Theory of Automata and Formal Language

Lecture-23

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Definition

A grammar G is said to be context free grammar if all the production rules of the grammar are of the following form:-

$$A \rightarrow \alpha \text{ where } \alpha \in (V \cup \Sigma)^* \text{ and } A \in V$$

Example: Consider the following grammar

S$\rightarrow$ 0B/1A

A$\rightarrow$ 0/0S/1AA

B$\rightarrow$ 1/1S/0BB

Derivation Tree

A tree is said to be derivation tree if it satisfies the following properties:-

1. All the nodes of the tree are labeled by variable, terminal or ϵ symbol.
2. The root node of the tree has labeled S (Starting symbol of the grammar).
3. All the internal nodes have labeled variable symbol.
4. All the leaf nodes have labeled terminal symbol or ϵ symbol.
5. If A $\rightarrow X_1X_2.....X_n$ be a production rule used in the derivation of the string, then in the tree, A will be at the parent node and $X_1, X_2, ..., X_n$ will be at the children of this node A.

Left Most Derivation

A derivation $A \overset{*}{\Rightarrow} w$ is said to be left most derivation if we apply the production rule in the derivation at the left most variable in every step.

Right Most Derivation

A derivation $A \overset{*}{\Rightarrow} w$ is said to be right most derivation if we apply the production rule in the derivation at the right most variable in every step.

Some Examples

Example: Consider the following grammar

S$\rightarrow$ 0B/1A

A$\rightarrow$ 0/0S/1AA

B$\rightarrow$ 1/1S/0BB

For the string 00110101, find the left most derivation, right most derivation and derivation tree.

Example: Consider the following grammar

S$\rightarrow$ AA

A$\rightarrow$ a/bA/Ab/AAA

Find parse tree for the string bbaaaab.

Example: Consider the following grammar

S$\rightarrow$ aAS/a

A$\rightarrow$ SbA/SS/ba

Find derivation tree for the string aabbaa.

Ambiguity in Grammar and Language

Ambiguous String

A string w $\in$ L(G) is said to be ambiguous string if there exists more than one derivation for the string.

Ambiguous Grammar

A grammar G is said to be ambiguous if there exists some string w $\in$ L(G) for which more than one derivation tree are possible.

Context Free Grammar

Example: Consider the following grammar:-

$S \rightarrow S+S/S*S/a/b$

Is this grammar ambiguous?

Solution:

Example: Consider the grammar G,

$S \rightarrow SbS/a.$

Show that grammar G is ambiguous.

Solution:

Context Free Grammar

Example: Consider the following grammar:-

S$\rightarrow$ a/abSb/aAb

A$\rightarrow$ bS/aAAb

Is this grammar ambiguous?

Solution:

Example: Consider the following grammar:-

S$\rightarrow$ aB/ab

A$\rightarrow$ aAB/a

B$\rightarrow$ ABb/b

Is this grammar ambiguous?

Solution:

Inherent Ambiguity

- If L is a context free language for which there exists an unambiguous grammar, then L is said to be unambiguous.
- If every grammar that generates L is ambiguous, then the language is said to be inherently ambiguous.

Example: Following language is inherent ambiguous
L = $\{a^n b^n c^m d^m ! n \geq 1, m \geq 1\} cup \{a^n b^m c^m d^n ! ngeq 1, m \geq 1\}$