Discrete Structures and Theory of Logic

Lecture-32

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Statement(Proposition)

All the declarative sentences to which it is possible to assign one and only one of the two possible truth values(True or False) are called statements. There are two types of statements. (i) Primitive Statement (ii) Compound Statement

Primitive Statement
The statement which do not contain any of the connective is called primitive statement.

Compound Statement:
The statement which contain more than one primitive statements is called compound statement.

Primitive statements are denoted by P, Q, R, S etc.

Example: Find out following sentences are statement or not.

1. Canada is a country.
2. Mumbai is the capital of India.
3. This statement is false.
4. $1+101 = 110$
5. Close the door.
6. Toronto is an old city.
7. Please go from here.

Solution:

Sentences (1), (2), (4), (6) are the statements.

Connective

Connectives are used to make compound sentences. Following are the main connectives:-

1. Negation($\neg$)
2. Conjunction($\wedge$)
3. Disjunction($\vee$)
4. Conditional($\rightarrow$)
5. Biconditional($\leftrightarrow$)

Negation

If P denotes a statement, then the negation of P is written as $\neg P$ and read as "not P". Truth table of negation is the following:-

P	$\neg P$
T	F
F	T

Example: Consider the statement

(1) P: London is a city.

Then $\neg P$ will be

London is not a city.

(2) Q: I went to my class yesterday.

$\neg Q$: I did not go to to my class yesterday.

Conjunction

The conjunction of two statements P and Q is denoted by $P \wedge Q$ which is read as "P and Q". Truth table for this is the following:-

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example: Form the conjunction of the following statements

P: It is raining today.

Q: There are 20 tables in this room.

Solution:

$P \wedge Q$: It is raining today and there are 20 tables in this room.

Example: Translate the following statement in to symbolic form

Jack and Jill went up the hill.

Solution:

P: Jack went up the hill.

Q: Jill went up the hill.

Symbolic form is $P \wedge Q$.

Example: Consider the following statements

(1) Roses are red and violets are blue.

(2) He opened the book and started to read.

(3) Jack and Jill are cousins.

Statement (1) can be written in the form of $\wedge$.

But (2) and (3) can not be written in the form of $\wedge$.

Disjunction

The disjunction of two statements P and Q is denoted by $P \vee Q$ which is read as "P or Q". Truth table for this is the following:-

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example: Consider the following statements
(1) I shall watch the game on television or go to the game.
(2) There is something wrong with the bulb or with the wiring.
(3) Twenty or thirty animals were killed in the fire today.
Statement (1) and (2) can be written in the form of $\vee$.
But (3) can not be written in the form of $\vee$.

Example: Using the following statements

R: Mark is rich.

H: Mark is happy.

Write the following statements in the symbolic form.

(a) Mark is poor but happy.

(b) Mark is rich but unhappy.

(c) Mark is neither rich nor happy.

(d) Mark is poor or he is both rich and unhappy.

Solution:

(a) $\neg R \wedge H$

(b) $R \wedge \neg H$

(c) $\neg R \wedge \neg H$

(d) $\neg R \vee (R \wedge \neg H)$

Conditional

If P and Q are two statements, then conditional of P and Q is denoted by 'P→Q'. It is read as "if P then Q".

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Example: Write the following statements in the symbolic form.
If either Jerry takes Calculus or Ken takes Sociology, then Larry will take English.
Solution:
P: Jerry takes Calculus.
Q: Ken takes Sociology.
R: Larry takes English.
Then symbolic form of given statement is
$(P \vee Q) \rightarrow R$

Biconditional

If P and Q are two statements, then biconditional of P and Q is denoted by 'P↔Q'. It is read as "P if and only if Q".

P	Q	P↔Q
T	T	T
T	F	F
F	T	F
F	F	T

Note: $P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)$

Example: Construct truth table for the following formula:-

$\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$