Big-O notations ?

TLE error ? [Time limit exceeded]

Worst Case / Best Case / Average Case TC ?

Why TLE occurs?

How to find T.C and S.C ?

↓ Time Complexity

↓ space Complexity

} → Next Class

Quiz 1 : Sum of first N natural numbers

$N = 5 \Rightarrow 1 + 2 + 3 + 4 + 5 = 15$

$$S = 1 + 2 + 3 + 4 + 5 + \cdots + (N-2) + (N-1) + N$$

$$+\ S = N + (N-1) + (N-2) + (N-3) + \cdots + 3 + 2 + 1$$

$$2S = (N+1) + (N+1) + (N+1) + (N+1) + \cdots + (N+1) + (N+1) + (N+1)$$

$$2S = (N+1) \cdot N$$

$$\boxed{S = \frac{N(N+1)}{2}}$$

$1 + 2 + 3 = 3 + 2 + 1$

Arithmetic Progression

⇒ Difference between any 2 terms in the series should be same

Ex: 1 4 7 10 14 17 20 ✗

3 3 3 4 3 3

2 8 14 20 26 ✓

6 6 6 6

Ex: 1 2 3 4 5 6 7

1 1 1 1 1

Generalize

a : 1st term

d : common difference

$a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ a+5d,\ \ldots$

$x - (a+d) = d$

$x - a - d = d \Rightarrow x = a+2d$

a : 7, d : 2 ⇒ 7 9 11 13 17 . . . . . .

$$\text{Sum}_k = \frac{k}{2}\left[2a + (k-1)d\right]$$ ✓

$a = 7,\ d = 2 \Rightarrow$ 7 9 11 13 15 .....

$k = 5$ $\quad S_5 = 7 + 9 + 11 + 13 + 15 = 55$

$a = 7$, $d = 2$

$$= \frac{5}{2}\left[2(7) + (5-1)\cdot 2\right]$$

$$= \frac{5}{2}\left[14 + 8\right] = \frac{5}{2} \times 22^{11} = 55$$

Geometric Progression

Ratio of consecutive terms should be equal

Ex: 2 6 20 60 180 ✗

$\frac{6}{2} = 3$ $\quad \frac{20}{6} = 3.33$

Ex: 2 6 18 54 162 ✓

3 3 3 3

Ex: 3 6 12 24 48 96 ✓

2 2 2 2 2

Generalized Form

a: 1st term

r: common ratio

$a \quad a \cdot r \quad ar^2 \quad ar^3 \quad ar^4 \quad ar^5 \ldots\ldots$

$\frac{ar}{a} = r \qquad \frac{ar^2}{ar} = r \qquad \frac{2xr}{xr}$

$$Sum_x = \frac{a(r^k - 1)}{r - 1}$$

GP: 3 6 12 24 48 Sum = 93

$a = 3$

$r = 2$

$k = 5$

$$Sum_5 = \frac{3(2^5 - 1)}{2 - 1} = 3 \times 31 = 93$$

Quiz

$[3, 10]$: 3, 4, 5, 6, 7, 8, 9, 10 $\Rightarrow$ 8

$10 - 3 + 1$

$[a, b] = b - a + 1$ ✓

Quiz:

$[4, 7]$: $7 - 4 + 1 = 4$

[$\Rightarrow$ closed bracket $\Rightarrow$ Number is included

($\Rightarrow$ Open bracket $\Rightarrow$ Number is excluded

$[5, 8) = \{5, 6, 7\}$

$[4, 10) = \{5, 6, 7, 8, 9\}$

$(3, 5] : \{4, 5\}$

Basics of Logarithms

$\log_b^a = c \Rightarrow$ To what power should we raise b to get a.

$$\boxed{b^c = a}$$

$\log_2^{64} = 6$ $\quad 2^6 = 64$

$\log_3^{27} = 3$ $\quad 3^3 = 27$

$\log_5^{25} = 2$ $\quad 5^2 = 25$

$\log_2^{32} = 5$ $\quad 2^5 = 32$

$\log_2^{10} = [3.xx]$ $\quad 2^{3.xx} = 10$

$2^3 = 8$ $\quad 2^4 = 16$

$\log_4^{28} = 2.xx$ $\quad 4^2 = 16$ $\quad 4^3 = 64$

$4^{2.xx} = 28$

$\log_2 2^{10} = x$ $\quad 2^x = 2^{10} \Rightarrow x = 10$

$\log_3 3^5 \quad 3^x = 3^5 \Rightarrow x = 5$

$$\log_a a^b = b$$

$\Rightarrow N = 2^k$

Apply $\log_2$ on both sides

$$\log_2 N = \log_2 2^k$$

$$K = \log_2 N$$

Quiz

```
for(i = 1; i <= 100; i++){
    s = s + i
}
```
100 iterations

$i : [1, 100] \Rightarrow 100 - 1 + 1 = 100$

$[a, b] = b - a + 1$

Quiz:

```
for(i=1; i<=N; i++){
    s = s + i
}
```

$i : [1, N] \Rightarrow N - 1 + 1 = \boxed{N}$

Quiz:

```
for(i=0; i<N; i++){
    s = s + i;
}
```

$N = 5 \Rightarrow i : [0, 4]$

$N \Rightarrow i : [0, N-1]$

$[a,b] = b - a + 1$

$[0, N-1] = N-1-0+1 = \boxed{N}$

Quiz:

```
for(i=1; i<=N; i++){
    print(i);
}
```

$i : [1, N] \Rightarrow N$

```
for(j=1; j<=M; j++){
    print(j);
}
```

$j : [1, M] \Rightarrow M$

$\boxed{\#\text{iters} = N + M}$

Quiz: `for(i=1; i <= 2^N; i++){ }`

$i : [1, 2^N]$ $\Rightarrow$ $2^N - 1 + 1 = 2^N$

Quiz: `for(i=1; i*i <= N; i++){ print("Hi"); }`

$i \times i \leq N$

$\sqrt{i^2} \leq \sqrt{N}$

$i \leq \sqrt{N}$

$i : [1, \sqrt{N}]$

#iters $= \sqrt{N} - 1 + 1 = \sqrt{N}$

Quiz:

```
i = N;
while( i > 1 ){
    i = i / 2
}
```

i before	# iteration	i after
N	1	N/2 => $\frac{N}{2^1}$
N/2	2	N/4 $\frac{N}{2^2}$
N/4	3	N/8 → $\frac{N}{2^3}$
N/8	4	N/16 → $\frac{N}{2^4}$
	⋮	
	K	$\frac{N}{2^K}$

Let's assume, this code will have K iterations

$$\frac{N}{2^K} = 1$$

$$N = 2^K \quad \Rightarrow \quad K =$$

$$\log_2^N = \boxed{\log_2^{2^K}}$$

$$\log_2^N = K \quad \Rightarrow \quad \boxed{K = \log_2^N}$$

i : $N, \frac{N}{2}, \frac{N}{4}, \frac{N}{8}, \frac{N}{16} \ldots\ldots 4, 8, 2, 1$

$\log_2^N$

Quiz:

```
for (i = 0; i <= N; i = i * 2) {
    print ("Hi");
}
```

i = 0

i = i * 2

i = 0 x 2 = 0

Infinite Iterations :

Quiz:

```
for (i = 1; i < N; i = i * 2) {
    print ("Hi");
}
```

8 : 32

1 2 4 8 . . . . . $\frac{N}{8}$ $\frac{N}{4}$ $\frac{N}{2}$ N

$\log_2 N$

i before	Iteration #	i after
1	1	2 $\Rightarrow 2^1$
2	2	4 $\Rightarrow 2^2$
4	3	8 $\Rightarrow 2^3$
8	4	16 $\Rightarrow 2^4$
	⋮	
	K	2^K

$$2^K = N$$

$$K = \log_2 N$$

Quiz:

```
for (i = 1; i ≤ 10; i++){
    for (j = 1; j ≤ N; j++){
        print(i)
    }
}
```

