Theory of Automata and Formal Language

Lecture-17

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Regular expression

Regular expression is defined recursively as follows:-

(1) ϕ, $\overline{\epsilon}$ and $\overline{a}$ are regular expression, where $a \in \Sigma$. These are called primitives regular expressions.

(2) If $\overline{r_1}$ and $\overline{r_2}$ are two regular expressions then $\overline{r_1} + \overline{r_2}$ and $\overline{r_1}\overline{r_2}$ are also regular expressions.

(3) If $\overline{r}$ are regular expression then $\overline{r}^*$ and $(\overline{r})$ are also regular expressions.

(4) Any expression derived from the step 1 to 3 are also regular expression.

Example: $(\overline{a} + \overline{b} + \overline{c})^*.(\overline{c} + \phi)$ is a regular expression.

Regular Expression

Regular language or Regular set
Each regular expression represents a set of elements. This set is said to be regular set.

Example:
Represent the following sets by regular expression:-

1. $\{1^{2n} \mid n > 0\}$
2. $\{w \in \{a, b\}^* \mid w \text{ has only one } a\}$
3. The set of all strings over $\{0,1\}$ which has most two 0's.
4. $\{a^2, a^5, a^8, \ldots\ldots\ldots\ldots\ldots\}$
5. $\{1^n \mid n \text{ is divisible by 2 or 3 or } n=5\}$
6. The set of all strings over $\{a,b\}$ beginning and ending with a.
7. The set of all strings over $\{a,b\}$ in which the number of occurrences of a is divisible by 3.
8. The set of all strings over $\{a,b\}$ with three consecutive b's.

Example:

Find the regular expressions for the following sets:-

1. $L = \{a^n b^m \ ! \ n \geq 1, m \geq 1, mn \geq 3\}$
2. $L = \{a^n \ ! \ n \geq 0, n \neq 3\}$
3. $L = \{a^{2n} b^{2m+1} \ ! \ n \geq 0, m \geq 0\}$
4. $L = \{a^n b^m \ ! \ n+m \text{ is even}\}$

Example:

Write regular expression for the following language over alphabet {0,1}.

1. All strings ending in 01.
2. All strings not ending in 01.
3. All strings containing an even number of 0's.
4. All strings with at most two occurrences of the substring 00.
5. All strings not containing the substring 10.

Example:

Write regular expression for the following language.

1. $L = \{w \in \{0,1\}^* \text{ ! } w \text{ has at least one pair of consecutive 0's } \}$
2. $L = \{w \in \{0,1\}^* \text{ ! } w \text{ has no pair of consecutive 0's } \}$
3. $L = \{w \text{ ! } w \in \{a,b\}^* \text{ and } |w| \bmod 3 = 0\}$
4. $L = \{w \text{ ! } w \in \{a,b\}^* \text{ and } n_a(w) \bmod 3 = 0\}$

Example:

Find the set corresponding to the following regular expressions:-

1. $(aa)^*(bb)^*b$
2. $(0+1)^*00(0+1)^*$
3. $((0+1)(0+1)^*)^*00(0+1)^*$
4. $(a+b)^*(a+bb)$
5. $(aa)^*(bb)^*b$