United College of Engineering and Research, Allahabad

Department of Computer Science & Engineering

Question Bank

Discrete Structure and Theory of Logic (KCS-303)

Unit-1

Q. No.	Question	CO	Bloom's level
1	Define various types of functions.	CO1	L1
2	How many symmetric and reflexive relations are possible from a set A containing 'n' elements?	CO1	L3
3	Prove that $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+$................$+\frac{1}{\sqrt{n}}>\sqrt{n}$for n ≥ 2 using principle of mathematical induction	CO1	L3
4	Find the numbers between 1 to 500 that are not divisible by any of the integers 2 or 3 or 5 or 7.	CO1	L4
5	Is the "divides" relation on the set of positive integers transitive? What is the reflexive and symmetric closure of the relation? R = {(a, b) | a > b} on the set of positive integers?	CO1	L3
6	Determine the power set P(A) of A = {a, b, c, d}.	CO1	L2
7	Define surjective function.	CO1	L1
8	Let f and g be the functions from the set of integers to the set of integers defined by f (x) = 2x + 3 and g(x) = 3x + 2. What is the composition of f and g? What is the composition of g and f?	CO1	L2

9	Consider the following relations on set {1, 2, 3, 4}: R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}, R2 = {(1, 1), (1, 2), (2, 1)}, R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)}, R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}, R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)}, R6 = {(3, 4)}. Which of these relations are reflexive?	CO1	L2
10	List all the ordered pairs in the relationR = {(a, b) | a divides b} on the set {1, 2, 3, 4,5, 6} and also display the graphical representation of the same.	CO1	L2
11	Prove the proposition P(n) that the sum of the first n positive integers is $\frac{n(n+1)}{2}$; that is, P(n) = 1 + 2 + 3+ · · · +n = $\frac{n(n+1)}{2}$	CO1	L3
12	Determine whether each of these statements is true or false. i) $0 \in \emptyset$ ii) $\emptyset \in \{0\}$ iii) $\{0\} \subset \emptyset$ iv) $\emptyset \subset \{0\}$ v) $\{0\} \in \{0\}$ vi) $\{0\} \subset \{0\}$ vii) $\{\emptyset\} \subseteq \{\emptyset\}$	CO1	L2
13	For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is anti-symmetric, and whether it is transitive i. {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)} ii. {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)} iii. {(2, 4), (4, 2)} iv. {(1, 2), (2, 3), (3, 4)} v. {(1, 1), (2, 2), (3, 3), (4, 4)} vi. {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}	CO1	L4
14	Let A and B be sets. Show that AXB≠ BXA. Under what condition AXB=BXA?	CO1	L2
15	Let R be a binary relation on the set of all positive integers such that: R= { (a, b) | a-b is an odd positive integers} Is R reflexive ? Symmetric? Transitive?	CO1	L2
16	Define Multiset and Power set. Determine the power set A={1,2}.	CO1	L2
17	Let f: X→Y and X=Y=R, the set of real number. Find f^{-1}if	CO1	L2

	i. $fr(x)=x^2$ ii. $f(x)=(2x-1)/5$		
18	Prove by using mathematical induction that: 7+77+777+............+777...........7=7/81[10^{n+1}-9n-10] for every n ϵ N.	CO1	L4
19	Let R be a relation on R, the set of real numbers, such that R={(x,y)│Ix-yI<1}. Is R an equivalence relation? Justify.	CO1	L3
20	Let R be a relation on the set of natural numbers N, as R = {(x, y): x, y ∈ N, 3x + y = 19}. Find the domain and range of R. Verify whether R is reflexive.	CO1	L2
21	Show that the relation R on the set Z of integers given by R = {(a, b): divides a – b}, is an equivalence relation.	CO1	L3
22	Proof by induction: $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\ldots\ldots\ldots+\frac{1}{n.(n+1)} = \frac{n}{(n+1)}$	CO1	L4

