Assignment

Q 1. Count of divisors

Problem Description

Given an array of integers **A**, find and return the **count of divisors** of each element of the array.

NOTE: The order of the resultant array should be the same as the input array.

Problem Constraints

1 <= length of the array <= 100000
1 <= A[i] <= 10^6

Input Format

The only argument given is the integer array A.

Output Format

Return the count of divisors of each element of the array in the form of an array.

Example Input

Input 1:

```
A = [2, 3, 4, 5]
```

Input 2:

```
A = [8, 9, 10]
```

Example Output

Output 1:

```
[2, 2, 3, 2]
```

Output 1:

```
[4, 3, 4]
```

Example Explanation

Explanation 1:

```
 The number of divisors of 2 : [1, 2], 3 : [1, 3], 4 : [1, 2, 4], 5 : [1, 5]
 So the count will be [2, 2, 3, 2].
```

Explanation 2:

```
 The number of divisors of 8 : [1, 2, 4, 8], 9 : [1, 3, 9], 10 : [1, 2, 5, 10]
 So the count will be [4, 3, 4].
```

Q 2. Prime Sum

Problem Description

Given an even number **A** (greater than 2), return two prime numbers whose sum will be equal to the given number.

If there is more than one solution possible, return the lexicographically smaller solution.

```
If [a, b] is one solution with a <= b, and [c,d] is another solution with c <= d, then
[a, b] < [c, d], If a < c OR a==c AND b < d.
```

NOTE: A solution will always exist. Read Goldbach's conjecture.

Problem Constraints

$4 <= A <= 2*10^7$

Input Format

First and only argument of input is an even number A.

Output Format

Return a integer array of size 2 containing primes whose sum will be equal to given number.

Example Input

```
 4
```

Example Output

```
[2, 2]
```

Example Explanation

```
There is only 1 solution for A = 4.
```

Q3. Lucky Numbers

Problem Description

A **lucky number** is a number that has exactly **2 distinct** prime divisors.

You are given a number **A,** and you need to determine the **count** of lucky numbers between the range 1 to A (both inclusive).

Problem Constraints

1 <= A <= 50000

Input Format

The first and only argument is an integer A.

Output Format

Return an integer i.e the count of lucky numbers between 1 and A, both inclusive.

Example Input

Input 1:

```
A = 8
```

Input 2:

```
A = 12
```

Example Output

Output 1:

```
1
```

Output 2:

```
3
```

Example Explanation

Explanation 1:

```
 Between [1, 8] there is only 1 lucky number i.e 6.
 6 has 2 distinct prime factors i.e 2 and 3.
```

Explanation 2:

```
 Between [1, 12] there are 3 lucky number: 6, 10 and 12.
```

Q 4. Prime Subsequence's

Given an array **A** having **N** positive numbers. You have to find the number of **Prime subsequences** of **A**.

A Prime subsequence is one that has **only prime** numbers, for example [2, 3], [5] are the Prime subsequence's where [2, 4] and [1, 2, 3, 4] are not.

Input Format

```
The first argument given is an Array A, having N integers.
```

Output Format

```
Return an integer X, i.e., number of Prime subsequences.
As X can be very large print X % (1000000007), here % is
modulus operator.
```

Constraints

```
1 <= N <= 1e3
1 <= A[i] <= 1e6
```

For Example

```
Input:
    A = [1, 2, 3]
Output:
     3

Explanation:
    no. Subsequences      Prime subsequences
    1.  [1]                     No
    2.  [1, 2]                  No
    3.  [1, 3]                  No
    4.  [1, 2, 3]               No
    5.  [2]                     Yes
```

```
    6.  [2, 3]                  Yes
    7.  [3]                     Yes
    8.  []                      No

    here we have 3 subsequences(5, 6, 7) those have only
prime number(s).
```

Q 5. Compute nCr % m

Problem Description

Given three integers **A**, **B,** and **C**,
where **A** represents **n**, **B** represents **r,** and **C** represents **m**, find and return the value of ${}^{n}C_{r}$ **% m** where ${}^{n}C_{r}$ **% m = (n!/((n-r)!*r!))% m**.

x! means factorial of x i.e. **x! = 1 * 2 * 3... * x.**

Problem Constraints

$1 <= A * B <= 10^{6}$

$1 <= B <= A$

$1 <= C <= 10^{6}$

Input Format

The first argument given is integer A (= n).
The second argument given is integer B (= r).
The third argument given is integer C (= m).

Output Format

Return the value of nCr % m.

