Theory of Automata and Formal Language

Lecture-10

Dharmendra Kumar
(Associate Professor)
Department of Computer Science and Engineering
United College of Engineering and Research, Prayagraj

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Non-deterministic Finite Automata (NFA)

A non-deterministic finite automata M is a 5-tuple, M = (Q, Σ, δ, q_0, F), where

Q $\rightarrow$ Finite set of states

$\Sigma \rightarrow$ Finite set of input symbols

$q_0 \in Q \rightarrow$ Initial state

F$\subseteq Q \rightarrow$ Set of final states

and $\delta \rightarrow$ Transition function

It is defined as following:-

$$\delta : Q \times \Sigma \rightarrow P(Q)$$

Where P(Q) is the power set of Q. That is,

$\delta(q, a) \subseteq Q, \qquad \forall q \in Q \text{ and } a \in \Sigma$

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Extended Transition Function

It is denoted by $\hat{\delta}$. It is defined as following:-

$$\hat{\delta} : Q \times \Sigma^* \rightarrow P(Q)$$

Properties of $\hat{\delta}$

1. $\hat{\delta}(q, \epsilon) = \{q\}$
2. $\hat{\delta}(q, a) = \delta(q, a), \qquad \forall a \in \Sigma$
3. $\hat{\delta}(q, wa) = \bigcup_{p \in \hat{\delta}(q,w)} \delta(p, a), \qquad$ where $q, p \in Q,\ a \in \Sigma$ *and* $w \in \Sigma^*$

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Another Extended Transition Function

It is denoted by $\hat{\hat{\delta}}$. It is defined as following:-

$$\hat{\hat{\delta}} : \mathrm{P(Q)x}\Sigma^* \rightarrow \mathrm{P(Q)}$$

Properties of $\hat{\hat{\delta}}$

1. $\hat{\hat{\delta}}(P, a) = \bigcup_{p \in P} \hat{\delta}(p, a)$
2. $\hat{\hat{\delta}}(P, w) = \bigcup_{p \in P} \hat{\delta}(p, w)$, where $P \subseteq Q,\ a \in \Sigma$ *and* $w \in \Sigma^*$

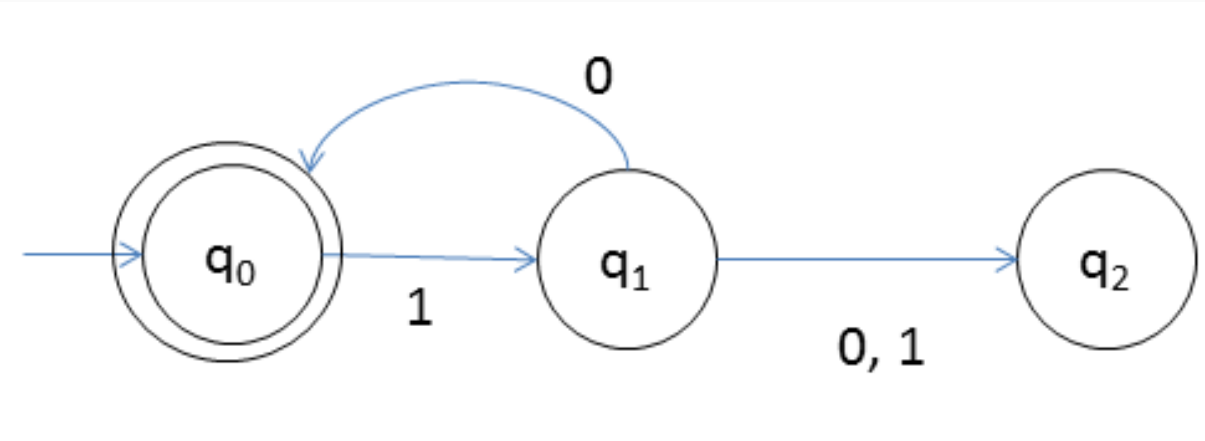
Language Accepted by NFA

Language accepted by NFA M is denoted by L(M). It is defined as following:-

$$L(M) = \{x \in \Sigma^* \mid \hat{\delta}(q_0, x) \cap F \neq \phi\}$$

Some Examples

Examples: Find the language accepted by following NFA:-



Non-deterministic Finite Automata (NFA)

