Theory of Automata and Formal Language

Lecture-21

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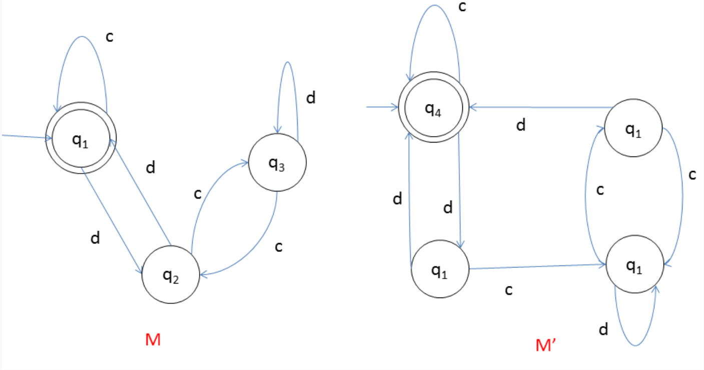
Construction of finite automata from regular expression

Example: Construct finite automata for the following regular expressions:-

1. $r = (a+b)^*(aa+bb)(a+b)^*$
2. $r = 10 + (0+11)0^*1$
3. $r = (a+b)^*b(a+bb)^*$
4. $r = aa^* + aba^*b^*$

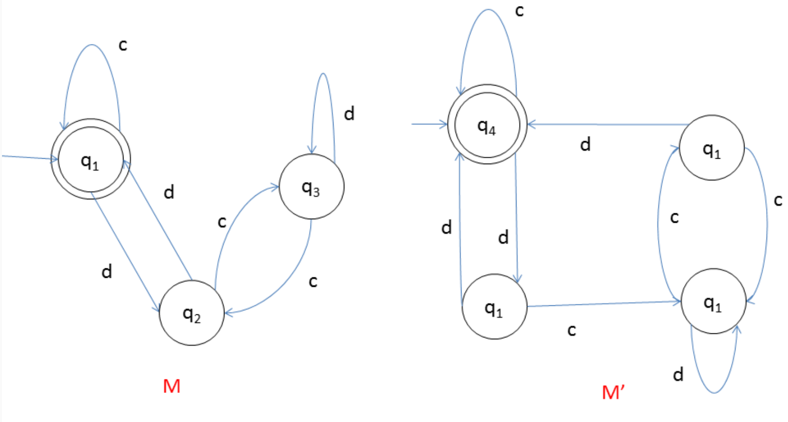
Equivalence of two finite automata

Example: Consider two DFA M and M':



Determine whether M and M' are equivalent.

Example: Show that following automata M_1 and M_2 are not equivalent.



Right and Left linear grammars

A grammar is said to be right linear grammar if all production rules are of the following form:-

$$A \rightarrow xB \text{ or } A \rightarrow x, \text{ where } A,B \in V \text{ and } x \in \Sigma^*$$

A grammar is said to be left linear grammar if all production rules are of the following form:-

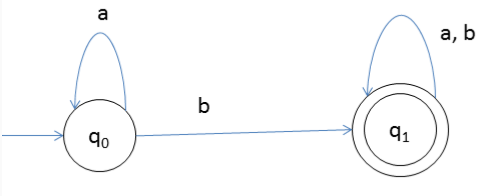
$$A \rightarrow Bx \text{ or } A \rightarrow x, \text{ where } A,B \in V \text{ and } x \in \Sigma^*$$

A regular grammar is one that is either right linear or left linear.

Construction of regular grammar from the given DFA

Suppose the given DFA is
$M = (\{q_0, q_1, ..., q_n\}, \Sigma, \delta, q_0, F)$
Now we construct the grammar G for M as
$\mathrm{G} = (\{Q_0, Q_1, ..., Q_n\}, \Sigma, Q_0, P)$
Where P is defined as
(i) $Q_i \to aQ_j \in P$ if $\delta(q_i, a) = q_j \notin F$
(ii) $Q_i \to aQ_j$ *and* $Q_i \to a \in P$ if $\delta(q_i, a) = q_j \in F$

Example: Find the regular grammar for the following DFA



Solution: Since the number of states are 2, therefore number of variables in the grammar will be 2. Let these variables are Q_0 and Q_1 corresponding to states q_0 and q_1. The starting symbol will be Q_0.
The production rules of the grammar are the following:-
$Q_0 \rightarrow aQ_0$
$Q_0 \rightarrow b/bQ_1$
$Q_1 \rightarrow a/b/aQ_1/bQ_1$

Construction of a FA from given regular grammar

$G = (\{A_0, A_1, ..., A_n\}, \Sigma, A_0, P)$

We construct finite automata M as

$M = (\{q_0, q_1, ..., q_n, q_f\}, \Sigma, \delta, q_0, \{q_f\})$

and δ is defined as

(i) If $A_i \to aA_j$ then $\delta(q_i, a) = q_j$

(i) If $A_i \to a$ then $\delta(q_i, a) = q_f$

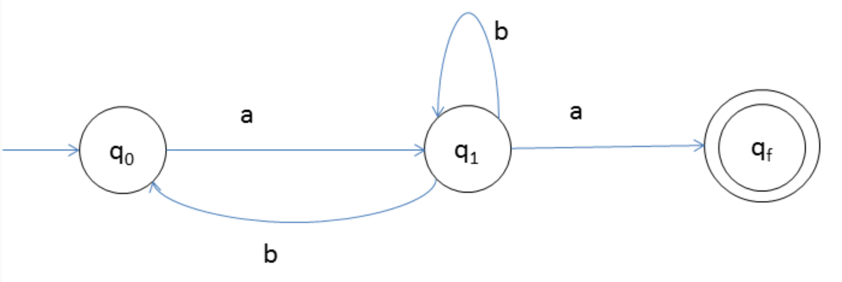
Example: Let $G = (\{A_0, A_1\}, \{a, b\}, A_0, P)$

Where P is

$A_0 \rightarrow aA_1,$ $A_1 \rightarrow bA_1,$ $A_1 \rightarrow a,$ $A_1 \rightarrow bA_0,$

Construct finite automata accepting L(G).

Solution:



Example: Let G $= (\{A_0, A_1, A_2, A_3\}, \{a, b\}, A_0, P)$

Where P is

$A_0 \rightarrow aA_0/bA_1,$ $A_1 \rightarrow aA_2/aA_3,$ $A_2 \rightarrow a/bA_1/bA_3,$

$A_3 \rightarrow b/bA_0,$

Construct finite automata accepting L(G).

Solution: