

Discrete Structures and Theory of Logic

Lecture-2

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Operations defined on set

Union operation

For any two sets A and B, the union of A and B is the set of all the elements which are belongs into A or B or both. It is denoted by $A \cup B$. Mathematically, it is defined as following:-

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, c \}$. Then union of A and B will be, $A \cup B = \{ a, b, c, d, e \}$.

Intersection operation

For any two sets A and B, the intersection of A and B is the set of all the elements which are belong into both A and B . It is denoted by $A \cap B$. Mathematically, it is defined as following:-

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, c \}$. Then intersection of A and B will be, $A \cap B = \{ c \}$.

Operations defined on set(cont.)

Set difference operation

For any two sets A and B, the set difference of A and B is the set of all the elements which are belong into A but not belong into B . It is denoted by A-B. Mathematically, it is defined as following:-

$$A-B = \{ x \mid x \in A \text{ and } x \notin B \}$$

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, c \}$. Then set difference of A and B will be, $A-B = \{a, b\}$.

Complement operation

Let U is the universal set. For any set A, the complement of A is the set of all the elements U, which are not belong into A. It is denoted by A^c or A' . Mathematically, it is defined as following:-

$$A' = U-A = \{ x \mid x \in U \text{ and } x \notin A \}$$

Example: Let $A = \{ a, b, c \}$ and $U = \{ a, b, c, d, e, f, g \}$. Then complement of A will be, $A' = \{ d, e, f, g \}$.

Operations defined on set(cont.)

Symmetric difference operation

For any two sets A and B, the symmetric difference of A and B is denoted by $A \oplus B$. Mathematically, it is defined as following:-

$$A \oplus B = (A - B) \cup (B - A)$$

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, c \}$. Then symmetric difference of A and B will be, $A \oplus B = \{ a, b, d, e \}$.

Disjoint and Mutually disjoint sets

Disjoint sets

Two sets A and B are said to be disjoint if there is no common elements between A and B . That is, A and B are disjoint iff $A \cap B = \phi$.

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, f \}$. Here A and B are disjoint because $A \cap B = \phi$.

Mutually disjoint sets

A collection of sets $S = \{ A_1, A_2, \dots, A_n \}$ is said to be mutually disjoint if each pair of A_i and A_j in S are disjoint. That is, S is mutually disjoint if $A_i \cap A_j = \phi$, $\forall i, j = 1, 2, \dots, n$. and $i \neq j$.

Example: Let $A = \{ \{1, 2\}, \{3\} \}$, $B = \{ \{1\}, \{2, 3\} \}$ and $C = \{ \{1, 2, 3\} \}$. These sets A , B and C are mutually disjoint because $A \cap B = \phi$, $B \cap C = \phi$, and $A \cap C = \phi$.

Some examples

Example: Show that $A \subseteq B \Leftrightarrow A \cap B = A$

Solution: In this question, we have to prove two parts.

First part: In this part, we have to show that if $A \subseteq B$ then $A \cap B = A$.

Suppose $A \subseteq B$.

Let $x \in A$. Since $A \subseteq B$ therefore $x \in B$. Clearly x is belong into both A and B . Therefore x also belongs into $A \cap B$. Therefore

$$A \subseteq A \cap B \dots\dots\dots(1)$$

Let $x \in A \cap B$. Therefore $x \in A$ and $x \in B$. Therefore we can say $x \in A$. Therefore

$$A \cap B \subseteq A \dots\dots\dots(2)$$

Using equations(1) and (2), $A \cap B = A$.

Some examples(cont.)

Second part: In this part we have to show that if $A \cap B = A$ then $A \subseteq B$.

Let $x \in A$. Since $A \cap B = A$ therefore $x \in A \cap B$. This imply that $x \in A$ and $x \in B$. Therefore we can say $x \in B$. Therefore

$$A \subseteq B.$$

Some examples(cont.)

Example: Show that

(a) $A-B = A \cap B'$

(b) $A \subseteq B \Leftrightarrow B' \subseteq A'$

Solution:

(a) Let $x \in A-B \Rightarrow x \in A$ and $x \notin B$
 $\Rightarrow x \in A$ and $x \in B'$
 $\Rightarrow x \in A \cap B'$

Therefore, $A-B \subseteq A \cap B'$ (1)

Now, let $x \in A \cap B' \Rightarrow x \in A$ and $x \in B'$
 $\Rightarrow x \in A$ and $x \notin B$
 $\Rightarrow x \in A-B$

Therefore, $A \cap B' \subseteq A-B$ (2)

Using equations (1) and (2), $A-B = A \cap B'$

Some examples(cont.)

(b) First part: Suppose $A \subseteq B$.

Let let $x \in B' \Rightarrow x \notin B$

$\Rightarrow x \notin A$ (Since $A \subseteq B$)

$\Rightarrow x \in A'$

Therefore, $B' \subseteq A'$

Second part: Suppose $B' \subseteq A'$.

Let let $x \in A \Rightarrow x \notin A'$

$\Rightarrow x \notin B'$ (Since $B' \subseteq A'$)

$\Rightarrow x \in B$

Therefore, $A \subseteq B$

Using first and second parts, we can say that

$$A \subseteq B \Leftrightarrow B' \subseteq A'$$

Some examples(cont.)

Example: Show that for any two sets A and B,

$$A-(A \cap B) = A-B$$

Solution: Let $x \in A-(A \cap B) \Leftrightarrow x \in A$ and $x \notin A \cap B$

$$\Leftrightarrow x \in A \text{ and } x \in (A \cap B)'$$

$$\Leftrightarrow x \in A \text{ and } (x \notin A \text{ or } x \notin B)$$

$$\Leftrightarrow (x \in A \text{ and } x \notin A) \text{ or } (x \in A$$

and $x \notin B)$

$$\Leftrightarrow \text{FALSE or } (x \in A \text{ and } x \notin B)$$

$$\Leftrightarrow (x \in A \text{ and } x \notin B)$$

$$\Leftrightarrow x \in A-B$$

Therefore, $A-(A \cap B) = A-B$