Theory of Automata and Formal Language

Lecture-30

Dharmendra Kumar
(Associate Professor)
Department of Computer Science and Engineering United College of Engineering and Research, Prayagraj March 30, 2021

Construction of PDA

In this section, we shall see how PDA's can be constructed.

Ex. Construct PDA to accept the language L={ $0^n 1^n$! n ≥ 1} by final state.

Solution: First we consider a string of a given language and check how it can accept.

Procedure:

In this language, since n numbers of 0' are followed by n numbers of 1's, therefore, to check equal number of 0 and 1, we have to push a symbol corresponding to 0 and pop that symbol corresponding to 1. Let that symbol is denoted by A.

Push stack symbol A in to the stack as long as scanned input symbol 0. When next scanned input symbol is 1, find top symbol of stack. If top symbol is A, then pop A from stack. When input pointer reaches at the end of string i.e. input string is empty, find top symbol of stack. If top symbol is Z_o, then machine goes to final state. And at this situation, machine accept string.

Ex. L={ $0^n 1^n$! $n \geq 1$} continue.

Step-1: Let q_o is the initial state and Z_o isthe bottom symbol of stack. We will push the stack symbol A into the stack if scanned input symbol 0 appears on the input tape. PDA will stay in this state q_o. The top symbol may be any thing.

The transition rules corresponding to this step are the following:-

$\delta(q_o, 0, Z_o) = \{(q_o, AZ_o)\}$

$\delta(q_o, 0, A) = \{(q_o, AA)\}$

Step-2: In state q_o, if the next scanned input symbol is 1 and if the top of stack is A, then PDA will pop the top symbol A from the stack and PDA changes its state to q_1.

The transition rule corresponding to this step is the following:-

$\delta(q_o, 1, A) = \{(q_1, \epsilon)\}$

Ex. L={ 0^n1^n ! n ≥ 1} continue.

Step-3: Now PDA is at state q_1. Now the input symbols in input string are 1's only. If current state is q_1, current input symbol is 1 and top symbol is A, then PDA will pop the top symbol A. This action continues till input string becomes empty or top symbol becomes Z_o.

The transition rule corresponding to this step is the following:-

$\delta(q_1, 1, A) = \{(q_1, \epsilon)\}$

Step-4: Now the sate is q_1 and input string is empty(ϵ). If top symbol is Z_o then PDA goes to final state without push or pop. Let the final state is q_2.

The transition rule corresponding to this step is the following:-

$\delta(q_1, \epsilon, Z_o) = \{(q_2, Z_o)\}$

Ex. L={ $0^n 1^n$! n ≥ 1} continue

Therefore final PDA is

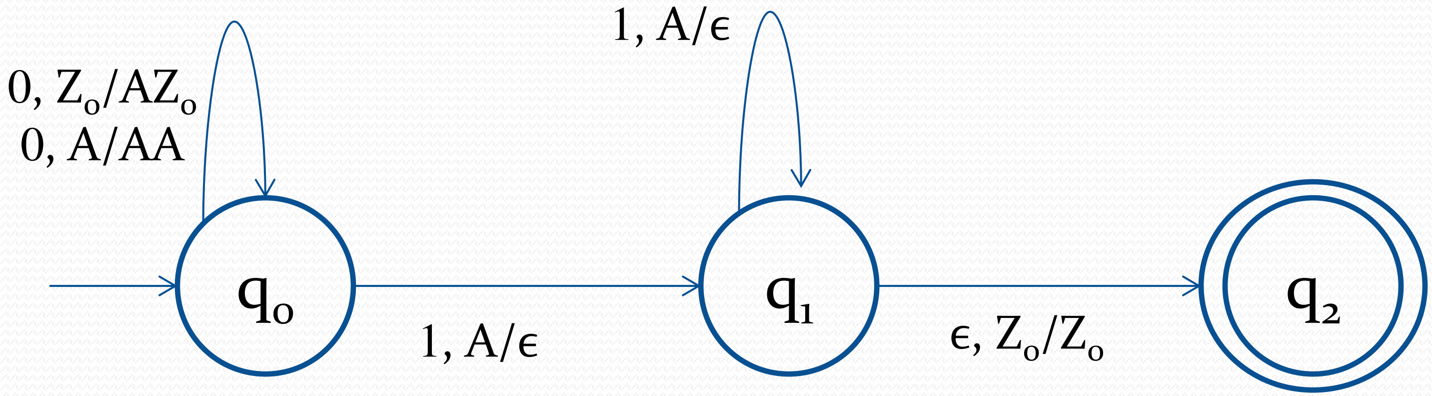
$M = (\{q_o, q_1, q_2\}, \{0, 1\}, \{A, Z_o\}, \delta, q_o, Z_o, \{q_2\})$