Example Input

Input 1:

```
 A = 5
 B = 2
 C = 13
```

Input 2:

```
 A = 6
 B = 2
 C = 13
```

Example Output

Output 1:

```
10
```

Output 2:

```
2
```

Example Explanation

Explanation 1:

```
The value of 5C2 % 11 is 10.
```

Explanation 2:

```
The value of 6C2 % 13 is 2.
```

Q 6. Compute nCr % p

Problem Description

Given three integers **A**, **B,** and **C**,
where **A** represents **n**, **B** represents **r,** and **C** represents **p** and **p** is a prime number greater than equal to **n**, find and return the value of **nCr % p** where **nCr % p = (n! / ((n-r)! * r!)) % p.**

x! means factorial of x i.e. **x! = 1 * 2 * 3... * x.**

NOTE: For this problem, we are considering 1 as a prime.

Problem Constraints

- $1 <= A <= 10^6$
- $1 <= B <= A$
- $A <= C <= 10^9+7$

Input Format

The first argument given is the integer A (= n).
The second argument given is the integer B (= r).
The third argument given is the integer C (= p).

Output Format

Return the value of nCr % p.

Example Input

Input 1:

```
A = 5
B = 2
C = 13
```

Input 2:

```
A = 6
B = 2
C = 13
```

Example Output

Output 1:

```
10
```

Output 2:

```
2
```

Example Explanation

Explanation 1:

```
nCr( n=5 and r=2) = 10.
p=13. Therefore, nCr%p = 10.
```

Q 7. Delete one

Problem Description

Given an integer array **A** of size **N**. You have to delete **one** element such that the GCD**(Greatest common divisor)** of the remaining array is maximum.

Find the **maximum** value of GCD.

Problem Constraints

$2 <= N <= 10^5$
$1 <= A[i] <= 10^9$

Input Format

First argument is an integer array A.

Output Format

Return an integer denoting the maximum value of GCD.

Example Input

Input 1:

```
A = [12, 15, 18]
```

Input 2:

```
A = [5, 15, 30]
```

Example Output

Output 1:

```
6
```

Output 2:

```
15
```

Example Explanation

Explanation 1:

```
If you delete 12, gcd will be 3.
If you delete 15, gcd will be 6.
If you delete 18, gcd will 3.
Maximum vallue of gcd is 6.
```

Explanation 2:

```
If you delete 5, gcd will be 15.
If you delete 15, gcd will be 5.
If you delete 30, gcd will be 5.
```

Q 8. Enumerating GCD

Problem Description

You are given a number **A** and a number **B**. Greatest Common Divisor (GCD) of all numbers between **A** and **B** inclusive is taken (GCD**(A, A+1, A+2 ... B)**).

As this problem looks a bit easy, it is given that numbers **A** and **B** can be in the range of $\mathbf{10^{100}}$.

You have to return the value of GCD found.

The greatest common divisor of 2 numbers, A and B, is the largest number, D that divides both A and B perfectly.

Problem Constraints

$1 <= A <= B <= 10^{100}$

Input Format

First argument is a string denoting A.

Second argument is a string denoting B.

Output Format

Return a string which contains the digits of the integer which represents the GCD. The returned string should not have any leading zeroes.

Example Input

```
A = "1"
B = "3"
```

Example Output

```
1
```

Example Explanation

```
Greatest divisor that divides both 1 and 3 is 1.
```

Q 9. Greatest Common Divisor

Problem Description

Given 2 non-negative integers `A` and `B`, find `gcd(A, B)`

GCD of 2 integers `A` and `B` is defined as the greatest integer 'g' such that 'g' is a divisor of both A and B. Both A and B fit in a 32 bit signed integer.

Note: DO NOT USE LIBRARY FUNCTIONS.

Problem Constraints

$0 <= A, B <= 10^9$

Input Format

First argument is an integer A.
Second argument is an integer B.

Output Format

Return an integer denoting the gcd(A, B).

Example Input

Input 1:

```
A = 4
B = 6
```

Input 2:

```
A = 6
B = 7
```

Example Output

Output 1:

```
 2
```

Output 2:

```
 1
```

Example Explanation

Explanation 1:

```
2 divides both 4 and 6
```

Explanation 2:

```
1 divides both 6 and 7
```

Q 10. Pubg

Problem Description

There are **N** players, each with strength **A[i]**. when player **i** attack player **j**, player **j** strength reduces to **max(0, A[j]-A[i])**. When a player's strength reaches **zero**, it loses the game, and the game continues in the same manner among other players until only **1** survivor remains.

Can you tell the **minimum health** last surviving person can have?

Problem Constraints

- 1 <= N <= 100000
- 1 <= A[i] <= 1000000

Input Format

First and only argument of input contains a single integer array A.

Output Format

Return a single integer denoting minimum health of last person.

