Theory of Automata and Formal Language

Lecture-15

Dharmendra Kumar
(Associate Professor)
Department of Computer Science and Engineering
United College of Engineering and Research, Prayagraj

April 21, 2021

Non-deterministic finite automata with null transition($\epsilon - move$)

A non-deterministic finite automata with $\epsilon - move$ M is a 5-tuple,

M = (Q, Σ, δ, q_0, F), where

Q $\rightarrow$ Finite set of states

Σ $\rightarrow$ Finite set of input symbols

$q_0 \in Q \rightarrow$ Initial state

F$\subseteq Q \rightarrow$ Set of final states

and $\delta \rightarrow$ Transition function

It is defined as following:-

$$\delta : \text{Q}\text{x}(\Sigma \cup \{\epsilon\}) \rightarrow \text{P(Q)}$$

Finite Automata (FA)

$\epsilon - closure(q)$

$\epsilon - closure$ of a state q is the set of all the states reachable from q by using only $\epsilon - move$.

$\epsilon - closure(P)$

Here P$\subseteq$Q.

$\epsilon - closure(P) = \bigcup_{p \in P} \epsilon - closure(p)$

Finite Automata (FA)

Extended Transition Function

It is denoted by $\hat{\delta}$. It is defined as following:-

$$\hat{\delta} : Q x \Sigma^* \rightarrow P(Q)$$

Properties of $\hat{\delta}$

1. $\hat{\delta}(q, \epsilon) = \epsilon - closure(q)$
2. $\hat{\delta}(q, a) = \epsilon - closure(\bigcup_{p \in \epsilon - closure(q)} \delta(p, a))$
3. $\hat{\delta}(q, wa) = \epsilon - closure(\bigcup_{p \in \hat{\delta}(q,w)} \delta(p, a))$, where $q, p \in Q,\ a \in \Sigma \ and \ w \in \Sigma^*$

Another Extended Transition Function

It is denoted by $\hat{\hat{\delta}}$. It is defined as following:-

$$\hat{\hat{\delta}} : \mathrm{P(Q)x}\Sigma^* \rightarrow \mathrm{P(Q)}$$

Properties of $\hat{\hat{\delta}}$

1. $\hat{\hat{\delta}}(P, a) = \underset{p \in P}{\cup} \hat{\delta}(p, a)$
2. $\hat{\hat{\delta}}(P, w) = \underset{p \in P}{\cup} \hat{\delta}(p, w),$ where $P \subseteq Q,\ a \in \Sigma\ and\ w \in \Sigma^*$

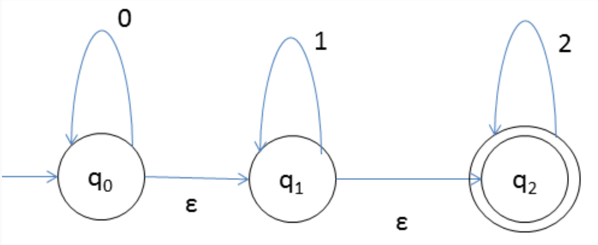
Language Accepted by NFA with null transition($\epsilon - move$)

Language accepted by NFA M is denoted by L(M). It is defined as following:-

$$L(M) = \{x \in \Sigma^* \;!\; \hat{\delta}(q_0, x) \cap F \neq \phi\}$$

Some Examples

Examples: Consider the following NFA:-



Determine the followings:-

(1) $\epsilon - closure\ of\ q_0,\ q_1,\ and\ q_2$

(2) $\hat{\delta}(q_0, 0)$, $\hat{\delta}(q_0, 00)$, $\hat{\delta}(q_0, 011)$, and $\hat{\delta}(q_1, 12)$.

(3) Language accepted by this NFA.

Conversion of NFA with $\epsilon - move$ into NFA without $\epsilon - move$

Suppose the given NFA with $\epsilon - move$ M is the following:-

M = (Q,Σ,δ,q_0,F)

Now, we construct M' as the following:-

M' = (Q,Σ,δ',q_0,F')

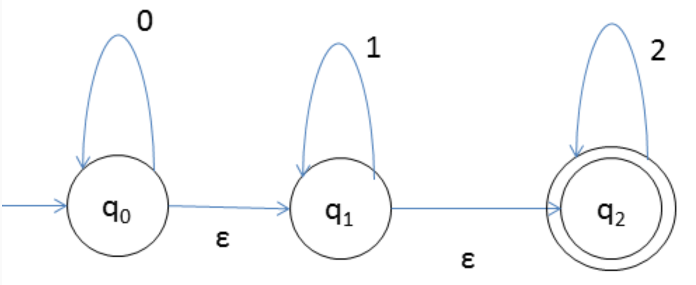
Where Q, Σ, q_0 are the same as M and

$$F' = \begin{cases} F \cup \{q_0\}\ , & if \quad \epsilon - closure(q_0) \cap F \neq \phi \\ F \quad , & otherwise \end{cases} \tag{1}$$

δ' is defined as following

$\delta'(q, a) = \hat{\delta}(q, a)$, for all q∈Q and a∈ Σ.

Examples: Consider the following NFA with $\epsilon - move$:-



Find NFA without $\epsilon - move$ equivalent to this.

Solution: