Discrete Structures and Theory of Logic

Lecture-9

Dharmendra Kumar

July 13, 2020

Some important examples

Example: How many reflexive relations are defined on the set with n elements? AKTU(2019)

Solution: According to reflexive property, each reflexive relation contains all the pairs like (a,a), where a belongs into the set. Total number of ordered pairs defined in the set with n elements is n^2. The number of ordered pairs like (a,a) will be n. Therefore, the remaining elements like (a,b) and a$\neq$b will be n^2 - n. Since the relation is a subset of set of ordered pairs, therefore total number of reflexive relations will be $2^{(n^2-n)}$.

Some important examples

Example: How many symmetric relations are defined on the set with n elements? AKTU(2019)

Solution: Consider the set is S with n elements. Relation is defined on the set S. The total number of relations defined on set S will be n^2, because relation is the subset of S$\times$S.

Now, if relation satisfies the symmetric property, then (a,b) and (b,a) belongs into the relation together. Therefore, the set whose all the subsets are reflexive relation contains $\frac{(n^2-n)}{2} + n = \frac{(n^2+n)}{2}$. Here, n is the number of ordered pairs like (a,a).

Therefore the total number of symmetric relations $= 2^{\frac{(n^2+n)}{2}}$.

Some important examples

Example: How many anti-symmetric relations are defined on the set with n elements?

Solution: Consider the set is S with n elements. Relation is defined on the set S. The total number of relations defined on set S will be n^2, because relation is the subset of S$\times$S.

The total number of ordered pairs related to itself = n. Clearly, all the subsets of these ordered pairs are anti-symmetric. Therefore, the total anti-symmetric relations defined on these ordered pairs = 2^n.

The remaining ordered pairs which are not related to itself $= n^2 - n$

Since both (a,b) and (b,a) can not belong into any anti-symmetric relations, Therefore, we consider only orederd pair $= \frac{(n^2-n)}{2}$.

Therefore, there are three possibilities for ordered pairs (a,b) and (b,a).

Solution(cont.)

First possibility: (a,b) and (b,a) both not belong.
Second possibility: (a,b) belong but (b,a) not belong.
Third possibility: (a,b) not belong but (b,a) belong.
Therefore, total number of anti-symmetric relations for these types of ordered pairs $= 3^{\frac{(n^2-n)}{2}}$.
Therefore, total number of anti-symmetric relations for the set S $= 2^n * 3^{\frac{(n^2-n)}{2}}$.

Example: Is the "divides" relation on the set of positive integers transitive? What is the reflexive and symmetric closure of the relation R $= \{$(a, b) — a $>$ b$\}$ on the set of positive integers? AKTU(2019)

Definition

Let X and Y are any two sets. A relation f from X to Y is called a function if for every x$\in$X, there is a unique element y$\in$Y such that (x,y) $\in$ f. It is denoted by f: X$\rightarrow$Y.

Example: Let X = $\{$1,2,3,4$\}$ and Y = $\{$x,y,w,z$\}$ and f = $\{$(1,x),(2,y),(3,w),(4,x)$\}$. Is f a function?
Solution: Clearly in function f, each element of set X has an image in set Y and that image has an unique. Therefore, f is a function.

Example: Let X = Y = R. Also let, f = { (x, x^2) ! x∈R } and g = { (x^2, x) ! x∈R }. Find out f and g is functions or not.
Solution: Here R is a set of real numbers. Clearly for f, each real number has a unique square because square of 2 is 4, 3 is 9, 4 is 16 etc. Therefore, f is a function.
For relation g, element 4 has two images 2 and -2. Similarly, 9 has two images 3 and -3. Therefore, g is not a function.

Domain, Range, and Co-domain

Consider a function f : X$\rightarrow$Y.

Domain of a function f is X. Co-domain of function f is Y. And range of f will be the set of second elements of all the ordered pairs in f i.e. range $\subseteq$ Y.

Example: Let X = {1,2,3,4} and Y = {x,y,w,z} and f = {(1,x), (2,y),(3,w),(4,x)}. Find domain, co-domain and range of f.

Solution:

Domain(f) = {1,2,3,4}

Co-domain(f) = {x,y,w,z}

Range(f) = {x,y,w}