Example Input

Input 1:

```
A = [6, 4]
```

Input 2:

```
A = [2, 3, 4]
```

Example Output

Output 1:

```
2
```

Output 2:

```
1
```

Example Explanation

Explanation 1:

```
Given strength array A = [6, 4]
Second player attack first player, A =  [2, 4]
First player attack second player twice. [2, 0]
```

Explanation 2:

```
Given strength array A = [2, 3, 4]
First player attack third player twice. [2, 3, 0]
First player attack second player. [2, 1, 0]
Second player attack first player twice. [0, 1, 0]
```

Homework

Q 1. Number Of Open Doors

Problem Description

Given an integer **A,** which denotes the number of doors in a row numbered 1 to A. All the doors are closed initially.

A person moves to and fro, changing the states of the doors as follows: the person opens a door that is already closed and closes a door that is already opened.

In the first go, he/she alters the states of doors numbered 1, 2, 3, … , A.
In the second go, he/she alters the states of doors numbered 2, 4, 6 ….
In the third go, he/she alters the states of doors numbered 3, 6, 9 …
This continues till the A'th go in, which you alter the state of the door numbered A.
Find and return the **number of open doors** at the end of the procedure.

Problem Constraints

$1 <= A <= 10^9$

Input Format

The only argument given is integer A.

Output Format

Return the number of open doors at the end of the procedure.

Example Input

Input 1:

```
A = 5
```

Input 2:

```
A = 6
```

Example Output

Output 1:

```
2
```

Output 2:

```
2
```

Example Explanation

Input 1:

```
 In the first go, he/she alters the states of doors numbered
1, 2, 3, 4, 5. Now, all doors are open.
 In the second go, he/she closes the doors numbered 2, 4.
 In the third go, he/she closes the door numbered 3.
 In the fourth go, he/she open the door numbered 4.
 In the fifth go, he/she closes the door numbered 5.
 Doors opened at the end are 1 and 4.
```

Input 2:

```
 In the first go, he/she alters the states of doors numbered
1, 2, 3, 4, 5, 6. Now, all doors are open.
 In the second go, he/she closes the doors numbered 2, 4, 6.
 In the third go, he/she closes the door numbered 3 and opens
door 6.
 In the fourth go, he/she open the door numbered 4.
 In the fifth go, he/she closes the door numbered 5.
 In the sixth go, he/she closes the door numbered 6.
 Doors opened at the end are 1 and 4.
```

Q 2. Aptitude 47

If two dice are thrown, what is the probability that the sum of the two dice will be a prime number?

Q 3. Prime Addition

You are given an `even` number `N` and you need to represent the given number as the sum of primes. The prime numbers do not necessarily have to be `distinct`. It is guaranteed that at least one possible solution exists.

You need to determine the `minimum` number of prime numbers needed to represent the given number.

Input

```
The first argument contains an integer N,the number you need
to represent (2<=N<=10^9).
```

Output

```
Return an integer which is the minimum number of prime
numbers needed to represent the given number N.
```

Examples

Input

```
26
```

Output

```
2
```

Explanation

Testcase 1-

```
You can represent 26 as: 13+13
So we require minimum of 2 prime numbers to represent the
number 26.
```

Q 4. Distinct Primes

You have given an array A having N integers. Let say G is the product of all elements of A. You have to find the number of distinct prime divisors of G.

Input Format

```
The first argument given is an Array A, having N integers.
```

Output Format

```
Return an Integer, i.e number of distinct prime divisors of
G.
```

Constraints

```
1 <= N <= 1e5
1 <= A[i] <= 1e5
```

For Example

```
Input:
    A = [1, 2, 3, 4]
Output:
     2

Explanation:
    here G = 1 * 2 * 3 * 4 = 24
    and distinct prime divisors of G are [2, 3]
```

Q 5. Sorted Permutation Rank

Problem Description

Given a string **A**. Find the rank of the string amongst its permutations **sorted lexicographically**.
Assume that no characters are repeated.

Note: The answer might not fit in an integer, so return `your answer % 1000003`

Problem Constraints

1 <= **|A|** <= 1000

Input Format

First argument is a string A.

Output Format

Return an integer denoting the rank of the given string.

Example Input

Input 1:

```
A = "acb"
```

Input 2:

```
A = "a"
```

Example Output

Output 1:

```
2
```

Output 2:

```
1
```

Example Explanation

Explanation 1:

```
Given A = "acb".
The order permutations with letters 'a', 'c', and 'b' :
abc
acb
bac
bca
cab
cba
So, the rank of A is 2.
```

Explanation 2:

```
Given A = "a".
Rank is clearly 1.
```

Q 6. Sorted Permutation Rank with Repeats

Problem Description

Given a string **A**, find the rank of the string amongst its permutations sorted lexicographically. Note that the characters might be repeated. If the characters are repeated, we need to look at the rank in unique permutations. Look at the example for more details.

