Discrete Structures and Theory of Logic

Lecture-10

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Types of function

Onto function (Surjective function)

A function f: $X \rightarrow Y$ is said to be onto function if every element of Y is the image of some element of X. That is, if range(f) = Y, then f is onto.

Into function

A function f: $X \rightarrow Y$ is said to be into function iff there exists at least one element in Y which is not the image of any element in X. That is, range(f) $\subset$ Y.

Types of function

One-one function (Injective function)

A function f: X$\rightarrow$Y is said to be one-one function if for all elements x_1, x_2 in X such that f(x_1) = f(x_2) then $x_1 = x_2$.

Many-one function

A function f: X$\rightarrow$Y is said to be many-one function iff two or more elements of X have same image in Y.

Bijective function)

A function f: X$\rightarrow$Y is said to be bijective function if f is both one-one and onto.

Exercise

Let N be the set of natural numbers including zero. Determine which of the following functions are one-one, onto and bijective.

1. f: $N \rightarrow N$, $f(j) = j^2 + 2$
2. f: $N \rightarrow N$, $f(j) = j \bmod 3$
3. f: $N \rightarrow N$, $f(j) = 1$, if j is odd
   $= 0$, if j is even
4. f: $N \rightarrow \{0,1\}$, $f(j) = 0$, if j is odd
   $= 1$, if j is even

Exercise(cont.)

Let I be the set of integers, I_+ the set of positive integers, and $I_p = \{0,1,2,3,\ldots\ldots\ldots\ldots\ldots\ldots,(p\text{-}1)\}$. Determine which of the following functions are one-one, onto and bijective.

1. f: I$\rightarrow$I, f(j) = (j-1)/2 , if j is odd
   = j/2 , if j is even
2. f: $I_+ \rightarrow I_+$, f(x) = greatest integer $\leq \sqrt{(}x)$
3. $I_7 \rightarrow I_7$, f(x) = 3x mod 7
4. $I_4 \rightarrow I_4$, f(x) = 3x mod 4

Exercise(cont.)

1. List all possible functions from X = {a,b,c} to Y = {0,1} and indicate in each case whether the function is one-one, onto and bijective.
2. Show that the functions f and g which both are from N×N to N given by f(x,y) = x+y and g(x,y) = xy are onto but not one-one.

Composition of functions

Let f: X$\rightarrow$Y and g: Y$\rightarrow$Z be two functions. Then composition of f and g is denoted by gof. It is defined as gof : X $\rightarrow$Z.
(gof)(x) = g(f(x))
Note: gof $\neq$ fog.
Example: Let X ={1,2,3}, Y = {p,q}, and Z = {a,b}. Also let f is a function from X to Y such that f = {(1,p), (2,p), (3,q)} and g is a function from Y to Z such that g = {(p,b), (q,b)}. Find gof.
Solution: gof = { (1,b), (2,b), (3,b)}

Composition of functions

Example: Let X =$\{1,2,3\}$, and f, g, h and s be functions from X to X given by

f = $\{(1,2),(2,3),(3,1)\}$, g = $\{(1,2), (2,1), (3,3)\}$, h = $\{(1,1), (2,2), (3,1)\}$, and s = $\{(1,1), (2,2),(3,3)\}$

Find fog, gof, fohog, sog, gos, sos and fos.

Solution:

fog = $\{((1,3),(2,2),(3,1)\}$

gof = $\{((1,1),(2,3),(3,2)\}$

fohog = $\{((1,1),(2,2),(3,2)\}$

Similarly, we can calculate others.

Example: Let f(x) = x+2, g(x) = x-2, and h(x) = 3x, $\forall$ x$\in$ R, where R is the set of real numbers. Find gof, fog, fof, hog and fohog.

Composition of functions

Solution:

gof(x) = g(f(x)) = g(x+2) = x+2-2 = x

fog(x) = f(g(x)) = f(x-2) = x-2+2 = x

fohog(x) = f(h(g(x))) = f(h(x-2)) = f(3(x-2)) = 3(x-2)+2 = 3x-4

Similarly, we can calculate others.

Example: Let f: R→R be given by f(x) = -x^2 and g: $R_+ \rightarrow R_+$ be given by g(x) = $\sqrt(x)$, where R_+ is the set of positive real numbers and R is the set of real numbers. Find fog. Is gof defined?

Solution:

fog(x) = f(g(x)) = f($\sqrt(x)$) = -($\sqrt(x)$)^2 = -x

gof can not be defined because square root of negative real number can not be a real number.