i	j	#iters
1	[1, N]	N + 1 + 1 = N
2	[1, N]	N
3	[1, N]	N
⋮	⋮	⋮
10	[1, N]	N

$10 \cdot N$

Quiz:

```
for(i=1; i<=N; i++){
    for(j=1; j<=N; j++){
        print("Hi");
    }
}
```

i	j	#iters
1	[1, N]	N
2	[1, N]	N
3	[1, N]	N
⋮	⋮	⋮
N	[1, N]	N

N + N + N + ⋯ + N (N−1 + 1 = N)

#iters : $N \times N = \boxed{N^2}$

Quiz:

```
for (i = 1; i <= N; i++) {
    for (j = 1; j < N; j = j * 2) {
        print("Hi");
    }
}
```

} $\log_2 N$ iters

i	j	#iters
1		$\log_2 N$ +
2		$\log_2 N$ +
3		$\log_2 N$ +
⋮		⋮ +
N		$\log_2 N$

#iters : $N \log_2 N$

```
for(i=0; i<N; i++){
    for(j=0; j<=i; j++){
        print("Hi");
    }
}
```

i	j	#iters
0	[0, 0]	0 − 0 + 1 = 1
1	[0, 1]	1 − 0 + 1 = 2
2	[0, 2]	3
⋮	⋮	
N−2	[0, N−2]	N − 2 − 0 + 1 = N − 1
N−1	[0, N−1]	N

$$\#iters = 1 + 2 + 3 + 4 + \dots + N-1 + N$$

$$= \frac{N(N+1)}{2}$$

Quiz:

```
for(i = 1; i < N; i++){
    for(j = 1; j <= 2^i; j++){
        print("Hi");
    }
}
```

j : [1, 2^i]

i	j	#Iters
1	[1, 2^1]	$2^1 - 1 + 1 = 2^1$
2	[1, 2^2]	$2^2 - 1 + 1 = 2^2$
3	[1, 2^3]	2^3
⋮	⋮	
N-2	[1, 2^{N-2}]	2^{N-2}
N-1	[1, 2^{N-1}]	2^{N-1}

Iters: $2^1 + 2^2 + 2^3 + \dots + 2^{N-2} + 2^{N-1}$

$\frac{2^2}{2^1} = 2$ $\quad$ $\frac{2^3}{2^2} = 2$

G.P

$a = 2$

$r = 2$

$K = N-1$

$$S_K = \frac{a(r^K - 1)}{r - 1} = \frac{2 \cdot (2^{N-1} - 1)}{2 - 1}$$

$$= 2 \cdot 2^{N-1} - 2$$

$$= \boxed{2^N - 2}$$

How to find Big-O

Steps

1) Calculate No. of iterations [Function of N]
2) Neglect the lower order / degree terms
3) Neglect the constant co-efficients

Ex1:

Iters: $11N^2 + 1000N + 10$

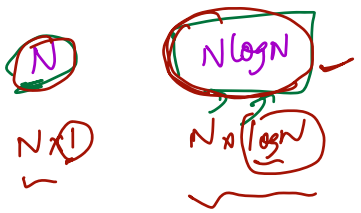
$\Rightarrow O(N^2)$

Ex2: $100N^2 + 5N + 1000000$

$O(N^2)$

Ex3: $5N + 6N\log N + 8\sqrt{N}$

$\Rightarrow O(N\log N)$



Ex: $N^2 + 6N\sqrt{N} + 10^4 N\log N$

$N \cdot N$ — $N \cdot \sqrt{N}$ — $N \cdot \log N$

N — $\sqrt{N}$ — $\log N$

$N = 2^{32}$

2^{32} — 2^{16} — $\log_2 2^{32} = 32$

$$\log N < \sqrt{N} < N < N\log N < N\sqrt{N} < N^2 < N^3 \ldots\ldots 2^N$$

lowest order — highest order

TA Help Request