Discrete Structures and Theory of Logic

Lecture-38

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Exercise

1. Show the validity of the following arguments, for which the premises are given on the left and the conclusion on the right.

(a) $\neg(P \wedge \neg Q)$, $\neg Q \vee R$, $\neg R$ C: $\neg P$

(b) $(A \to B) \wedge (A \to C)$, $\neg(B \wedge C)$, $D \vee A$ C: D

(c) $\neg J \to (M \vee N)$, $(H \vee G) \to \neg J$, $H \vee G$ C: $M \vee N$

(d) $(P \to Q)$, $(\neg Q \vee R) \wedge \neg R$, $\neg(\neg P \wedge S)$ C: $\neg S$

2. Derive the following using rule CP if necessary.

(a) $\neg P \vee Q$, $\neg Q \vee R$, $R \to S \Rightarrow P \to S$

(b) P, $P \to (Q \to (R \wedge S))$, $\Rightarrow Q \to S$

(c) $(P \vee Q) \to R \Rightarrow (P \wedge Q) \to R$

(d) $P \to (Q \to R)$, $Q \to (R \to S) \Rightarrow P \to (Q \to S)$

3. Show that the following sets of premises are inconsistent.

(a) $P \to Q$, $P \to R$, $Q \to \neg R$, P

(b) $A \to (B \to C)$, $D \to (B \wedge \neg C)$, $A \wedge D$

4. Show the following (use indirect method if needed).

(a) $R \rightarrow \neg Q$, $R \vee S$, $S \rightarrow \neg Q$, $P \rightarrow Q \Rightarrow \neg P$

(b) $S \rightarrow \neg Q$, $R \vee S$, $\neg R$, $\neg R \leftrightarrow Q \Rightarrow \neg P$

(c) $\neg(P \rightarrow Q) \rightarrow \neg(R \vee S)$, $((Q \rightarrow P) \vee \neg R)$, $R \Rightarrow P \leftrightarrow Q$

Predicate Calculus

Predicate

Consider the statements.

John is a bachelor.

Smith is a bachelor.

The part "is a bachelor" is called a predicate.

Now, we denote the predicate as following:- B: is a bachelor.

Therefore the statement in predicate form will be:-

B(John), B(Smith).

Example: Write the following statements in predicate form.

(a) Jack is taller than Jill.

(b) Canada is to the north of the United States.

Solution:

(a) Let P: is taller than

Therefore, statement in predicate form will be

P(Jack, Jill)

(b)Let Q: is to the north of the

Therefore, statement in predicate form will be

Q(Canada, United States)

The statement function, variables and quantifiers

A simple statement function of one variable is defined to be an expression consisting of a predicate symbol and an individual variable. Such a statement function becomes a statement when the variable is replaced by the name of any object.

Let M(x) : x is a man.

H(x): x is a mortal.

The compound statement functions are

$M(x) \wedge H(x)$, $M(x) \rightarrow H(x)$, $\neg H(x)$, $M(x) \vee \neg H(x)$ etc.

Example: Consider the following statements

(a) All men are mortal.

(b) Every apple is red.

(c) Any integer is either positive or negative.

Write these statements in predicate form.

Solution:

(a) Let M(x) : x is a man.

H(x): x is a mortal.

Therefore, the predicate form of the statement will be

$(\forall x)(M(x) \rightarrow H(x))$

(b) Let A(x) : x is an apple.

R(x): x is a red.

Therefore, the predicate form of the statement will be

$(\forall x)(A(x) \rightarrow R(x))$

(c) Let I(x) : x is an integer.
P(x): x is either positive or negative integer.
Therefore, the predicate form of the statement will be
$(\forall x)(I(x) \rightarrow P(x))$
The symbol $(\forall x)$ or (x) is said to be universal quantifier. It is used in the statement which contains for all, every and for any.

Example: Find the predicate form of the following statement.
For any x and y, if x is taller than y, then y is not taller than x.
Solution:
$(\forall x)(\forall y)(G(x, y) \rightarrow \neg G(y, x))$

Example: Consider the following statements

(a)There exists a man.

(b)Some men are clever.

(c) Some real numbers are rational.

Write these statements in predicate form.

Solution:

Let M(x): x is a man.

C(x): x is clever.

R(x): x is real number.

Q(x): x is rational number.

(a) $(\exists x)M(x)$

(b) $(\exists x)(M(x) \wedge C(x))$

(c) $(\exists x)(R(x) \wedge Q(x))$

The symbol $(\exists x)$ is said to be existential quantifier. It is used in the statement which contains some or there exists.