Theory of Automata and Formal Language

Lecture-11

Dharmendra Kumar
(Associate Professor)
Department of Computer Science and Engineering
United College of Engineering and Research, Prayagraj

April 21, 2021

Theorem: For a given NFA M, there exists a DFA M' such that L(M) = L(M').

Proof:

Suppose the given NFA M is

$$M = (Q,\Sigma,\delta,q_0,F)$$

Now, we construct a DFA M' as following

$$M' = (Q',\Sigma,\delta',q_0',F')$$

Where $Q' = P(Q)$, q_0'=$\{q_0\}$

$F' = \{P \subseteq Q \,!\, P \cap F \neq \phi\}$

and $\delta' \equiv \hat{\hat{\delta}}$

i.e. $\delta'(P, a) = \hat{\hat{\delta}}(P, a) = \underset{p \in P}{\cup} \hat{\delta}(p, a) = \underset{p \in P}{\cup} \delta(p, a)$

Now, we have to show that

$$\mathrm{L(M)} = \mathrm{L}(M') \text{ .... } (1)$$

To prove the equation (1), first we will prove that

$$\hat{\delta}'(q_0', w) = \hat{\delta}(q_0, w),\ \forall w \in \Sigma^* \text{................}(2)$$

We will prove this by using induction method on the length of string w.

For $\mid w \mid = 0$, i.e. w $= \epsilon$

$\hat{\delta}'(q_0', \epsilon) = q_0' = \{q_0\} = \hat{\delta}(q_0, \epsilon)$

Therefore, $\hat{\delta}'(q_0', \epsilon) = \hat{\delta}(q_0, \epsilon)$

Therefore, it is proved for w $= \epsilon$.

For $\mid w \mid = 1$, i.e. w = a, where a$\in \Sigma$
$\hat{\delta'}(q_0', a) = \delta'(q_0', a)$
$= \hat{\hat{\delta}}(\{q_0\}, a)$
$= \hat{\delta}(q_0, a)$
Therefore, $\hat{\delta'}(q_0', a) = \hat{\delta}(q_0, a)$
Therefore, it is proved for w = a, i.e. $\mid w \mid = 1$.
Suppose equation (2) is true for string w = x.
Therefore, $\hat{\delta'}(q_0', x) = \hat{\delta}(q_0, x)$, ...(3)

Finite Automata (FA)

Now, we will prove for w = xa.

$$\begin{aligned}\hat{\delta}'(q_0', xa) &= \delta'(\hat{\delta}'(q_0', x), a)\\ &= \hat{\hat{\delta}}(\hat{\delta}(q_0, x), a)\\ &= \bigcup_{p \in \hat{\delta}(q_0, x)} \hat{\delta}(p, a)\\ &= \bigcup_{p \in \hat{\delta}(q_0, x)} \delta(p, a)\\ &= \hat{\delta}(q_0, xa)\end{aligned}$$

Therefore, $\hat{\delta}'(q_0', xa) = \hat{\delta}(q_0, xa)$

Therefore, it is proved for w = xa.

Therefore, equation (2) is proved.

Now, we prove the equation (1) by using equation (2).

$$\begin{aligned}\text{Let } w \in L(M') &\Leftrightarrow \hat{\delta}'(q_0', w) \in F'\\ &\Leftrightarrow \hat{\delta}'(q_0', w) \cap F \neq \phi\\ &\Leftrightarrow \hat{\delta}(q_0, w) \cap F \neq \phi\\ &\Leftrightarrow w \in L(M)\end{aligned}$$

Therefore, L(M) = L(M')

Now, it is proved.

Some Examples

Example: Find DFA equivalent to the following NFA:-
M = $(\{q_0, q_1, q_2\}, \{a,b\}, \delta, q_0, \{q_2\})$,
where δ is given by

δ	a	b
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
q_2		q_0, q_1

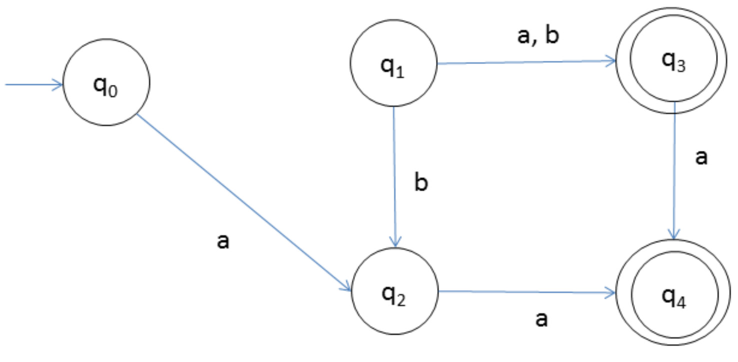
Example: Find DFA equivalent to the following NFA:-

M = $(\{q_0, q_1, q_2, q_3\}, \{a,b\}, \delta, q_0, \{q_3\})$,

where δ is given by

δ	a	b
$\rightarrow q_0$	q_0, q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_3
q_3		q_2

Example: Construct DFA equivalent to the following NFA:-



Example: Construct DFA equivalent to the following NFA:-

δ	a	b
$\rightarrow q_0$	q_1, q_3	q_2, q_3
q_1	q_1	q_3
q_2	q_3	q_2
q_3		