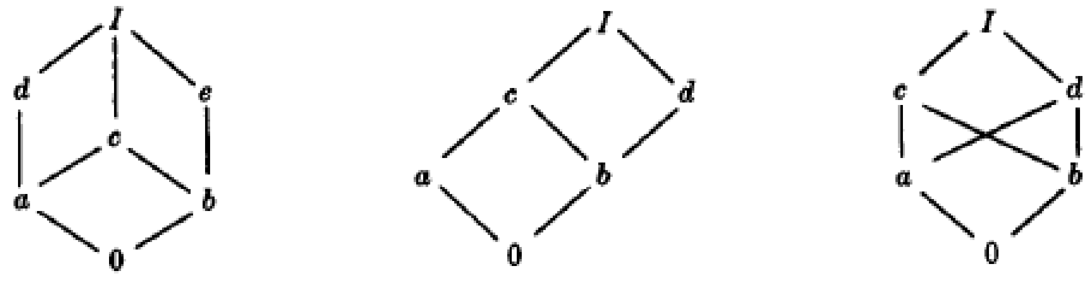
Unit-2

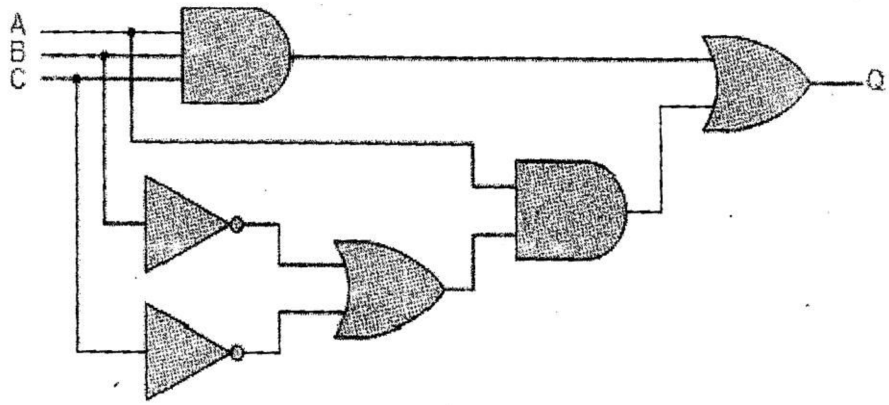
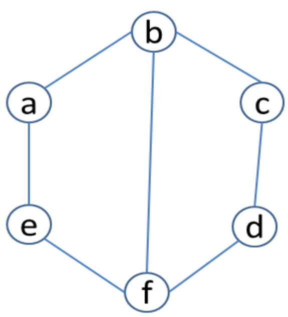
Q. No.	Question	CO	Bloom's level
1	Let Z be the group of integers with binary operation * defined by a*b = a + b − 2, for all a, b$\in$ Z . Find the identity element of the group (Z,*).	CO2	L3
2	Justify that "If a,b are the arbitrary elements of a group G then $(ab)^2 = a^2b^2$ if and only if G is abelian.	CO2	L2
3	What do you mean by cosets of a subgroup? Consider the group Z of integers under addition and the subgroup H = {…., -12, -6, 0, 6 12, ……} considering of multiple of 6 i. Find the cosets of H in Z ii. What is the index of H in Z.	CO2	L3
4	What is Ring? Define elementary properties of Ring with example.	CO2	L2
5	Prove or disprove that intersection of two normal subgroups of a groupG is again a normal subgroup of G.	CO2	L3
6	Consider the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7. (i) Find the multiplication table of G. (ii) Find 2^{-1}, 3^{-1}, 6^{-1}. (iii) Find the orders and subgroups generated by 2 and 3. (iv) Is G cyclic?	CO2	L3
7	Define the Subgroup of a group.	CO2	L1
8	Explain Cyclic group. Let H be a subgroup of a finite group G. Justify the statement "the order of H is a divisor of the order of G".	CO2	L3
9	Prove that (R,+,*) is a ring with zero divisors, where R is 2x2 matrix and + and * are usual addition and multiplication operations.	CO2	L3
10	Let (A,*) be a monoid such that for every x in A , x*x=e, where e is the identity element.Show that (A,*) is an abelian group.	CO2	L3
11	Define ring and give an example of a ring with zero divisors.	CO2	L2
12	State and prove Lagrange's theorem for group.	CO2	L3

