Theory of Automata and Formal Language

Lecture-25

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June 4, 2021

Elimination of null productions

Null Production: A production rule is said to be null production if it is of the following form:-

$$A \to \epsilon$$

Nullable Variable: A variable A is said to be nullable if it derives empty string i.e.

$$\text{A} \Rightarrow \epsilon, \ \forall A \in V$$

Procedure:

Consider a grammar G = (V, Σ, S, P). Let G' is a grammar having no null productions such that L(G') = L(G)-$\{\epsilon\}$.

G' is constructed as following:-

G' = (V, Σ, S, P')

Step-1: Determination of the set of nullable variables

Let W_i is the set of nullable variables. W_i is calculated as following:-

$W_1 = \{A \mid A \rightarrow \epsilon \in P\}$

$W_2 = W_1 \cup \{A \mid \exists$ a production $A \rightarrow \alpha \in P$ and $\alpha \in W_1^*\}$

....

....

$W_{i+1} = W_i \cup \{A \mid \exists$ a production $A \rightarrow \alpha \in P$ and $\alpha \in W_i^*\}$

Repeat this process until $W_{i+1} = W_i$

Now, we terminate this process.

Step-2: Determination of P'

(i) We add the production rules of P into P' whose RHS does not include any nullable variable.

(ii) Consider the remaining production rules of P.

If A$\rightarrow$ $X_1X_2.............X_n \in P$ then we add A$\rightarrow$ $\alpha_1\alpha_2.............\alpha_n$ into P', where $\alpha_i = X_i$ or ϵ but not all α_i equal to ϵ if X_i is nullable variable otherwise put $\alpha_i = X_i$.

Example: Consider the following grammar

S$\rightarrow$ aS/AB

A$\rightarrow$ ϵ

B$\rightarrow$ ϵ

D$\rightarrow$ b

Eliminate the null productions.

Elimination of Unit Productions

Unit Production: A production rule is said to be unit production if it is of the following form:-
$A \rightarrow B$, where A, B $\in$ V.

Example: Consider the following grammar
$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow C/b$
$C \rightarrow D$
$D \rightarrow E$
$E \rightarrow a$
Eliminate the unit productions.

Chomosky Normal Form (CNF)

A grammar G is said to be in Chomosky nomal form if every production rules are of the following form:-

$A \rightarrow BC$ or $A \rightarrow a$,

where A,B,C $\in$V and a $\in \Sigma$.

Reduction into Chomosky Normal Form

Step-1: Elimination of null productions and unit productions

In this step, we eliminate the null production and unit productions from the grammar. Let the resultant grammar is G=(V,Σ,S,P).

Step-2: Elimination of terminals from RHS if rule is not in CNF

Let $G_1 = (V_1, \Sigma, S, P_1)$ be the grammar obtained in this step.

All the productions of P which are in CNF, must also belong into P_1 and all the variables of V must also belong into V_1.

Consider the remaining production rules.

Add the new variables into V_1 equal to the number of terminals on the right hand side of these production rules. And if the terminals are $a_1, a_2, ..., a_n$, then add variables $X_1, X_2, ..., X_n$ into V_1 and add $X_1 \rightarrow a_1$, $X_2 \rightarrow a_2$,................,$X_n \rightarrow a_n$ rules into P_1.

Step-3: Restricting the number of variables on the RHS

Let $G_2 = (V_2, \Sigma, S, P_2)$ be the grammar obtained in this step.

Add all the elements of V_1 into V_2. Add the production rules of P_1 into P_2 which are in CNF.

Consider those production rules of P_1 which are not in CNF. These production rules contains at least three elements on the right hand side.

Consider production rules $A \rightarrow X_1X_2..........X_n$, where n$\geq$3. Then we add n-2 variable into V_2. Let these are $Y_1, Y_2, ..., Y_{n-2}$.

Add production rules $A \rightarrow X_1Y_1$, $Y_{\rightarrow}X_2Y_2$,..........,$Y_{n-2} \rightarrow X_{n-1}X_n$ to P_2.

Now, the grammar G_2 is the resultant grammar which is in CNF.

Example: Reduce the following grammar into CNF:-

(1) $S \rightarrow aAD$, $A \rightarrow aB/bAB$, $B \rightarrow b$, $D \rightarrow d$.

(2) $S \rightarrow aAbB$, $A \rightarrow a/aA$, $B \rightarrow b/bB$.

(3) $S \rightarrow \tilde{}S/[S\supset)S]/p/q$.