NOTE:

- The answer might not fit in an integer, so return `your answer % 1000003` where 1000003 is a prime number.
- String A can consist of both lowercase and uppercase letters. Characters with lesser ASCII values are considered smaller, i.e., 'a' > 'Z'.

Problem Constraints

1 <= len(A) <= 1000000

Input Format

First argument is a string A.

Output Format

Return an integer denoting the rank.

Example Input

Input 1:

```
A = "aba"
```

Input 2:

```
A = "bca"
```

Example Output

Output 1:

```
2
```

Output 2:

```
4
```

Example Explanation

Explanation 1:

```
The order permutations with letters 'a', 'a', and 'b' :
   aab
   aba
   baa
So, the rank is 2.
```

Explanation 2:

```
The order permutations with letters 'a', 'b', and 'c' :
   abc
   acb
   bac
   bca
   cab
   cba
So, the rank is 4.
```

Q 7. GCD_CMPL

In the following C++ function, let n >= m.

```
    int gcd(int n, int m) {
            if (n%m ==0) return m;
            if (n < m) swap(n, m);
            while (m > 0) {
                n = n%m;
```

```
                swap(n, m);
            }
            return n;
    }
```

What is the time complexity of the above function assuming `n > m?`.
Θ symbol represents **theta** notation and **Ω** symbol represents **omega** notation.

Q 8. Largest Coprime Divisor

Problem Description

You are given two positive numbers **A** and **B** . You need to find the maximum valued integer **X** such that:

- **X** divides **A** i.e. A % X = 0
- **X** and **B** are co-prime i.e. gcd(X, B) = 1

Problem Constraints

1 <= A, B <= 10^9

Input Format

First argument is an integer A.
Second argument is an integer B.

Output Format

Return an integer maximum value of X as descibed above.

Example Input

Input 1:

```
A = 30
B = 12
```

Input 2:

```
A = 5
B = 10
```

Example Output

Output 1:

```
5
```

Output 2:

```
1
```

Example Explanation

Explanation 1:

```
All divisors of A are (1, 2, 3, 5, 6, 10, 15, 30).
The maximum value is 5 such that A%5 == 0 and gcd(5,12) = 1
```

Explanation 2:

```
1 is the only value such that A%5 == 0 and gcd(1,10) = 1
```

Q 9. Divisor game

Problem Description

Scooby has 3 three integers **A, B, and C**.

Scooby calls a positive integer special if it is divisible by B and it is divisible by **C**. You need to tell the number of special integers less than or equal to **A**.

Problem Constraints

$1 <= A, B, C <= 10^9$

Input Format

First argument is a positive integer A
Second argument is a positive integer B
Third argument is a positive integer C

Output Format

One integer corresponding to the number of special integers less than or equal to A.

Example Input

Input 1:

```
A = 12
B = 3
C = 2
```

Input 2:

```
A = 6
B = 1
C = 4
```

Example Output

Output 1:

```
2
```

Output 2:

```
1
```

Example Explanation

Explanation 1:

```
The two integers divisible by 2 and 3 and less than or equal to 12 are 6,12.
```

Explanation 2:

```
Only 4 is a positive integer less than equal to 6 which is divisible by 1 and 4.
```

Q 10. All GCD Pair

Problem Description

Given an array of integers **A** of size **N** containing GCD of every possible pair of elements of another array.

Find and return the original numbers used to calculate the GCD array in any order. For example, if original numbers are **{2, 8, 10}** then the given array will be **{2, 2, 2, 2, 8, 2, 2, 2, 10}.**

Problem Constraints

$1 <= N <= 10000$
$1 <= A[i] <= 10^9$

Input Format
The first and only argument given is the integer array A.

Output Format

Find and return the original numbers which are used to calculate the GCD array in any order.

Example Input

Input 1:

```
A = [2, 2, 2, 2, 8, 2, 2, 2, 10]
```

Input 2:

```
A = [5, 5, 5, 15]
```

Example Output

Output 1:

```
[2, 8, 10]
```

Output 2:

```
[5, 15]
```

Example Explanation

Explanation 1:
```
Initially, array A = [2, 2, 2, 2, 8, 2, 2, 2, 10].

2 is the gcd between 2 and 8, 2 and 10.
8 and 10 are the gcds pair with itself.
Therefore, [2, 8, 10] is the original array.
```

Explanation 2:

```
Initially, array A = [5, 5, 5, 15].
5 is the gcd between 5 and 15.
15 is the gcds pair with itself.
Therefore, [5, 15] is the original array.
```