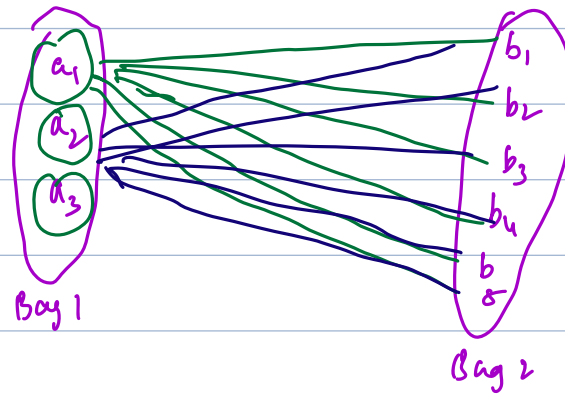


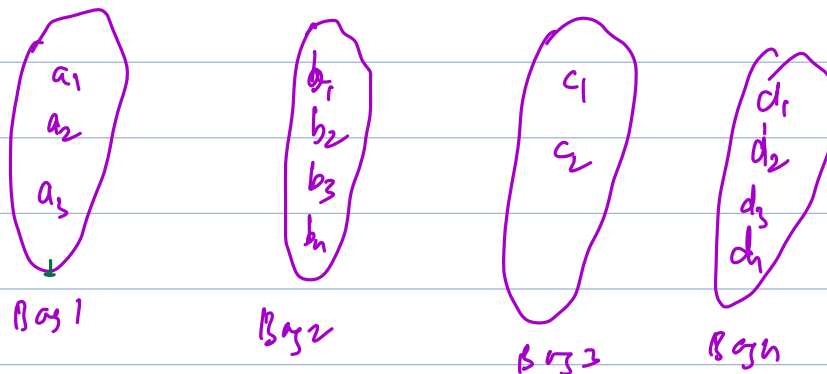
### Question: Bag Problem

If there are 2 bags with 3 and 5 items in each bag, find no. of ways we can select one item from Bag1 and other from Bag2?



$$\begin{aligned} 5 &\Rightarrow (a_1, b_1) & (a_1, b_2) & (a_1, b_3) & (a_1, b_4) & (a_1, b_5) \\ 5 &\Rightarrow (a_2, b_1) & (a_2, b_2) & (a_2, b_3) & (a_2, b_4) & (a_2, b_5) \\ &(a_3, b_1) & (a_3, b_2) & (a_3, b_3) & (a_3, b_4) & (a_3, b_5) \end{aligned}$$

$$5 + 5 + 5 + \dots = 5 \times 3 = 15$$



Ans:

$$3 \times 4 \times 2 \times 4 = 96 \text{ ways}$$

Question: Find total sum of all subarrays  
 $N \leq 10^6$

$A = [2, 8, 9, 4]$

<u>S</u>	<u>e</u>	<u>Subarray</u>	<u>Sum</u>
0	0	$[2]$ ✓	+ 2
0	1	$[2, 8]$ ✓	+ 10
0	2	$[2, 8, 9]$ ✓	+ 19
0	3 $\Rightarrow$	$[2, 8, 9, 4]$ ✓	+ 23
1	1	$[8]$	+ 8
1	2	$[8, 9]$	+ 17
1	3	$[8, 9, 4]$ ✓	+ 21
2	2	$[9]$	+ 9
2	3	$[9, 4]$	+ 13
3	3	$[4]$	+ 4
			<hr/>
			106

Approach 1:

```

long totalSum = 0;
for (start = 0; start < N; start++) {
    long sum = 0;
    for (end = start; end < N; end++) {
        sum += A[end];
        totalSum += sum;
    }
}
return totalSum;

```

T-C:  $O(N^2)$

S-C:  $O(1)$

$N \leq 10^6$

#Subarrays =  $\frac{N(N+1)}{2} \Rightarrow O(N^2)$

$N^2 = (10^6)^2 = 10^{12}$

## Approach:

If we iterate over all subarrays, we can never do less than  $O(N^2)$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

$$N=3$$

$$\# \text{subarray} = \frac{3 \times 4}{2} = 6$$

<u>S</u>	<u>E</u>	<u>Sum</u>	
0	0	$A[0]$	$\Rightarrow 1$
0	1	$A[0] + A[1]$	$\Rightarrow 3$
0	2	$A[0] + A[1] + A[2]$	$\Rightarrow 6$
1	1	$A[1]$	$\Rightarrow 2$
1	2	$A[1] + A[2]$	$\Rightarrow 5$
2	2	$A[2]$	$\Rightarrow 3$

$3 \times A[0] + 4 \times A[1] + 3 \times A[2]$

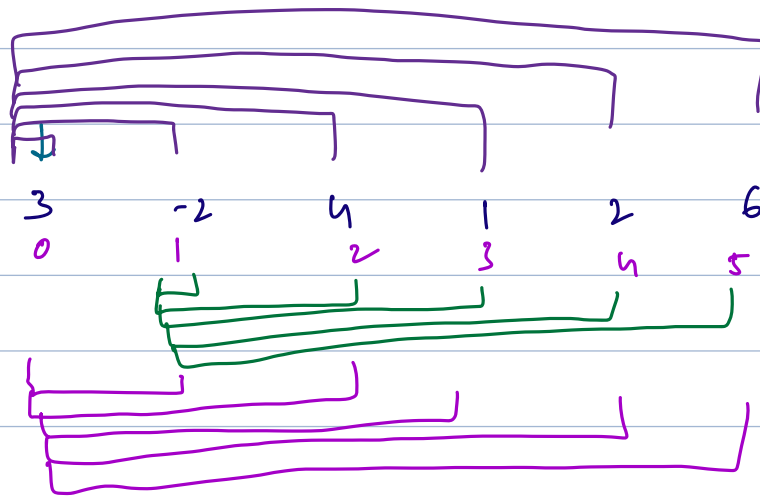
$= 3(1) + 4(2) + 3(3) = 20$

what are these elements 3, 4, 3?

It denotes no. of subarrays in which  $A[i]$  is present

**Task:** For each element  $A[i]$ , find no. of subarray in which it is a part of.

A:

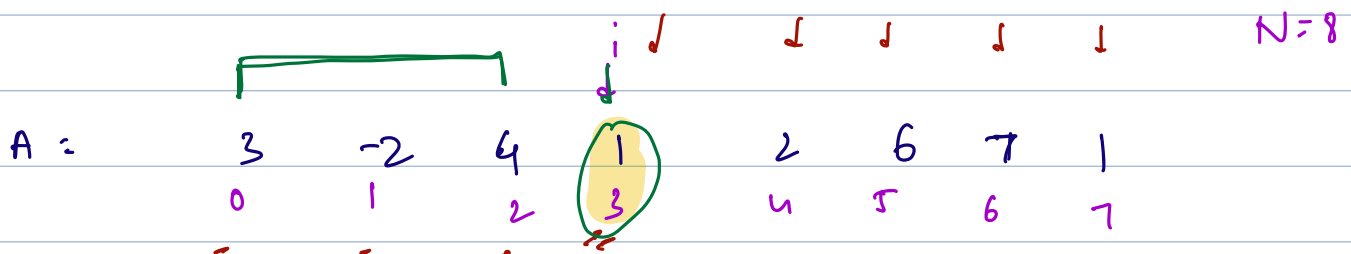


$$N = 6$$

$$N - i = 6 - 1 = 5$$

#subarrays in which  $A[0]$  exist = 6  $(A[0] \times 6)$

#subarrays in which  $A[i]$  exist =  $5 + 5 = 10$   $(A[i] \times 10)$



Question: No. of subarrays in which  $A[3]$  is a part of

Can  $A[3]$  be a part of any subarray which starts after index 3? NO!

