# Discrete Structures and Theory of Logic

## Lecture-11

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# Inverse function

Let f: X→Y is a function. If f is a bijective function then we can define the inverse function of f. It is denoted by $f^{-1}$. It is defined as $f^{-1}$: Y→ X. If f(a) = b then $f^{-1}$(b) = a.

**Example:** Let X = {1,2,3} and Y = {p,q,r}. f: X→ Y be given by

f = {(1,p), (2,q), (3,q)}

Is inverse of this function possible?

**Solution:** Inverse of this function is not possible because this function is not bijective. This function is not bijective because f is not onto function.

**Example:** Let R be the set of real numbers and let f: R$\rightarrow$ R be given by

f = $\{$(x, $x^2$ ) ! x $\in$ R$\}$

Is inverse of this function possible?

**Solution:** Inverse of this function is not possible because this function is not bijective. This function is not bijective because f is not onto function.

**Example:** Let R be the set of real numbers and let f: R$\rightarrow$ R be given by

f = $\{$(x, x+2) ! x $\in$ R$\}$

Is inverse of this function possible?

**Solution:** Inverse of this function is possible because this function is bijective.

# Identity function and Invertible function

**Identity function**

A function $I_X$: X$\rightarrow$ X is called an identity function if $I_X$(x) = x, $\forall$ x $\in$ X.

**Invertible function**

A function f is said to be invertible function if there exists an inverse function of this function.

**Note:**

(1) If f: X$\rightarrow$Y is invertible then $f^{-1}$of $= I_X$ and fo$f^{-1} = I_Y$

(2) Let f: X$\rightarrow$ Y and g: Y$\rightarrow$ X are two functions. The function g is equal to $f^{-1}$ only if gof $= I_X$ and fog $= I_Y$.

## Some examples

**Example:** Show that the functions f(x) $= x^3$ and g(x) $= x^{1/3}$, for x $\in$ R are inverses of one another.
**Solution:** These functions will be inverse of each other if gof = I = fog.
gof(x) = g(f(x)) = g($x^3$) = $(x^3)^{1/3} = x^{3/3}$ = x = I(x).
fog(x) = f(g(x)) = f($x^{1/3}$) = $(x^{1/3})^3 = x^{3/3}$ = x = I(x).
Therefore these functions are inverses of each others.

## Some examples

**Example:** Let $F_X$ be the set of all bijective functions from X to X, where X $= \{1,2,3\}$. Find all the elements of $F_X$ and also find the inverse of each element.

**Solution:** Since the number of elements in set X is 3, therefore the number of bijective functions will be $3! = 6$. These functions are:-

$f_1 = \{(1,1),(2,2),(3,3)\}$ $\quad$ $f_2 = \{(1,1),(2,3),(3,2)\}$

$f_3 = \{(1,2),(2,1),(3,3)\}$ $\quad$ $f_4 = \{(1,2),(2,3),(3,1)\}$

$f_5 = \{(1,3),(2,2),(3,1)\}$ $\quad$ $f_6 = \{(1,3),(2,1),(3,2)\}$

Inverse of these functions is determined by interchanging values in each ordered pairs of corresponding functions.

**Note:** If a set X has n elements, then the number of bijective functions from X to X is n!.

# Exercise

1. Let f: R $\rightarrow$R and g: R $\rightarrow$R are two functions such that f(x) = $x^2 - 2$ and g(x) = x+4, where R is the set real numbers. Find fog and gof. State wheteher these functions are injective, surjective and bijective.
2. If f: X $\rightarrow$Y and g: Y$\rightarrow$ Z and both f and g are onto, show that gof is also onto. Is gof one-one if both g and f are one-one?
3. Let f: R $\rightarrow$R be given by f(x) = $x^2$-2. Find $f^{-1}$.

## Exercise(cont.)

How many functions are there from X to Y for the sets given below? Find also the number of functions which are one-one, onto and bijective.

1. $X = \{1,2,3\}$, $Y = \{1,2,3\}$
2. $X = \{1,2,3,4\}$, $Y = \{1,2,3\}$
3. $X = \{1,2,3\}$, $Y = \{1,2,3,4\}$
4. $X = \{1,2,3,4,5\}$, $Y = \{1,2,3\}$
5. $X = \{1,2,3\}$, $Y = \{1,2,3,4,5\}$

# Exercise(cont.)

1. Let X = {1,2,3,4}. Define a function f: X $\rightarrow$X such that f$\neq I_X$ an is one-one. Find $f^2$, $f^3$, $f^{-1}$ and $fof^{-1}$. Can you find another one-one function g: X $\rightarrow$X such that g $\neq I_X$ but gog = $I_X$?
2. Let f: X$\rightarrow$Y and X=Y=R, the set of real numbers. Find $f^{-1}$ if
   2.1 f(x) = $x^2$
   2.2 f(x) = $\frac{(2x-1)}{5}$
3. Let f and g be the functions from the set of integers to the set of integers defined by f (x) = 2x + 3 and g(x) = 3x + 2. What is the composition of f and g? What is the composition of g and f?