intervals vs. Confidence Levels

- Confidence levels are expressed as a percentage (for example, a 95% confidence level).
- It means that should you repeat an experiment or survey over and over again, 95 percent of the time your results will match the results you get from a population (in other words, your statistics would be sound!).
- Confidence intervals are your results, usually the range in which you expect to find the results in .

Confidence Intervals vs. Confidence Levels

- For example, you survey a group of pet owners to see how many cans of dog food they purchase a year.

- You test your statistics at the 99 percent confidence level and get a confidence interval of (200,300). That means you think they buy between 200 and 300 cans a year. You're super confident (99% is a very high level!) that your results are sound, statistically.



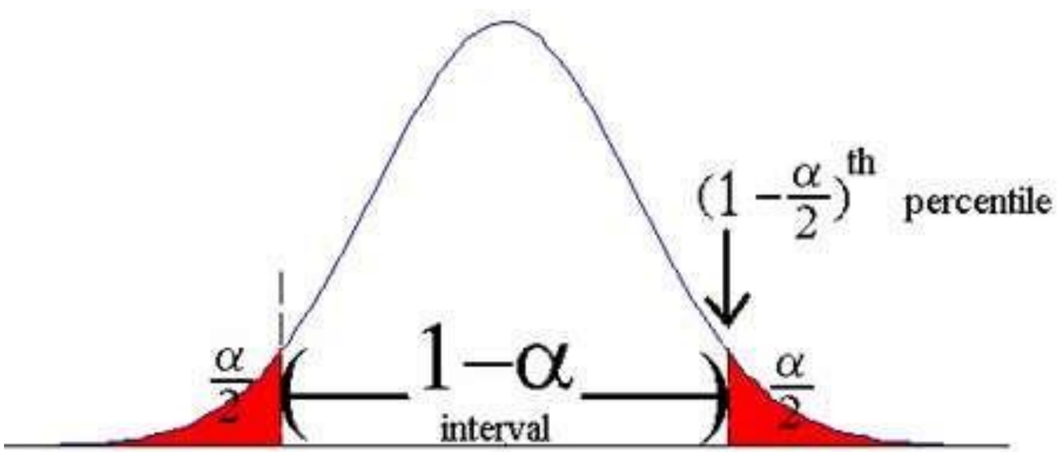
Confidence Coefficient

- The confidence coefficient is the confidence level stated as a proportion, rather than as a percentage.
- For example, if **confidence level = 99%, confidence coefficient = 0.99.**
- In general, the **higher the coefficient**, the **more certain you are that your results are accurate.**
- For example, a .99 coefficient is more accurate than a coefficient of .89.
- It's extremely rare to see a coefficient of 1 (meaning that you are positive without a doubt that your results are completely, 100% accurate).
- A coefficient of zero means that you have no faith that your results are accurate at all.

Significance level

- Alpha(α) is called as the significance level.
- Confidence level is taken as $(1-\alpha)*100\%$

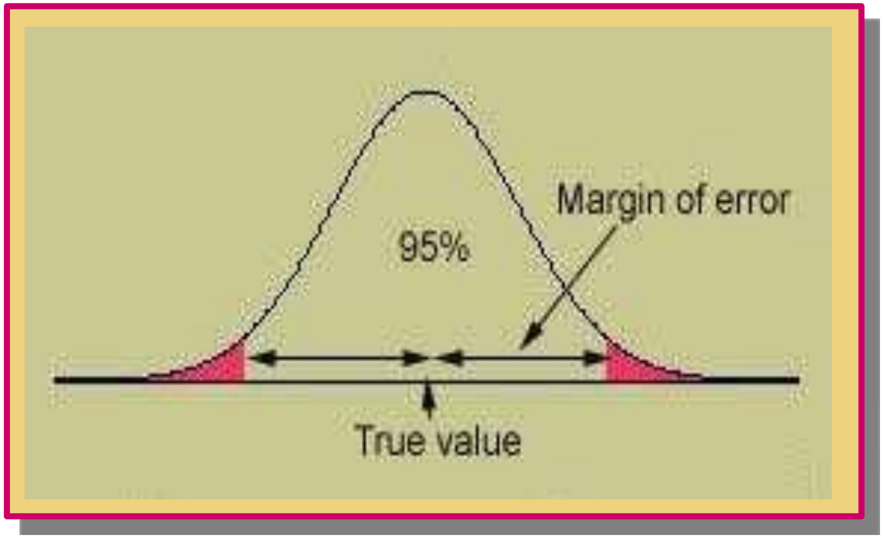
Confidence coefficient $(1 - \alpha)$	Confidence level $(1 - \alpha * 100\%)$	
0.90	90%	α = ?
0.95	95%	α = ?
0.99	99%	α = ?



