Discrete Structures and Theory of Logic

Lecture-9

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Some important examples

Example: How many reflexive relations are defined on the set with n elements? AKTU(2019)

Solution: According to reflexive property, each reflexive relation contains all the pairs like (a,a), where a belongs into the set. Total number of ordered pairs defined in the set with n elements is n^2. The number of ordered pairs like (a,a) will be n. Therefore, the remaining elements like (a,b) and a$\neq$b will be n^2 - n. Since the relation is a subset of set of ordered pairs, therefore total number of reflexive relations will be $2^{(n^2-n)}$.

Example: How many symmetric relations are defined on the set with n elements? AKTU(2019)

Solution: Consider the set is S with n elements. Relation is defined on the set S. The total number of relations defined on set S will be n^2, because relation is the subset of S$\times$S.

Now, if relation satisfies the symmetric property, then (a,b) and (b,a) belongs into the relation together. Therefore, the set whose all the subsets are reflexive relation contains $\frac{(n^2-n)}{2} + n = \frac{(n^2+n)}{2}$. Here, n is the number of ordered pairs like (a,a).

Therefore the total number of symmetric relations $= 2^{\frac{(n^2+n)}{2}}$.

Some important examples

Example: How many anti-symmetric relations are defined on the set with n elements?

Solution: Consider the set is S with n elements. Relation is defined on the set S. The total number of relations defined on set S will be n^2, because relation is the subset of S$\times$S.

The total number of ordered pairs related to itself = n. Clearly, all the subsets of these ordered pairs are anti-symmetric. Therefore, the total anti-symmetric relations defined on these ordered pairs = 2^n.

The remaining ordered pairs which are not related to itself = $n^2 - n$

Since both (a,b) and (b,a) can not belong into any anti-symmetric relations, Therefore, we consider only orederd pair = $\frac{(n^2-n)}{2}$.

Therefore, there are three possibilities for ordered pairs (a,b) and (b,a).

Solution(cont.)

First possibility: (a,b) and (b,a) both not belong.
Second possibility: (a,b) belong but (b,a) not belong.
Third possibility: (a,b) not belong but (b,a) belong.
Therefore, total number of anti-symmetric relations for these types of ordered pairs $= 3^{\frac{(n^2-n)}{2}}$.
Therefore, total number of anti-symmetric relations for the set S $= 2^n * 3^{\frac{(n^2-n)}{2}}$.
Example: Is the "divides" relation on the set of positive integers transitive? What is the reflexive and symmetric closure of the relation R $= \{$(a, b) — a $>$ b$\}$ on the set of positive integers? AKTU(2019)