Theory of Automata and Formal Language

Lecture-36

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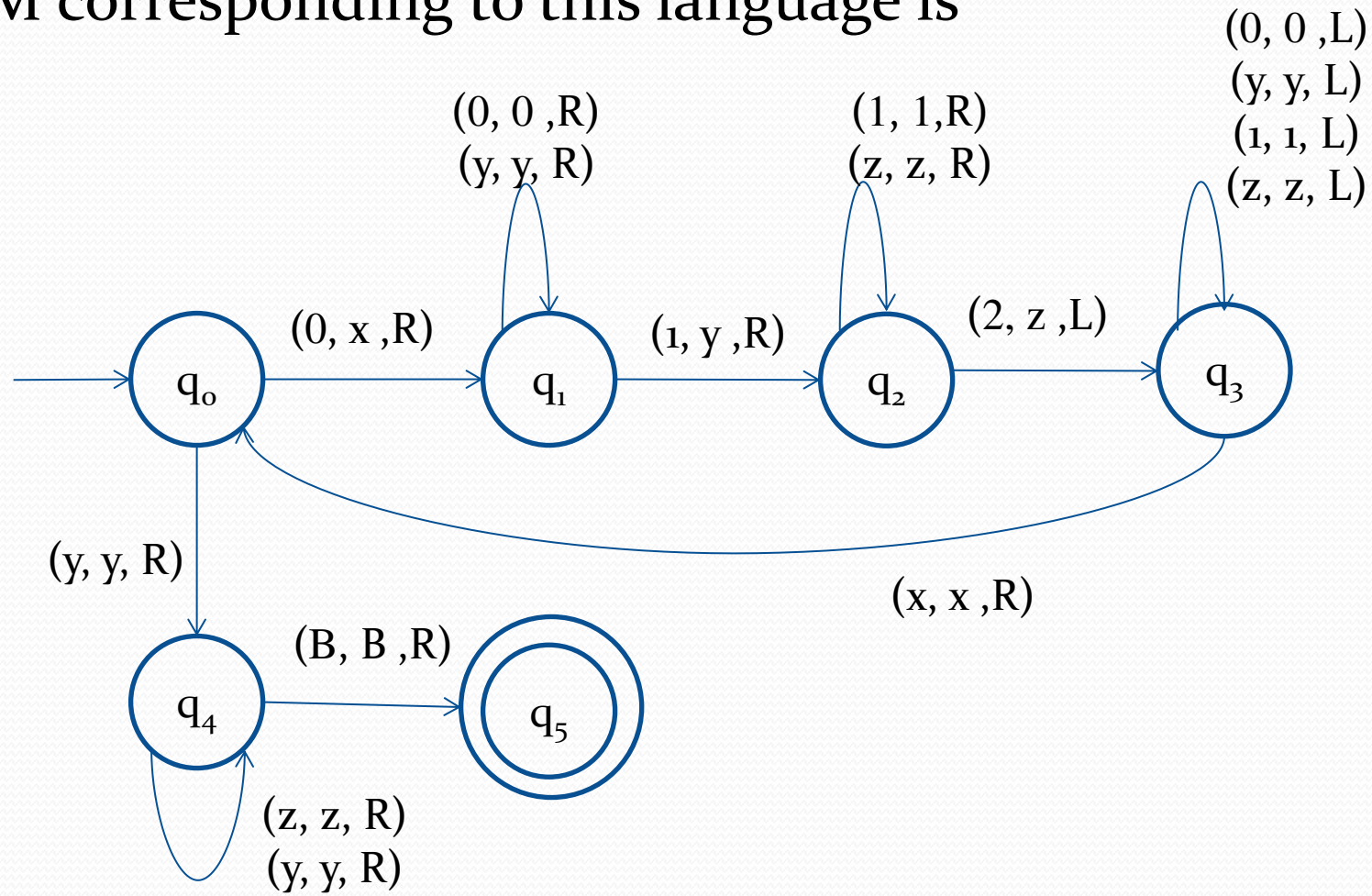
Ex. Construct Turing machine for the language

$$L = \{ 0^n 1^n 2^n ! \; n \geq 1 \}.$$

Solution:

The TM corresponding to this language is



Processing and Verification of TM

Acceptance

Consider string w = 001122 .

