Discrete Structures and Theory of Logic

Lecture-6

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Properties of binary relations defined on a set

There are following properties which can be defined on a set. Consider the set is A.

Reflexive property

A binary relation R defined on set A is said to be satisfies reflexive property if every element of set A is related to itself. That is, aRa , $\forall$ a $\in$ A. That is, (a,a) $\in$ R, $\forall$ a $\in$ A.

Symmetric property

A binary relation R defined on set A is said to be satisfies symmetric property if (a,b) $\in$ R then (b,a) $\in$ R, $\forall$ a,b $\in$ A. That is, if aRb then bRa, $\forall$ a,b $\in$ A.

Properties of binary relations(cont.)

Transitive property

A binary relation R defined on set A is said to be satisfies transitive property if (a,b) $\in$ R and (b,c) $\in$ R then (a,c) $\in$ R, $\forall$ a,b,c $\in$ A.

Irreflexive property

A binary relation R defined on set A is said to be satisfies irreflexive property if no element of set A is related to itself. That is, (a,a) $\notin$ R, $\forall$ a $\in$ A.

Properties of binary relations(cont.)

Anti-symmetric property

A binary relation R defined on set A is said to be satisfies anti-symmetric property if (a,b) $\in$ R and (b,a) $\in$ R then a=b, $\forall$ a,b $\in$ A.

Asymmetric property

A binary relation R defined on set A is said to be satisfies asymmetric property if (a,b) $\in$ R then (b,a) $\notin$ R, $\forall$ a,b $\in$ A.

Note: A relation which satisfies reflexive property is said to be reflexive relation. A relation which satisfies symmetric property is said to be symmetric relation. A relation which satisfies transitive property is said to be transitive relation. A relation which satisfies irreflexive property is said to be irreflexive relation. A relation which satisfies anti-symmetric property is said to be anti-symmetric relation. A relation which satisfies asymmetric property is said to be asymmetric relation.

Some examples

Example: Consider the following relations defined on set $A = \{1,2,3,4\}$. Find out which of these satisfies which of the above properties i.e. reflexive, symmetric, transitive, irreflexive, anti-symmetric, and asymmetric.

1. $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$
2. $\{(1,1),(2,2),(2,1),(1,2),(3,3),(4,4)\}$
3. $\{(2,4),(4,2)\}$
4. $\{(1,2),(2,3),(3,4)\}$
5. $\{(1,1),(2,2),(3,3),(4,4)\}$
6. $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

Some examples(cont.)

Solution:

1. Transitive.
2. Reflexive, symmetric, transitive.
3. Symmetric, irreflexive.
4. Irreflexive, anti-symmetric, asymmetric.
5. Reflexive, symmetric, transitive, anti-symmetric.
6. Irreflexive.

Example: Give an example of a relation which satisfies corresponding properties.

1. Neither reflexive nor irreflexive.
2. Both symmetric and anti-symmetric.
3. Reflexive, transitive but not symmetric.
4. Symmetric, transitive but not reflexive.
5. Reflexive, symmetric but not transitive.
6. Reflexive, transitive but neither symmetric nor anti-symmetric.

Some examples(cont.)

Example: Which of the following relations are transitive?
$R_1 = \{(1,1)\}$, $R_2 = \{(1,2),(2,2)\}$, $R_3 = \{(1,2),(2,3),(1,3),(2,1)\}$
solution: R_1 and R_2 are transitive but R_3 is not transitive. R_3 is not transitive because for pairs (1,2) and (2,1), its transitie Pair (1,1) not belong into R_3.

Some examples(cont.)

Example: Given S = $\{1,2,3,4\}$, and a relation R on S defined by

$$R = \{(1,2),(4,3),(2,2),(2,1),(3,1)\}$$

Show that R is not transitive. Find a relation $R_1 \supseteq R$ such that R_1 is transitive. Can you find another relation $R_2 \supseteq R$ which is also transitive?

Example: Given S = $\{1,2,3,.........,10\}$, and a relation R on S defined by

$$R = \{ (a,b) ! \ a+b = 10\}$$

Which of the properties of a relation satisfy R?