Theory of Automata and Formal Language

Lecture-38

Dharmendra Kumar
(Associate Professor)
Department of Computer Science and Engineering United College of Engineering and Research, Prayagraj March 30, 2021

Ex. Show that following function is Turing computable:-

$$f(m, n) = m-n \qquad m \geq n$$
$$= 0 \qquad \text{otherwise}$$

Solution: Clearly this function is proper subtraction function.

We have to find TM corresponding to this function.

There are two cases of this function.

Case-1: If $m \geq n$ then value of the function is m-n. i.e. if m= 6 and n=4 then value = 2.

Case-2: If $m < n$ then value of the function is 0. i.e. if m= 4 and n=6 then value = 0.

Before constructing TM for this function, first we process the input and develop rules through which machine move from initial ID to final ID.

Case-1: when m ≥ n.

$q_0 1111011 \vdash 1q_0 111011 \vdash 11q_0 11011 \vdash 111q_0 1011 \vdash$
$1111q_0 011 \vdash 11110q_1 11 \vdash 1111q_2 0y1 \vdash 111q_2 10y1 \vdash$
$111yq_0 0y1 \vdash 111y0q_1 y1 \vdash 111y0yq_1 1 \vdash 111y0q_2 yy \vdash 111yq_2 0yy$
$\vdash 111q_2 y0yy \vdash 11q_2 1y0yy \vdash 11yq_0 y0yy \vdash 11yyq_0 0yy$

$\vdash 11yy0q_1 yy \vdash 11yy0yq_1 y \vdash 11yy0yyq_1 B \vdash 11yy0yq_3 yB$

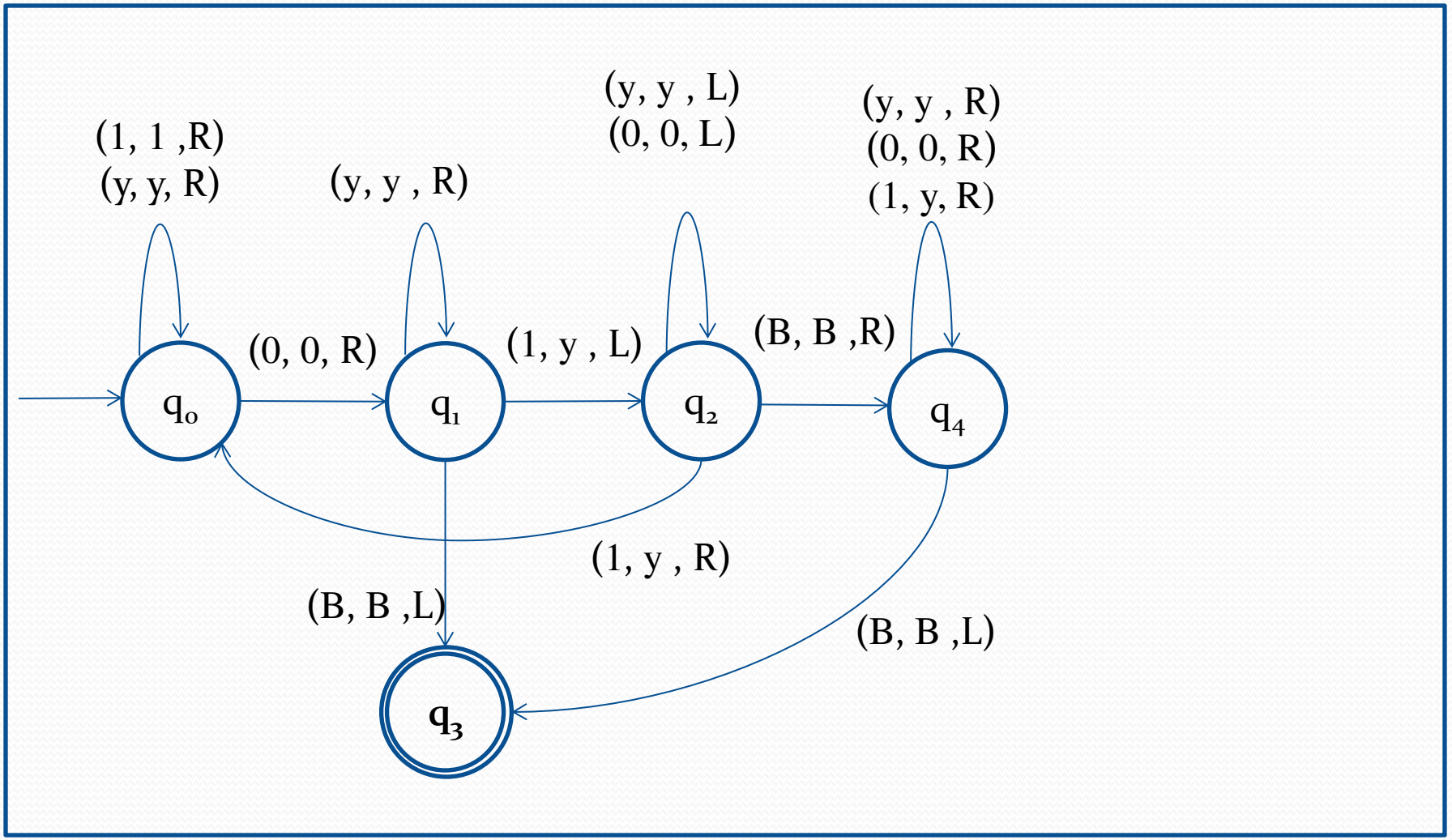
(machine halts at final state)

Case-2: when m < n.

$q_0 1101111 \vdash^* yy0yyq_1 11 \vdash yy0yq_2 yy1 \vdash^* q_2 Byy0yyy1 \vdash$
$Bq_4 yy0yyy1 \vdash^* Byy0yyyq_4 1 \vdash Byy0yyyyq_4 B \vdash Byy0yyyq_3 yB$

(machine halts at final state)

Therefore, the TM corresponding this function will be constructed as following:-



Ex. Construct Turing machine for the following function

$f(m,n) = m*n \qquad m, n \in N$

Solution: This function multiply two numbers.

If inputs are 2 and 3 then output will be 6.

Processing:

$q_0110111 \vdash xq_110111 \vdash x1q_10111 \vdash x10q_2111 \vdash x10yq_211 \vdash$

$x10yyq_21 \vdash x10yyyq_2B \vdash x10yyq_3yB \vdash x10yy1q_4B \vdash x10yy1B\ q_5B \vdash$
$x10yy1q_6B1 \vdash x10yyq_31B1 \vdash x10yq_3y1B1$

$\vdash x10y1q_41B1 \vdash x10y11q_4B1 \vdash x10y11Bq_51 \vdash x10y11B1q_5B \vdash$
$x10y11Bq_611 \vdash x10y11q_6B11 \vdash x10y1q_31B11 \vdash x10yq_311B11 \vdash$
$x10q_3y11B11 \vdash x101q_411B11 \vdash x1011q_41B11 \vdash x10111q_4B11 \vdash$
$x10111Bq_511 \vdash x10111B1q_51 \vdash x10111B11q_5B \vdash x10111B1q_611 \vdash$
$x10111Bq_6111 \vdash x10111q_6B111 \vdash x1011q_31B111 \vdash x101q_311B111 \vdash$
$x10q_3111B111 \vdash x1q_30111B111 \vdash xq_310111B111 \vdash q_3x10111B111 \vdash$
$xq_010111B111 \vdash *xxq_00111B111111 \vdash xxBq_7111B111111$
$\vdash xxBBq_711B111111 \vdash xxBBBq_71B111111 \vdash xxBBBBq_7B111111$
$\vdash xxBBBBBq_8111111$ (machine halts at final state)

Therefore, the TM corresponding this function will be constructed as following:-