$q_0001122 \vdash xq_101122 \vdash x0q_1122 \vdash x0yq_2122 \vdash x0y1q_222 \vdash$
$x0yq_31z2 \vdash x0q_3y1z2 \vdash xq_30y1z2 \vdash q_3x0y1z2 \vdash xq_00y1z2 \vdash$
$xxq_1y1z2 \vdash xxyq_11z2 \vdash xxyyq_2z2 \vdash xxyyzq_22 \vdash xxyyq_3zz \vdash$
$xxyq_3yzz \vdash xxq_3yyzz \vdash xq_3xyyzz \vdash xxq_0yyzz \vdash xxyq_4yzz \vdash$
$xxyyq_4zz \vdash xxyyzq_4z \vdash xxyyzzq_4B \vdash xxyyzzBq_5B$

(machine halts at final state)

Since machine halts at final state, therefore this string is accepted by TM.

Rejection

Consider string w = 00112 .

$q_0 00112 \vdash x q_1 0112 \vdash x0 q_1 112 \vdash x0y q_2 12 \vdash x0y1 q_2 2 \vdash x0y q_3 1z \vdash x0 q_3 y1z \vdash x q_3 0y q_1 1z \vdash q_3 x0y1z \vdash x q_0 0y1z \vdash xx q_1 y1z \vdash xxy q_1 1z \vdash xxyy q_2 z \vdash xxyyz q_2 B$

(machine halts at non-final state)

Since machine halts at non-final state, therefore this string is not accepted by TM.

Ex. Construct TM to accept the language

$$L = \{ wcw^R \mid w \in \{a, b\}^* \}$$

Solution:

Some strings of this set are c, aca, bcb, abcba, bacab, aabcbaa etc.

Clearly all these strings are palindrome. That is, first symbol and last symbol are same. Similarly, second symbol and second last symbol are same , and so on.

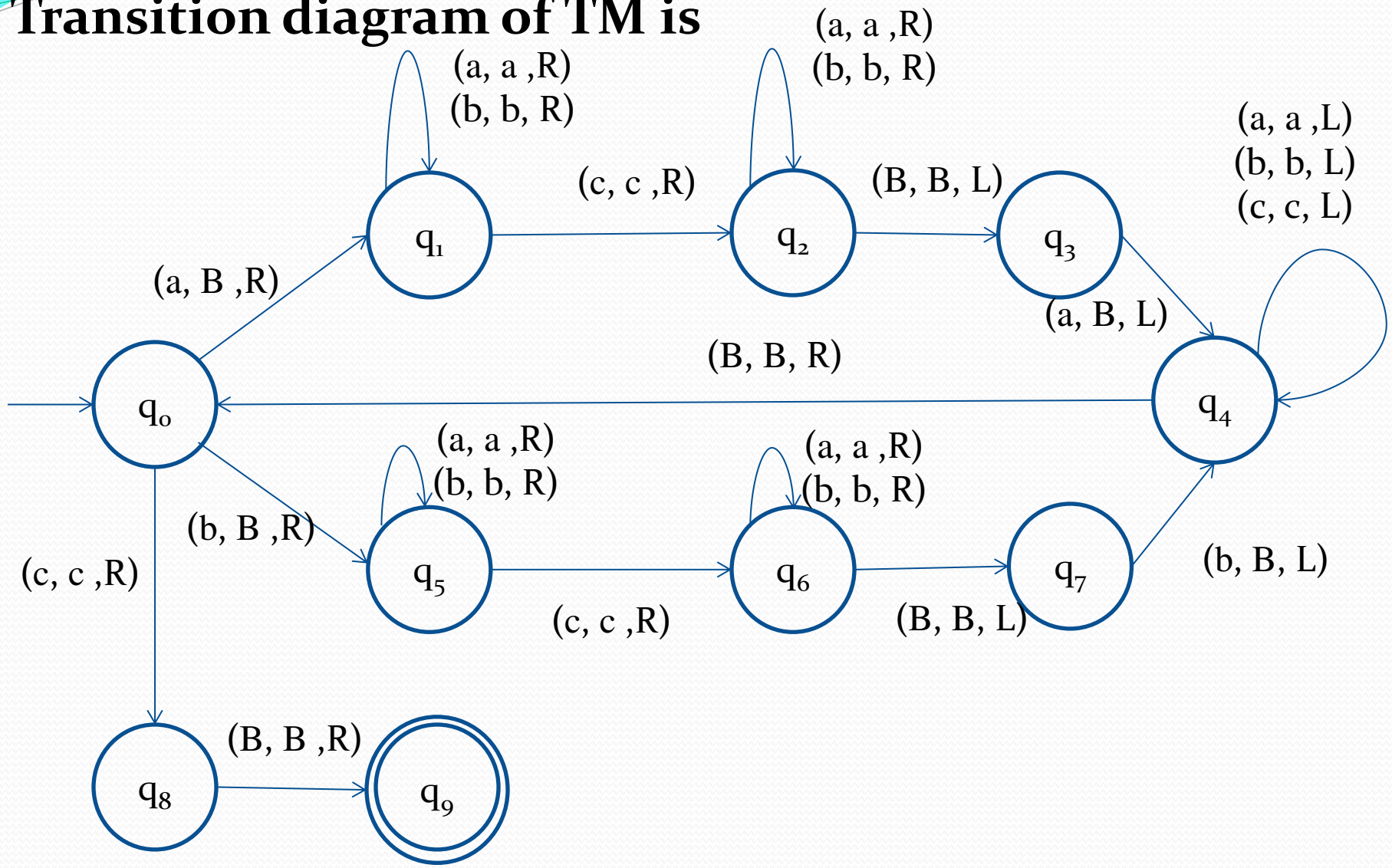
Procedure: TM is constructed in following steps. Let q_o is the initial state.

