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# Kinematics

This chapters explains the kinematics, i.e. the computation of the tool centre point out of joint angles and vice versa. First is demonstrated in chapter 1.1 (simple), latter in chapter 1.2 (rather tricky).

But before starting any kinematics, it is necessary to define all coordinate systems.

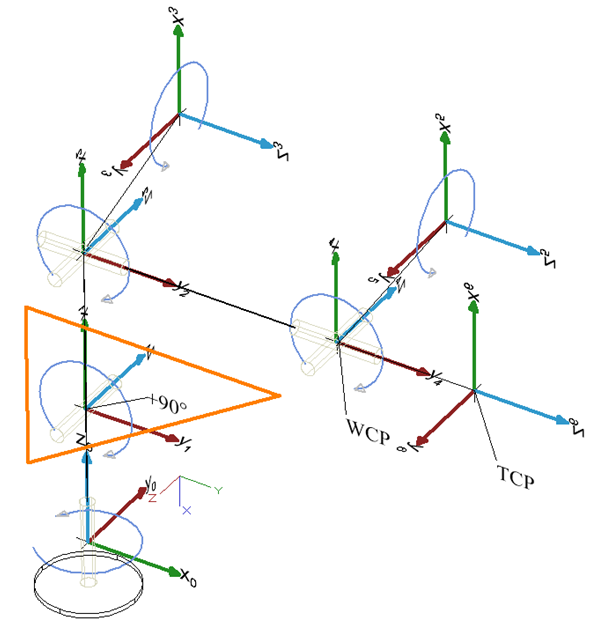


Figure ‑ Coordinate Systems in default position

The picture shows the used coordinate systems in the default position of the bot, having all angles at 0°, starting from the base (angle0) and ending with the coordinate system of the hand (angle6). For convenience the forearm (angle1) adds +90° to the real angle in order to have the base position at 0°of the bot, although the illustrated actually is -90°. The coordinate systems are arranged according to the Denavit Hardenberg convention, which is:

* The angle rotates along the z-axis
* The z-axis points on the direction of the next joint
* The transformation from anglei to anglei+1 is done via

1. rotating around the x-axis by 
2. translation by *a* along the x-axis
3. translation by *d* along the z-axis, and a
4. rotation around the z-axis induced by the joint angle

So, the Denavit Hardenberg parameters are:

|  |  |  |  |
| --- | --- | --- | --- |
| Joint | a[°] | a[mm] | d[mm] |
| hip | -90° | 0 | d0 |
| upperarm | 0 | a1 | 0 |
| forearm | -90° | 0 | 0 |
| ellbow | 90° | 0 | d3 |
| wrist | -90° | 0 | 0 |
| hand | 0 | 0 | d5 |

Table Denavit Hardenberg parameters

The general definition of a Denavit Hardenberg transformation is

|  |  |
| --- | --- |
|  | (1‑1) |

Combined with the DH parameters, the following DH matrixes define the transformation from one joint to its successor:

|  |  |
| --- | --- |
|  | (1‑2) |
|  | (1‑3) |
|  | (1‑4) |
|  | (1‑5) |
|  | (1‑6) |
|  | (1‑7) |

## Forward Kinematics

With the DH transformation matrixes at hand, computation of the bot’s pose out of the joint angles is straight forward. The matrix representing the gripper’s pose is

|  |  |
| --- | --- |
|  | (1‑8) |

By multiplying the transformation matrix with the origin (as homogeneous vector), we get the absolute coordinates of the tool centre point in world coordinate system (i.e. relative to the bot’s base).

|  |  |
| --- | --- |
|  | (1‑9) |

The orientation in terms of roll/nick/yaw of the tool centre point can be derived out of by taking the part representing the rotation matrix (). [[1]](#footnote-1)

|  |  |
| --- | --- |
|  | (1‑10) |
|  | (1‑11) |
|  | (1‑12) |
|  | (1‑13) |

Due to singularities, we need to consider and use

|  |  |
| --- | --- |
|  | (1‑14) |
|  | (1‑15) |

Instead. if

|  |  |
| --- | --- |
|  | (1‑16) |
|  | (1‑17) |

**Note:** Unfortunately, the gripper’s coordinate system is not appropriate for human interaction, since the default position as illustrated in Figure 1‑1 is not. So, it is handy to rotate the gripper matrix such that the default position becomes. The according rotation matrix represents a rotation of -90° along x,y, and z, which results in a simple rotation matrix of

|  |  |
| --- | --- |
|  | (1‑18) |

In the following, this is not considered, but we stop at the coordinate system to simplify the computation.

## Inverse Kinematics

Inverse kinematics denotes the computation of all joint angles out of the tool-centre-point’s position and orientation. In general it is tricky, in this case it is only possible since the upper three joint angles point to one point, the so-called wrist centre point (Figure 1‑1).

1. <https://de.wikipedia.org/wiki/Roll-Nick-Gier-Winkel#Berechnung_aus_Rotationsmatrix> [↑](#footnote-ref-1)