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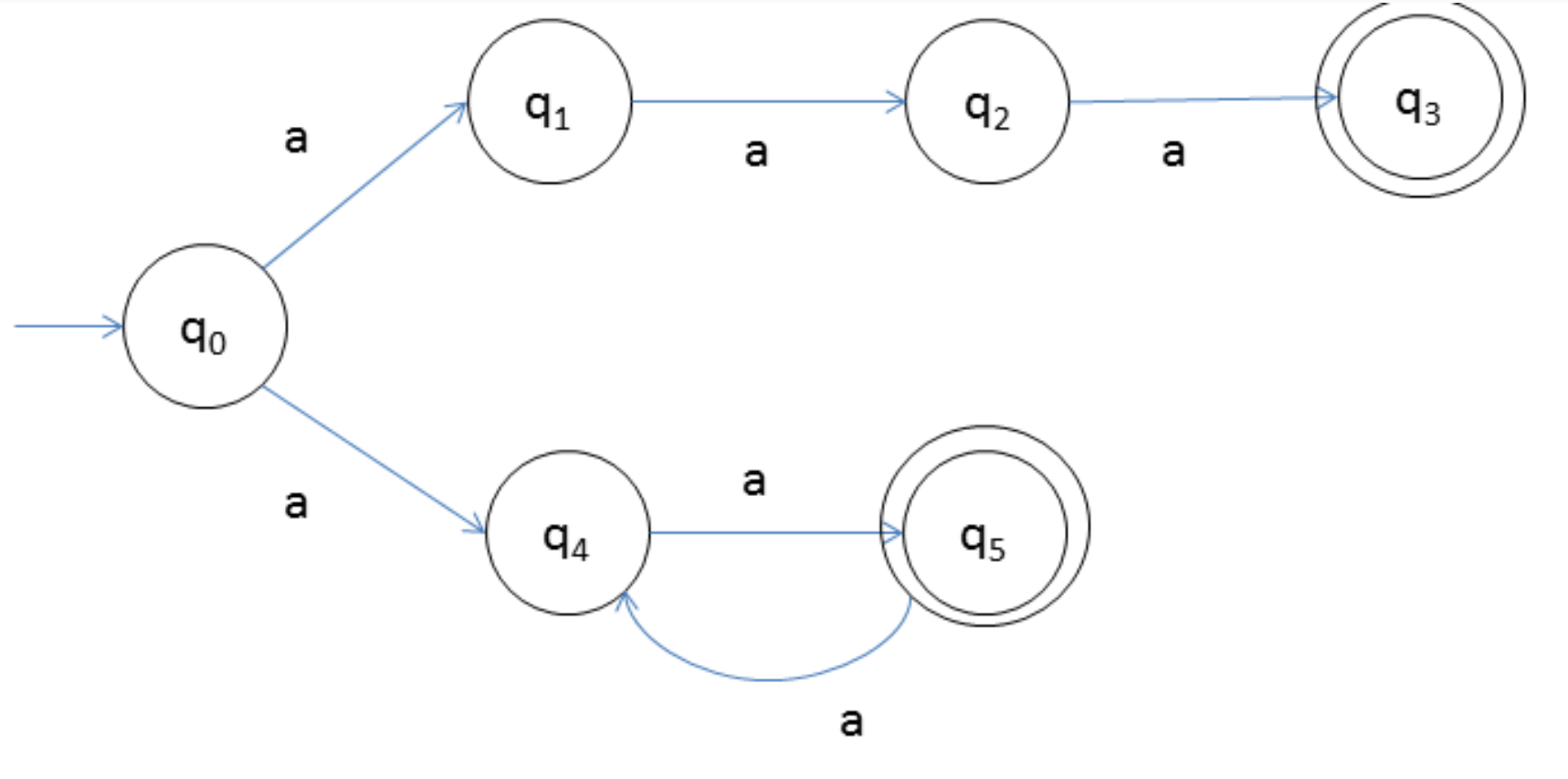

