Discrete Structures and Theory of Logic

Lecture-42

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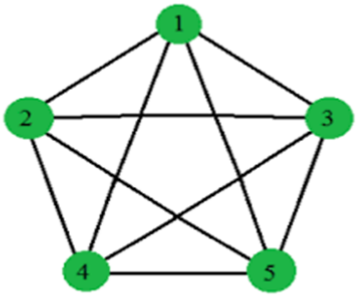
Regular Graph

A graph is called a regular if all the vertices of the graph have the same degree. If degree of each vertex is k, then the graph is called k-regular graph.

Example: Following graph is regular.



4 Regular

Example: Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. And also draw this graph.

Solution:

Let e is the number of edges in the graph. Since the sum of degree of all vertices is 2e, therefore

$2*4+4*2 = 2e \Rightarrow e = 8$

Hence the number of edges in the graph is 8.

Example: Does there exists a 4-regular graph with 6 vertices? If so, construct a graph.

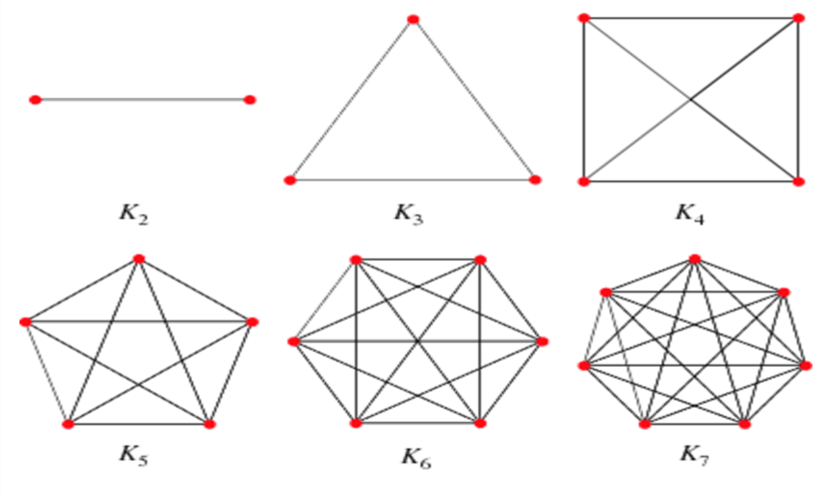
Solution:

Complete Graph

A graph G is said to be complete if every vertex in G is connected with every other vertex. A complete graph is denoted by K_n, where n is the number of vertex in G. Number of edges in complete graph K_n is exactly n(n-1)/2 edges.

Example: Draw the complete graph for n=2 to 7.

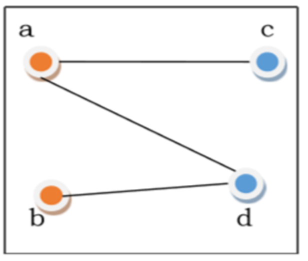
Solution:



Bipartite Graph

If the vertex-set of a graph G can be split into two disjoint sets, V_1 and V_2, in such a way that each edge in the graph joins a vertex in V_1 to a vertex in V_2, and there are no edges in G that connect two vertices in V_1 or two vertices in V_2, then the graph G is called a bipartite graph.

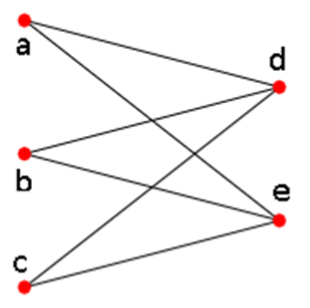
Example: Following graph is bipartite.



Complete Bipartite Graph

A complete bipartite graph is a bipartite graph in which each vertex in the first set is joined to every single vertex in the second set. The complete bipartite graph is denoted by $K_{m,n}$ where the graph G contains m vertices in the first set and n vertices in the second set.

Example: Draw the complete bipartite graph for m=3 and n=2.



Isomorphism of Graphs

Suppose G=(V,E) and G'=(V',E') are two graphs. A function f: V$\rightarrow$ V' is called a graph isomorphism if

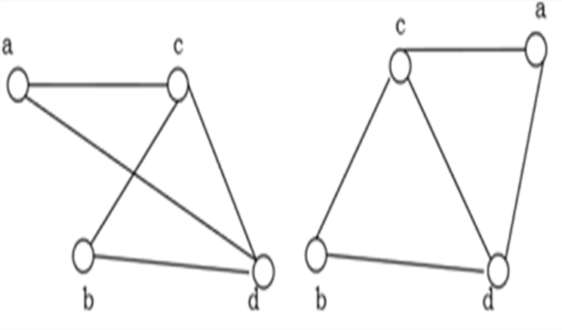
1. f is bijective.
2. For all a,b $\in$ V, (a,b)$\in$ E iff (f(a),f(b))$\in$ E'.

If such function exists then graph G and G' are said to be isomorphic to each other.

Conditions for Graph Isomorphism

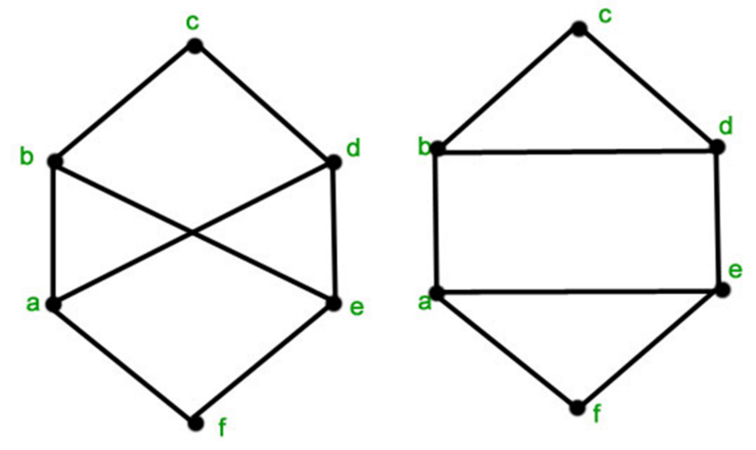
1. Both graphs G and G' must have the same number of vertices.
2. Both graphs G and G' must have the same number of edges.
3. Degree sequence of both graphs are same.

Example: Is the following graphs isomorphism?



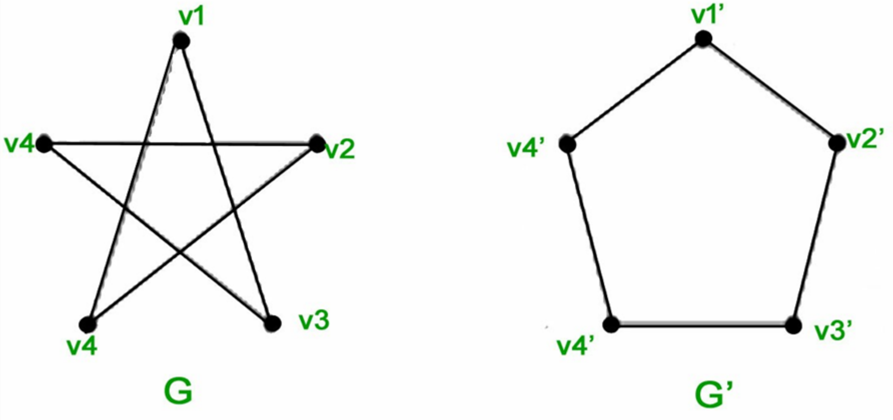
Solution:

Example: Is the following graphs isomorphism?



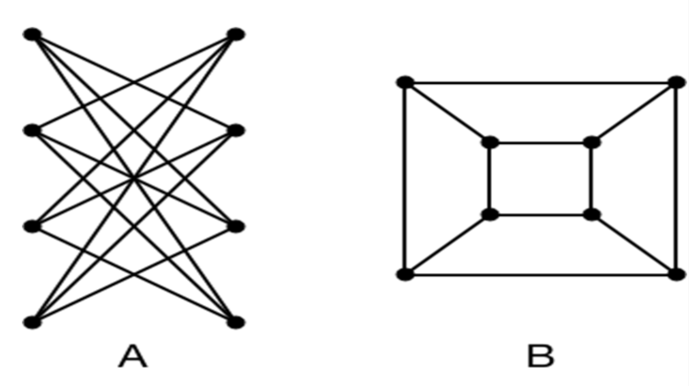
Solution:

Example: Is the following graphs isomorphism?



Solution:

Example: Is the following graphs isomorphism?



Solution:

Homomorphism of graph

Suppose G=(V,E) and G'=(V',E') are two graphs. A function f: V$\rightarrow$ V' is called a graph homomorphism if for all a,b $\in$ V, if (a,b)$\in$ E then (f(a),f(b))$\in$ E'.
If such function exists then graph G and G' are said to be homomorphic to each other.

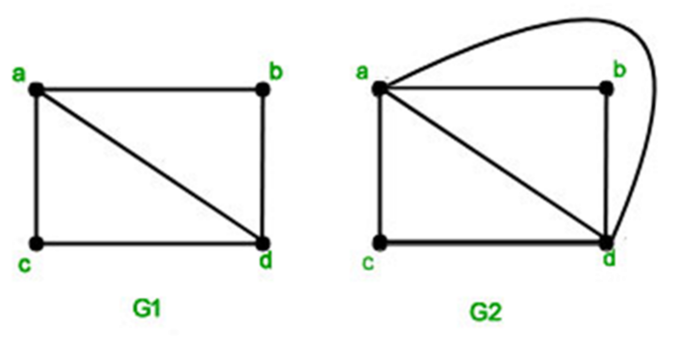
Euler Graphs

A graph G is called Euler graph if it contains an Euler cycle.

Euler cycle: An Euler cycle is a cycle which contains every edges of the graph and no edge is repeated.

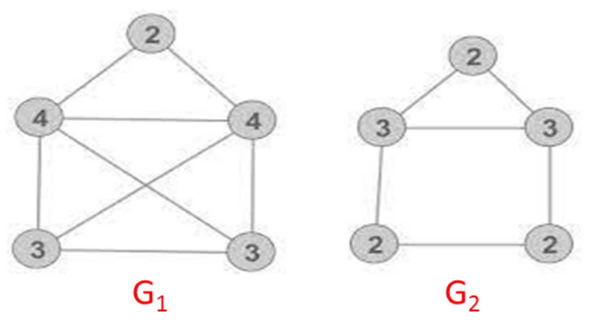
Euler path: A path is called an Euler path if it contains every edges of the graph and no edge is repeated.

Example: Which graphs shown below an Euler?



Solution:

Example: Which graphs shown below an Euler?



Solution:

Note: A graph is an Euler iff every vertex has an even degree.

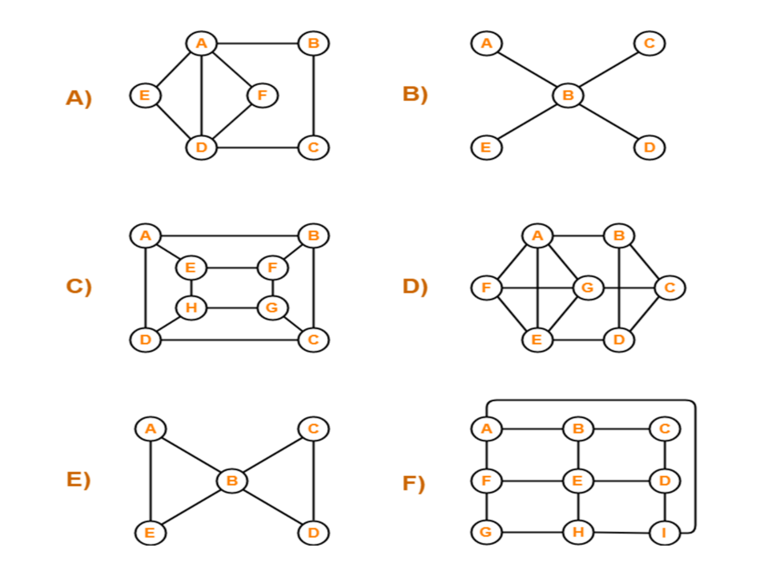
Hamiltonian Graphs

A graph G is called Hamiltonian graph if it contains an Hamiltonian cycle.

Hamiltonian cycle: An Hamiltonian cycle is a cycle which contains every vertex of the graph and no vertex is repeated.

Hamiltonian path: A path is called an Hamiltonian path if it contains every vertex of the graph and no vertex is repeated.

Example: Which graphs shown below a Hamiltonian?



Note: A simple connected graph G of order n$\geq$3 vertices is Hamiltonian if deg(v)$\geq$ n/2 for every v in G.

Note: Let G be a simple graph with n vertices and m edges where m is atleast 3. If $m \geq \frac{(n-1)(n-2)}{2} + 2$, then G is Hamiltonian.