If the first input symbol is a, then remove it and change its state to q_1. After this, machine move to the last input symbol, if last input symbol is a, then machine remove it and back to first input symbol of string. This process continue.

If the first input symbol is b, then remove it and change its state to q_5. After this, machine move to the last input symbol, if last input symbol is b, then machine remove it and back to first input symbol of string. This process continue.

$L = \{ wcw^R \mid w \in \{a, b\}^* \}$

Transition diagram of TM is



Processing and Verification of TM

Acceptance

Consider string w = aabcbaa .

$q_0aabcbaa \vdash Bq_1abcbaa \vdash Baq_1bcbaa \vdash Babq_1cbaa \vdash Babcq_2baa \vdash Babcbq_2aa \vdash Babcbaq_2a \vdash Babcbaaq_2B \vdash Babcbaq_3aB \vdash Babcbq_4aB \vdash Babcq_4baB \vdash Babq_4cbaB \vdash Baq_4bcbaB \vdash Bq_4abcbaB \vdash q_4BabcbaB \vdash Bq_0abcbaB \vdash BBq_1bcbaB \vdash BBbq_1cbaB \vdash BBbcq_2baB \vdash BBbcbq_2aB \vdash BBbcbaq_2B \vdash BBbcbq_3aB \vdash BBbcq_4bBB \vdash BBbq_4cbBB \vdash BBq_4bcbBB \vdash Bq_4BbcbBB \vdash BBq_0bcbBB \vdash BBBq_5cbBB \vdash BBBcq_6bBB \vdash BBBcbq_6BB \vdash BBBcq_7bBB \vdash BBBq_4cBBB \vdash BBq_4BcBBB \vdash BBBq_0cBBB \vdash BBBcq_8BBB \vdash BBBcBq_9BB$

(machine halts at final state)

Since machine halts at final state, therefore this string is accepted by TM.

Rejection

Consider string w = abcaa .

