Discrete Structures and Theory of Logic

Lecture-27

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Example: Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

Solution:

(1) $P \rightarrow M$ By rule P

(2) $\neg M$ By rule P

(3) $\neg P$ By rule T, (1), (2) and modus tollens

(4) $P \vee Q$ By rule P

(5) Q By rule T, (3), (4) and disjunctive syllogism

(6) $Q \rightarrow R$ By rule P

(7) R By rule T, (3), (4) and modus ponens

(8) $R \wedge (P \vee Q)$ By rule T, (4), (7) and conjunction

Example: Show that $R \to S$ can be derived from the premises $\neg R \vee P$, $P \to (Q \to S)$, and Q.

Solution:

(1) $P \to (Q \to S)$

(2) $\neg R \vee P$

(3) $R \to P$

(4) $R \to (Q \to S)$

(5) $\neg R \vee \neg Q \vee S$

(6) $Q \to (\neg R \vee S)$

(4) Q

(5) $\neg R \vee S$

(9) $R \to S$

Example: "If there was a ball game, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time. Therefore there was no ball game." Show that these statements constitute a valid argument.

Solution:

Let

P: There was a ball game.

Q: Traveling was difficult.

R: They arrived on time.

Therefore sentences in symbolic form will be:

Premises: $P \rightarrow Q$, $R \rightarrow \neg Q$, R and Conclusion: $\neg P$

(1) $R \rightarrow \neg Q$

(2) R

(3) $\neg Q$

(4) $P \rightarrow Q$

(5)$\neg P$

Rule CP: If we can derive S from r and a set of premises, then we can derive $R \to S$ from the set of premises alone.

Example: If A works hard, then either B or C will enjoy themselves. If B enjoy himself, then A will not work hard. If D enjoys himself, then C will not. Therefore, if A works hard, then D will not enjoy himself.

Solution:

Let

A: A works hard.

B: B will enjoy himself.

C: C will enjoy himself.

D: D will enjoy himself.

Therefore sentences in symbolic form will be:

Premises: $A \to (B \vee C)$, $B \to \neg A$, $D \to \neg C$ and Conclusion: $A \to \neg D$

Mathematical Logic

(1) A

(2) $A \to B \vee C$

(3) $B \vee C$ (4) $\neg C \to B$

(5) $D \to \neg C$

(6) $D \to B$

(7) $B \to \neg A$

(8) $D \to \neg A$

(9) $A \to \neg D$

(10)$\neg D$

Consistency of premises and Indirect method of proof

Consistency of premises

- A set of premises $H_1, H_2, ..., H_m$ is said to be consistent if their conjunction has the truth value T for some assignment of the truth values of the atomic variables appearing in $H_1, H_2, ..., H_m$.
- If for every assignment of the truth values to the atomic variables, at least one of the formulas $H_1, H_2, ..., H_m$ is false, so that their conjunction is identically false, then the formulas $H_1, H_2, ..., H_m$ are called inconsistent.
- Alternatively, a set of formulas $H_1, H_2, ..., H_m$ is inconsistent if their conjunction implies a contradiction, that is, $H_1 \wedge H_2 \wedge .... \wedge H_m \Rightarrow R \wedge \neg R$, Where R is any formula.

Indirect method of proof

- In order to show that a conclusion C follows logically from the premises $H_1, H_2, ..., H_m$, we assume that C is false and $\neg C$ as an additional premises.
- If the new set of premises is inconsistent, then the assumption that $\neg C$ is true does not hold simultaneously with $H_1, H_2, ..., H_m$ being true. Therefore, C is true whenever $H_1, H_2, ..., H_m$ is true. Thus, C follows logically from the premises $H_1, H_2, ..., H_m$.

Example: Show that the following premises are inconsistent.
(1) If Jack misses many classes through illness, then he fails high school.
(2) If Jack fails high school, then he is uneducated.
(3) If Jack reads a lot of books, then he is not uneducated.
(4) Jack misses many classes through illness and reads a lot of books.

Solution:

E: Jack misses many classes.

S: Jack fails high school.

A: Jack reads a lot of books.

H: Jack is uneducated.

The premises are $E \to S$, $S \to H$, $A \to \neg H$, and $E \wedge A$.

(1) $E \rightarrow S$

(2) $S \rightarrow H$

(3) $E \rightarrow H$ (4) $A \rightarrow \neg H$

(5) $H \rightarrow \neg A$

(6) $E \rightarrow \neg A$

(7) $\neg E \vee \neg A$

(8) $\neg(E \wedge A)$

(9) $E \wedge A$

(10)$\neg(E \wedge A) \wedge (E \wedge A)$ Contradiction