Design and Analysis of Algorithms

Lecture-30

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Dijkstra's algorithm

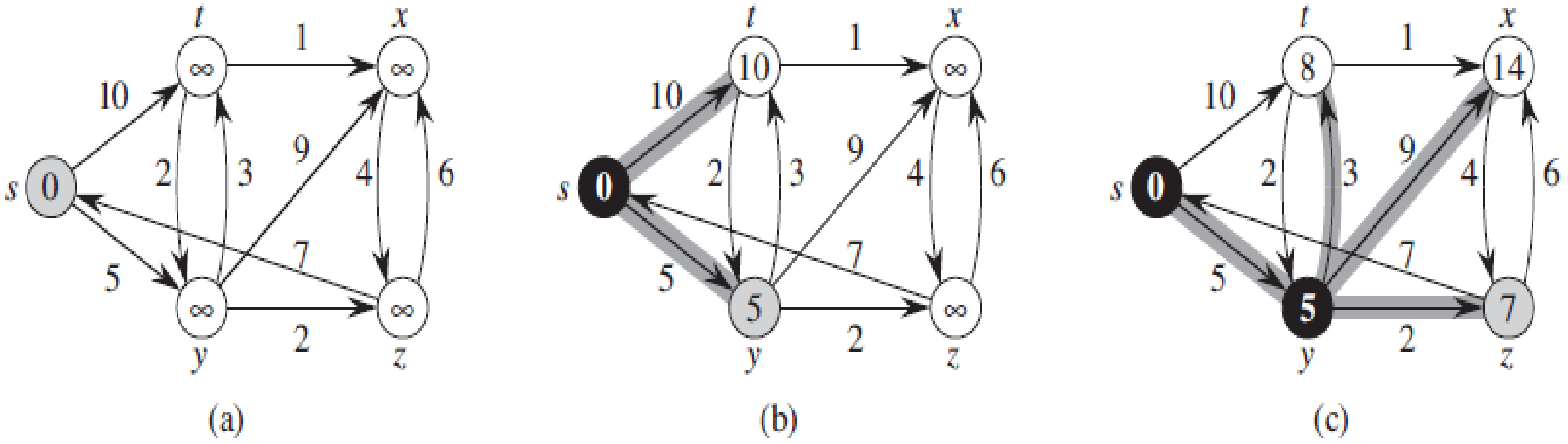
- Dijkstra's algorithm solves the single-source shortest-paths problem on a weighted, directed graph $G = (V, E)$ for the case in which all edge weights are nonnegative.

- Dijkstra's algorithm maintains a set S of vertices whose final shortest-path weights from the source s have already been determined. The algorithm repeatedly selects the vertex $u \in V-S$ with the minimum shortest-path estimate, adds u to S, and relaxes all edges leaving u.

- In the following implementation, we use a min-priority queue Q of vertices, keyed by their d values.

