Theory of Automata and Formal Language

Lecture-18

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Equivalent regular expressions

Two regular expressions $\bar{p}$ and $\bar{q}$ are said to be equivalent if they represent the same set of strings.

Identities for regular expressions

Let p, q, r are all the regular expressions. Then

(1) $\phi + \mathrm{r} = \mathrm{r}$

(2) ϕ . $\mathrm{r} = \phi = \mathrm{r}.\phi$

(3) $\epsilon.\mathrm{r} = \mathrm{r}.\epsilon = \mathrm{r}$

(4) $\epsilon^* = \epsilon$ and $\phi^* = \epsilon$ (5) $\mathrm{r}+\mathrm{r} = \mathrm{r}$

(6) $r^* r^* = r^*$

(7) $rr^* = r^* r$

(8) $(r^*)^* = r^*$

(9) $\epsilon \ + \ rr^* = r^* = \epsilon \ + \ r^* r$

(10) $(p.q)^* p = p.(p.q)^*$

(11) $(p+q)^* = (p^* q^*)^* = (p^* + q^*)^*$

(12) $(p+q).r = p.r + q.r$ and $r.(p+q). = r.p + r.q$

ARDEN's Theorem

Statement: Let P and Q be two regular expressions over Σ. If P does not contain ϵ , then the following equation in R,

R = Q + RP ....(1)

has a unique solution R = QP^*.

Proof:

Q+RP = Q + QP*P

= Q(ϵ + P^*P)

= QP*

= R

Therefore equation (1) is satisfied when R = QP*.

Therefore R = QP* is a solution of equation (1).

Regular Expression

To prove uniqueness of (1), replacing R by Q+RP.

$$
\begin{aligned}
R = Q + RP &= Q + (Q+RP)P \\
&= Q + QP + RP^2 \\
&= Q + QP + (Q + RP)P^2 \\
&= Q + QP + QP^2 + RP^3 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&= Q + QP + QP^2 + \ldots\ldots + QP^i + RP^{i+1} \\
&= Q(\epsilon + P + P^2 + \ldots\ldots + P^i) + RP^{i+1}
\end{aligned}
$$

Therefore, $R = Q(\epsilon + P + P^2 + \ldots\ldots + P^i) + RP^{i+1}$ ....(2)

Regular Expression

Let w$\in$ R and $\mid w \mid$ = i.
From equation (2), w will belong into right hand side of equation (2).
Therefore, w$\in$ Q($\epsilon +$ P $+P^2$ $+$........$+$ P^i) $+$ RP^{i+1}
Clearly, w$\notin$ RP^{i+1} since $\mid w \mid$ = i.
Therefore, w $\in$ Q($\epsilon +$ P $+P^2$ $+$........$+$ P^i)
Therefore, w $\in$ QP^*
Therefore, R$\subseteq$ QP^* ....(3)

Let w $\in$ QP^*.

Then w$\in$QP^k for some k$\geq$ 0.

Therefore, w $\in$ Q(ϵ + P +P^2 +........+ P^k)

Therefore, w belong into the right hand side of equation (2).

Therefore, w$\in$ R.

Therefore, $QP^* \subseteq$ R ....(4)

From (3) and (4),

R = QP^*

Example:

(a) Give a regular expression for representing the set L of strings in which every 0 is immediately followed by at least two 1's.
(b) Prove that the regular expression
r $= \epsilon + 1^*(011)^*(1^*(011)^*)^*$
also describes the same set of strings.

Regular Expression

Example: Prove that

$$(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1) = 0^*1(0 + 10^*1)^*$$