Design and Analysis of Algorithms

Lecture-38

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N-Queen Problem

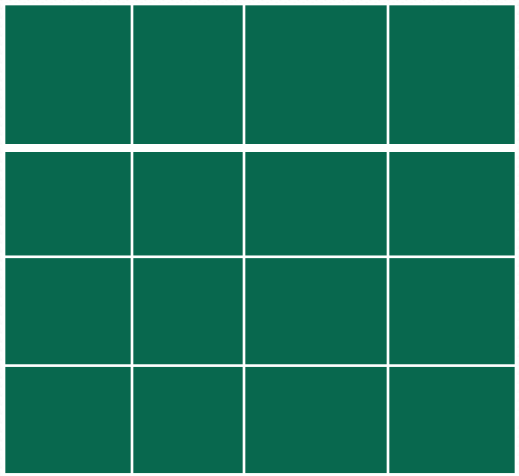
Statement:

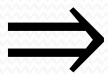
In this problem, we have given N queens and a NxN chessboard. The objective of the problem is to place all the N queens on this chessboard in such way no two queens lie on the same row, column or diagonal.

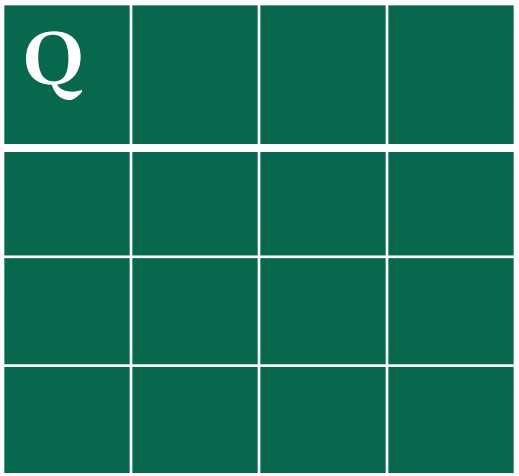
N-Queen Problem

Example: Solve the 4-Queen Problem.

Solution:

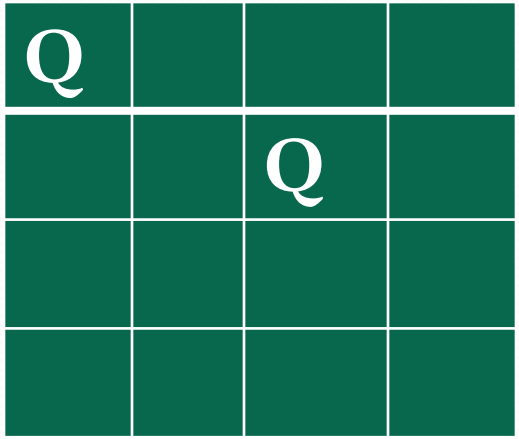






Initial position of chessboard

N-Queen Problem



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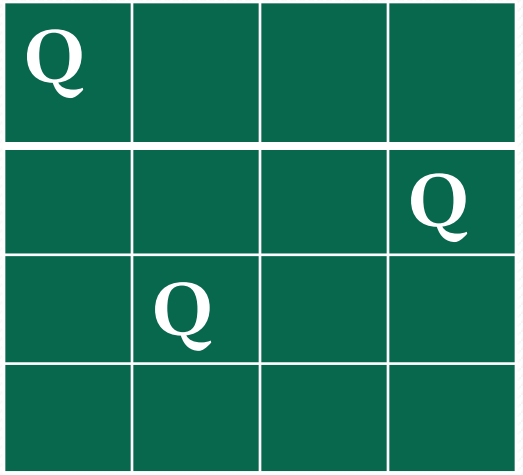

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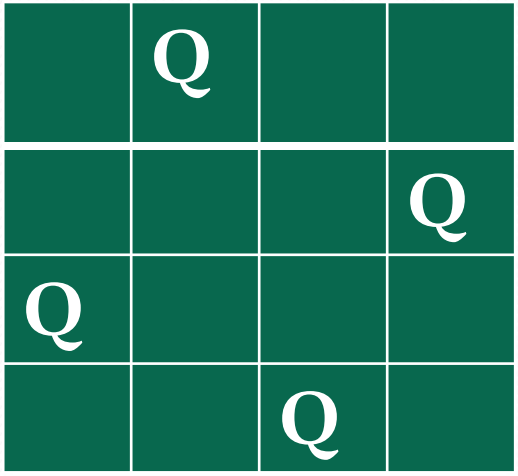


Final solution

Backtracking algorithm for N-Queen

- According to this approach, we put 1st queen in 1st row, 2nd queen in 2nd row and so on. We have to only find column number of the queen.
- Therefore, the solution vector will be (x_1, x_2, ……., x_n). Here x_i is the column number of i^{th} queen.
- The explicit constrains for this problem are that no two x_i's can be same and no two queens can be on the same diagonal.
- Using these two constraints, the size of solution space are n!.
- Suppose two queens are placed at positions (i,j) and (k,l). They are on the same diagonal only if

$$i-j = k-l \quad \text{or} \quad i+j = k+l$$

$$\Rightarrow \quad i-k = j-l \quad \text{or} \quad i-k = -(j-l)$$

Therefore, two queens lie on the same diagonal iff

$$|i-k| = |j-l|$$

Backtracking algorithm for N-Queen

An algorithm f or determining the solution of n-queens problem is:-

```
        N-Queen(k, n)
1.      for i ← 1 to n
2.              do if  Place(k,i) = True
3.                      then  x[k] ← I
4.                              if k=n
5.                                      then for  j←1 to n
6.                                                      do print x[j]
7.                              else
8.                                              N-Queen(k+1, n)
```

