Discrete Structures and Theory of Logic

Lecture-1

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Syllabus

Unit-1

Set Theory: Introduction, Combination of sets, Multisets, Ordered pairs. Proofs of some general identities on sets. Relations: Definition, Operations on relations, Properties of relations, Composite Relations, Equality of relations, Recursive definition of relation, Order of relations.

Functions: Definition, Classification of functions, Operations on functions, Recursively defined functions. Growth of Functions.

Natural Numbers: Introduction, Mathematical Induction, Variants of Induction, Induction with Nonzero Base cases. Proof Methods, Proof by counter – example, Proof by contradiction.

Unit-2

Algebraic Structures: Definition, Groups, Subgroups and order, Cyclic Groups, Cosets, Lagrange's theorem, Normal Subgroups, Permutation and Symmetric groups, Group Homomorphisms, Definition and elementary properties of Rings and Fields.

Unit-3

Lattices: Definition, Properties of lattices –Bounded, Complemented, Modular and Complete lattice. Boolean Algebra: Introduction, Axioms and Theorems of Boolean algebra, Algebraic manipulation of Boolean expressions. Simplification of Boolean Functions, Karnaugh maps, Logic gates, Digital circuits and Boolean algebra.

Unit-4

Propositional Logic: Proposition, well formed formula, Truth tables, Tautology, Satisfiability, Contradiction, Algebra of proposition, Theory of Inference.

Predicate Logic: First order predicate, well formed formula of predicate, quantifiers, Inference theory of predicate logic.

Syllabus cont.

Unit-5

Trees: Definition, Binary tree, Binary tree traversal, Binary search tree.

Graphs: Definition and terminology, Representation of graphs, Multigraphs, Bipartite graphs, Planar graphs, Isomorphism and Homeomorphism of graphs, Euler and Hamiltonian paths, Graph coloring, Recurrence Relation Generating function: Recursive definition of functions, Recursive algorithms, Method of solving recurrences.

Combinatorics: Introduction, Counting Techniques, Pigeonhole Principle

Reference books

1. Kenneth H. Rosen, Discrete Mathematics and Its Applications, 6/e, McGraw-Hill, 2006.
2. B. Kolman, R.C. Busby, and S.C. Ross, Discrete Mathematical Structures, 5/e, Prentice Hall, 2004.
3. Trembley, J.P R. Manohar, “Discrete Mathematical Structure with Application to Computer Science”, McGraw Hill.
4. Sarkar, Swapan Kumar, Discrete Mathematics, S. Chand.

Course Outcome

CO1	Describe and Identify concepts of set theory, relation, function and Apply proof of induction method to proof the statements
CO2	Describe and Recognize group, ring and field and Solve the problem related with Group theory
CO3	Describe Lattice and Boolean algebra and Identify different types of lattice.
CO4	Formulate logic statements in terms of predicates, quantifiers, and logical connectives and Derive an expression equivalent to another expression.
CO5	Explain Trees, Graphs and Identify types of Graph and Solve problems related with Combinatory, Recurrence relations and Generating functions.

Set

- A well-defined collection of distinct objects can be considered to be a set.
- A set is typically expressed by curly braces, $\{\}$ enclosing its elements.
- If A is a set and a is an element of it, then we write $a \in A$. The fact that a is not an element of A is written as $a \notin A$.
- For instance, if A is the set $\{1, 2, 4, 9\}$, then $1 \in A$; $4 \in A$; $2 \in A$ and $9 \in A$. But $7 \notin A$; $10 \notin A$, the English word 'four' is not in A, etc.

Representation of sets

We can represent sets in two ways.

1. Tabular form or roster form: Listing the elements of a set inside a pair of braces { } is called the roster form.

2. Set builder form: In the set builder form, all the elements of the set, must possess a single property to become the member of that set.

Examples

1. Let X $=$ {apple, tomato, orange}. Here, orange $\in$ X, but potato $\notin$ X.
2. X $= \{a_1, a_2, ..., a_{100}\}$. Then, $a_{100} \in$ X.
3. Observe that the sets $\{1, 2, 3\}$ and $\{3, 1, 2\}$ are equal.
4. Let X $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then X is the set of first 10 natural numbers. Or equivalently, X is the set of integers between 0 and 11.
5. X $=$ {x : x is a prime number}.
6. X $=$ {x : $0 < x \leq 10$ and x is an even integer }

Clearly examples 1, 2, 3, and 4 are in roster form, but 5 and 6 are in set builder form.

Cardinality of a set

The number of elements in a set is said to be cardinality of a set. It is denoted by $||$ symbol. That is, if A is a set then cardinality of set A is denoted by $|A|$.

Example: X $= \{$x : $0 < x \leq 10$ and x is an even integer $\}$

The cardinality of this set, $|X| = 5$.

Types of set

Finite and Infinite sets

A set is said to be finite if the number of elements in the set is finite otherwise it is said to be infinite.

For example, a set of days in a week, set of months in a year, and a set of integer lie between 1 and 100 are finite sets. But set of integers, set of real numbers, and set of stars in sky are infinite sets.

Null or Empty set

A set which does not contain any element, is said to be null set. It is denoted by ϕ.

Example: set A $=$ $\{$a ! a is an integer lie between 4 and 5$\}$

Types of set(Cont.)

Singleton set

A set is said to be singleton set if it contains only one element.

Universal set

A universal set is the set of all elements under consideration, denoted by capital U or sometimes capital E.

Example: If we consider the elements are integers, then universal set will be the set of integer numbers. Similarly, if the elements are days of a week, then the set of all days in a week will be the universal set.

Types of set(Cont.)

Subset

Consider two sets A and B. Set B is said to be subset of A if all the elements of B belong into A. It is denoted by $\subseteq$ symbol. That is, B $\subseteq$ A.

Example: Consider three sets A, B and C such that A= $\{2, 3, 5, 8\}$, B= $\{3, 5\}$, C=$\{2, 9, 5, 8\}$. Clearly B is a subset of A but B is not a subset of C. Similarly, neither A is a subset of A nor C is a subset of A.

Note: (1) Every set A is a subset of itself i.e. A $\subseteq$ A.

(2) The null set ϕ is a subset of any set.

(3) If A $\subseteq$ B and B $\subseteq$ C then A $\subseteq$ C.

Types of set(Cont.)

Superset

Consider two sets A and B. Set A is said to be superset of B if all the elements of B belong into A. It is denoted by $\supseteq$ symbol. That is, A $\supseteq$ B.

Example: Consider three sets A, B and C such that A$=\{$ 2, 3, 5, 8$\}$, B$=\{$ 3, 5$\}$, C$=\{$ 2, 9, 5, 8$\}$. Clearly A is a superset of B but C is not a superset of B. Similarly, neither A is a superset of A nor C is a superset of A.

Types of set(Cont.)

Proper and Improper subsets

A set B is said to proper subset of set A if B is a subset of A and not equal to A that is B $\subseteq$ A and A $\neq$ B. It is denoted by $\supset$. Therefore, we can represent proper subset as B $\subset$ A.

A set B is said to improper subset of set A if B is a subset of A and equal to A that is B $\subseteq$ A and A $=$ B.

Example: Consider four sets A, B, C and D such that A$=\{$ 2, 3, 5, 8$\}$, B$=\{$ 3, 5$\}$, C$=\{$ 2, 3, 5$\}$, D$=\{$ 2, 3, 5, 8$\}$. Clearly, B and C are the proper subsets and D is an improper subset.

Types of set(Cont.)

Equal set

Two sets are said to be equal if both contains same elements. That is, if A $\subseteq$ B and B $\subseteq$ A then A = B.

Power set

The power set of a set A is the set of all the subsets of set A. It is denoted by P(A) or 2^A.

Example: (1) Consider set A = { a, b, c}. Then the power set of A is, P(A) = $\{\phi$,{a},{b},{c},{a,b},{b,c},{a,c},{a,b,c}}.

(2) The power set of null or empty set will be $\{\phi\}$.

Note: If set A has n elements then number of elements in the power set of A will be 2^n.