Theory of Automata and Formal Language

Lecture-32

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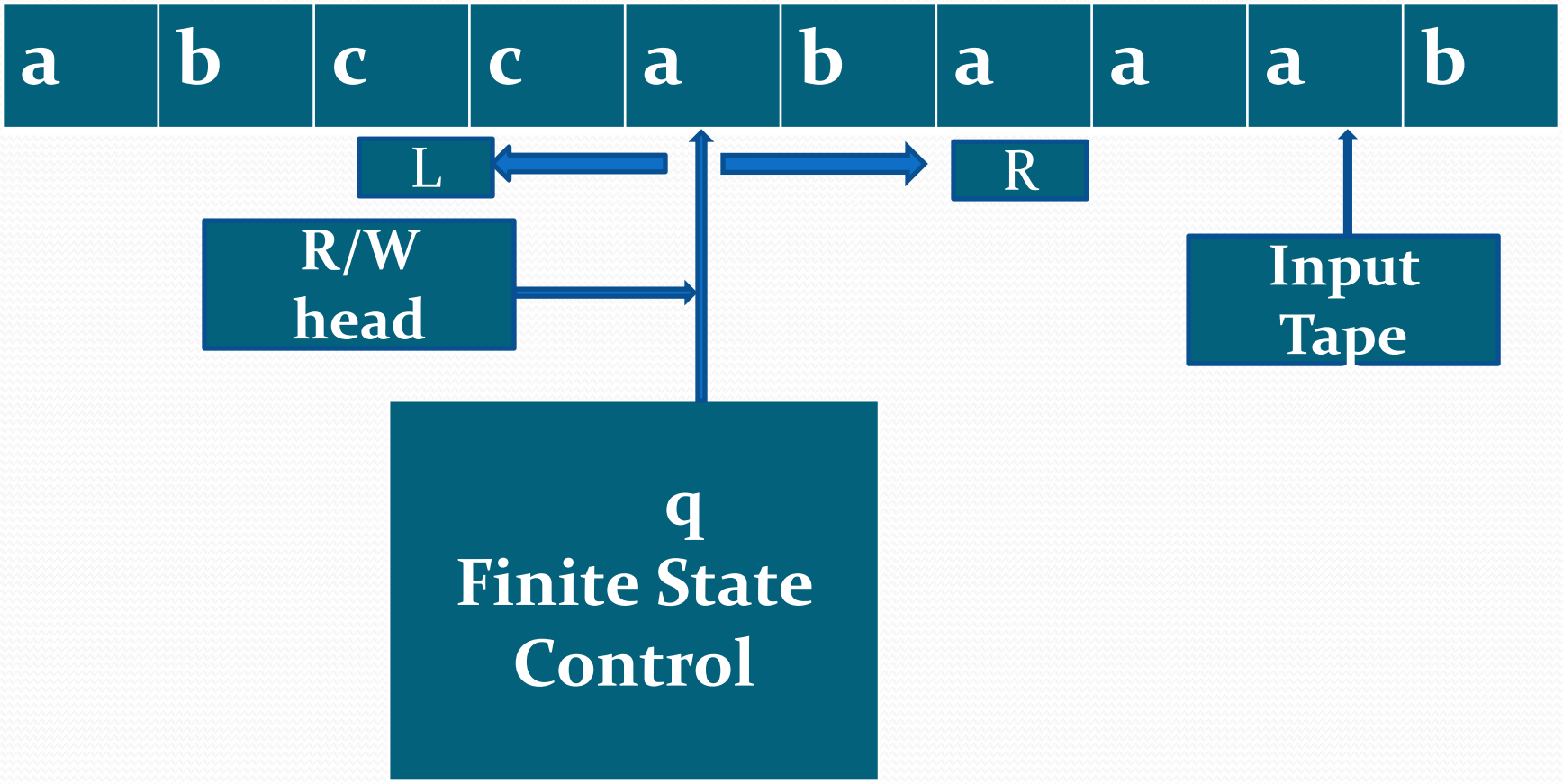
Unit-5

Turing Machine

What is Turing machine?