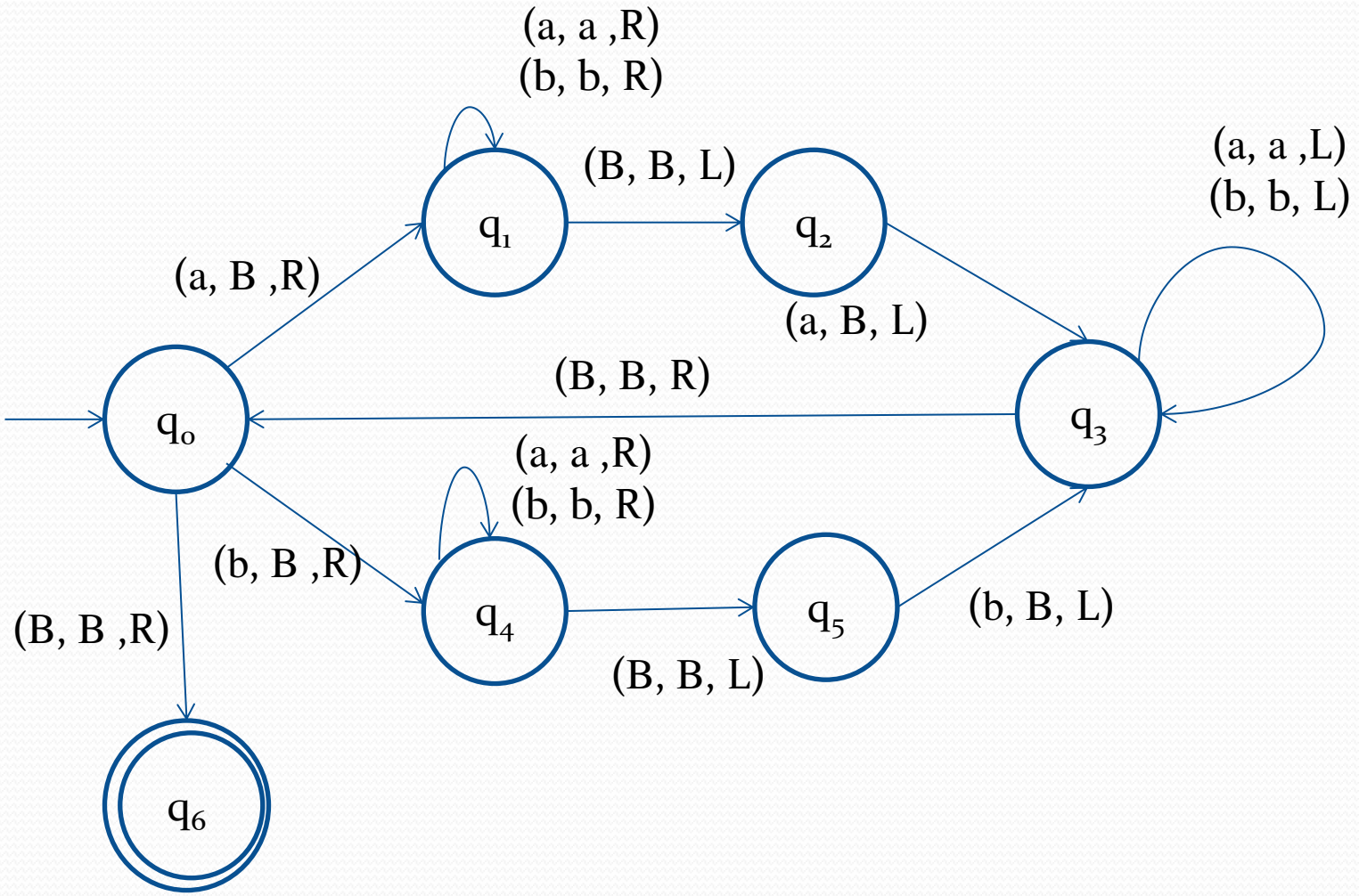
$q_0abcaa \vdash Bq_1bcaa \vdash Bq_1bcaa \vdash Bbq_1caa \vdash Bbcq_2aa \vdash Bbcaq_2a \vdash Bbcaaq_2B \vdash Bbcaq_3aB \vdash Bbcq_4aB \vdash Bbq_4caB \vdash Bq_4bcaB \vdash q_4BbcaB \vdash Bq_0bcaB \vdash BBq_5caB \vdash BBcq_6aB \vdash BBcaq_6B \vdash BBcq_7aB$ (machine halts at non-final state)

Since machine halts at non-final state, therefore this string is not accepted by TM.

Ex. Construct TM to accept the language

$$L = \{ ww^R \mid w \in \{a, b\}^* \}$$

Transition diagram of TM is



Processing and Verification of TM

Acceptance

Consider string w = abba.

$q_0abba \vdash Bq_1bba \vdash Bbq_1ba \vdash Bbbq_1a \vdash Bbbaq_1B \vdash Bbbq_2aB \vdash Bbq_3bBB \vdash Bq_3bbBB \vdash q_3BbbBB \vdash Bq_0bbBB \vdash BBq_4bBB \vdash BBbq_4BB \vdash BBq_5bBB \vdash Bq_3BBBB \vdash BBq_0BBB \vdash BBBq_6BB$

(machine halts at final state)

Since machine halts at final state, therefore this string is accepted by TM.

Rejection

Consider string w = abaa .

$q_0abaa \vdash Bq_1baa \vdash Bbq_1aa \vdash Bbaq_1a \vdash Bbaaq_1B \vdash Bbaq_2aB \vdash Bbq_3aBB \vdash Bq_3baBB \vdash q_3BbaBB \vdash Bq_0baBB \vdash BBq_4aBB \vdash BBaq_4BB \vdash BBq_5aBB$ (machine halts at non-final state)

Since machine halts at non-final state, therefore this string is not accepted by TM.