

# Discrete Structures and Theory of Logic

## Lecture-18

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September 18, 2023

## Exercise(cont.)

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Let  $I$  be the set of integers,  $I_+$  the set of positive integers, and  $I_p = \{0, 1, 2, 3, \dots, (p-1)\}$ . Determine which of the following functions are one-one, onto and bijective.

1.  $f: I \rightarrow I$ ,  
$$f(j) = \begin{cases} (j-1)/2, & \text{if } j \text{ is odd} \\ j/2, & \text{if } j \text{ is even} \end{cases}$$
2.  $f: I_+ \rightarrow I_+$ ,  
 $f(x) = \text{greatest integer} \leq \sqrt{x}$
3.  $f: I_7 \rightarrow I_7$ ,  
 $f(x) = 3x \bmod 7$
4.  $f: I_4 \rightarrow I_4$ ,  
 $f(x) = 3x \bmod 4$

## Exercise(cont.)

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1. List all possible functions from  $X = \{a,b,c\}$  to  $Y = \{0,1\}$  and indicate in each case whether the function is one-one, onto and bijective.
2. Show that the functions  $f$  and  $g$  which both are from  $\mathbb{N} \times \mathbb{N}$  to  $\mathbb{N}$  given by  $f(x,y) = x+y$  and  $g(x,y) = xy$  are onto but not one-one.

## Composition of functions

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Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be two functions. Then composition of  $f$  and  $g$  is denoted by  $\text{gof}$ . It is defined as  $\text{gof} : X \rightarrow Z$ .

$$(\text{gof})(x) = g(f(x))$$

**Note:**  $\text{gof} \neq \text{fog}$ .

**Example:** Let  $X = \{1,2,3\}$ ,  $Y = \{p,q\}$ , and  $Z = \{a,b\}$ . Also let  $f$  is a function from  $X$  to  $Y$  such that  $f = \{(1,p), (2,p), (3,q)\}$  and  $g$  is a function from  $Y$  to  $Z$  such that  $g = \{(p,b), (q,b)\}$ . Find  $\text{gof}$ .

**Solution:**  $\text{gof} = \{(1,b), (2,b), (3,b)\}$

## Composition of functions

**Example:** Let  $X = \{1,2,3\}$ , and  $f$ ,  $g$ ,  $h$  and  $s$  be functions from  $X$  to  $X$  given by

$f = \{(1,2), (2,3), (3,1)\}$ ,  $g = \{(1,2), (2,1), (3,3)\}$ ,  $h = \{(1,1), (2,2), (3,1)\}$ , and  $s = \{(1,1), (2,2), (3,3)\}$

Find  $f \circ g$ ,  $g \circ f$ ,  $f \circ h \circ g$ ,  $s \circ g$ ,  $g \circ s$ ,  $s \circ s$  and  $f \circ s$ .

**Solution:**

$f \circ g = \{((1,3), (2,2), (3,1))\}$

$g \circ f = \{((1,1), (2,3), (3,2))\}$

$f \circ h \circ g = \{((1,1), (2,2), (3,2))\}$

Similarly, we can calculate others.

**Example:** Let  $f(x) = x+2$ ,  $g(x) = x-2$ , and  $h(x) = 3x$ ,  $\forall x \in \mathbb{R}$ , where  $\mathbb{R}$  is the set of real numbers. Find  $g \circ f$ ,  $f \circ g$ ,  $f \circ f$ ,  $h \circ g$  and  $f \circ h \circ g$ .

# Composition of functions

## Solution:

$$\text{gof}(x) = g(f(x)) = g(x+2) = x+2-2 = x$$

$$\text{fog}(x) = f(g(x)) = f(x-2) = x-2+2 = x$$

$$\text{fohog}(x) = f(h(g(x))) = f(h(x-2)) = f(3(x-2)) = 3(x-2)+2 = 3x-4$$

Similarly, we can calculate others.

**Example:** Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = -x^2$  and  $g: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  be given by  $g(x) = \sqrt{x}$ , where  $\mathbb{R}_+$  is the set of positive real numbers and  $\mathbb{R}$  is the set of real numbers. Find  $\text{fog}$ . Is  $\text{gof}$  defined?

## Solution:

$$\text{fog}(x) = f(g(x)) = f(\sqrt{x}) = -(\sqrt{x})^2 = -x$$

$\text{gof}$  can not be defined because square root of negative real number can not be a real number.