Theory of Automata and Formal Language

Lecture-5

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Chomsky Hierarchy

According to Chomsky's classification, all the grammars are divided into following four categories:-

Type 0 Grammar(Unrestricted Grammar)

If there is no restriction on the production rules, then grammar is said to be type 0 grammar or unrestricted grammar.

Type 1 Grammar(Context Sensitive Grammar)

A grammar is said to be type 1 grammar or context sensitive grammar if every production rules are of the following form:-

$\phi_1 A \phi_2 \rightarrow \phi_1 \psi \phi_2$, where $\phi_1, \phi_2, \psi \in (V \cup \Sigma)^*$ and A $\in$ V.

Grammar and Language

Type 2 Grammar(Context Free Grammar)

A grammar is said to be type 2 grammar or context free grammar if every production rules are of the following form:-

$$A \to \psi, \qquad \text{where } \psi \in (V \cup \Sigma)^* \text{ and A} \in \text{V}.$$

Type 3 Grammar(Regular Grammar)

A grammar is said to be type 3 grammar or regular grammar if every production rules are of the following form:-

$$A \to aB \text{ or } A \to a, \qquad \text{where } a \in \Sigma \text{ and A, B} \in \text{V}$$

Grammar and Language

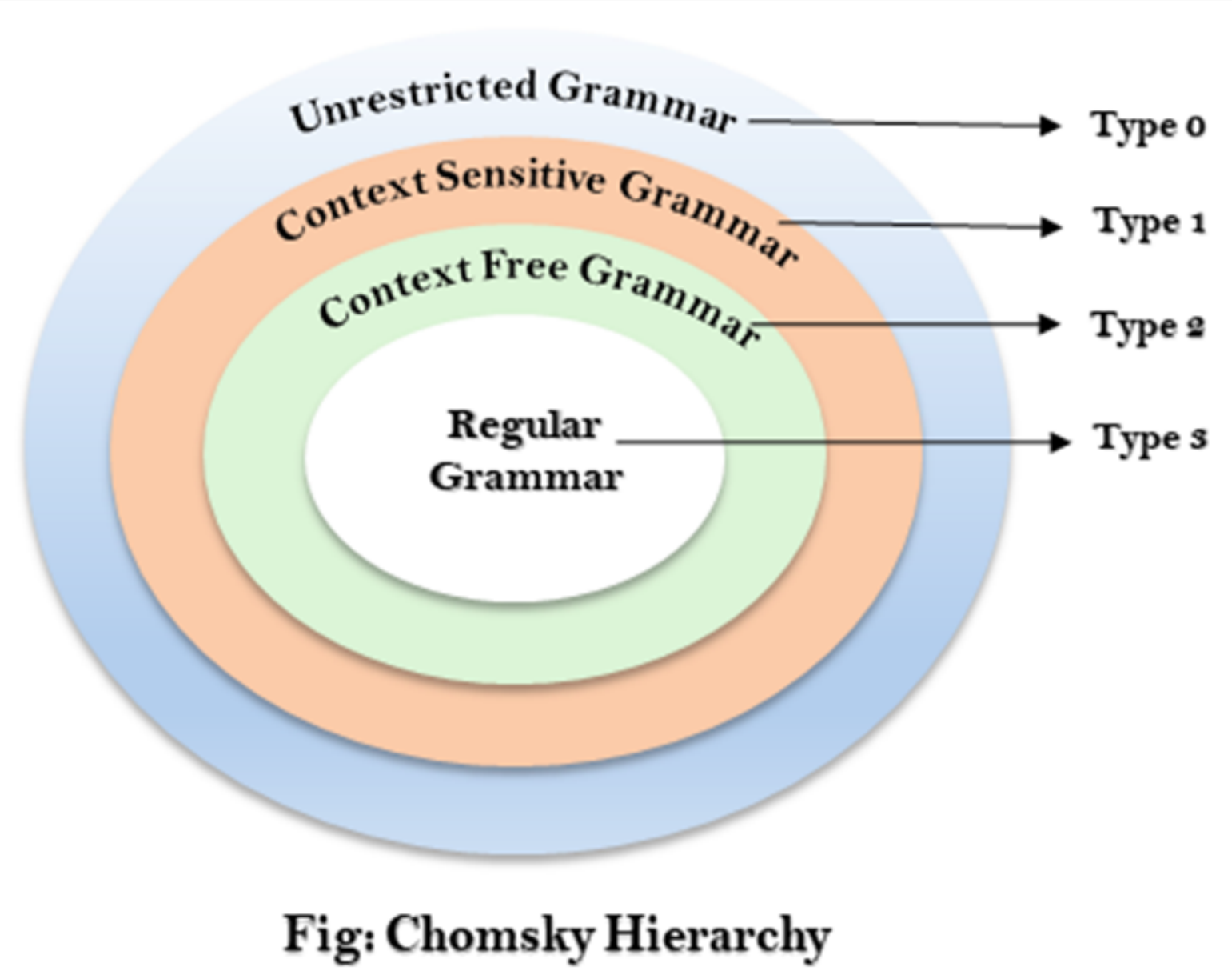

Fig: Chomsky Hierarchy

The Hierarchy

Class	Grammars	Languages	Automaton
Type-0	Unrestricted	Recursive Enumerable	Turing Machine
Type-1	Context Sensitive	Context Sensitive	Linear-Bound
Type-2	Context Free	Context Free	Pushdown
Type-3	Regular	Regular	Finite

Exercise

Determine the highest type of grammar in the following grammars:-

1. $S \to 0S1/0A1, \quad A \to 1A/1$
2. $S \to 0S1/0A/0/1B/1, \quad A \to 0A/0, \quad B \to 1B/1$
3. $S \to 0SBA/01A, \quad AB \to BA, \quad 1B \to 11, \quad 1A \to 10, \quad 0A \to 00$
4. $S \to 0S1/0A1, \quad A \to 1A0/10$
5. $S \to 1S/0A/0/1, \quad A \to 1A/1S/1$

Exercise

Construct context free grammars to generate the following languages:-

1. L = $\{0^m 1^n \mid m \neq n \text{ and } m, n \geq 1\}$
2. L = $\{a^l b^m c^n \mid$ one of l,m,n equals 1 and the remaining two are are equal $\}$
3. L = $\{0^m 1^n \mid 1 \leq m \leq n\}$
4. L = $\{a^l b^m c^n \mid l + m = n\}$
5. The set of all strings over $\{0,1\}$ containing twice as many 0's as 1's.

Exercise

Construct regular grammars to generate the following languages:-

1. $\mathrm{L} = \{a^{2n} \mid n \geq 1\}$
2. The set of all strings over {a,b} ending in a.
3. The set of all strings over {a,b} beginning with a.
4. $\mathrm{L} = \{a^l b^m c^n \mid l, m, n \geq 1\}$
5. $\mathrm{L} = \{(ab)^n \mid n \geq 1\}$

AKTU Examination Questions

1. Construct the CFG for the language $L = \{a^{2n}b^n \mid n \geq 3\}$.
2. Design the CFG for the following language:
   2.1 $L = \{0^m1^n \mid m \neq n \text{ and } m, n \geq 1\}$
   2.2 $L = \{a^l b^m c^n \mid l + m = n \text{ and } l, m \geq 1\}$
3. Define alphabet, string and language.
4. Define Chomsky hierarchy.
5. Define and give the difference between positive closure and Kleene closure.
6. Determine the grammar for language $L = \{a^n b^m \mid n \neq m\}$. Also explain the type of this language.
7. Identify the language generated by context free grammar $S \rightarrow (S)/SS/(\ )$
8. Describe Chomsky hierarchy of languages with proper example.