

Discrete Structures and Theory of Logic

Lecture-24

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Duality law

Two formulas P and Q are said to be dual of each others if either one can be obtained from the other by replacing \wedge by \vee and \vee by \wedge . The connectives \wedge and \vee are also called dual of each other. If the formula P contains the special symbols T or F , then Q , its dual, is obtained by replacing T by F and F by T .

Example: Dual of $(P \wedge Q) \vee T$ is $(P \vee Q) \wedge F$.

Converse, Inverse and Contrapositive

For any statement formula $P \rightarrow Q$, the statement formula $Q \rightarrow P$ is called the converse, $\neg P \rightarrow \neg Q$ is called its inverse and $\neg Q \rightarrow \neg P$ is called its contrapositive.

Note: $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$ and $Q \rightarrow P \Leftrightarrow \neg P \rightarrow \neg Q$

Tautological Implication

A statement A is said to be tautologically imply a statement B iff $A \rightarrow B$ is tautology. We shall denote this by $A \Rightarrow B$, which is read as "A implies B".

$A \Rightarrow B$ guarantees that B has the truth value T whenever A has the truth value T.

Exercise

1. Show the following implications.

(a) $(P \wedge Q) \Rightarrow (P \rightarrow Q)$

(b) $P \Rightarrow (Q \rightarrow P)$

(c) $(P \rightarrow (Q \rightarrow R)) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$

2. Show the following equivalences.

(a) $P \rightarrow (Q \rightarrow P) \Leftrightarrow \neg P \rightarrow (P \rightarrow \neg Q)$

(b) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

(c) $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$

(d) $\neg(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$

Exercise

3. Show the following implications without constructing truth tables.

(a) $(P \rightarrow Q) \Rightarrow P \rightarrow (P \wedge Q)$

(b) $(P \rightarrow Q) \rightarrow Q \Rightarrow (P \vee Q)$

(c) $((P \vee \neg P) \rightarrow Q) \rightarrow ((P \vee \neg P) \rightarrow R) \Rightarrow (Q \rightarrow R)$

(d) $(Q \rightarrow (P \wedge \neg P)) \rightarrow (R \rightarrow (P \wedge \neg P)) \Leftrightarrow (R \rightarrow Q)$

Formulas with distinct truth tables

A statement formula containing n variables must have as its truth table one of the 2^{2^n} possible truth tables, each of them having 2^n rows.

Functionally complete set of connectives

A set of connectives by which every formula can be expressed in terms of an equivalent formula containing the connectives from this set, is called a functionally complete set of connectives.

Minimal functionally complete set:

A functional complete set is said to be minimal functionally complete set if its proper subset is not functionally complete.

Example:

1. Are the sets $\{\wedge, \vee, \neg\}$, $\{\wedge, \neg\}$, *and* $\{\vee, \neg\}$ functionally complete?
2. Is the set $\{\wedge, \vee, \neg\}$ minimal functionally complete?
3. Are the sets $\{\wedge, \neg\}$, and $\{\vee, \neg\}$ minimal functionally complete?
4. Are the sets $\{\neg\}$, $\{\wedge\}$, $\{\vee\}$ *and* $\{\wedge, \vee\}$ functionally complete?

Example: Write the formulas which are equivalent to the formulas given below and which contain the connectives \wedge and \neg .

1. $\neg(p \leftrightarrow (Q \rightarrow (R \vee P)))$
2. $((P \vee Q) \wedge R) \rightarrow (P \vee R)$