Discrete Structures and Theory of Logic

Lecture-17

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Unit-2

Function and Boolean Algebra

Definition of Function

Let X and Y are any two sets. A relation f from X to Y is called a function if for every $x \in X$, there is a unique element $y \in Y$ such that $(x,y) \in f$. It is denoted by f: $X \to Y$.

Example: Let $X = \{1,2,3,4\}$ and $Y = \{x,y,w,z\}$ and $f = \{(1,x), (2,y),(3,w),(4,x)\}$. Is f a function?

Solution: Clearly in function f, each element of set X has an image in set Y and that image has an unique. Therefore, f is a function.

Function

Example: Let X = Y = R. Also let, f = { (x, x^2) ! x∈R } and g = { (x^2, x) ! x∈R }. Find out f and g is functions or not.
Solution: Here R is a set of real numbers. Clearly for f, each real number has a unique square because square of 2 is 4, 3 is 9, 4 is 16 etc. Therefore, f is a function.
For relation g, element 4 has two images 2 and -2. Similarly, 9 has two images 3 and -3. Therefore, g is not a function.

Domain, Range, and Co-domain of a Function

Consider a function $f : X \rightarrow Y$.

Domain of a function f is X. Co-domain of function f is Y. And range of f will be the set of second elements of all the ordered pairs in f i.e. range $\subseteq$ Y.

Example: Let $X = \{1,2,3,4\}$ and $Y = \{x,y,w,z\}$ and $f = \{(1,x),(2,y),(3,w),(4,x)\}$. Find domain, co-domain and range of f.

Solution:

Domain(f) $= \{1,2,3,4\}$

Co-domain(f) $= \{x,y,w,z\}$

Range(f) $= \{x,y,w\}$

Domain

Question: Find the domain of the following functions

1. f(x)$=\frac{2x}{(x^2-4)}$
2. f(x)$=\frac{x^2-1}{(x-1)x}$
3. f(x)$=\sqrt{\frac{(x^2-4)}{(x-5)}}$
4. f(x)$=\sqrt{(2x+3)}+\sqrt{\frac{(3-x^2)}{x}}$

Range

Question: Find the range of the following functions

1. f(x)=$2x^2 - 5x + 3$
2. f(x)=$\sqrt{x^2 + 2x + 3}$
3. f(x)=$\sqrt{\frac{(2-x)}{x}}$

Types of function

Onto function (Surjective function)

A function f: $X \to Y$ is said to be onto function if every element of Y is the image of some element of X. That is, if range(f) $=$ Y, then f is onto.

Into function

A function f: $X \to Y$ is said to be into function iff there exists at least one element in Y which is not the image of any element in X. That is, range(f) $\subset$ Y.

Types of function

One-one function (Injective function)

A function f: X$\rightarrow$Y is said to be one-one function if for all elements x_1, x_2 in X such that f(x_1) $=$ f(x_2) then $x_1 = x_2$.

Many-one function

A function f: X$\rightarrow$Y is said to be many-one function iff two or more elements of X have same image in Y.

Bijective function)

A function f: X$\rightarrow$Y is said to be bijective function if f is both one-one and onto.

Exercise

Let N be the set of natural numbers including zero. Determine which of the following functions are one-one, onto and bijective.

1. f: N$\rightarrow$N, f(j) = j^2+2
2. f: N$\rightarrow$N, f(j) = j mod 3
3. f: N$\rightarrow$N, f(j) = 1 , if j is odd
   = 0 , if j is even
4. f: N$\rightarrow\{0,1\}$, f(j) = 0 , if j is odd
   = 1 , if j is even