Discrete Structures and Theory of Logic

Lecture-3

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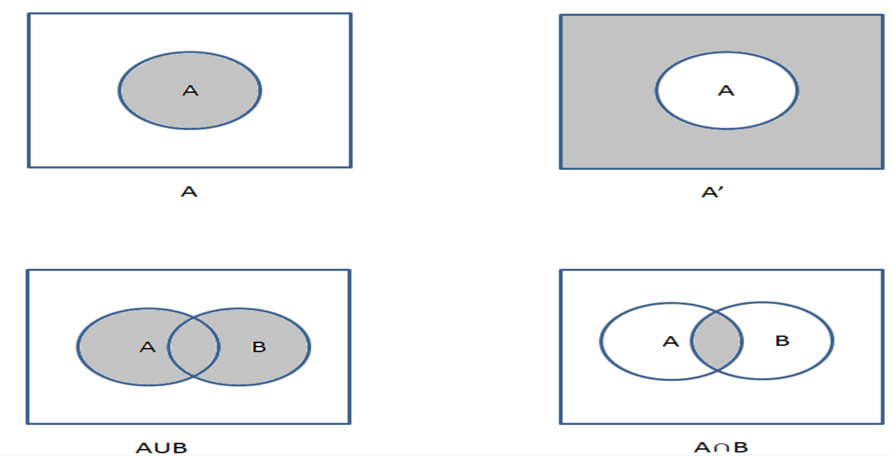
Venn Diagram

Venn Diagram is a diagram representing mathematical or logical sets pictorially as circles or closed curves within an enclosing rectangle (the universal set), common elements of the sets being represented by intersections of the circles.
The universal set U is represented by a set of points in a rectangle and a subset A of U is represented by a circle or some other closed curve inside the rectangle.

Venn Diagram(Cont.)



Examples

Example: Using Venn diagram, show that

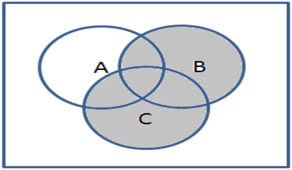
(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

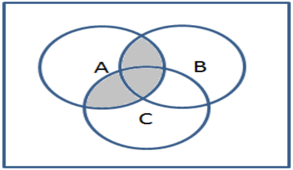
Solution:

(a)

LHS =



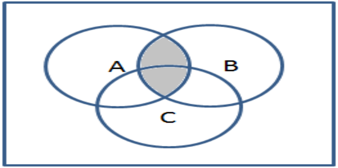
BUC



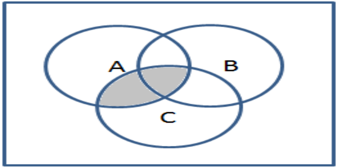
A ∩(BUC)

Examples

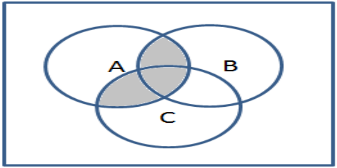
RHS $=$



A ∩B



A ∩C



(A ∩B) U (A ∩C)

Some examples

Example: In a group of 60 people, 40 speak Hindi, 20 speak both English and Hindi and all people speak at least one of the two languages. How many people speak only English and not Hindi? How many speak English?

Solution:

Total people = 60

Hindi speaking people = 40

Both English and Hindi speaking = 20

Let A = The set of Hindi speaking people

and B = The set of English speaking people

Therefore, $n(A) = 40$, $n(A \cup B) = 60$, and $n(A \cap B) = 20$.

Number of people that speak only English and not Hindi is

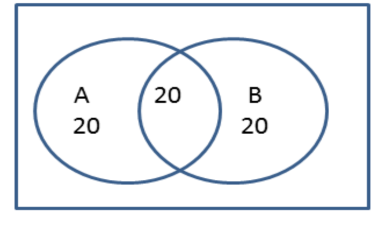
$n(B-A) = n(A \cup B) - n(A) = 60-40 = 20$

Number of people that speak English is

$n(B) = n(A \cup B) - n(A) + n(A \cap B) = 60-40+20 = 40$

Solution(cont.)

By Venn diagram, we can also compute these values.



From above diagram,

Number of people that speak only English and not Hindi $= 20$

Number of people that speak English $= 40$

Some examples(cont.)

Example: A class has 175 students. In which, the number of students studying subjects are the following:-
Mathematics: 100, Physics: 70, Chemistry: 46; Mathematics and Physics: 30, Mathematics and Chemistry: 28, Physics and Chemistry: 23, Mathematics, Physics and Chemistry: 18. Find the following:-
(1) How many students are enrolled in Mathematics alone; Physics alone and Chemistry alone.
(2) The number of students who have not offered any of these subjects.

Some examples(cont.)

Solution:

Total students = 175

Let M, P, C denote the sets of students enrolled in Mathematics, Physics and Chemistry. Therefore,

n(M) = 100, n(P) = 70, n(C) = 46, n(M∩P) = 30, n(P∩C) = 23, n(M∩C) = 28, n(M∩$P \cap C$) = 18.

Therefore, n(M∪P∪C) = n(M)+n(P)+n(C)-n(M∩P)-n(P∩C)-n(M∩C) +n(M∩$P \cap C$)

= 100+70+46-30-23-28+18

= 153

Solution(cont.)

(1) Number of students enrolled in Mathematics alone is

$n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C) = 100 - 30 - 28 + 18 = 60$

Number of students enrolled in Physics alone is

$n(P) - n(M \cap P) - n(P \cap C) + n(M \cap P \cap C) = 70 - 30 - 23 + 18 = 35$

Number of students enrolled in Chemistry alone is

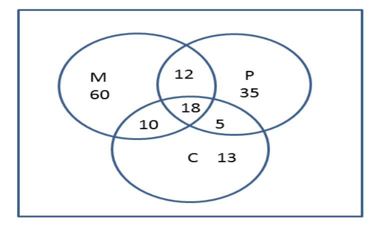
$n(C) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C) = 46 - 23 - 28 + 18 = 13$

(2) Number of students who have not offered any subjects is

Total students - $n(M \cup P \cup C) = 175 - 153 = 22$

Solution(cont.)

By Venn diagram:



From above diagram,

Number of students enrolled in Mathematics alone = 60

Number of students enrolled in Physics alone = 35

Number of students enrolled in Chemistry alone = 13

Number of students who have not offered any subjects = total students - (60+35+13+12+10+5+18)

= 175 - 153 = 22

Some examples(cont.)

Example: Find the number of integers between 1 and 250 that are not divisible by any of the integers 2, 3, and 5.

Solution: Let A, B and C denotes the set integers between 1 and 250 that are divisible by 2, 3 and 5 respectively. Therefore,

$\text{n(A)} = \left\lfloor \frac{250}{2} \right\rfloor = 125,$ $\text{n(B)} = \left\lfloor \frac{250}{3} \right\rfloor = 83$

$\text{n(C)} = \left\lfloor \frac{250}{5} \right\rfloor = 50,$ $\text{n(A}\cap\text{B)} = \left\lfloor \frac{250}{6} \right\rfloor = 41$

$\text{n(A}\cap\text{C)} = \left\lfloor \frac{250}{10} \right\rfloor = 25,$ $\text{n(B}\cap\text{C)} = \left\lfloor \frac{250}{15} \right\rfloor = 16$

$\text{n(A}\cap B \cap C) = \left\lfloor \frac{250}{30} \right\rfloor = 8$

Number of integers divisible by any of 2,3, and 5 is

$= \text{n(A}\cup B \cup C)$

$= \text{n(A)+n(B)+n(C)-n(A}\cap\text{B)-n(B}\cap\text{C)-n(A}\cap\text{C)+n(A}\cap B \cap C)$

$= 125+83+50-41-25-16+8 = 266-82 = 184$

Therefore, number of integers not divisible by any of 2,3, and 5 is

$= \text{Total integers between 1 and 250 - n(A}\cup B \cup C)$

$= 250-184 = 66$

Equivalence laws in set theory

(1) Idempotent laws

(a) $A \cup A = A$

(b) $A \cap A = A$

(2) Associative laws

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

(3) Commutative laws

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cup A$

(4) Distributive laws

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Equivalence laws in set theory(cont.)

(5) Absorption laws

(a) $A \cup (A \cap B) = A$

(b) $A \cap (A \cup B) = A$

(6) Identity laws

(a) $A \cup \phi = A$ (b) $A \cap \phi = \phi$

(c) $A \cup U = U$ (d) $A \cap U = A$

(7) Complement laws

(a) $A \cup A' = U$ (b) $A \cap A' = \phi$

(c) $U' = \phi$ (d) $\phi' = U$

(8) DeMorgan's laws

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

Partition of a set

Let S be a given set and A = $\{A_1, A_2, A_3, \ldots\ldots\ldots\ldots, A_n\}$, where each A_i, for i = 1,2,3,......,n, is a subset of S.

A is called the partition of set S if it satisfies the following two conditions:-

(1) $\cup_{i=1}^{n} A_i = S$

(2) $A_i \cap A_j = \phi$, $\forall$ i,j = 1,2,3,.......,n and i$\neq$j.

Example:

Consider set S = { a, b, c, d } and A = {{a, b }, { c, d }}.

In this example, A is the partition of S because

$$\{a, b\} \cup \{c, d\} = S \text{ and } \{a, b\} \cap \{c, d\} = \phi.$$