Discrete Structures and Theory of Logic

Lecture-43

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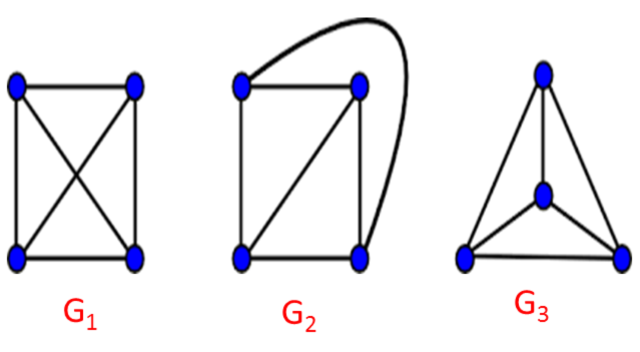
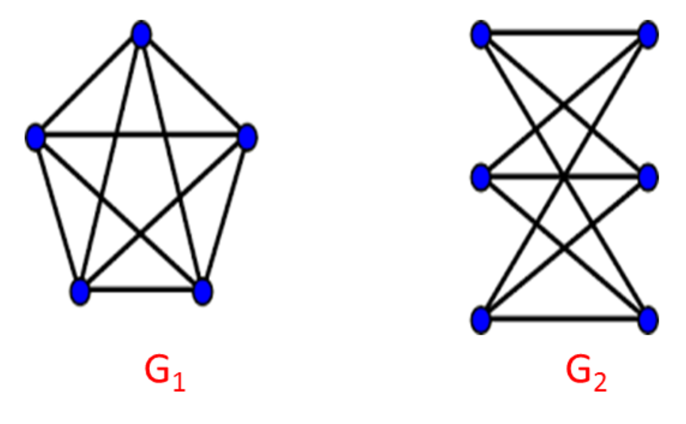
Planar Graph

A graph is said to be planar if it can be drawn on a plane without crossing their edges.

Example: Are the following graphs planar?



Example: Are the following graphs planar?



Properties of Planar Graphs:

1. If a connected planar graph G has e edges and r regions, then r $\geq$ (2/3)e.
2. If a connected planar graph G has e edges and v vertices, then 3v-e $\geq$ 6.
3. A complete graph K_n is a planar if and only if n $<$ 5.
4. A complete bipartite graph K_{mn} is planar if and only if m $<$ 3 or n $<$ 3.

Example: Prove that complete graph K_4 is planar.
Solution: The complete graph K_4 contains 4 vertices and 6 edges. We know that for a connected planar graph 3v-e $\geq$6. Hence for K_4, we have 3x4-6 $=$ 6 which satisfies the property. Thus K_4 is a planar graph. Hence Proved.

Euler's Formula

Let G be a connected planar graph and let n, e and r denote respectively the number of vertices, edges and region in a plane representation of G, then $n\text{-}e+r = 2$.

Matrix representation of Graphs

There are two principal ways to represent a graph G with the matrix, i.e., adjacency matrix and incidence matrix representation.

Adjacency Matrix Representation

If an undirected Graph G consists of n vertices, then the adjacency matrix of a graph is an n x n matrix A $= [a_{ij}]$ and defined by

$a_{ij} = 1$, if there exists an edge between vertex v_i and v_j

$= 0$, otherwise

Incidence Matrix Representation

If an undirected Graph G consists of n vertices and m edges, then the incidence matrix is an n x m matrix $C = [c_{ij}]$ and defined by
$c_{ij} = 1$, if the vertex v_i incident by edge e_j
$= 0$, otherwise

Note: The number of ones in an incidence matrix of the undirected graph (without loops) is equal to the sum of the degrees of all the vertices in a graph.

Graph Coloring

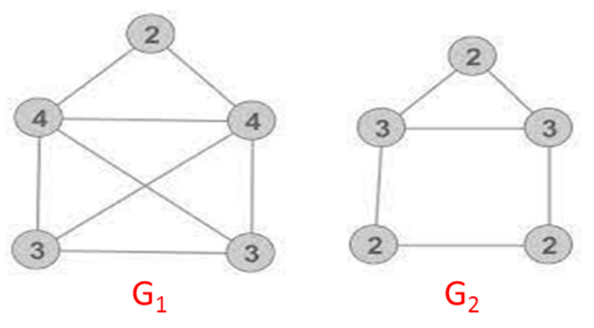
Vertex Coloring

Vertex coloring is an assignment of colors to the vertices of a graph ‘G’ such that no two adjacent vertices have the same color.

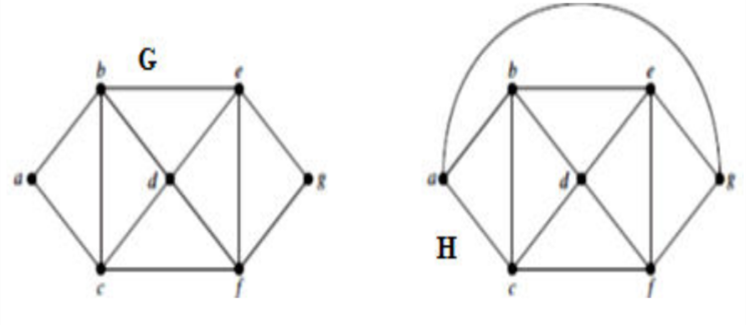
Chromatic Number

The minimum number of colors required for vertex coloring of graph ‘G’ is called as the chromatic number of G, denoted by X(G).

Example: Find the chromatic number of the following graphs



Example: Find the chromatic number of the following graphs



Note: $\chi(G) = 1$, if and only if 'G' is a null graph. If 'G' is not a null graph, then $\chi(G) \geq 2$.

Note: A graph 'G' is said to be n-coverable if there is a vertex coloring that uses at most n colors, i.e., X(G) $\geq$ n.

Note: The chromatic number of K_n is n.

Region Coloring

Region coloring is an assignment of colors to the regions of a planar graph such that no two adjacent regions have the same color. Two regions are said to be adjacent if they have a common edge.