Design and Analysis of Algorithms

Lecture-23

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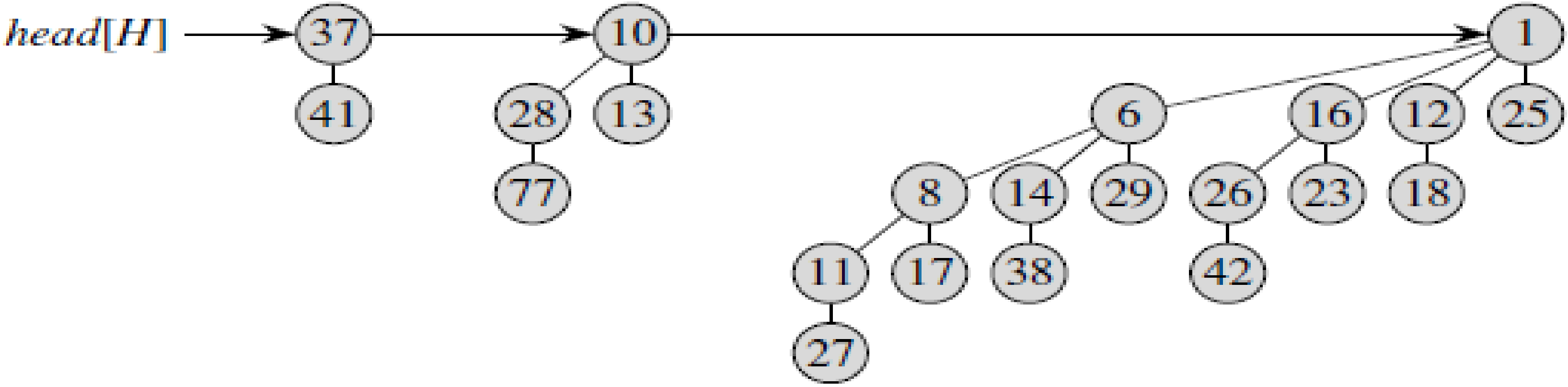
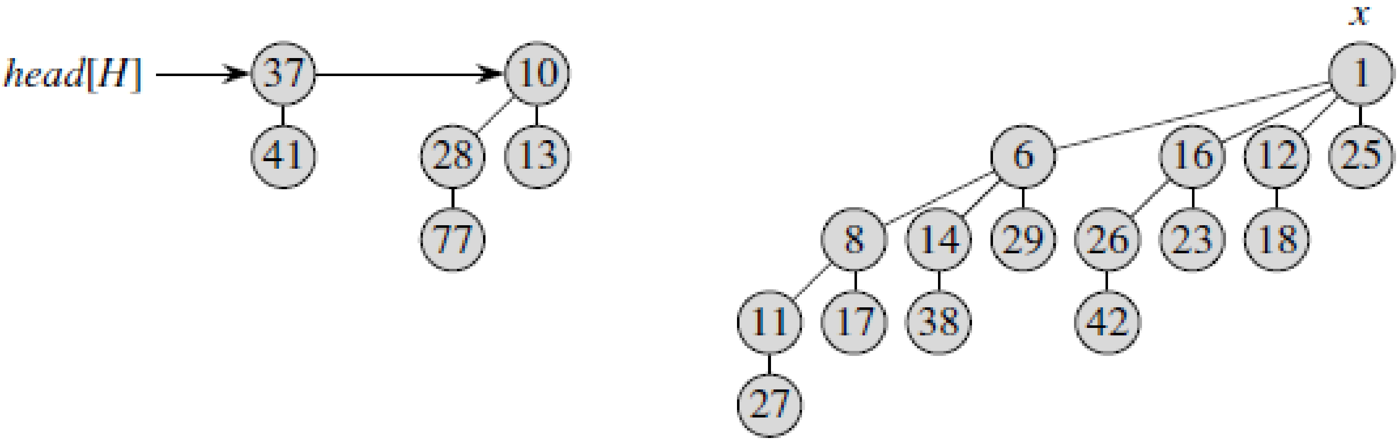
Extracting the node with minimum key

Example: Extract the node with the minimum key from the following binomial heap H:-

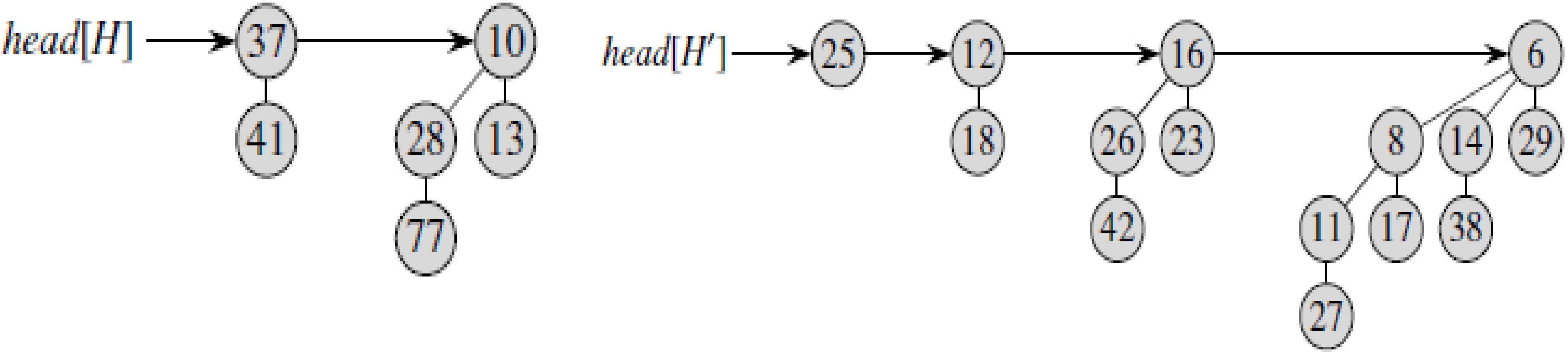


Solution: Step-1: Find the node with minimum key and remove that node.