Backtracking algorithm for N-Queen

An algorithm f or determining the solution of n-queens problem is:-

Place(k, i)

1. for j ← 1 to k-1
2. do if (x[j]=i) or (abs(x[j]-i) = abs(j-k))
3. then return (False)
4. return(True)

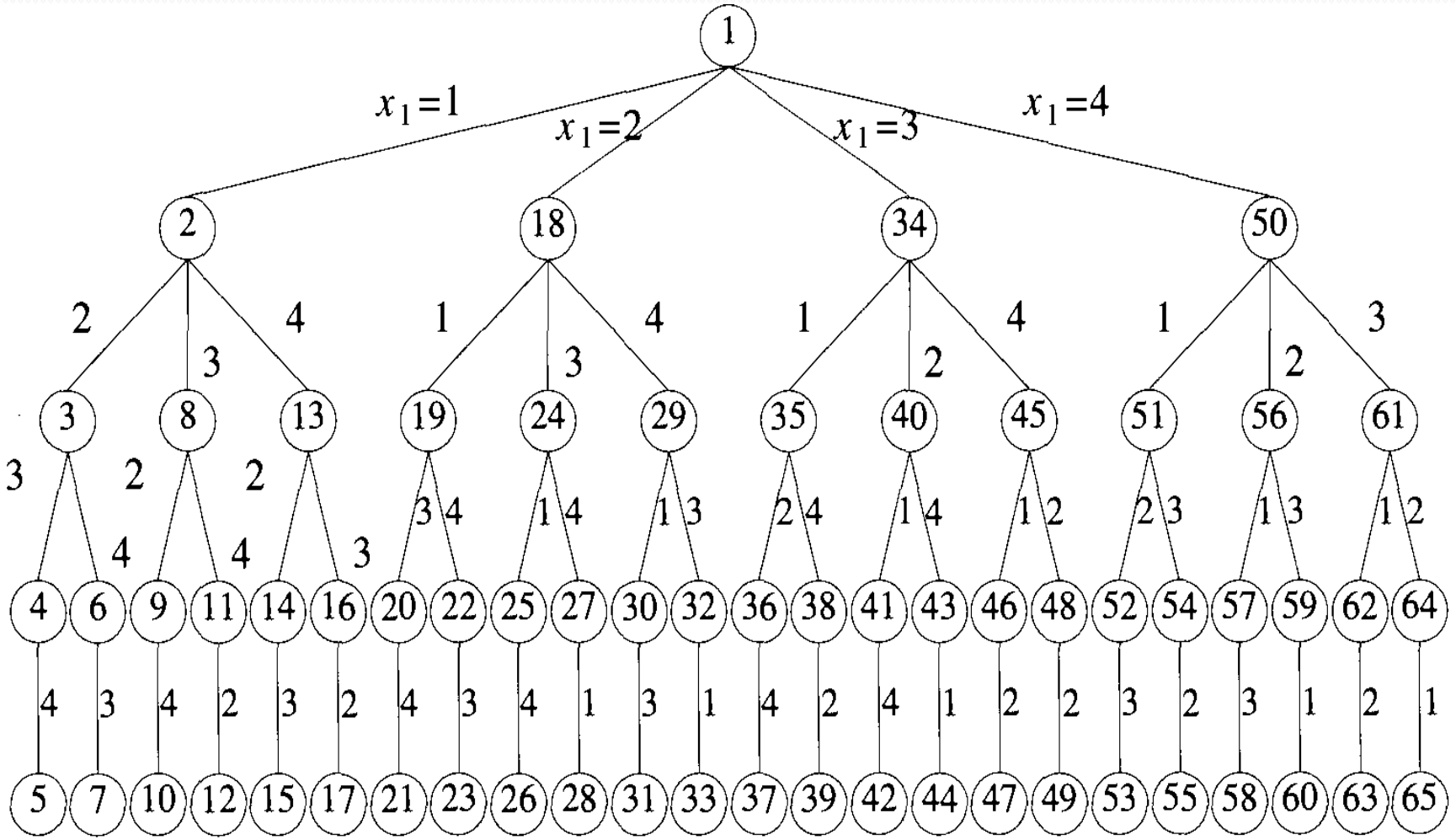
The initial call is N-Queen(1, n).

Running time of this algorithm is O(n!).

Backtracking algorithm for N-Queen

Example: Draw the state space tree for 4-queen problem.

Solution: State space tree for 4-queen problem is the following:-



Backtracking algorithm for N-Queen

Example: Solve the 8-queens problem.

Solution: