Discrete Structures and Theory of Logic

Lecture-40

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Example: Show that $(\forall x)(P(x) \to Q(x)) \wedge (\forall x)(Q(x) \to R(x)) \Rightarrow (\forall x)(P(x) \to R(x))$.

Solution:

(1) $(\forall x)(P(x) \to Q(x))$, By rule P

(2) $P(y) \to Q(y)$, By rule US and (1)

(3) $(\forall x)(Q(x) \to R(x))$, By rule P

(4) $Q(y) \to R(y)$, By rule US and (3)

(5) $P(y) \to R(y)$, By rule T, (2), (3) and hypothetical syllogism

(6) $(\forall x)(P(x) \to R(x))$, By rule UG and (5)

Example: Show that $(\exists x)M(x)$ follows logically from the premises.
$(\forall x)(H(x) \rightarrow M(x))$ and $(\exists x)H(x)$

Solution:

(1) $(\forall x)(H(x) \rightarrow M(x))$, By rule P

(2) $H(y) \rightarrow M(y)$, By rule US and (1)

(3) $(\exists x)H(x)$, By rule P

(4) H(y) , By rule ES and (3)

(5) M(y) , By rule T, (2), (4) and modus ponens

(6) $(\exists x)M(x)$, By rule EG and (5)

Example: Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$

Solution:

(1) $(\exists x)(P(x) \wedge Q(x))$, By rule P

(2) $P(y) \wedge Q(y)$, By rule ES

(3) P(y)

(4) Q(y)

(5) $(\exists x)P(x)$, By rule EG

(6) $(\exists x)Q(x)$, By rule EG

(7) $(\exists x)P(x) \wedge (\exists x)Q(x)$

Example: Show that from

(a) $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y))$

(b) $(\exists y)(M(y) \wedge \neg W(y))$

the conclusion $(\forall x)(F(x) \rightarrow \neg S(x))$ follows.

Solution:

(1) $(\exists y)(M(y) \wedge \neg W(y))$, By rule P

(2) $(M(z) \wedge \neg W(z))$, By rule ES and (1)

(3) $\neg(M(z) \rightarrow W(z))$, By rule T and (2)

(4) $(\exists y)\neg(M(y) \rightarrow W(y))$, By rule EG

(5) $\neg(\forall y)\neg(M(y) \rightarrow W(y))$

(6) $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y))$

(7) $\neg(\exists x)(F(x) \wedge S(x))$, By rule T, (5), (6) and modus ponens

(8) $(\forall x)\neg(F(x) \wedge S(x))$

(9) $\neg(F(x) \wedge S(x))$, By rule US and (8)

(10) $(F(x) \rightarrow \neg S(x))$

(11) $(\forall x)(F(x) \rightarrow \neg S(x))$, By rule UG and (10)

Example: Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x))$

Solution:

We shall use the indirect method of proof by assuming $\neg((\forall x)P(x) \vee (\exists x)Q(x))$ as an additional premise.

(1) $\neg((\forall x)P(x) \vee (\exists x)Q(x))$, By rule P(assumed)

(2) $\neg(\forall x)P(x) \wedge \neg(\exists x)Q(x))$, By rule T

(3) $\neg(\forall x)P(x)$

(4) $(\exists x)\neg P(x))$

(5) $\neg(\exists x)Q(x))$

(6) $(\forall x)\neg Q(x)$

(7) $\neg P(y)$, By rule ES and (4)

(8) $\neg Q(y)$, By rule US and (6)

(9) $\neg P(y) \land \neg Q(y)$
(10) $\neg(P(y) \lor Q(y))$
(11) $(\forall x)(P(x) \lor Q(x))$, By rule P
(12) $(P(y) \lor Q(y))$, By rule US and (11)
(13) $\neg(P(y) \lor Q(y)) \land (P(y) \lor Q(y))$, By rule T and (10),(12)
This is contradiction. Therefore the given statement is proved.

AKTU Examination Questions

1. Verify that the given propositions are tautology or not.
   (a) $p \vee \neg(p \wedge q)$
   (b) $\neg p \wedge q$
2. Write the contra positive of the implication: "if it is Sunday then it is a holiday".
3. Show that the propositions $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent
4. Show that $((P \vee Q) \wedge \neg(\neg Q \vee \neg R)) \vee (\neg P \vee \neg Q) \vee (\neg P \vee \neg Q)$ is a tautology by using equivalences.
5. Obtain the principle disjunctive and conjunctive normal forms of the formula $(P \rightarrow R) \wedge (Q \leftrightarrow P)$.
6. Explain various Rules of Inference for Propositional Logic.

7. Prove the validity of the following argument "if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed."

8. Prove that $(P \vee Q) \rightarrow (P \wedge Q)$ is logically equivalent to $P \leftrightarrow Q$.

9. Express this statement using quantifiers:
"Every student in this class has taken some course in every department in the school of mathematical sciences".

10. Construct the truth table for the following statements:

(a) $(P \rightarrow \neg Q) \rightarrow \neg P$
(b) $P \leftrightarrow (\neg P \vee \neg Q)$