- A **Turing machine** is a mathematical model of computation that defines an abstract **machine**, which manipulates symbols on a strip of tape according to a table of rules.
- It is a generalized machine which can accept all the type of languages i.e. regular , context free, context sensitive, recursive and recursive enumerable languages .
- There are two purposes for a **Turing machine**: deciding formal languages and solving mathematical functions.

Model of Turing Machine



Mathematical Definition of Turing Machine (TM)

A Turing machine is described by a 7-tuple
$M=(Q, \Sigma, \Gamma, \delta, q_o, B, F)$ where,

- Q is the finite set of states,
- $\Sigma \subseteq \Gamma$ is the set of input symbols
- Γ is the set of tape symbols,
- $q_o \in Q$ is the initial state,
- $B \in \Gamma$ is a blank symbol
- F is the set of final states, and
- δ is a transition function which is defined as following:-
- $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

where,
L represents left direction and R represents right direction.

Instantaneous Description(ID)

An instantaneous description of TM is a string of the following form:-

$$\alpha\, q\, \beta$$

Where, $\alpha, \beta \in \Gamma^*$, $q \in Q$.

$\alpha\beta$ denotes the whole contents of the tape.

q is a current state.

R/W head of machine will be at the leftmost symbol of β.

Initial ID will be $q_o w$. where $w \in \Sigma^*$

Move relation

This relation exist between two consecutives ID's. It is dented by $\vdash_M$.

Consider an ID of a TM at any instant is

$a_1a_2a_3.....a_{i-1}qa_ia_{i+1}.....a_n$.

(1) If $\delta(q, a_i) = (p, y, R)$ then move of machine will be

$a_1a_2a_3.....a_{i-1}qa_ia_{i+1}.....a_n \vdash_M a_1a_2a_3.....a_{i-1}\, ypa_{i+1}.....a_n$

(2) If $\delta(q, a_i) = (p, y, L)$ then move of machine will be

$a_1a_2a_3.....a_{i-1}qa_ia_{i+1}.....a_n \vdash_M a_1a_2a_3.....pa_{i-1}\, ya_{i+1}.....a_n$

Language accepted by TM

The language accepted by Turing machine M is defined as following:-

$L(M) = \{ w \,!\, q_0 w \;\vdash_M^* \alpha\, p\, \beta$ where $w \in \Sigma^*$,

$p \in F$ and $\alpha, \beta \in \Gamma^*$ and machine halts on the final state.$\}$

Representation of TM

Two representations are used for TM.

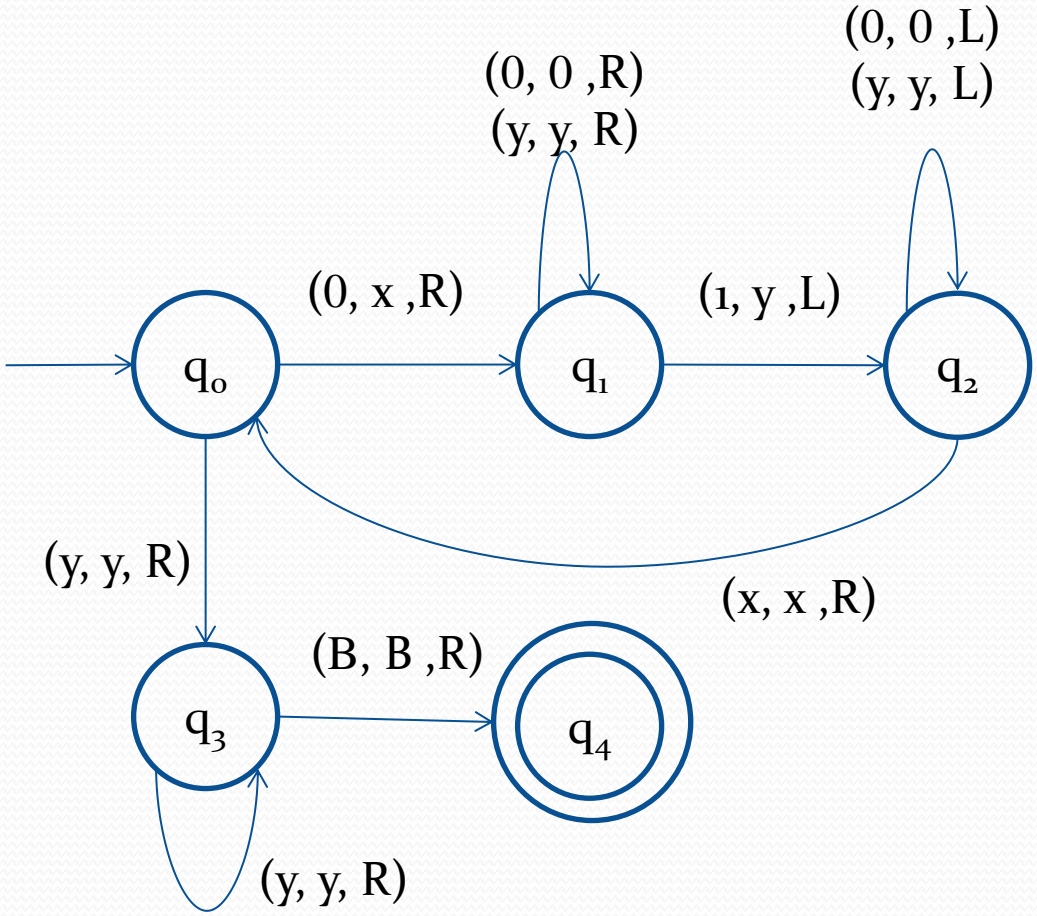
(1) By transition table (2) By transition diagram

By transition table

δ	Tape symbols				
States	**0**	**1**	**x**	**y**	**B**
$\longrightarrow q_0$	(q_1, x, R)			(q_3, y, R)	
q_1	$(q_1, 0, R)$	(q_2, y, L)		(q_1, y, R)	
q_2	$(q_2, 0, L)$		(q_0, x, R)	(q_2, y, L)	
q_3				(q_3, y, R)	(q_4, B, R)
q_4					

Representation of TM(continue)

By transition diagram



Processing or working of TM

Ex. Check the acceptability of following strings

(1) 0011 (2) 011 (3) 00101

by the TM in the previous example.

Solution:

(1) For string 0011

$q_0 0011 \vdash x q_1 011 \vdash x 0 q_1 11 \vdash x q_2 0 y 1 \vdash q_2 x 0 y 1 \vdash x q_0 0 y 1 \vdash$
$x x q_1 y 1 \vdash x x y q_1 1 \vdash x x q_2 y y \vdash x q_2 x y y \vdash x x q_0 y y \vdash x x y q_3 y \vdash$
$x x y y q_3 B \vdash x x y y B q_4 B$ (machine halts at final state)

Since machine halts at final state, therefore this string is accepted by TM.

Processing or working of TM(continue)

(2) For string 011

$q_0011 \vdash xq_111 \vdash q_2xy1 \vdash xq_0y1 \vdash xyq_31$ (machine halts at non-final state)

Since machine halts at non-final state, therefore this string is not accepted by TM.

(3) For string 00101

$q_000101 \vdash xq_10101 \vdash x0q_1101 \vdash xq_20y01 \vdash q_2x0y01 \vdash xq_00y01 \vdash xxq_1y01 \vdash xxyq_101 \vdash xxy0q_11 \vdash xxyq_20y \vdash xxq_2y0y \vdash xq_2xy0y \vdash xxq_0y0y \vdash xxyq_30y$ (machine halts at non-final state)

Since machine halts at non-final state, therefore this string is not accepted by TM.