(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 0111   **Roll No.** ☐☐☐☐☐☐☐☐☐☐

B. Tech.

(SEM. III) ODD SEMESTER THEORY
EXAMINATION 2012–13
DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours   *Total Marks : 100*

Note :– (i)   Attempt **all** questions.

           (ii)   Make suitable assumptions wherever necessary.

1. Attempt any **four** parts of the following :–   **(5×4=20)**

     (a)   Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, and the ordering of elements of U has the elements in increasing order; that is, a_i = i. What bit strings represent the subset of all odd integers in U, the subset of all even integers in U, and the subset of integers not exceeding 5 in U ?

     (b)   If $f : A \to B$ and $g : B \to C$ and both f and g are onto, show that $g \circ f$ is also onto. Is $g \circ f$ one-to-one if both g and f are one-to-one ?

     (c)   Let R be a binary relation on the set of all positive integers such that :

           R = {(a, b) | a – b is an odd positive integer}.

     Is R reflexive ? Symmetric ? Antisymmetric ? Transitive ? An equivalence relation ?

(d) Explain the relationship between equivalence relations on a set and partitions of this set.

(e) Show by induction that any positive integer n greater than or equal to 2 is either a prime or product of primes.

(f) What are the different proof methods ? Explain the proof by contradiction by an example.

2. Attempt any **four** parts of the following :   (5×4=20)

(a) Let (A, *) be a semigroup. Furthermore for every a and b in A, if $a \neq b$, then $a * b \neq b * a$ i.e. if $a * b = b * a$, then $b = a$. Show that :

   (i)   For every a in A, $a * a = a$.

   (ii) For every a, b in A $a * b * a = a$

   (iii) For every a, b, c in A $a * b * c = a * c$

(b) Define identity and zero elements of a set under a binary operation *. What do you mean by an inverse element ?

(c) Let (A, *) be a monoid such that for every x in A, $x * x = e$, where e is the identity element. Show that (A, *) is an abelian group.

(d) Define the subgroup of a group. Let (G, o) be a group. Let $H = \{a \mid a \, \varepsilon \, G$ and $a \, o \, b = b \, o \, a$ for all $b \, \varepsilon \, G\}$. Show that H is a normal subgroup.

(e) Let f_1 and f_2 be homomorphisms from an algebraic system (A, o) to another algebraic system (B, *). Let g be a function from A to B such that $g(a) = f_1(a) * f_2(a)$, for all a in A. Show that g is a homomorphism from (A, o) to (b, *) if (B, *) is a commutative semigroup.

(f) Give the definitions of rings, integral domains and fields.

3. Attempt any **two** parts of the following : (10×2=20)

   (a) Define a partial ordering. Explain how to construct the Hasse diagram of a partial order on a finite set. Draw the Hasse diagram for inclusion on the set P(S), where S = {a, b, c}.

   (b) Give an example of a finite lattice where at least one element has more than one complement and at least one element has no complement. Show that the lattice (P(S), $\subseteq$) where P(S) is the power set of a finite set S is complemented.

   (c) Define a Boolean algebra. Show that in a Boolean algebra meet and join operations are distributive to each other.

4. Attempt any **two** parts of the following : (10×2=20)

   (a) Show the following implications without constructing the truth tables :

   (i) $(P \wedge Q) \Rightarrow (P \rightarrow Q)$

   (ii) $P \rightarrow Q \Rightarrow P \rightarrow (P \wedge Q)$

   (iii) $(P \rightarrow Q) \rightarrow Q \Rightarrow P \vee Q$.

   (b) (i) Find a compound proposition involving the propositional variables p, q, r and s that is true when exactly three of these propositional variables are true and is false otherwise.

   (ii) Show that the hypothesis "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip." and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

(c) (i) Express this statement using quantifiers : "Every student in this class has taken some course in every department in the school of mathematical sciences".

(ii) If $\forall x \exists y\, P(x, y)$ is true, does it necessarily follow that $\forall y\, P(x, y)$ is true ? Justify your answer.

5. Attempt any **two** parts of the following : $\qquad$ (10×2=20)

(a) Define preorder, inorder, and postorder tree traversal. Give an example of preorder, postorder, and inorder traversal of a binary tree of your choice with at least 12 vertices.

(b) What do you mean by generating function ? Solve the recurrence relation :

$$a_n = 2\,a_{n-1} - a_{n-2}, \quad n \geq 2 \text{ given } a_0 = 3,\ a_1 = -2.$$

using generating function.

(c) Write short notes on the following :

(i) Planar Graphs

(ii) Isomorphism of graphs

(iii) Pigeon hole principle.