The Margin of Error

- **Margin of error: provides a bound to the difference between a particular estimate and the parameter that it estimates.**
- From the Central Limit Theorem, the sampling distributions of $\overline{x}$ and $\hat{p}$ will be approximately normal under certain assumptions
- For unbiased estimators with normal sampling distributions**, 95% of all point estimates will lie within 1.96 standard deviations of the parameter of interest.**

Note:
Here
we assume that the sample sizes are large.



So here,
Margin of error = 1.96 x std error of estimator

Why 1.96 ?

Probability VS. Confidence

- A 95% confidence interval for the population mean μ was computed to be (12.304, 12.696).
- It is tempting to say that the probability is 95% that μ is between 12.304 and 12.696.
- **The term probability refers to random events, which can come out differently when experiments are repeated**.
- The numbers 12.304 and 12.696 are fixed, not random.
- The population mean is also fixed. The mean is either in the interval or it is not.
- There is no randomness involved. Therefore we say that we have 95% confidence (not probability) that μ is in this interval

Example

- A 90% confidence interval for the mean diameter (in cm) of steel rods manufactured on a certain extrusion machine is computed to be (14.73, 14.91). True or false: The probability that the mean diameter of rods manufactured by this process is between 14.73 and 14.91 is 90%.

- Solution
- False. A specific confidence interval is given. The mean is either in the interval or it isn't. We are 90% confident that the population mean is between 14.73 and 14.91.
- The term probability is inappropriate.

Probability VS. Confidence

Let’s say that we are discussing
a *method* used to compute a 95% confidence interval.

The method will succeed in covering the population mean 95% of the time,
and fail the other 5% of the time.

In this case, **whether the population mean is covered or not is a random event**,
because it can vary from experiment to experiment.

Therefore it *is* **correct** to say that
a *method* for computing a 95% confidence interval has probability 95% of covering the population mean.

Textbook: Page 328 onwards

Probability VS. Confidence

- Probability and confidence, while related, represent different concepts in statistics.

- Probability refers to the likelihood of a specific event occurring, while confidence reflects the degree of certainty that a statistical estimate (like a confidence interval) contains the true population parameter.

- A 95% confidence interval means that if you were to repeat the sampling process many times, 95% of the calculated confidence intervals would contain the true population parameter.

  Confidence focuses on the reliability of an interval estimate, not on the probability of a single event.

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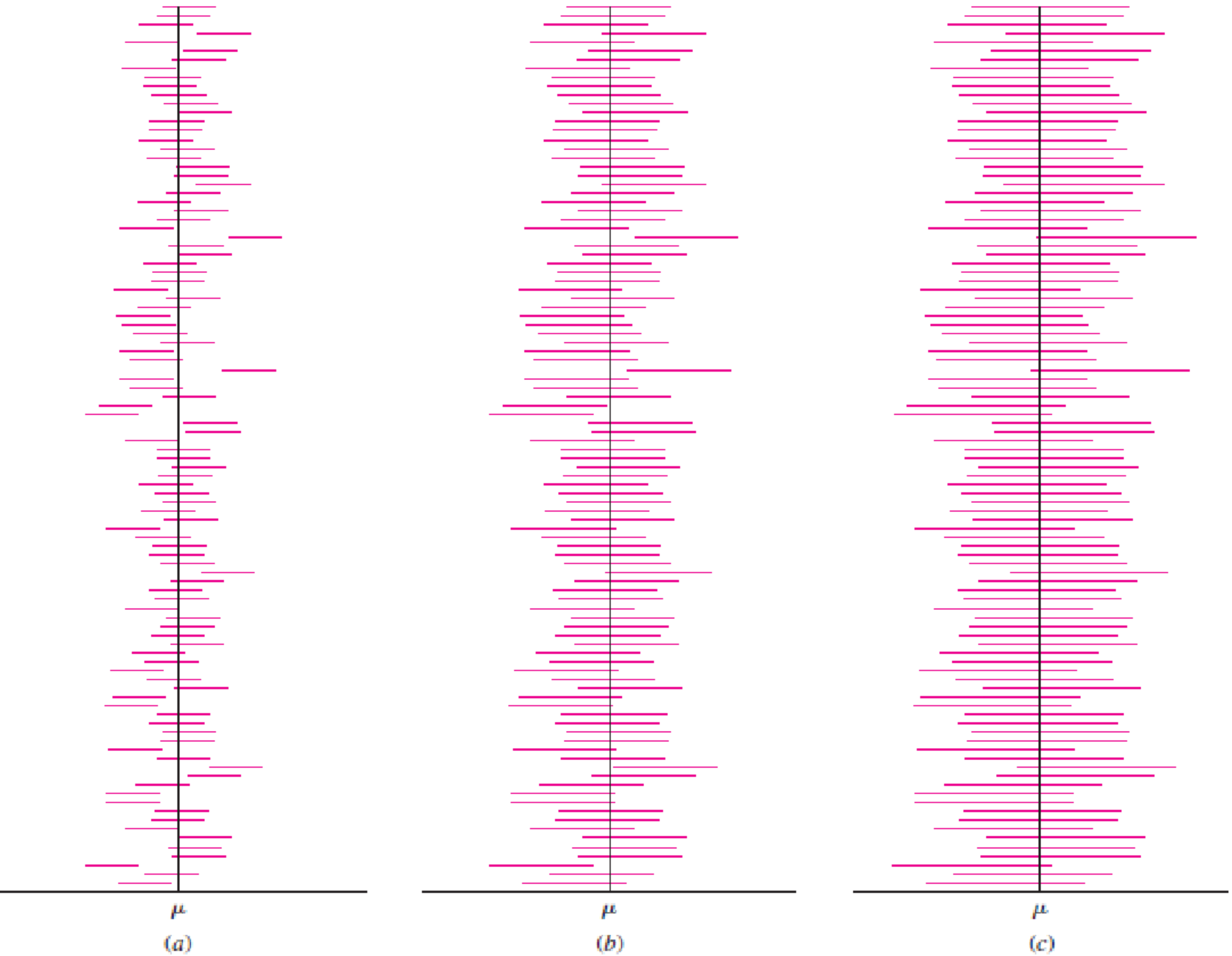
Probability VS. Confidence



Probability VS. Confidence

(a) One hundred 68% confidence intervals for a population mean, each computed from a different sample. Although precise, they fail to cover the population mean 32% of the time. This high failure rate makes the 68% confidence interval unacceptable for practical purposes.

(b) One hundred 95% confidence intervals computed from these samples. This represents a good compromise between reliability and precision for many purposes.

(c) One hundred 99.7% confidence intervals computed from these samples. These intervals fail to cover the population mean only three times in 1000. They are extremely reliable, but imprecise.

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Example

An engineer plans to compute a 90% confidence interval for the mean diameter of steel rods. She will measure the diameters of a large sample of rods, compute $\overline{X}$ and s, and then compute the interval $\overline{X} \pm 1.645s/\sqrt{n}$.

True or false:

The probability that the population mean diameter will be in this interval is 90%.

Solution

- **True**. What is described here is a method for computing a confidence interval, rather than a specific numerical value. It is correct to say that a method for computing a 90%
- confidence interval has probability 90% of covering the population mean.

References

- “Statistics for Engineers and Scientists”, William Navidi, McGraw Hill Education, India, 6th Edition, 2024

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