13	Prove that every cyclic group is an abelian group.	CO2	L4
14	Obtain all distinct left cosets of { 0, 3 } in the group (Z_6, $+_6$) and find their union.	CO2	L2
15	Let (G,*) be a group, where * is usual multiplication operation on G. Show that for any a, b ϵ G, i. $(a^{-1})^{-1} = a$ ii. $(ab)^{-1} = b^{-1}a^{-1}$	CO2	L3
16	Prove that the set S={0,1,2,3} forms a ring under addition and multiplication modulo 4 but not a field.	CO2	L3
17	Define the following with suitable example: (i) Cyclic group (ii) Zero divisor of ring.	CO2	L2
18	Let G={1,-1,i,-i} be the multiplicative group, where i= $\sqrt{-1}$ i. Determine whether G is an abelian. ii. If G is a cyclic group, then determine generator of G.	CO2	L4
19	Prove that intersection of two subgroups is also a subgroup.	CO2	L3
20	Show that every group of order 3 is cyclic group.	CO2	L2

Unit-3

<table>
<tr><th>Q. No.</th><th>Question</th><th>CO</th><th>Bloom’s level</th></tr>
<tr><td>1</td><td>Prove that a lattice with 5 elements is not a Boolean algebra.</td><td>CO3</td><td>L3</td></tr>
<tr><td>2</td><td>Show that the following are equivalent in a Boolean algebra
$a \le b \Leftrightarrow a*b' = 0 \Leftrightarrow b' \le a' \Leftrightarrow a' \oplus b = 1$</td><td>CO3</td><td>L3</td></tr>
<tr><td>3</td><td>Let $(L,\vee,\wedge,\le)$ be a distributive lattice and $a, b \in L$. if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ then show that $b = c$</td><td>CO3</td><td>L3</td></tr>
<tr><td>4</td><td>Find the values of the Boolean function represented by
$F (x, y, z) = xy + z'$.</td><td>CO3</td><td>L3</td></tr>
<tr><td>5</td><td>Prerequisites in college is a familiar partial ordering of available classes. We write $A \prec B$ if course A is a prerequisite for course B. Let C be the ordered set consisting of the mathematics courses and their prerequisites appearing in figure.
<table>
<tr><th>Class</th><th>Prerequisites</th></tr>
<tr><td>Math 101</td><td>None</td></tr>
<tr><td>Math 201</td><td>Math 101</td></tr>
<tr><td>Math 250</td><td>Math 101</td></tr>
<tr><td>Math 251</td><td>Math 250</td></tr>
<tr><td>Math 340</td><td>Math 201</td></tr>
<tr><td>Math 341</td><td>Math 340</td></tr>
<tr><td>Math 450</td><td>Math 201, Math 250</td></tr>
<tr><td>Math 500</td><td>Math 450, Math 251</td></tr>
</table>
i. Draw the Hasse diagram for the partialordering C of these classes.
ii. Find all minimal and maximal elements of C.
iii. Does C have a first element or a last element?</td><td>CO3</td><td>L4</td></tr>
<tr><td>6</td><td>Answer these questions for the poset({3, 5, 9, 15,24, 45}, |).
i. Find the maximal elements. ii. Find the minimal elements.
iii. Is there a greatest element? iv. Is there a least element?
v. Find all upper bounds of {3, 5}.vi. Find the least upper bound of {3, 5}.</td><td>CO3</td><td>L4</td></tr>
</table>

	vii. Find all lower bounds of {15, 45}. viii.Find the greatest lower bound of {15, 45},if it exists.		
7	Which of the partially ordered sets in Fig are lattices?  i. ii. iii.	CO3	L2
8	Define a Partial Ordering .	CO3	L1
9	Describe the Boolean duality principle. Write the dual of each Boolean equations: i. x+x’y=x+y ii. (x.1)(0+x’)=0.	CO3	L2
10	Draw the Haase diagram of [p(a,b,c),≤], Find greatest element , least element ,minimal element & maximal element.	CO3	L3
11	Simplify the following Boolean function using three variables maps: i. f(x,y,z)=Σ(0,1,5,7) ii. f(x,y,z)=Σ(1,2,3,6,7)	CO3	L3
12	Show that the “greater than or equal” relation (>=) is a partial ordering on the set of integers.	CO3	L3
13	Distinguish between bounded lattice and complemented lattice.	CO3	L2
14	In a Lattice if a≤b≤c , then show that i. a∨b=b∧c ii. (a∨b)∨(b∧c) = (a∨b)∧ (a∨c) = b	CO3	L3
15	Prove that every finite subset of a lattice has an LUB and a GLB.	CO3	L3
16	Give an example of a lattice which is a modular butnot a distributive.	CO3	L2
17	Find the Boolean algebra expression for thefollowing system.	CO3	L2