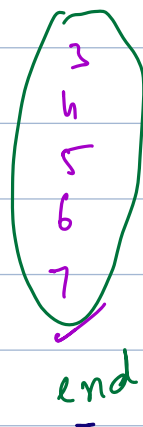
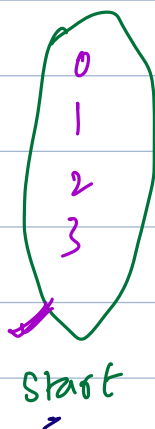
$$\text{start} \leq i \Rightarrow [0, i]$$

Will  $A[3]$  be a part of every subarray which starts before index 3? NO!

$$\text{end} \geq i \Rightarrow \text{end} = [i, N-1]$$

$$i = 3$$

✓



$$= 4 \times 5 = \boxed{20}$$

$$i \Rightarrow \text{start: } [0, i] \Rightarrow i+1$$

$$\text{end: } [i, N-1] \Rightarrow N - i + 1 = N - i$$

#Subarrays which have  $A[i] = (i+1) \times (N-i)$

	$i$	$i$				$N = 6$
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$A =$	3	-2	4	-1	2	6
	0	1	2	3	4	5

$$(i+1)|S| = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}$$

$$(N-i)|e| = \begin{matrix} 6 & 5 & 4 & 3 & 2 & 1 \end{matrix}$$

$$\text{\#Subarrays} \quad \begin{matrix} \overline{6} & \overline{10} & \overline{12} & \overline{12} & \overline{10} & \overline{6} \end{matrix}$$

$$\begin{aligned} & (6 \times 3) + (10 \times -2) + (12 \times 4) + (12 \times -1) + \\ & (10 \times 2) + (6 \times 1) = \end{aligned}$$

```

long sumSubarrays (int[] A, int N) {
    long totalSum = 0;
    for (i = 0; i < N; i++) {
        totalSum += (long) A[i] * (i+1) * (N-i);
    }
    return totalSum;
}

```

TC:  $O(N)$   
 SC:  $O(1)$

$$N \leq 10^6$$

$$A[i] \leq 10^9$$

$$\boxed{10^9 \mid 10^9 \mid 10^9 \mid \dots \mid 10^9}$$

$$10^6 \times 10^9 = 10^{15} \rightarrow \text{long value of array}$$

Contribution Technique

Question: Given an array of size  $N$ , print start and end indices of all subarrays of length  $K$

$A =$ 

0	1	2	3	4	5	6	7	8	9	10	11
3	4	2	-1	6	7	8	9	3	2	-1	4

$K = 6$

$K=5$	$K=6$	$K$
$[0, 4]$	$[0, 5]$	$[0, K-1]$
$[1, 5]$	$[1, 6]$	$[1, K]$
$[2, 6]$	$[2, 7]$	$[2, K+1]$
$\vdots$	$[3, 8]$	$\vdots$
$\vdots$	$[4, 9]$	$\vdots$
$\vdots$	$[5, 10]$	$\vdots$
$[x, 11]$	$[x, 11]$	$[x, N-1]$
$(N-1)$	$(N-1)$	

$11 - x + 1 = 5$   
 $x = 12 - 5$   
 $= 7$

$Start: \{0, 1, 2, \dots, 7\}$   
 $\{0, 7\} = 7$

$N - x + x = K$   
 $x = N - K$

$[7, 11]$

$[N-K, N-1]$

Start:  $\{0, 1, 2, \dots, N-K\}$

$[0, N-K] \Rightarrow N-K-0+1 = N-K+1$

$[a, b] = b - a + 1$

$N-K-0+1 = N-K+1$

★★★★★

1<sup>st</sup> Subarray of size  $K$  :  $[0, K-1]$   
last Subarray of size  $K$  :  $[N-K, N-1]$   
No. of Subarrays of size  $K$  :  $N-K+1$

```
printSubarraysK ( int[] A , int N, int K ) {
```

```
    s = 0 ;
```

```
    e = K-1 ;
```

```
    while ( s ≤ N-K ) < // e ≤ N-1
```

```
        print ( s, e );
```

```
        s++;
```

```
        e++;
```

```
    }
```

```
}
```



Question: Given an array, find the maximum subarray sum of length  $k$

$$N = 10$$
$$A = \begin{matrix} -3 & 4 & -2 & 5 & 3 & -2 & 8 & 2 & -1 & 4 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{matrix}$$
$$k = 5$$

$\Sigma$	$\underline{C}$	$\underline{\text{Sum}}$
0	4	7
1	5	8
2	6	12
3	7	16
4	8	10
5	9	11

3