Discrete Structures and Theory of Logic

Lecture-2

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Operations defined on set

Union operation

For any two sets A and B, the union of A and B is the set of all the elements which are belongs into A or B or both. It is denoted by $A \cup B$. Mathematically, it is defined as following:-

$A \cup B = \{ x ! x \in A \text{ or } x \in B\}$

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, c \}$. Then union of A and B will be, $A \cup B = \{ a, b, c, d, e \}$.

Intersection operation

For any two sets A and B, the intersection of A and B is the set of all the elements which are belong into both A and B . It is denoted by $A \cap B$. Mathematically, it is defined as following:-

$A \cap B = \{ x ! x \in A \text{ and } x \in B\}$

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, c \}$. Then intersection of A and B will be, $A \cap B = \{ c \}$.

Operations defined on set(cont.)

Set difference operation

For any two sets A and B, the set difference of A and B is the set of all the elements which are belong into A but not belong into B . It is denoted by A-B. Mathematically, it is defined as following:-

A-B = $\{ x ! x \in A$ and $x \notin B\}$

Example: Let A = { a, b, c } and B = { d, e, c }. Then set difference of A and B will be, A-B = {a, b}.

Complement operation

Let U is the universal set. For any set A, the complement of A is the set of all the elements U, which are not belong into A. It is denoted by A^c or A'. Mathematically, it is defined as following:-

A'= U-A = $\{ x ! x \in U$ and $x \notin A\}$

Example: Let A = { a, b, c } and U = { a, b, c, d, e, f, g }. Then complement of A will be, A' = { d, e, f, g}.

Operations defined on set(cont.)

Symmetric difference operation

For any two sets A and B, the symmetric difference of A and B is denoted by $A \bigoplus B$. Mathematically, it is defined as following:-

$A \bigoplus B = (A\text{-}B) \cup (B\text{-}A)$

Example: Let $A = \{ a, b, c \}$ and $B = \{ d, e, c \}$. Then symmetric difference of A and B will be, $A \bigoplus B = \{a, b, d, e \}$.

Disjoint and Mutually disjoint sets

Disjoint sets

Two sets A and B are said to be disjoint if there is no common elements between A and B. That is, A and B are disjoint iff A∩B $=\phi$.

Example: Let A $=$ { a, b, c } and B $=$ { d, e, f }. Here A and B are disjoint because A∩B $=\phi$.

Mutually disjoint sets

A collection of sets S={ $A_1, A_2, ..., A_n$} is said to be mutually disjoint if each pair of A_i and A_j in S are disjoint. That is, S is mutually disjoint if $A_i \cap A_j = \phi$, $\forall$ i,j $=$ 1,2,.....,n. and i$\neq$j.

Example: Let A $=$ { {1, 2}, { 3} }, B $=$ { {1}, {2, 3}} and C $=$ { {1, 2, 3}}. These sets A, B and C are mutually disjoint because A∩B $=\phi$, B∩C $=\phi$, and A∩C $=\phi$.

Some examples

Example: Show that $A \subseteq B \Leftrightarrow A \cap B = A$

Solution: In this question, we have to prove two parts.

First part: In this part, we have to show that if $A \subseteq B$ then $A \cap B = A$.

Suppose $A \subseteq B$.

Let $x \in A$. Since $A \subseteq B$ therefore $x \in B$. Clearly x is belong into both A and B. Therefore x also belongs into $A \cap B$. Therefore

$$A \subseteq A \cap B \text{ ....}(1)$$

Let $x \in A \cap B$. Therefore $x \in A$ and $x \in B$. Therefore we can say $x \in A$. Therefore

$$A \cap B \subseteq A \text{ ....}(2)$$

Using equations(1) and (2), $A \cap B = A$.

Some examples(cont.)

Second part: In this part we have to show that if $A \cap B = A$ then $A \subseteq B$.
Let $x \in A$. Since $A \cap B = A$ therefore $x \in A \cap B$. This imply that $x \in A$ and $x \in B$. Therefore we can say $x \in B$. Therefore

$$A \subseteq B.$$

Some examples(cont.)

Example: Show that

(a) A-B = A$\cap$B'

(b) A $\subseteq$ B $\Leftrightarrow$ B' $\subseteq$ A'

Solution:

(a) Let x $\in$ A-B $\Rightarrow$ x $\in$ A and x $\notin$ B

$\Rightarrow$ x $\in$ A and x $\in$ B'

$\Rightarrow$ x $\in$ A$\cap$B'

Therefore, A-B $\subseteq$ A$\cap$B' ....(1)

Now, let x $\in$ A$\cap$B' $\Rightarrow$ x $\in$ A and x $\in$ B'

$\Rightarrow$ x $\in$ A and x $\notin$ B

$\Rightarrow$ x $\in$ A-B

Therefore, A$\cap$B' $\subseteq$ A-B ....(2)

Using equations (1) and (2), A-B = A$\cap$B'

Some examples(cont.)

(b) First part: Suppose $A \subseteq B$.

Let let $x \in B' \Rightarrow x \notin B$

$$\Rightarrow x \notin A \text{ (Since } A \subseteq B \text{)}$$

$$\Rightarrow x \in A'$$

Therefore, $B' \subseteq A'$

Second part: Suppose $B' \subseteq A'$.

Let let $x \in A \Rightarrow x \notin A'$

$$\Rightarrow x \notin B' \text{ (Since } B' \subseteq A' \text{)}$$

$$\Rightarrow x \in B$$

Therefore, $A \subseteq B$

Using first and second parts, we can say that

$$A \subseteq B \Leftrightarrow B' \subseteq A'$$

Some examples(cont.)

Example: Show that for any two sets A and B,

$$A\text{-}(A \cap B) = A\text{-}B$$

Solution: Let $x \in A\text{-}(A \cap B) \Leftrightarrow x \in A$ and $x \notin A \cap B$

$\Leftrightarrow x \in A$ and $x \in (A \cap B)'$

$\Leftrightarrow x \in A$ and ($x \notin A$ or $x \notin B$)

$\Leftrightarrow$ ($x \in A$ and $x \notin A$) or ($x \in A$ and $x \notin B$)

$\Leftrightarrow$ FALSE or ($x \in A$ and $x \notin B$)

$\Leftrightarrow$ ($x \in A$ and $x \notin B$)

$\Leftrightarrow x \in A\text{-}B$

Therefore, $A\text{-}(A \cap B) = A\text{-}B$