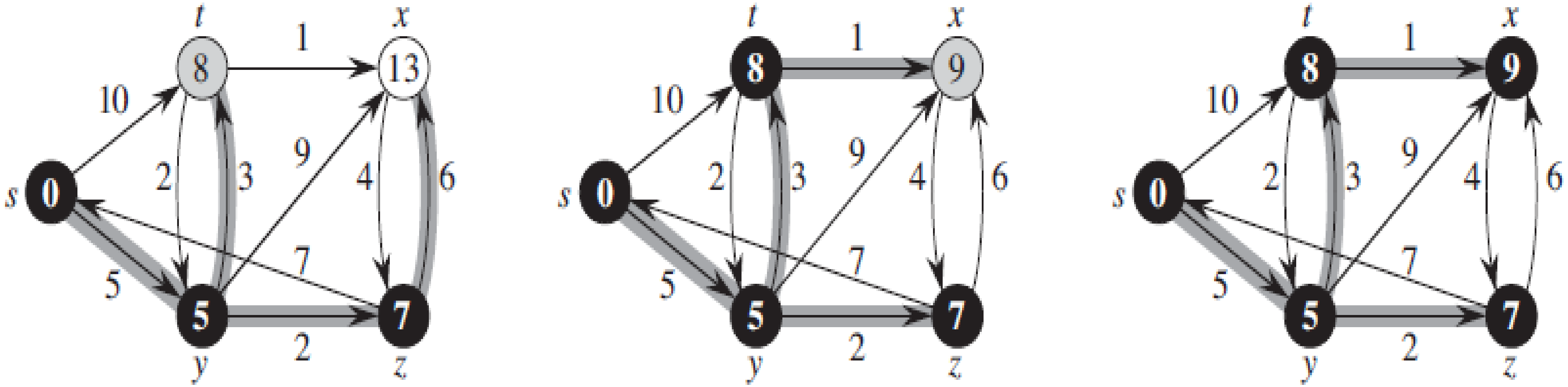
Dijkstra’s algorithm

Example:



The execution of Dijkstra’s algorithm. s is the source vertex. The shortest-path estimates appear within the vertices, and shaded edges indicate predecessor values. Black vertices are in the set S, and white vertices are in the min-priority queue Q = V – S.

Dijkstra’s algorithm



(d)

(e)

(f)

Dijkstra's algorithm

```
DIJKSTRA(G, w, s)
INITIALIZE-SINGLE-SOURCE(G, s)
S = ∅
Q = G.V
while Q ≠ ∅
    u = EXTRACT-MIN(Q)
    S = S ∪ {u}
    for each vertex v ∈ G.Adj[u]
        RELAX(u, v, w)
```

Time complexity of this algorithm is O(E logV).

AKTU Examination Questions

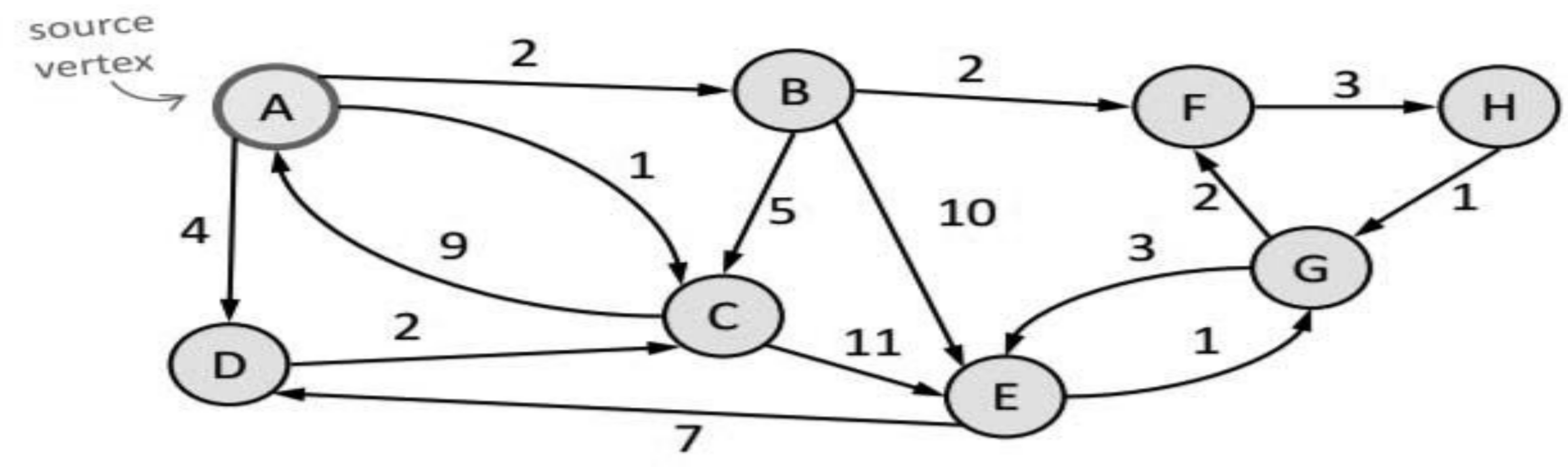
1. Consider the following instance for knapsack problem. Find the solution using Greedy method:

N= 10, W=130

P [] = {21, 31, 43, 53, 41, 63, 65, 75}

V [] = {11, 21, 31, 33, 43, 53, 65, 65}

2. Apply the greedy single source shortest path algorithm on the following graph:

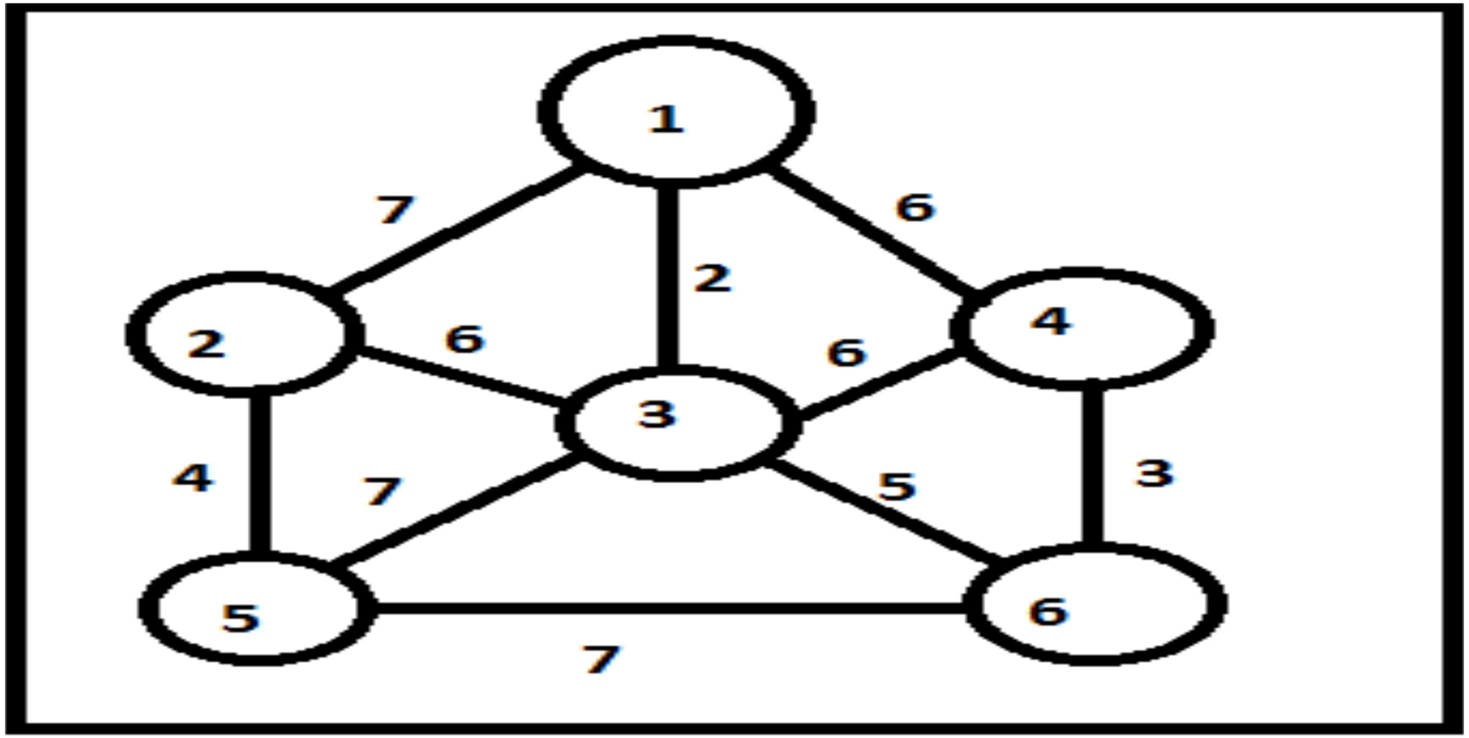


AKTU Examination Questions

3. Describe Activity selection problem.
4. Write an algorithm for minimum spanning tree with example.
5. What are greedy algorithms? Explain their characteristics?
6. Define feasible and optimal solution.
7. Define spanning tree. Write Kruskal's algorithm for finding minimum cost spanning tree. Describe how Kruskal's algorithm is different from Prim's algorithm for finding minimum cost spanning tree.
8. Explain Single source shortest path.

