Discrete Structures and Theory of Logic

Lecture-25

Dr. Dharmendra Kumar
(Associate Professor)
United College of Engineering and Research, Prayagraj

October 2, 2023

Some other connectives

NAND Connective

It is denoted by $\uparrow$.

$$P \uparrow Q \Leftrightarrow \neg(P \wedge Q)$$

Truth table for this is the following

P	Q	$P \uparrow Q$
T	T	F
T	F	T
F	T	T
F	F	T

NOR Connective

It is denoted by $\downarrow$.

$$P \downarrow Q \Leftrightarrow \neg(P \vee Q)$$

Truth table for this is the following

P	Q	$P \downarrow Q$
T	T	F
T	F	F
F	T	F
F	F	T

Example: Express the connectives $\neg, \wedge and \vee$ in the terms of $\uparrow$ only.

Solution:

1. $\neg P \Leftrightarrow \neg P \vee \neg P \Leftrightarrow \neg(P \wedge P) \Leftrightarrow P \uparrow P$
2. $P \wedge Q \Leftrightarrow \neg\neg(P \wedge Q) \Leftrightarrow \neg(P \uparrow Q) \Leftrightarrow (P \uparrow Q) \uparrow (P \uparrow Q)$
3. $P \vee Q \Leftrightarrow \neg\neg(P \vee Q) \Leftrightarrow \neg(\neg P \wedge \neg Q) \Leftrightarrow (\neg P) \uparrow (\neg Q) \Leftrightarrow (P \uparrow P) \uparrow (Q \uparrow Q)$

Example: Express the connectives $\neg, \wedge and \vee$ in the terms of $\downarrow$ only.

Solution:

1. $\neg P \Leftrightarrow \neg P \wedge \neg P \Leftrightarrow \neg(P \vee P) \Leftrightarrow P \downarrow P$
2. $P \vee Q \Leftrightarrow \neg\neg(P \vee Q) \Leftrightarrow \neg(P \downarrow Q) \Leftrightarrow (P \downarrow Q) \downarrow (P \downarrow Q)$
3. $P \wedge Q \Leftrightarrow \neg\neg(P \wedge Q) \Leftrightarrow \neg(\neg P \vee \neg Q) \Leftrightarrow (\neg P) \downarrow (\neg Q) \Leftrightarrow (P \downarrow P) \downarrow (Q \downarrow Q)$

Note: NAND or NOR is functionally complete.

Exercise

1. Express $P \to (\neg P \to Q)$ in terms of $\uparrow$ only. Express same formula in terms of $\downarrow$ only.
2. Express $P \uparrow Q$ in terms of $\downarrow$ only.
3. Show the following:-
   (a) $\neg(P \uparrow Q) \Leftrightarrow \neg P \downarrow \neg Q$
   (b) $\neg(P \downarrow Q) \Leftrightarrow \neg P \uparrow \neg Q$
4. Write a formula which is equivalent to the formula
   $$P \wedge (Q \leftrightarrow R)$$
   and contains the connective NAND($\uparrow$) only. Obtain an equivalent formula which contains the connective NOR($\downarrow$) only.
5. Show the following equivalences.
   (a) $(P \to C) \wedge (Q \to C) \Leftrightarrow (P \vee Q) \to C$
   (b) $((Q \wedge A) \to C) \wedge (A \to (P \vee C)) \Leftrightarrow (A \wedge (P \to Q)) \to C$
   (c) $((P \wedge Q \wedge A) \to C) \wedge (A \to (P \vee Q \vee C)) \Leftrightarrow (A \wedge (P \leftrightarrow Q)) \to C$

Normal Form

There are following types of normal form.

Disjunctive normal form A statement formula is said to be in disjunctive normal form if it is the disjunction of conjunction.
Example: $(P \wedge Q) \vee (\neg Q \wedge R)$

Conjunctive normal form A statement formula is said to be in conjunctive normal form if it is the conjunction of disjunction.
Example: $(P \vee Q) \wedge (\neg Q \vee R)$

Principal disjunctive normal form A statement formula is said to be in principal disjunctive normal form if it is the disjunction of minterms only.
Example: $(P \wedge Q) \vee (\neg P \wedge Q)$

Principal conjunctive normal form A statement formula is said to be in principal conjunctive normal form if it is the conjunction of maxterms only.
Example: $(P \vee Q) \wedge (\neg P \vee Q)$

Exercise

1. Obtain disjunctive normal form of the followings:-
   (a) $P \wedge (P \rightarrow Q)$
   (b) $\neg(P \vee Q) \leftrightarrow (P \wedge Q)$
   Also find the conjunctive normal form of above formulas.
2. Obtain the principal disjunctive normal form of the followings:-
   (a) $\neg P \vee Q$
   (b) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$
   (c) $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$
3. Obtain the principal conjunctive normal form of the followings:-
   (a) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$
   (b) $P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$
   (c) $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$