Theory of Automata and Formal Language

Lecture-22

Dharmendra Kumar
(Associate Professor)
Department of Computer Science and Engineering
United College of Engineering and Research, Prayagraj

May 6, 2021

Pumping Lemma

Let L be an infinite regular language. Then there eists some positie integer n such that any x∈ L with x≥n, can be decomposed as

x=uvw

with $\mid uv \mid \leq n$ and $\mid v \mid \geq 1$,

such that

$uv^i w \in$ L, ∀ i = 0,1,2,3,......................

Application of Pumping Lemma

It is used to show that a language is not regular.

Note: Pumping lemma is based on the principle of pigeonhole principle.

Pigeonhole Principle

If we put n objects into m boxes and n$>$m, then at least one box must have more than one item in it.

Regular Expression

Some Examples

1. Show that L $= \{a^{i^2} ! \ i \geq 1\}$ is not regular.
2. Show that L $= \{a^n b^n ! \ i \geq 1\}$ is not regular.
3. Show that L $= \{ww^R ! \ w \in \{a, b\}^*\}$ is not regular.
4. Show that L $= \{a^p !$ p is a prime number $\}$ is not regular.
5. Show that L $= \{a^{n!} ! \ n \geq 0\}$ is not regular.
6. Show that L $= \{(ab)^n a^k ! \ n > k, \ k \geq 0\}$ is not regular.

Closure properties of regular languages

Theorem: Show that class of regular languages is closed under union operation.

or

If L_1 and L_2 are two regular set then $L_1 \cup Ł_2$ is also regular set.

Theorem: Show that class of regular languages is closed under concatenation operation.

or

If L_1 and L_2 are two regular set then $L_1.Ł_2$ is also regular set.

Theorem: Show that class of regular languages is closed under Kleene closure operation.

or

If L is regular set then L^* is also regular set.

Regular Expression

Theorem: Show that class of regular languages is closed under complement operation.

or

If L is regular set then $\overline{L}$ is also regular set.

Proof:

Suppose L is a regular set. Since L is regular set then there exists a finite automata. Let this finite automata is

M $= (Q, \Sigma, \delta, q_0, F)$.

Now we construct the finite automata M' as following:-

$M' = (Q, \Sigma, \delta, q_0, F')$.

Where $F' =$ Q-F

Now we have to show that L$(M') = \overline{L}(M) = \Sigma^* - L(M)$

Regular Expression

Let x$\in L(M') \;\Leftrightarrow\; \delta(q_0, x) \in F'$

$$\Leftrightarrow\; \delta(q_0, x) \in Q - F$$
$$\Leftrightarrow\; \delta(q_0, x) \notin F$$
$$\Leftrightarrow\; x \notin L(M)$$
$$\Leftrightarrow\; x \in \overline{L}(M)$$

Therefore L$(M') = \overline{L}(M)$

Clearly M' is a finite automata accepting $\overline{L}$. Therefore $\overline{L}$ is a regular set.

Regular Expression

Theorem: Show that class of regular languages is closed under intersection operation.

or

If L_1 and L_2 are two regular set then $L_1 \cap L_2$ is also regular set.

Proof:

$L_1 \cap \text{Ł}_2 = (\overline{\bar{L}_1 \cup \bar{L}_1}) = \Sigma^* - ((\Sigma^* - L_1) \cup (\Sigma^* - L_2))$ ....(1)

Since L_1 and L_2 are two regular set therefore $(\Sigma^* - L_1)$ and $(\Sigma^* - L_1)$ are also regular sets according to the previous theorem.

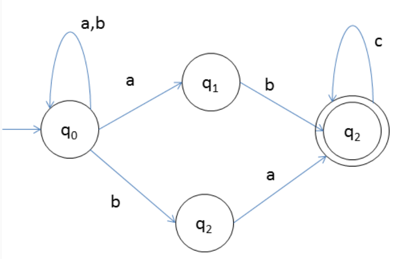
Since $(\Sigma^* - L_1)$ and $(\Sigma^* - L_1)$ is regular therefore $(\Sigma^* - L_1) \cup (\Sigma^* - L_1)$ is also regular set.

According to the previous theorem, $\Sigma^* - ((\Sigma^* - L_1) \cup (\Sigma^* - L_2))$ is also regular.

Now from equation (1), $L_1 \cap L_2$ is also regular set. Now it is proved.

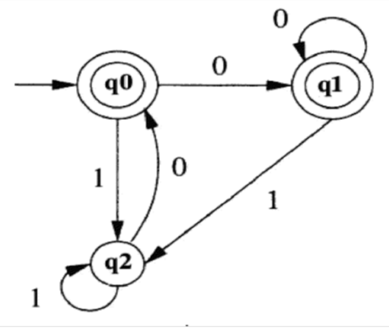
AKTU Examination Questions

1. For the given language $L_1 = \epsilon$, $L_2 = \{a\}$, $L_3 = \phi$. Compute $L_1 L_2^* \cup L_3^*$.
2. Write regular expression for set of all strings such that number of a's divisible by 3 over $\Sigma = \{a,b\}$.
3. Find the regular expression corresponding to the finite automata given bellow:



Regular Expression

4. Prove that the following Language L $= \{a^n b^n\}$ is not regular.
5. Explain the Closure properties of regular expression.
6. Design a regular expression that accepts all the strings for input alphabet {a,b} containing exactly 2 a's.
7. State recursive definition of regular expression and construct regular expression corresponding to the following state transition diagram:-



Regular Expression

8. State pumping lemma for regular sets. Show that L $= \{a^p$! p is a prime number$\}$ is not regular set.

9. Discuss closure properties of regular languages under the operations: concatenation, union, intersection and complement.

10. Give the regular expression for set of all strings over 0,1 containing exactly three 0's.

11. Write a regular expression to denote a language L which accepts all the strings that begin or end with either 00 or 11.

12. State closure properties of regular languages. Also Prove that regular languages are closed under intersection and difference.

13. State Arden's theorem and construct regular expression for the following FA using Arden's theorem:

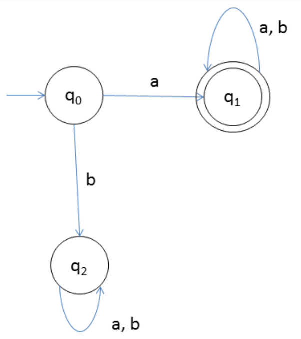
State	Input	
	0	1
A	{A, B}	ϕ
B	C	{A, B}
C	B	ϕ
A is the initial state and C is Final State		

14. Using pumping lemma, prove that the language L $= \{a^{i^2} \; ! \; i \geq 1\}$ is not regular.

15. State the pumping lemma theorem for regular languages.

Regular Expression

16. Convert the FA given below to left linear grammar.



17. Write regular expression for a language containing strings of 0's and 1's which does not end in '01'.