Theory of Automata and Formal Language

Lecture-28

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Pumping Lemma for Context Free Languages

Let L be an infinite context free language. Then there exists some positive integer n such that any x$\in$ L with x$\geq$n, can be decomposed as

x=uvwxy

with $\mid vxy \mid \leq n$ and $\mid vx \mid \geq 1$,

such that

$uv^iwx^iy \in$ L, $\forall$ i = 0,1,2,3,......................

Application: This lemma is used to show a given language is not context free.

Example: Show that the language
$L = \{ a^n b^n c^n \mid n \geq 0\}$
is not context free.
Solution:

Exercise

1. L $= \{$ww ! w$\in\{$a,b$\}$*$\}$
2. L $= \{a^{n!} ~!~ n \geq 0\}$
3. L $= \{a^n b^j ~!~ n = j^2\}$
4. L $= \{a^{n^2} ~!~ n \geq 0\}$
5. L $= \{a^p$! p is a prime number$\}$
6. L $= \{a^n b^j c^k ~!~ k > n,~ k > j\}$

Decision Properties of Regular and Context Free Languages

Theorem: Given a context free grammar G=(V,Σ,S,P), there exists an algorithm for deciding whether or not L(G) is empty.

Proof: For the simplicity, we assume that $\epsilon \notin L(G)$. We use the algorithm for removing useless symbols and productions. If S is found to be useless, then L(G) is empty otherwise L(G) contains at least one element.

Context Free Grammar

Theorem: Given a context free grammar G=(V,Σ,S,P), there exists an algorithm for deciding whether or not L(G) is infinite.

Proof: We assume that G contains no $\epsilon - \textit{poductions}$, no unit-productions, and no useless symbols.

Convert the grammar into CNF.

We draw a directed graph whose vertices are variables in G. If A→BC is a production, then there are directed edges from A to B and A to C.

L is finite iff the directed graph has no cycles.

Theorem: Show that there exists an algorithm for deciding whether a regular language, L is empty.

Proof: Construct a deterministic finite automata M accepting L. Determine the set of all the states reachable from q_0. If this set contains a final state, then L is non-empty otherwise L is empty.

Theorem: Show that there exists an algorithm for deciding whether a regular language, L is infinite.

Proof: Construct a deterministic finite automata M accepting L. L is infinite iff M has a cycle.

AKTU Examination Questions

1. Convert the following CFG to its equivalent GNF:
   S $\rightarrow$ AA a, A $\rightarrow$ SS b.
2. Prove that the following Language L = $\{a^n b^n c^n ! \ n \geq 1\}$ is not Context Free.
3. Is context free language closed under union? If yes, give an example.
4. Remove useless productions from the following grammar
   S$\rightarrow$AB/ab, A$\rightarrow$a/aA/B, B$\rightarrow$D/E
5. Reduce the given Grammar G = ({S,A,B},{a,b},S,P) to Chomosky Norma Form, where P is the following
   S$\rightarrow$bA/aB, A$\rightarrow$a/aS/bAA, B$\rightarrow$b/bS/aBB

Context Free Grammar

6. Discuss the inherent ambiguity of context free languages with suitable example. Construct the context free grammar that accept the following language

L $= \{a^i b^j c^k \ ! \ i = j \ or \ j = k\}$

7. Define the parse tree. Construct the parse tree for the string abbcde considering the productions

S→aAcBe, A→b/Ab, B→d

Is this ambiguous? Justify.

8. Prove or disprove that union and concatenation of two context free languages is also context free.

9. Prove that the language L $= \{a^n b^n c^n \ ! \ n \geq 1\}$ is neither regular nor context free.

10.Determine the language generated by grammar $S \rightarrow Sab|aSb|abS|baS|b$

11. What is inherent ambiguity? Explain with the help of suitable example.

12. Remove the Unit productions from the following grammar: $S \rightarrow aSb|A$, $A \rightarrow cAd|cd$

13. Write the procedure to convert a given CFG into equivalent grammar in CNF. Apply the procedure and convert the grammar with following production into CNF: $S \rightarrow bA|aB$, $A \rightarrow bAA|aS|a$, $B \rightarrow aBB|bS|b$

14. Define Greibach normal form for a CFG. Reduce the following CFG into GNF:

S $\rightarrow$ AB, $A \rightarrow BS|a$, $B \rightarrow A|b$

15. Let G be the grammar $S \rightarrow 0B|1A$, $A \rightarrow 0|0S|1AA$, $B \rightarrow 1|1S|0BB$

For the string 00110101, find: (i) The leftmost derivation. (ii) The rightmost derivation. (iii) The derivation tree.

Context Free Grammar

16. Check whether the grammar is ambiguous or not.

R$\rightarrow$ R+R/ RR/ R*/ a / b / c. Obtain the string w = a+b*c

17. Eliminate unit productions in the grammar. S$\rightarrow$A/bb A$\rightarrow$B/b B$\rightarrow$S/a

18. Find out whether the language L = $\{x^n y^n z^n | n \geq 1\}$ is context free or not.

19. Convert the following CFG into CNF

S $\rightarrow$ XY / Xn / p

X $\rightarrow$ mX / m

Y $\rightarrow$ Xn / o

20. Convert the following CFG into CNF S $\rightarrow$ ASA / aB, A $\rightarrow$ B / S, B $\rightarrow$ b / ϵ

21. Write CFG for language L = $\{a^n b^n \, !n \geq 0\}$. Also convert it into CNF.

Context Free Grammar

22. Define ambiguity. Show that the grammar G with following production is ambiguous.
$S \rightarrow a$ / aAb / $abSb$, $A \rightarrow aAAb$ / bS
23. Convert the following grammar in GNF: $S \rightarrow AB$, $A\rightarrow BS$ / a , $B\rightarrow SA$ / b
24. Define derivation Tree. Show the derivation tree for string 'aabbbb' with the following grammar $S\rightarrow AB/\epsilon$, $A\rightarrow aB$, $B\rightarrow Sb$.