Theory of Automata and Formal Language

Lecture-2

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Definition of Grammar

A grammar G is defined as a quadruple G = (V,Σ,S,P), where

V → Finite set of variables or non-terminal symbols

Σ → Finite set of terminal symbols

S ∈ V → Starting symbol of the Grammar

P → Finite set of production rules

And P is defined as following:-

$P \subseteq (V \cup \Sigma)^* X (V \cup \Sigma)^*$

(u,v)∈ P is denoted by

u → v, where u,v ∈ $(V \cup \Sigma)^*$

u always contains at least one variable.

Direct derivation or derivation in one step

Let $\alpha, \beta \in (V \cup \Sigma)^*$. α directly derives β if $\alpha \to \beta \in P$. It is denoted by $\alpha \Rightarrow \beta$.

Derivation in many steps

Let $\alpha, \beta \in (V \cup \Sigma)^*$. α derives β if there exists production rules $\alpha \to A_1$, $A_1 \to A_2$,$A_2 \to A_3$,.....$A_n \to \beta$, such that $\alpha \Rightarrow A_1 \Rightarrow A_2 \Rightarrow A_3 \Rightarrow \ldots\ldots\ldots\ldots \Rightarrow A_n \Rightarrow \beta$. It is denoted by $\alpha \overset{*}{\Rightarrow} \beta$.

Sentential Form

Let $\alpha \in (V \cup \Sigma)^*$. The string α is said to be in the sentential form if
$S \stackrel{*}{\Rightarrow} \alpha$, where S is the starting symbol.

Language generated by a Grammar

The set of all the sentental forms consisting of only terminal symbols is said to be language generated Grammar. That is,

$$L(G) = \{ x \in \Sigma^* \mid S \stackrel{*}{\Rightarrow} x \}$$

Equivalent Grammars

Two grammars G_1 and G_2 are said to be equivalent grammar if the languages generated by both grammars are the same. That is,

$$\mathrm{L}(G_1) = \mathrm{L}(G_2).$$

Examples

Determine the languages generated by the following grammars:-

1. $S \rightarrow aS/a$
2. $S \rightarrow 0S1/\epsilon$
3. $S \rightarrow aCa$, $C \rightarrow aCa/b$
4. $S \rightarrow aS/bS/a/b$
5. $S \rightarrow 0SA2$, $S \rightarrow 012$, $2A \rightarrow A2$, $1A \rightarrow 11$