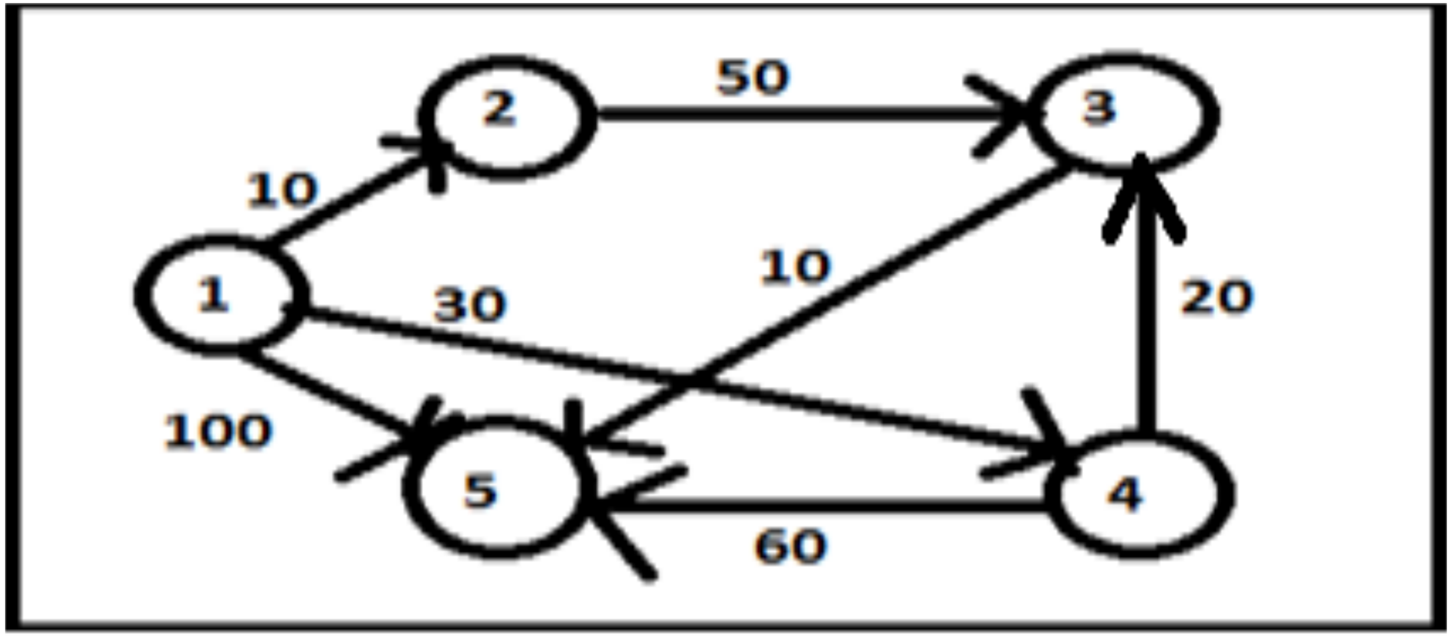
AKTU Examination Questions

9. What is Minimum Cost Spanning Tree? Explain Kruskal’s Algorithm and Find MST of the Graph. Also write its Time-Complexity.



AKTU Examination Questions

10. Find the shortest path in the below graph from the source vertex 1 to all other vertices by using Dijkstra's algorithm.



AKTU Examination Questions

11. Given the six items in the table below and a Knapsack with Weight 100, what is the solution to the Knapsack problem in all concepts. I.e. explain greedy all approaches and find the optimal solution.

ITEM ID	WEIGHT	VALUE	VALUE/WEIGHT
A	100	40	.4
B	50	35	.7
C	40	20	.5
D	20	4	.2
E	10	10	1
F	10	6	.6

12. Prove that if the weights on the edge of the connected undirected graph are distinct then there is a unique Minimum Spanning Tree. Give an example in this regard. Also discuss Prim's Minimum Spanning Tree Algorithm in detail.

AKTU Examination Questions

13. Consider the weights and values of items listed below. Note that there is only one unit of each item. The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy}. Find the value of $V_{opt} - V_{greedy}$.

Item	I_1	I_2	I_3	I_4
W	10	7	4	2
V	60	28	20	24

AKTU Examination Questions

14. When do Dijkstra and the Bellman-Ford algorithm both fail to find a shortest path? Can Bellman ford detect all negative weight cycles in a graph? Apply Bellman Ford Algorithm on the following graph: