Theory of Automata and Formal Language

Lecture-12

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Minimization of Finite Automata

Equivalent states

Two states q_1 and q_2 are said to be equivalent if both $\hat{\delta}(q_1, x)$ and $\hat{\delta}(q_2, x)$ are final states or both of them are non-final states for all $x \in \Sigma^*$.

K-equivalent states

Two states q_1 and q_2 are said to be k-equivalent if both $\hat{\delta}(q_1, x)$ and $\hat{\delta}(q_2, x)$ are final states or both of them are non-final states for all $x \in \Sigma^*$ and $| x | \leq k$.

Construction of Minimum Automata

Step 1: First we find a set Π_0 that consists of two sets. First is the set of final states and second is the set of non-final states. That is,

$$\Pi_0 = \{ F, Q\text{-}F \}$$

$\Pi_k \rightarrow$ Set of k-equivalence classes

$Q \rightarrow$ Set of states

$F \rightarrow$ Set of final states

Finite Automata (FA)

Step 2 (Construction of Π_{k+1} from Π_k):

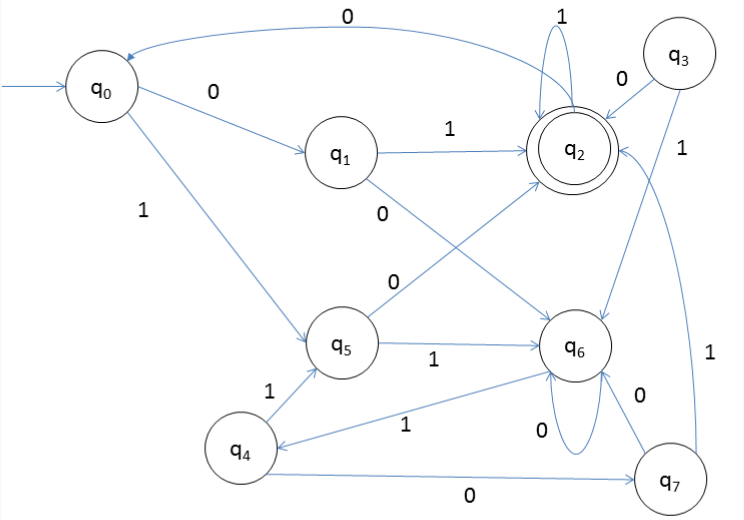
1. Put all the equivalence classes or sets of Π_k into Π_{k+1} as it as if it consists of single states.
2. Let S be a set belong into Π_k. Let q_i and q_j are the two states belong into S.
3. Compute states q_i and q_j (k+1)-equivalent or not.
4. If they are (k+1)-equivalent then put these two states in the same set of Π_{k+1}, otherwise both states belong into different sets in Π_{k+1}.
5. Similarly, we check all pairs of states in S. And put the states either in same set or in different set in Π_{k+1}.
6. Similarly, we apply above procedure for all the sets belong into Π_k.

Finite Automata (FA)

Step 3: Construct Π_n for n= 1,2,3,4,............, until $\Pi_n = \Pi_{n+1}$.

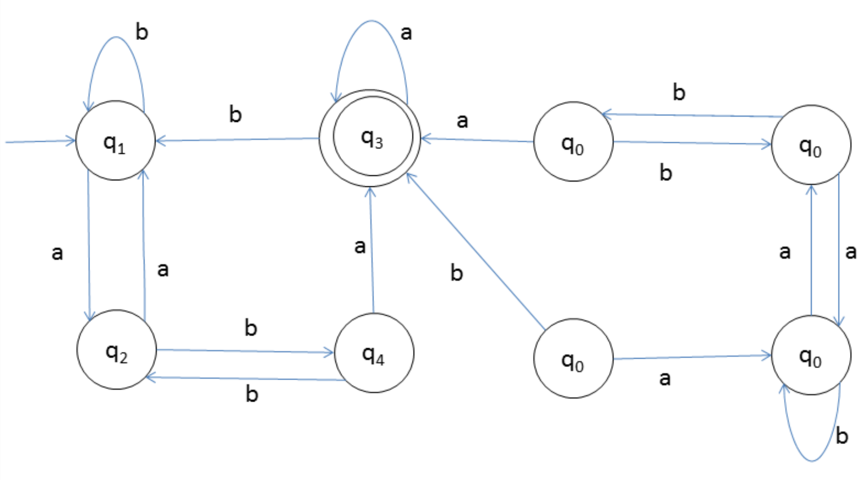
Step 4: For the required minimum state automata, the states are the equivalence classes obtained in step 3 i.e. the elements of Π_n. The state table is obtained by replacing a state q by the corresponding equivalence class [q].

Example: Construct a minimum state automata equivalent to the following finite automata:



Finite Automata (FA)

Example: Minimize the following automata:-



Finite Automata (FA)

Example: Minimize the following automata:-

δ	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_4	q_3
q_2	q_4	q_3
q_3	q_5	q_6
q_4	q_7	q_6
q_5	q_3	q_6
q_6	q_6	q_6
q_7	q_4	q_6