Theory of Automata and Formal Language

Lecture-24

Dharmendra Kumar
(Associate Professor)
Department of Computer Science and Engineering
United College of Engineering and Research, Prayagraj

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Simplification of Context Free Grammar

Simplification of grammar means to remove the useless symbols (variables and terminals) and production rules from the grammar.

Active Variable A variable A is said to be active if it derives a terminal string i.e

$A \rightarrow x$, where $x \in \Sigma^*$

Reachable symbols A symbol is said to be reachable if it appears in a string derives from starting symbol of the grammar i.e.
If $S \Rightarrow X$, then every symbols in X are said to be reachable.

Useful Variable A variable is said to be useful if it is active and reachable both.

Construction of a Grammar in which all the variables are active (Elimination of Non-Active Variables from a Grammar)

Suppose the given context free grammar is

G = (V,Σ,S,P)

Now we construct a context free grammar G' in which all the variables are active.

G' = (V',Σ, S, P')

Step-1: Determination of V'

Let A_i denote the set of active variables. A_i is determined recursively as following:-

$A_1 = \{$A $\in$ V ! if A$\rightarrow x \in P$, where $x \in \Sigma^*$ $\}$

$A_2 = A_1 \cup \{$A $\in$ V ! if A$\rightarrow \alpha \in P$, where $\alpha \in (A_1 | cup \Sigma)^*$ $\}$

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$A_{i+1} = A_i \cup \{$A $\in$ V ! if A$\rightarrow \alpha \in P$, where $\alpha \in (A_i | cup \Sigma)^*$ $\}$

Repeat this process until $A_{i+1} = A_i$

Now, we terminate this process. Now, V' = A_i

Step-2: Determination of P'

P' is obtained from P by removing those production rules in which non-active variables belong.

Example: Consider the grammar

G = ({S,A,B,C,E}, {a,b,c}, P, S)

where P = { $S \rightarrow AB$, $A \rightarrow a$, $B \rightarrow b$, $B \rightarrow C$, $E \rightarrow c/\epsilon$

Eliminate the non-active variables from this grammar.

Construction a Grammar in which all the symbols are reachable (Elimination of Non-Reachable Symbols from a Grammar)

Suppose the given context free grammar is

G = (V,Σ,S,P)

Now we construct a context free grammar G' in which all the symbols are reachable.

G' = (V',Σ', S, P')

Step-1: Determination of V' and ,Σ'

Let R_i denote the set of reachable symbols. R_i is determined recursively as following:-

$R_1 = \{$S $\}$

$R_2 = R_1 \cup \{$x ! A$\rightarrow \alpha \in P$, where $A \in R_1$ and $\alpha \in (V \cup \Sigma)^*$ and α contains x$\}$

....

....

$R_{i+1} = R_i \cup \{$x ! A$\rightarrow \alpha \in P$, where $A \in R_i$ and $\alpha \in (V \cup \Sigma)^*$ and α contains x$\}$

Repeat this process until $R_{i+1} = R_i$

Now, we terminate this process.

Now, V' $= R_i \cap V$

Now, Σ' $= R_i \cap \Sigma$

Step-2: Determination of P'

P' is obtained from P by removing those production rules in which non-reachable symbols belong.

Example: Consider the grammar

G = ({S,A,B,C,E}, {a,b,c}, P, S)

where P = { S$\rightarrow$AB, A$\rightarrow$a, B$\rightarrow$b, B$\rightarrow$C, E$\rightarrow$c/ϵ

Eliminate the non-reachable symbols from this grammar.

Construction of Reduced Grammar

Reduced Grammar: A grammar is said to be reduced grammar if all the symbols and production rules are useful.

Procedure:
Suppose the given grammar is G.
This methods consists of two steps. They are:-
Step-1: Find the grammar G' equivalent to G in which all the variables are active.
Step-2: Find the grammar G" equivalent to G' which includes only reachable symbols.
This grammar G" is a reduced grammar equivalent to G.

Context Free Grammar

Example: Find the reduced grammar equivalent to the following grammar

$S \rightarrow AB/CA$

$B \rightarrow BC/AB$

$A \rightarrow a$

$C \rightarrow aB/b$

Example: Reduce the following grammar

$S \rightarrow aAa$

$A \rightarrow Sb/bCC/DaA$

$C \rightarrow abb/DD$

$E \rightarrow aC$

$D \rightarrow aDA$