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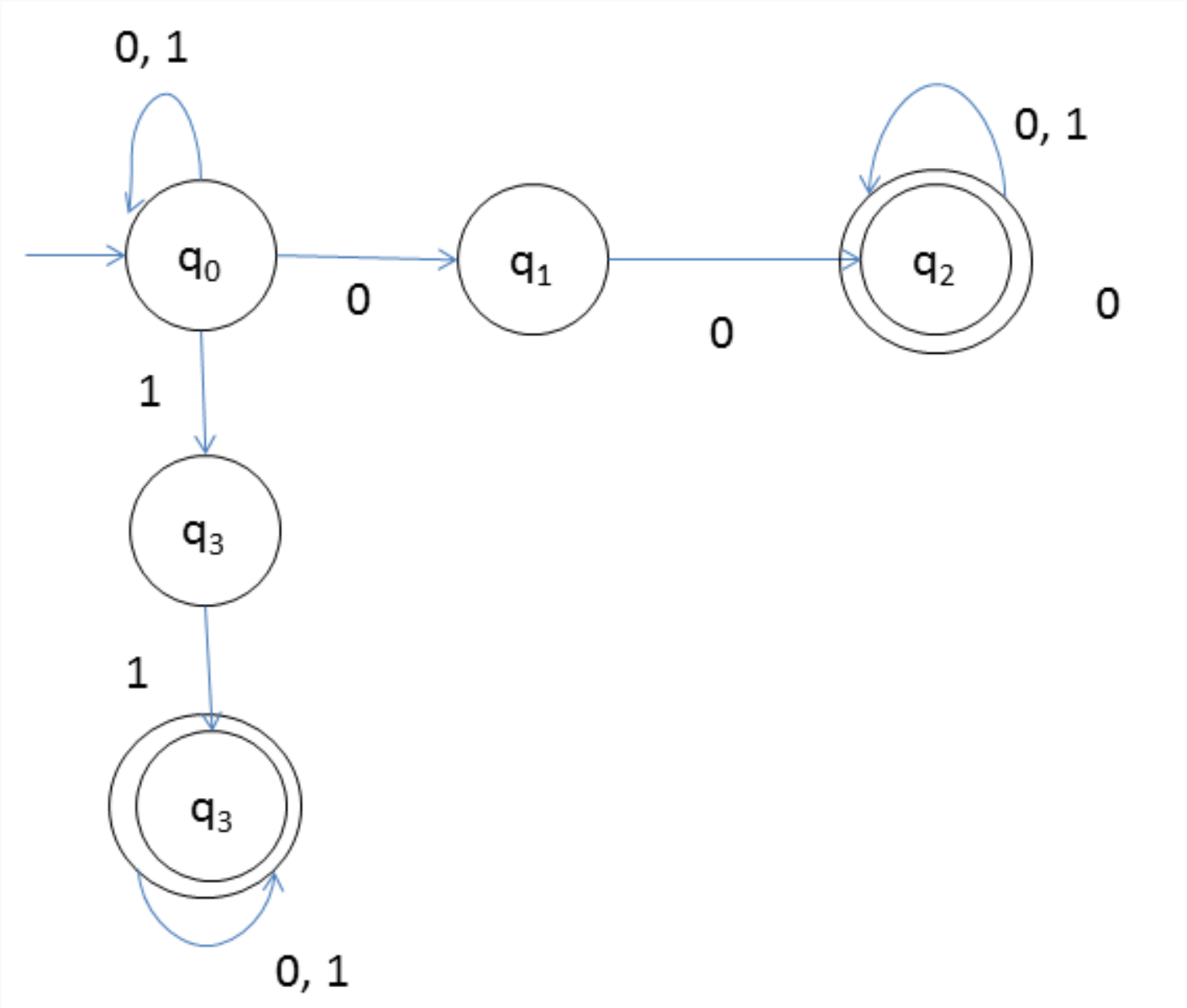
Examples: Find the language accepted by following NFA:-



Exercise

1. Design an NFA with no more than five states for the set $\{abab^n \mid n \geq 0\} \cup \{aba^n \mid n \geq 0\}$.
2. Construct an NFA with three states that accepts the language $\{ab, abc\}^*$.
3. Find an NFA with three states that accepts the language

$$L = \{a^n \mid n \geq 1\} \cup \{b^m a^k \mid m.k \geq 0\}$$