PRINCIPLES OF POINT ESTIMATION

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MATHEMATICS FOR COMPUTER SCIENCE ENGINEERS

Point Estimation

Computer Science and Engineering

Topics to be covered...

- ✓Point Estimator and properties .
- ✓Properties of Point Estimator.
- ✓Measuring Goodness of an Estimator.
- ✓Introduction to MSE(Mean Square Error)

Sample Statistics & Population Parameters

- Statistics are used to estimate parameters.

- Descriptive statistics effectively describes the data and it does not make any inference from the data.

- Descriptive refers to the numerical summary of the population which is referred as parameter and for sample it is called as statistic.

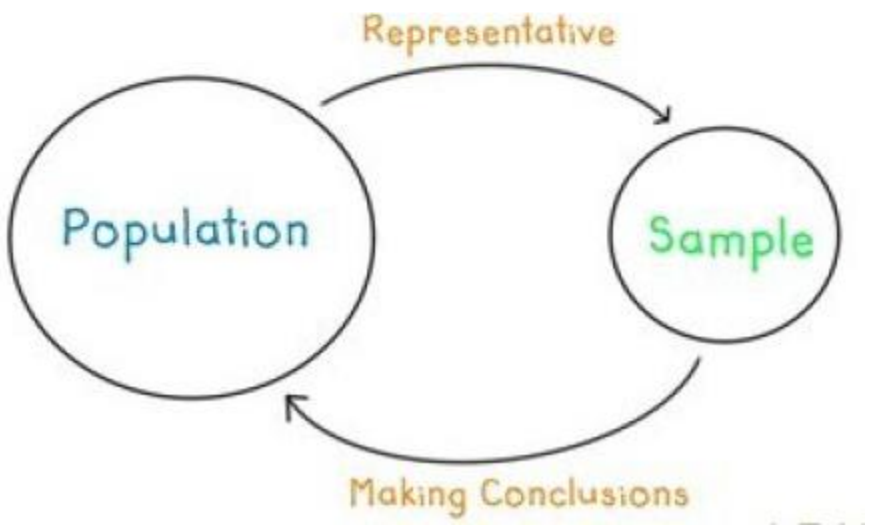
Inferential Statistics

- Draw inferences and make conclusion or evaluation about population using the evidence provided by the sample.

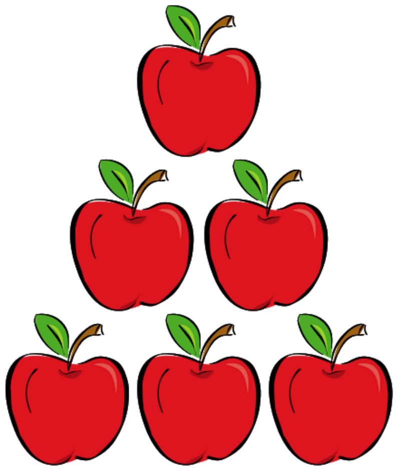
- It helps to estimate the parameters of the population.

- The sample may not provide a complete depiction of the population.

- There will always be an uncertainty when drawing conclusions about the population from the sample.



Understanding Point Estimation



- Anna is interested in finding the mean weight of the apples that are imported from Kashmir. However a survey claims that the average weight of an apple is around 90g.

What can we do now?

- It is well known that we cannot weigh every apple (population). So, we need to take samples.

- From those samples we can make inferences about the entire population.

- There are also chances that the samples what we examine will have some errors.

Image Source: http://clipart-library.com/

Sampling

- We know that all the apples cannot be weighted, so she decided to take collection of four samples of size 20 each.

- After performing the test, the sample means are displayed below.

$$Sample\ 1:\ \bar{x} = 88g \qquad Sample\ 3:\ \bar{x} = 91g$$

$$Sample\ 2:\ \bar{x} = 89.5g \qquad Sample\ 4:\ \bar{x} = 89.9g$$

What is Point Estimate?

- We have to understand that there is a claim that the average weight of an apple is around 90g.
- From samples we understand that Sample 4 having 89.9g is pretty close to 90g.
- Since this is a sample we cannot expect it be exactly 90g. So, can we claim that these samples accurately reflect the weight parameter of the overall population?
- On the other hand, 89.9g is close enough to 90g and it is plausible to accept. **This is referred as Point Estimate.**

Point Estimate

- It is a single numeric value specified for the data which is also referred as sample statistic.

- We collect data for the purpose of estimating some numerical characteristic of the population from which they come.

- A quantity calculated from the data is called a statistic, and a statistic that is used to estimate an unknown constant, or parameter, is called a point estimator. Once the data has been collected, we call it a point estimate.

- Point estimate infers about the population parameters.

Inferences from Point Estimator

Point Estimators	
Sample Mean	($\bar{X}$)
Sample Standard Deviation	(s)
Sample Proportion	($\hat{p}$)

Inference $\rightarrow$

Unknown Population Parameters	
Population Mean	(μ)
Population Standard Deviation	(σ)
Population Proportion	(p)

Point estimate is used to make an estimation of unknown population parameters including population mean, standard deviation and proportion .

Example – Point Estimation of Population Mean

- Point Estimate of Population mean μ is Sample mean $\bar{X}$.

- **Example:** Sample of heights of 34 male freshman students in a class was obtained.

185 161 174 175 202 178 202 139 177
170 151 176 197 214 283 184 189 168
188 170 207 180 167 177 166 231 176
184 179 155 148 180 194 176

$$\bar{X} = 182.44$$

- This can be inferred as the single numeric point estimate for the population mean (true mean) of all the freshman students.

Example – Point Estimation of Population Proportion

- An point estimate of population proportion, p, is the sample proportion

$$\hat{p} = \frac{X}{n}$$

where X is the number of successes in the sample and n is the sample size.

Example: A sample of 100 people were selected in a particular locality to estimate the proportion of them go for walking the park everyday. In this sample 40 do.

$$\hat{p} = \frac{40}{100} = 0.4$$

Properties of Point Estimator

- For a large population, when sampling technique is used, it is not going to be perfect always. There will always be some uncertainty in estimation.

Property : 1 - Bias

- When the expected value of an estimator is different from the value of the parameter that is being estimated.

- When they are equal, we call it as unbiased.

Property: 2 - Consistency

- This portrays how close the point estimator can be to the true value of the parameter even if it increases in size.
- The consistency and accuracy of point estimator can be achieved by using large samples.
- This can be exercised by mean and the variance.
- To be more consistent the mean of the sample should move towards the true value of the population parameter.

Property: 3 - Efficiency

- A very efficient point estimator should have the following,

  a) smallest variance.
  b) unbiased observation.
  c) consistent.

- All these parameters can be achieved from a normally distributed population.

Introduction to MSE

How can we measure the goodness of an estimator?

- Given a point estimator, how do we determine how good it is?

  Goodness Measure - Mean Squared Error(MSE)

- What methods can be used to construct good point estimators?

  Good Method to construct Point Estimator - Maximum Likelihood Estimate (MLE)

Measuring Goodness of Estimator – Mean Squared Error (MSE)

- A good estimator should be both accurate and precise.
- Accuracy of an estimator is measured by bias.
- Precision is measured by standard deviation or uncertainty.
- It can be measured by a quantity called Mean Squared Error (MSE).
- MSE combines both bias and uncertainty.

Mean Squared Error (MSE) - Bias & Uncertainty

- The **bias** of the estimator is denoted by,

$$\hat{\theta} = \mu_{\hat{\theta}} - \theta$$

- The difference between the mean of the estimator and true value.

- θ is the unknown parameter

- $\hat{\theta}$ denote the estimator of θ

- The **uncertainty** is the standard deviation $\sigma_{\hat{\theta}}$, defined as the standard error of the estimator.

Mean Squared Error (MSE)

- MSE is found by adding the variance to square of the bias.

$$MSE_{\hat{\theta}} = Var(\hat{\theta}) + (Bias\ of\ \hat{\theta})^2$$

- By definition,

Let θ be a parameter, and $\hat{\theta}$ be the estimator of θ.

The mean squared error (MSE) of $\hat{\theta}$ is

$$MSE_{\hat{\theta}} = (\mu_{\hat{\theta}} - \theta)^2 + \sigma_{\hat{\theta}}{}^2$$

- An equivalent expression is,

$$MSE_{\hat{\theta}} = \mu_{(\hat{\theta} - \theta)^2}$$

Note: $(\hat{\theta} - \theta)^2$ is the difference between estimated value and true value and it is called as error.

Derivation of Mean Squared Error (MSE)

$$MSE_{\hat{\theta}} = E(\hat{\theta} - \theta)^2$$

$$= E(\hat{\theta} - \theta)^2$$

$$= E(\hat{\theta}^2 + \theta^2 - 2\hat{\theta}\theta)$$

$$= E\left(\hat{\theta}^2\right) + \theta^2 + E\left(2\hat{\theta}\theta\right)$$

Add and subtract by $\left(E(\hat{\theta})\right)^2$

$$= \left(E(\hat{\theta})\right)^2 + \theta^2 - 2.\theta.E\left(\hat{\theta}\right) + E\left(\hat{\theta}^2\right) - E\widehat{(\theta)}^2$$

$$MSE(\hat{\theta}) = \left(E(\hat{\theta}) - \theta\right)^2 + E(\hat{\theta}^2) - \left(E(\hat{\theta})\right)^2$$

Where:

- First term $\left(E(\hat{\theta}) - \theta\right)^2$ = **(Bias)^2**
- Second term $E(\hat{\theta}^2) - \left(E(\hat{\theta})\right)^2$ = **Variance**

Problem

Let x, y be independent random variables with:

$$E[x] = E[y] = \mu, \quad Var(x) = 1, \quad Var(y) = 4$$

Consider the estimator:

$$T = \frac{2x + y}{3} + 1$$

a) Find the bias of T.

b) Compute $Var(T)$.

c) Find the MSE of T.

Problem

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a) Find the bias of T

$$E[T] = E\left[\frac{2x+y}{3}\right] + 1 = \frac{2\mu+\mu}{3} + 1 = \mu + 1$$

$$\text{Bias} = E[T] - \mu = (\mu+1) - \mu = 1$$

b) Compute $Var(T)$

The constant +1 does not affect variance:

$$Var(T) = Var\left(\frac{2x+y}{3}\right)$$

$$= \frac{4Var(x) + Var(y)}{9} = \frac{4\cdot 1 + 4}{9} = \frac{8}{9}$$

Problem

c) Find the MSE of T

$$MSE(T) = Var(T) + \text{Bias}^2$$

$$= \frac{8}{9} + (1)^2 = \frac{8}{9} + 1 = \frac{17}{9} \approx 1.89$$

Problem : Do it Yourself

Let x and y be independent normal random variables with mean μ, $Var(x) = 1$, $Var(y) = 9$.

Consider the estimators:

$$T_1 = \frac{x + 2y}{3}, \quad T_2 = \frac{3x + y}{4}$$

a) Are T_1 and T_2 unbiased?

b) Find the variance of each estimator.

c) Compute the MSE of each estimator.

d) Which estimator would you prefer at $\mu = 0$? Why?

THANK YOU

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