			
18	Define a Boolean function of degree n. Simplify the following Boolean expression using K-map: $xyz + xy'z + x'y'z + x'yz + x'y'z'$	CO3	L3
19	Draw Hasse diagram on divisibility relation on the following set A = {3, 4, 12, 24, 48, 72}	CO3	L2
20	If the lattice is represented by the Hasse diagram given below  i. Find all the complements of ‘e’. ii. Prove that this lattice is bounded complemented lattice.	CO3	L3

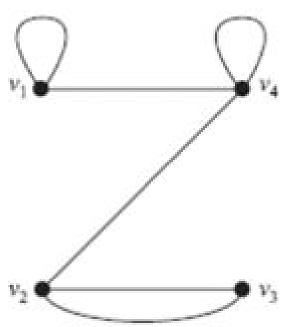
Unit-4

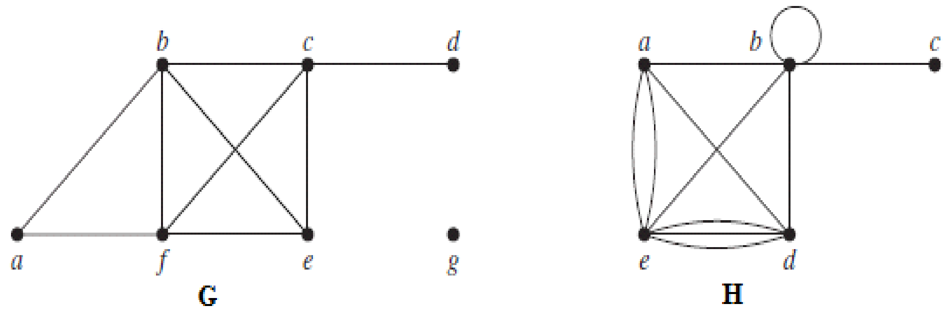
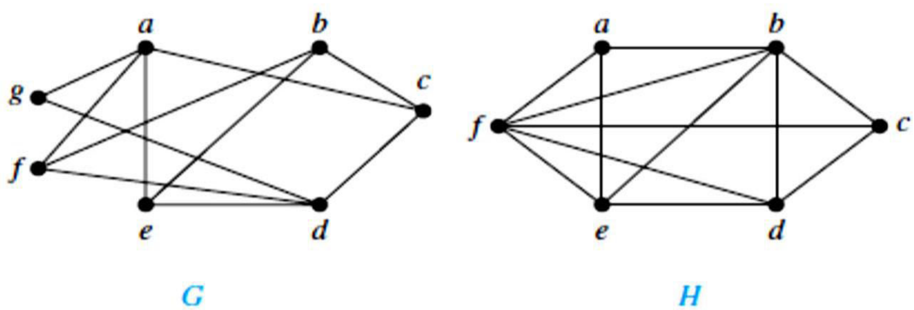
Q. No.	Question	CO	Bloom's level
1	Write the contra positive of the implication: "if it is Sunday then it is a holiday".	CO4	L2
2	Show that the propositions $p \rightarrow q$ *and* $\neg p \vee q$ are logically equivalent.	CO4	L2
3	Show that $((P \vee Q) \wedge \neg(\neg Q \vee \neg R)) \vee (\neg P \vee \neg Q) \vee (\neg P \vee \neg R)$ is a tautology by using equivalences.	CO4	L3
4	Obtain the principle disjunctive and conjunctive normal forms of the formula $(p \rightarrow r) \wedge (q \leftrightarrow p)$	CO4	L3
5	Explain various Rules of Inference for Propositional Logic.	CO4	L1
6	Prove the validity of the following argument "if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy.Therefore, the races are not fixed.	CO4	L3
7	Verify that the given propositions are tautology or not. i. $p \vee \neg (p \wedge q)$ ii. $\neg p \wedge q$	CO4	L2
8	Prove that $(P \vee Q) \rightarrow (P \wedge Q)$ is logically equivalent to $P \leftrightarrow Q$.	CO4	L3
9	Express this statement using quantifiers: "Every student in this class has taken some course in every department in the school ofmathematical sciences".	CO4	L3
10	Constructed the truth table for the following statements: i. $(P \rightarrow Q') \rightarrow P'$ ii. $P \leftrightarrow (P' \vee Q')$.	CO4	L3
11	Show the implications without constructing the truth table $(P \rightarrow Q) \rightarrow Q \Rightarrow P \vee Q$.	CO4	L3

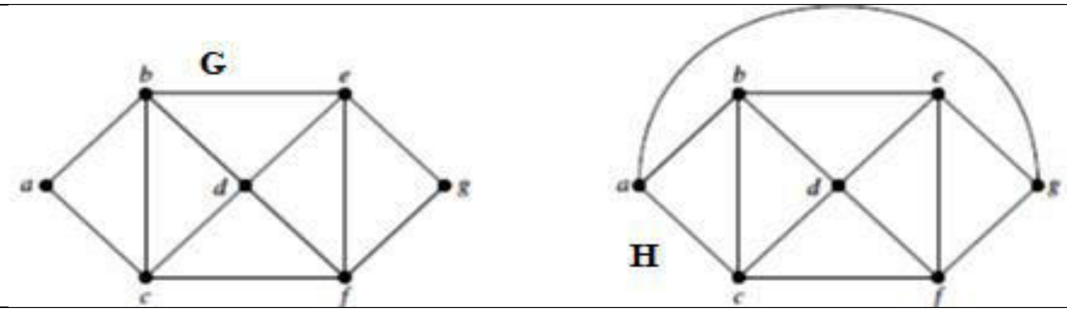
12	Write the symbolic form and negate the following statements : • Everyone who is healthy can do all kinds of work. • Some people are not admired by everyone. • Everyone should help his neighbors, or his neighborswill not help him.	CO4	L2
13	Differentiate between tautology and contradiction with suitable examples.	CO4	L2
14	Show that statements P→Q and Q' → P' are logically equivalent.	CO4	L2
15	Prove the validity of the following argument:- If I get the job and work hard then I will get promoted. If I will get promoted, then I will be happy. I will not be happy. Therefore I will not get the job or I will not work hard.	CO4	L3
16	The contra-positive of a statement S is given as " If x<2 then x+4 < 6" . Write the statement S and its converse.	CO4	L2
17	Show that $((P\vee Q)\wedge \sim(\sim P\wedge (\sim Q\vee\sim R)))\vee (\sim P\wedge \sim R)\vee (\sim P \vee R)$ is tautology without using truth table.	CO4	L4
18	Rewrite the following arguments using quantifiers, variables and predicate symbols. i. All birds can fly. ii. Some men are genius. iii. Some numbers are not rational. iv. There is a student who likes mathematics but not geography.	CO4	L3
19	"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it always the case that wages for such persons are not equal therefore the labour market is not perfect". Test the validity of this argument using truth table.	CO4	L3
20	Express the following statements using quantifiers and logical connectives. i. Mathematics book that is published in India has ablue cover. ii. AII animals are mortal. All human being are animal.Therefore, ail human being are mortar. iii. There exists a mathematics book with a cover thatis not	CO4	L3

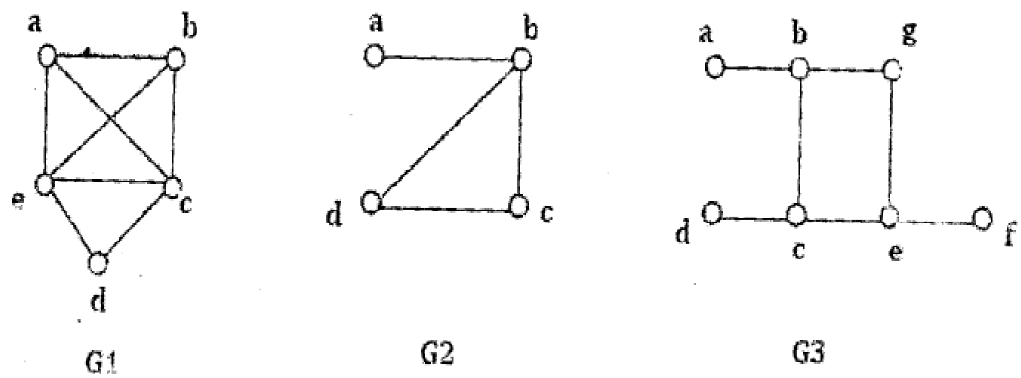
	blue. iv. He eats crackers only ifhe drinks milk. v. There are mathematics books that are publishedoutside India. vi. Not all books have bibliographies.		

Unit-5

Q. No.	Question	CO	Bloom's level
1	Show that there does not exist a graph with 5 vertices with degrees 1, 3,4, 2, 3 respectively.	CO5	L2
2	Obtain the generating function for the sequence 4, 4, 4, 4, 4, 4, 4.	CO5	L3
3	Define Pigeon-hole principle.	CO5	L1
4	Define planar graph. Prove that for any connected planar graph, v – e+ r = 2 Where v, e, r is the number of vertices, edges, and regions of the graph respectively.	CO5	L2
5	Solve the following recurrence equation using generating function G (K) -7 G (K-1) + 10 G (K-2) = 8K + 6	CO5	L4
6	A collection of 10 electric bulbs contain 3 defective ones (i) In how many ways can a sample of four bulbs be selected? (ii) In how many ways can a sample of 4 bulbs be selected which contain 2 good bulbs and 2 defective ones? (iii) In how many ways can a sample of 4 bulbs be selected so that either the sample contains 3 good ones and 1 defectives ones or 1 good and 3defectives ones?	CO5	L4
7	Draw all trees with exactly six vertices.	CO5	L2
8	Find the adjacency matrix A = [a_{ij}] of graph given in following figure:-  	CO5	L2
9	What are the degrees and what are the neighborhoods of the vertices in	CO5	L2

	the graphs G and H displayed in Figure ? 		
10	For which values of n do these graphs have an Euler path but no Euler circuit? i. K_n ii. C_n iii. W_n iv. Q_n	CO5	L3
11	Are the graphs G and H displayed in the figure bipartite? 	CO5	L3
12	Represent the expressions (x + xy) + (x/y) and x + ((xy + x)/y) using binary trees. Write these expressions in: i. Prefix notation. ii. Postfix notation. iii. Infix notation.	CO5	L3
13	Construct the ordered rooted tree whose preorder traversal is a, b, f, c, g, h, i, d, e, j, k, l, where a has four children, c has three children, j has two children, b and e have one child each, and all other vertices are leaves.	CO5	L4
14	What are the chromatic numbers of the graphs G and H shown in figure	CO5	L2

			
15	How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?	CO5	L3
16	What is a binary Search tree? Explain with example.	CO5	L2
17	Determine the value of each of their prefix expressions: i. -*2/933 ii. +-*335/↑232	CO5	L2
18	Define preorder, inorder and postorder tree traversal. Give an example of preorder, postorder&inorder traversal of a binary tree of your choice with at least 12 vertices.	CO5	L4
19	Solve the recurrence relation by the method of generating function. $a_r-7a_{r-1}+10a_{r-2}=0, r\geq 2$, Given $a_0=3$ and $a_1=3$.	CO5	L4
20	Find the recurrence relation from $y_n = A2^n + B(-3)^n$.	CO5	L3
21	State the applications of binary search tree.	CO5	L2
22	Define Multigraph. Explain with example in brief.	CO5	L2
23	Let G be a graph with 10 vertices. If 4 vertices has degree 4and 6 vertices has degree 5, then find the number of edges of G.	CO5	L3
24	Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.	CO5	L3
25	Solve the recurrence relation $y_{n+2} - 5y_{n+1} + 6y_n = 5^n$ subject to the condition $y_0 = 0$, $y_1 = 2$.	CO5	L3
26	Prove that a connected graph G is Euler graph if and only if every vertex of G is of even degree.	CO5	L4

27	Which of the following simple graph have a Hamiltonian circuit or, if not a Hamiltonian path? 	CO5	L2
28	Solve the recurrence relation using generating function: $a_n - 7a_{n-1} + 10a_{n-2} = 0$ with $a_0=3$, and $a_1=3$.	CO5	L4
29	Solve the recurrence relation $a_{r+2} - 5a_{r+1} + 6a_r = (r+1)^2$	CO5	L3
30	Explain the following terms with examples. i. Homomorphism and Isomorphism graphs ii. Euler and Hamiltonian Graph iii. Planar and Complete bipartite graph	CO5	L2