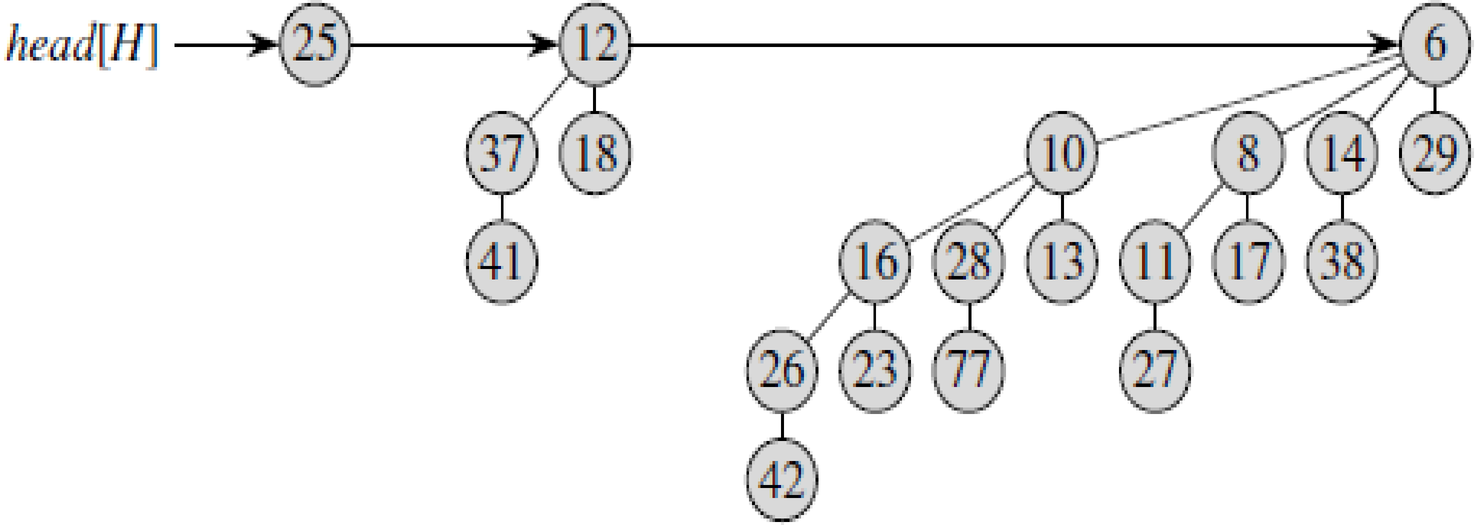


Extracting the node with minimum key

Step-2: Make the binomial heap H’ from children of minimum node.

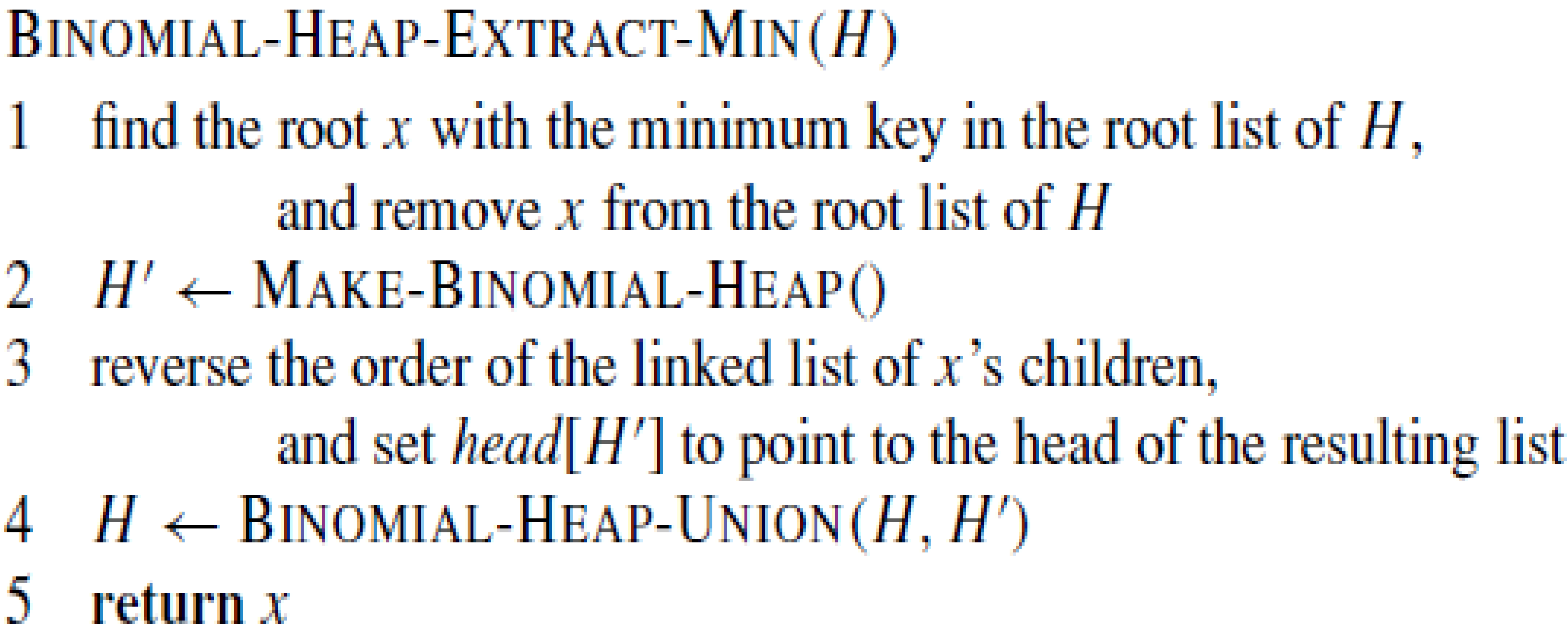


Step-3: Find the union of H and H’.



Extracting the node with minimum key

The following procedure extracts the node with the minimum key from binomial heap H and returns a pointer to the extracted node.



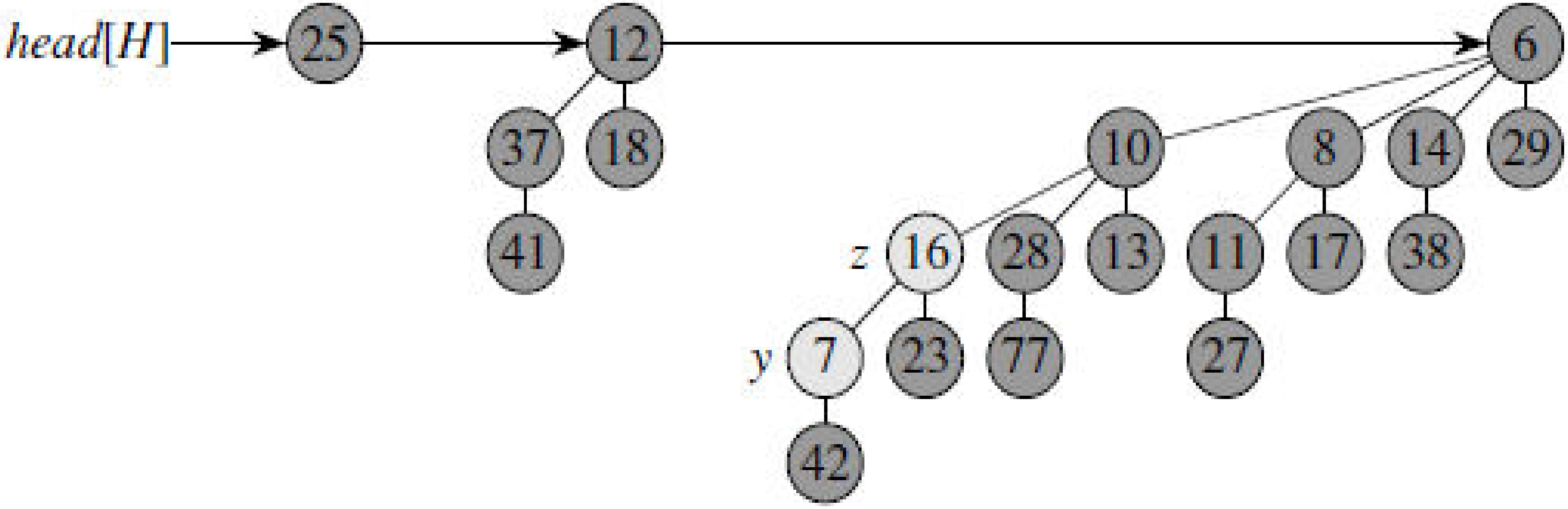
```
BINOMIAL-HEAP-EXTRACT-MIN($H$)
find the root $x$ with the minimum key in the root list of $H$,
        and remove $x$ from the root list of $H$
$H' \leftarrow$ MAKE-BINOMIAL-HEAP()
reverse the order of the linked list of $x$'s children,
        and set $head[H']$ to point to the head of the resulting list
$H \leftarrow$ BINOMIAL-HEAP-UNION($H, H'$)
return $x$
```

Time Complexity:

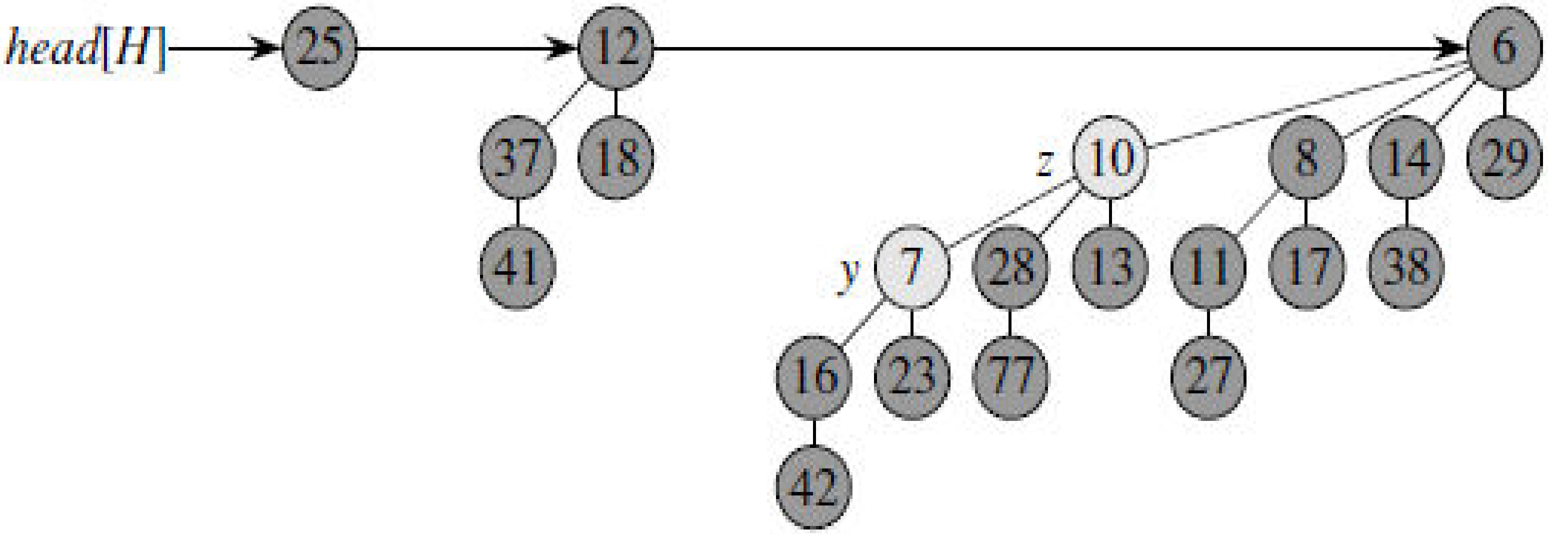
Time complexity of this algorithm is O(lg n).

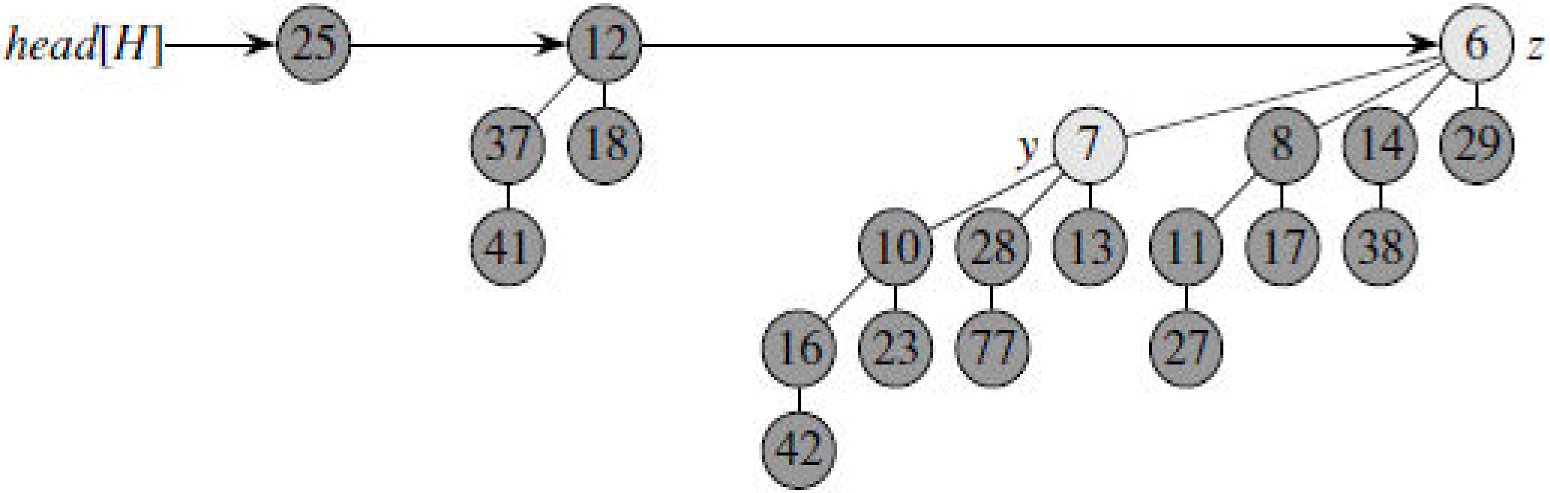
Decreasing a key

Example: Decrease the value of a node y to be 7.



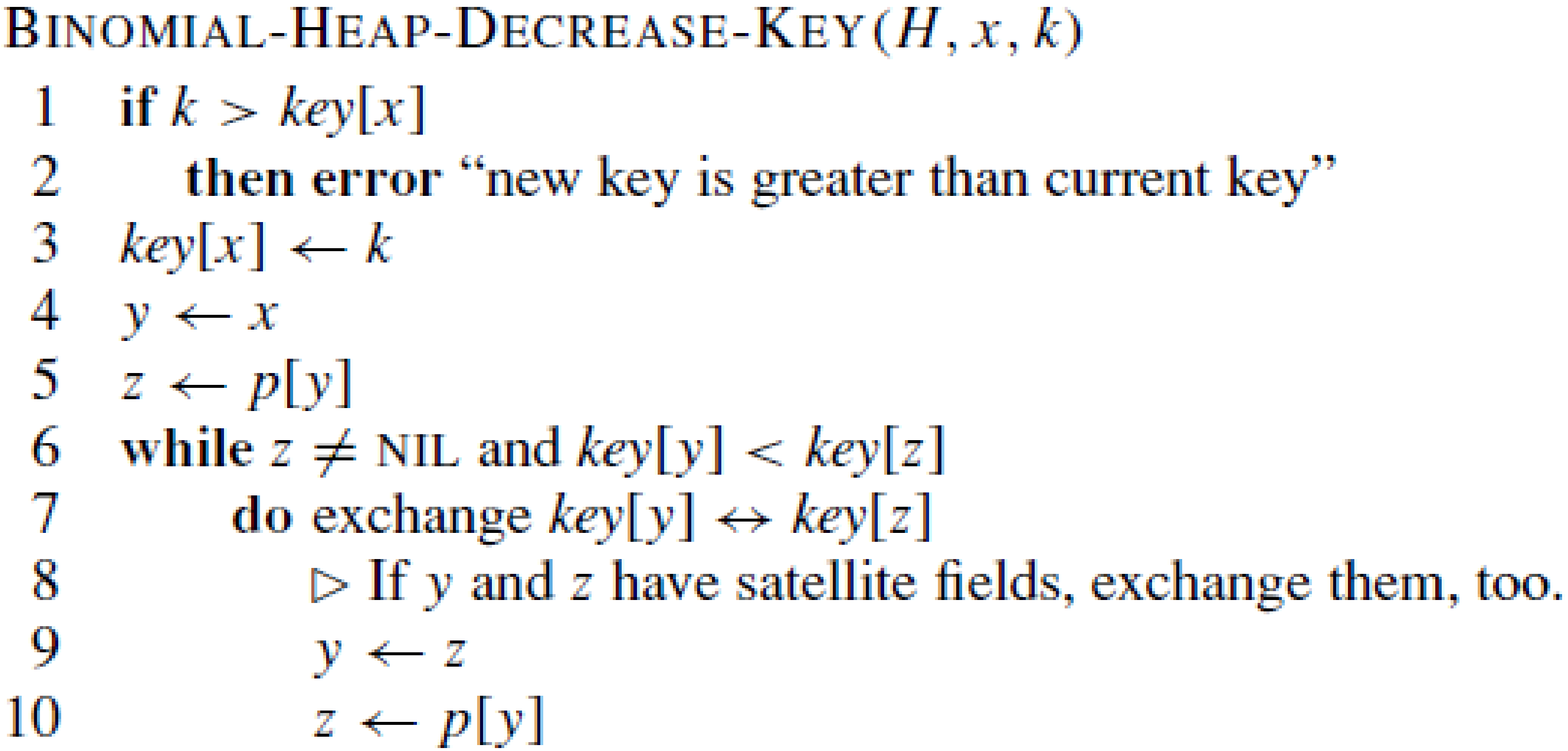
Decreasing a key





Decreasing a key

The following procedure decreases the key of a node x in a binomial heap H to a new value k. It signals an error if k is greater than x’s current key.



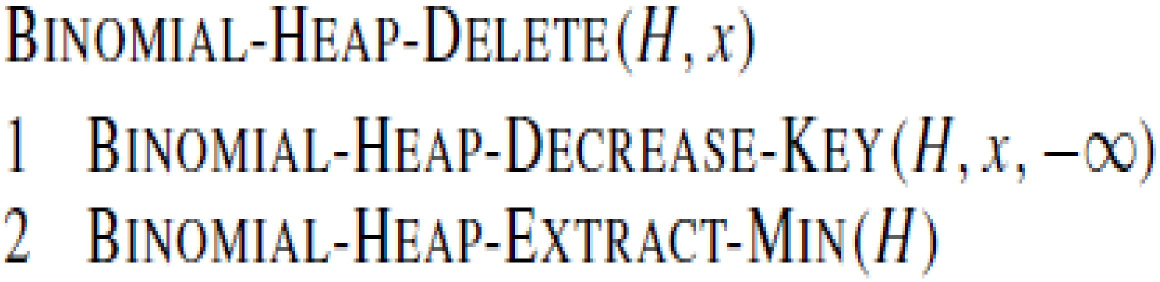
```
BINOMIAL-HEAP-DECREASE-KEY$(H, x, k)$
if $k > key[x]$
    then error “new key is greater than current key”
$key[x] \leftarrow k$
$y \leftarrow x$
$z \leftarrow p[y]$
while $z \neq$ NIL and $key[y] < key[z]$
      do exchange $key[y] \leftrightarrow key[z]$
         ▷ If $y$ and $z$ have satellite fields, exchange them, too.
         $y \leftarrow z$
         $z \leftarrow p[y]$
```

Time Complexity:

Time complexity of this algorithm is O(lg n).

Deleting a key

Following procedure is used to delete the key value of a node. This implementation assumes that no node currently in the binomial heap has a key of −∞.



```
BINOMIAL-HEAP-DELETE($H, x$)
    BINOMIAL-HEAP-DECREASE-KEY($H, x, -\infty$)
    BINOMIAL-HEAP-EXTRACT-MIN($H$)
```

Time Complexity:

Time complexity of this algorithm is O(lg n).

AKTU examination questions

1. Explain various properties of Binomial Tree.
2. Prove that maximum degree of any node in an n node binomial tree is log n.
3. Explain the algorithm to extract the minimum elements in a binomial Heap. Give an example for the same.
4. Define Binomial Heap with example.
5. Explain the algorithm to delete a given element in a binomial Heap. Give an example for the same.
6. Explain properties of Binomial Heap. Write an algorithm to perform uniting two Binomial Heaps. And also to find Minimum Key.
7. Explain the different conditions of getting union of two existing binomial Heaps. Also write algorithm for union of two Binomial Heaps. What is its complexity?

Thank You.