δ is defined as following:-

$\delta(q_o, 0, Z_o) = \{(q_o, AZ_o)\}$ $\delta(q_o, 0, A) = \{(q_o, AA)\}$

$\delta(q_o, 1, A) = \{(q_1, \epsilon)\}$ $\delta(q_1, 1, A) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, Z_o) = \{(q_2, Z_o)\}$



Processing and Verification of above PDA

Acceptance

Consider string w = 000111 .

Processing of this string by PDA

$(q_0, 000111, Z_0) \vdash (q_0, 00111, AZ_0) \vdash (q_0, 0111, AAZ_0)$
$\vdash (q_0, 111, AAAZ_0) \vdash (q_1, 11, AAZ_0) \vdash (q_1, 1, AZ_0) \vdash (q_1, \epsilon , Z_0)$
$\vdash (q_2, \epsilon, Z_0)$ (Final configuration)

Rejection

Consider string w = 00111.

Processing of this string by PDA

$(q_0, 00111, Z_0) \vdash (q_0, 0111, AZ_0) \vdash (q_0, 111, AAZ_0)$
$\vdash (q_1, 11, AZ_0) \vdash (q_1, 1, Z_0)$ (Non-final configuration)

PDA examples continue

Ex. Construct PDA to accept the language

$L = \{ wcw^R \ ! \ w \in \{a, b\}^* \}$ by final state.

Solution:

In this language, w is any string of a and b. w^R is the reverse string of w.

If w= abb, then string abbcbba ∈ L. Clearly all the strings belong in to L are palindrome.

Some strings belong in to this set are c, aca, bcb, abcba, bacab etc.

Procedure: In this PDA, we push symbol A and B in to the stack corresponding to input symbol a and b in input string. PDA will stay at the q_0. when c appears in input string, it changes its state to other state(Let it be q_1) without push or pop. At q_1 state, it only pop.

- If current input symbol is a and top symbol is A, then pop the top symbol A.
- Similarly, If current input symbol is b and top symbol is B, then pop the top symbol B.
- At last if input string is empty and top symbol is Z_0, then machine goes to final state.

