Design and Analysis of Algorithms

Lecture-4

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Asymptotic Notations

Example: Show that using definition of notations

(a) $3n^3-10n+50 = \theta(n^3)$

(b) $5n^2-100n \neq \theta(n^3)$

(c) $3n^3-10n+50 = O(n^3)$

(d) $5n^2-100n \neq O(n)$

(e) $3n^3-10n+50 = \boldsymbol{\Omega}(n^3)$

(f) $5n^2-100n \neq \boldsymbol{\Omega}(n^3)$

Asymptotic Notations

Limit based method to compute notations for a function

First compute $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$.

(1) If c is a constant such that $0 < c < \infty$, then $f(n) = \theta(g(n))$.

(2) If c is a constant such that $0 \leq c < \infty$, then $f(n) = O(g(n))$.

(3) If c is a constant such that $0 < c \leq \infty$, then $f(n) = \boldsymbol{\Omega}(g(n))$.

(4) If c is a constant such that $c = 0$, then $f(n) = o(g(n))$.

(5) If c is a constant such that $c = \infty$, then $f(n) = \boldsymbol{\omega}(g(n))$.

Asymptotic Notations

Exercises

1. Let f(n) and g(n) be asymptotically non-negative functions. Using the basic definition of ,θ-notation, prove that **max(f(n), g(n)) = θ(f(n)+g(n)).**

2. Show that for any real constants a and b, where b > 0,

$(n+a)^b = \theta(n^b)$

3. Solve the followings:-

(a) Is $2^{n+1} = O(2^n)$?

(b) Is $2^{2n} = O(2^n)$?

Asymptotic Notations

Exercise(cont.)

7. Arrange the following in ascending order of growth or rank the following functions by order of growth.

n^3, $(3/2)^n$, 2^n, n^2, $\log(n)$, 2^{2n}, $\log\log(n)$, $n!$, e^n.

8. Let f(n) and g(n) be two asymptotically positive functions. Prove or disprove the following:-

(a) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.

(b) $f(n) + g(n) = \theta(\min(f(n), g(n)))$.

(c) $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n.

(d) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.

(e) $f(n) = O((f(n))^2)$

Asymptotic Notations

AKTU questions

1. Take the following list of functions and arrange them in ascending order of growth rate. That is, if function g(n) immediately follows function f(n) in your list, then it should be the case that f(n) is O(g(n)). $f_1(n) = n^{2.5}$, $f_2(n) = \sqrt{2^n}$, $f_3(n) = n + 10$, $f_4(n) = 10^n$, $f_5(n) = 100^n$, and $f_6(n) = n^2 \log n$

2. Rank the following by growth rate: n, $2\lg \sqrt{n}$, log n, log (logn), $\log^2 n$, (lgn)lgn, 4, $(3/2)^n$, n!