Discrete Structures and Theory of Logic

Lecture-3

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Partition of a set

Let S be a given set and A = $\{A_1, A_2, A_3, \ldots\ldots\ldots\ldots, A_n\}$, where each A_i, for i = 1,2,3,......,n, is a subset of S.

A is called the partition of set S if it satisfies the following two conditions:-
(1) $\cup_{i=1}^{n} A_i$ = S
(2) $A_i \cap A_j = \phi$, $\forall$ i,j = 1,2,3,.......,n and i$\neq$j.
Example: Consider set S = { a, b, c, d } and A = {{a, b }, { c, d }}.
In this example, A is the partition of S because

$$\{a, b\} \cup \{c, d\} = S \text{ and } \{a, b\} \cap \{c, d\} = \phi.$$

Multiset

A multiset is an unordered collection of elements, in which the multiplicity of an element may be one or more than one or zero. The multiplicity of an element is the number of times the element repeated in the multiset. In other words, we can say that an element can appear any number of times in a set.

Example:

$A = \{l, l, m, m, n, n, n, n\}$

$B = \{a, a, a, a, a, c\}$

Union of Multisets

The Union of two multisets A and B is a multiset such that the multiplicity of an element is equal to the maximum of the multiplicity of an element in A and B and is denoted by $A \cup B$.

Example:

Let $A = \{l, l, m, m, n, n, n, n\}$

$B = \{l, m, m, m, n\}$,

Therefore, $A \cup B = \{l, l, m, m, m, n, n, n, n\}$

Intersection of Multisets

The intersection of two multisets A and B, is a multiset such that the multiplicity of an element is equal to the minimum of the multiplicity of an element in A and B and is denoted by $A \cap B$.

Example:

Let $A = \{l, l, m, n, p, q, q, r\}$

$B = \{l, m, m, p, q, r, r, r, r\}$

Therefore, $A \cap B = \{l, m, p, q, r\}$

Difference of Multisets

The difference of two multisets A and B, is a multiset such that the multiplicity of an element is equal to the multiplicity of the element in A minus the multiplicity of the element in B if the difference is +ve, and is equal to 0 if the difference is 0 or negative.

Example:

Let A = {l, m, m, m, n, n, n, p, p, p}

B = {l, m, m, m, n, r, r, r}

A - B = {n, n, p, p, p}

Sum of Multisets

The sum of two multisets A and B, is a multiset such that the multiplicity of an element is equal to the sum of the multiplicity of an element in A and B.

Example:

Let $A = \{l, m, n, p, r\}$

$B = \{l, l, m, n, n, n, p, r, r\}$

$A + B = \{l, l, l, m, m, n, n, n, n, p, p, r, r, r\}$

Countable and Uncountable sets

A set is said to be countable if:
(1) It is finite, or
(2) It has the same cardinality (size) as the set of natural numbers.
Equivalently, a set is countable if it has the same cardinality as some subset of the set of natural numbers. Otherwise, it is uncountable.
For example, the set of integers, the set of rational numbers or the set of algebraic numbers are countable set. An uncountable set is the set of real numbers.
For example, the set of real numbers between 0 and 1 is an uncountable set because no matter what, you'll always have at least one number that is not included in the set. This set does not have a one-to-one correspondence with the set of natural numbers.

Exercise

Give another description of the following sets and indicate those which are infinite sets.

1. $\{\, x \mid x$ is an integer and $5 \leq x \leq 12\}$
2. $\{\, 2, 4, 8, \ldots\ldots\ldots\ldots \,\}$
3. All the countries of the world.

Exercise

Given S = { 2, a, { 3 }, 4 } and R = { { a }, 3, 4, 1 }, indicate whether the following are true or false.

1. $\{a\} \in S$
2. $\{a\} \in R$
3. $\{ a, 4, \{3\}\} \subseteq S$
4. $\{\{a\}, 1, 3, 4 \} \subset R$
5. $R = S$
6. $\{a\} \subseteq S$
7. $\{a\} \subseteq R$
8. $\phi \subset R$
9. $\phi \subseteq \{\{a\}\} \subseteq R \subseteq U$
10. $\{\phi\} \subseteq S$
11. $\{\phi\} \in R$
12. $\{\phi\} \subseteq \{\{3\}, 4\}$

Exercise

1. Show that
   $(R \subseteq S) \wedge (S \subset Q) \Rightarrow R \subset Q$
   Is it correct to replace $R \subset Q$ by $R \subseteq Q$? Explain your answer.
2. Determine the power sets of the followings:-
   2.1 $\{ a, \{ b \} \}$
   2.2 $\{ 1, \phi \}$
   2.3 $\{ 1, 2, 3, 4 \}$
3. What is the power set of the empty set? What is the power set of the set $\{\phi\}$?

Exercise

1. Find the numbers between 1 to 500 that are not divisible by any of the integers 2 or 3 or 5 or 7.
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2. Determine whether each of these statements is true or false.
   2.1 $0 \in \phi$
   2.2 $\phi \in 0$
   2.3 $0 \subset \phi$
   2.4 $\phi \subset 0$
   2.5 $0 \in 0$
   2.6 $0 \subset 0$
   2.7 $\phi \subseteq \phi$