Ex. L={ wcw^R ! w ∈ {a, b}* } continue

Therefor e the PDA corresponding to above language is constructed as following:-

$M = (\{q_o, q_1, q_2\}, \{a, b, c\}, \{A, B, Z_o\}, \delta, q_o, Z_o, \{q_2\})$

δ is defined as following:-

$\delta(q_o, a, Z_o) = \{(q_o, AZ_o)\}$ $\delta(q_o, b, Z_o) = \{(q_o, BZ_o)\}$

$\delta(q_o, a, A) = \{(q_o, AA)\}$ $\delta(q_o, b, B) = \{(q_o, BB)\}$

$\delta(q_o, a, B) = \{(q_o, AB)\}$ $\delta(q_o, b, A) = \{(q_o, BA)\}$

$\delta(q_o, c, Z_o) = \{(q_1, Z_o)\}\delta(q_o, c, A) = \{(q_1, A)\}$

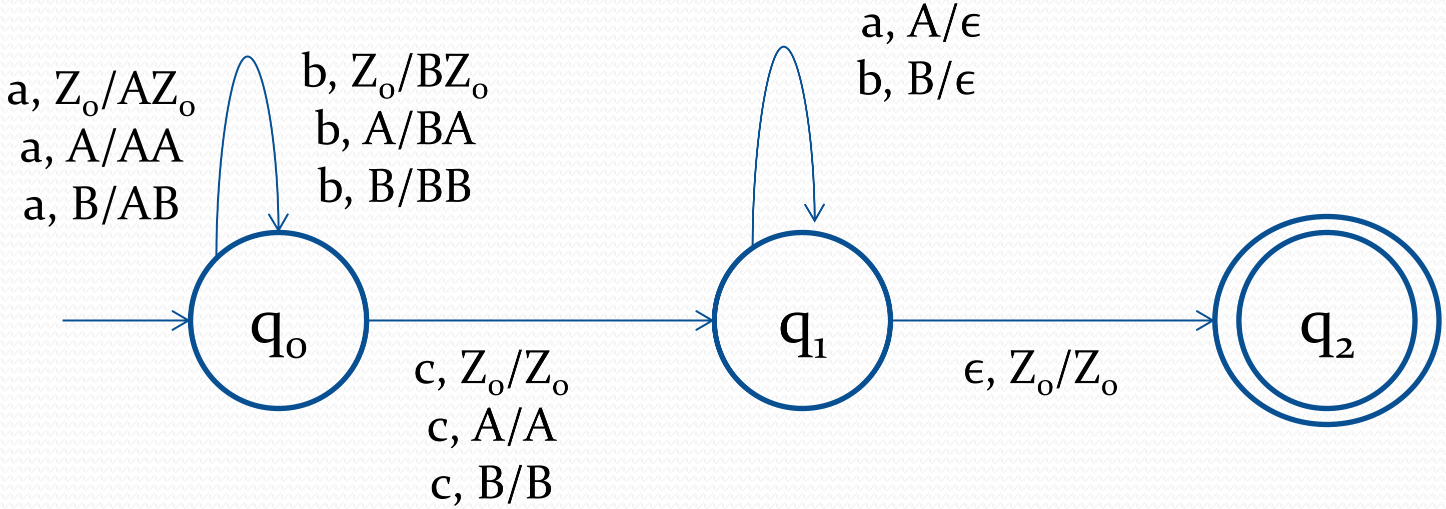
$\delta(q_o, c, B) = \{(q_1, B)\}$

$\delta(q_1, a, A) = \{(q_1, \epsilon)\}\delta(q_1, b, B) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, Z_o) = \{(q_2, Z_o)\}$

Ex. L={ wcw^R ! w ∈ {a, b}* } continue

Transition diagram of PDA is the following:-



Processing and Verification of above PDA

Acceptance

Consider string x= abbcbba.

Processing of this string by PDA

$(q_0, abbcbba, Z_0) \vdash (q_0, bbcbba, AZ_0) \vdash (q_0, bcbba, BAZ_0)$
$\vdash (q_0, cbba, BBAZ_0) \vdash (q_1, bba, BBAZ_0) \vdash (q_1, ba, BAZ_0)$
$\vdash (q_1, a, AZ_0) \vdash (q_1, \epsilon, Z_0) \vdash (q_2, \epsilon, Z_0)$ (Final configuration)

Rejection

Consider string x = abbcba.

Processing of this string by PDA

$(q_0, abbcba, Z_0) \vdash (q_0, bbcba, AZ_0) \vdash (q_0, bcba, BAZ_0)$
$\vdash (q_0, cba, BBAZ_0) \vdash (q_1, ba, BBAZ_0) \vdash (q_1, a, BAZ_0)$
(Non-final configuration)