# Theory of Automata and Formal Language

## Lecture-37

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# Some additional problems

Construct TM for the following languages:-

(1) $L = \{ a^{n+2}b^n \mid n \geq 1\}$

(2) $L = \{ a^n b^n c^m \mid m, n \geq 0\}$

(3) $L = \{a^n b^n c^n \mid n \geq 1\}$

# Turing computable function

Def. A function f : $N^n \rightarrow N$ is said to be Turing computable function if there exist a Turing machine which compute this function.

Here $N^n = N \times N \times N \ldots\ldots\ldots\ldots\ldots\ldots\ldots \times N$(upto n times)

Note:

1) In the designing of Turing machine, we use unary number to represent a number. Here we use the unary number as a string of 1's.

Ex. 4 = 1111, 3 = 111 and so on.

2) If the function has multiple arguments, then we separate the arguments by 0.

Ex. Construct Turing machine for the following function

1) $f(n) = n+2$ $\quad n \in N$

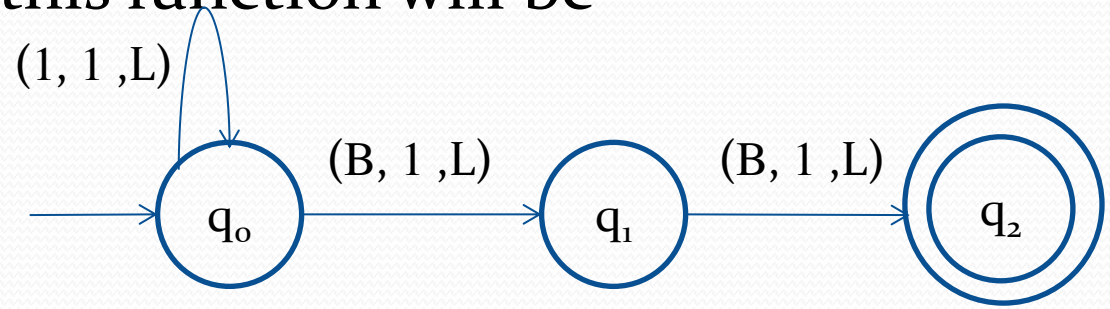
2) $f(m,n) = m+n$ $\quad m, n \in N$

Solution:

1) In this function, if input is 111 then output will be 11111.

i.e. $q_0 1111 \vdash^* q_f 111111$

TM for this function will be




Computation by this machine

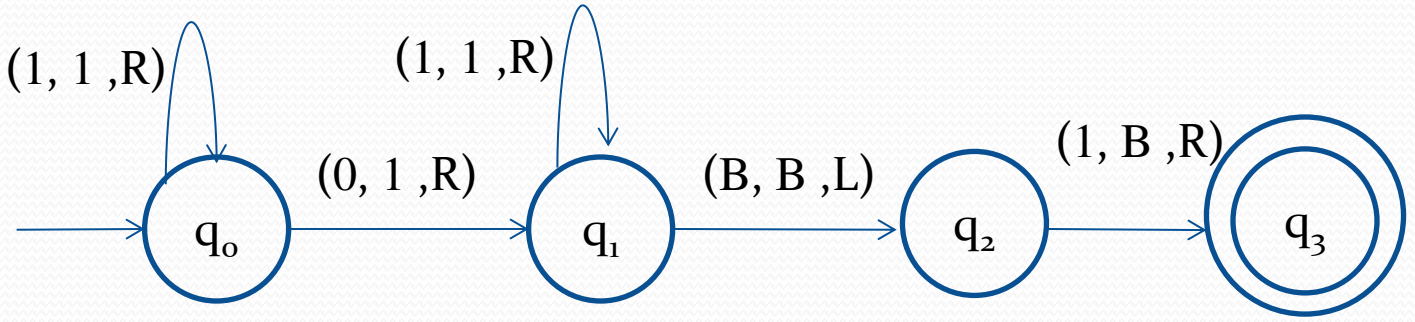
$q_0 1111 \vdash q_0 B111 \vdash q_1 B1111 \vdash q_2 B11111$ (machine halt at final state)

## (2) f(m,n) = m+n  m, n ∈ N

Solution:

In this function if the input is 110111 then output will be 11111.

TM for this function will be




Computation by this machine

$q_0110111 \vdash 1q_010111 \vdash 11q_00111 \vdash 111q_1111 \vdash 1111q_111 \vdash 11111q_11 \vdash 111111q_1B \vdash 11111q_21\ B \vdash 11111Bq_3B$ (machine halt at final state)

Ex. Show that following function is Turing computable:-

$f(n) = 3*n \qquad n \geq 1$

Solution:

A function is said to be Turing computable if there exist a TM for this.

Therefore, we shall construct TM for this function.

In this function, if the input is 2 then output will be 6. That is if input is 11 then output will be 111111.

First, we shall show that how string 11 is converted into 111111. After this, we construct Turing machine for this.

Suppose initial state is $q_o$.

$q_o11 \vdash yq_11 \vdash y1q_1B \vdash y1Bq_2B \vdash y1B1q_3B \vdash y1B11q_4B \vdash y1B1q_511$
$\vdash y1Bq_5111 \vdash y1q_5B111 \vdash yq_51B111 \vdash q_5y1B111 \vdash yq_01B111$
$\vdash yyq_1B111 \vdash yyBq_2111 \vdash yyB1q_211 \vdash yyB11q_21 \vdash yyB111q_2B$
$\vdash yyB1111q_3B \vdash yyB11111q_4B \vdash yyB1111q_511B \vdash yyB111q_5111B$
$\vdash yyB11q_51111B \vdash yyB1q_511111B \vdash yyBq_5111111B$
$\vdash yyq_5B111111B \vdash yq_5yB111111B \vdash yyq_0B111111B \vdash yyBq_6111111B$

Therefore, the Turing machine corresponding above function is constructed as following:-