HW 4 - ML Principles

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1 Question 1 - Original LeNet5

Files

- LeNet5_1.pth - The model
  - BOX LINK: https://rutgers.box.com/s/j8ptygcabbqvy9xoba3ck1ea4exlkx8h
- test1.py
  - **used for grading**
  - has the proper **prepossessing transforms**
  - has the proper **test code**
- LeNet5_data.py
- LeNet5_train.py
- LeNet5_test.py
- MNIST Data Problem 1 Folder
  - outputed data from LeNet5_data.py
  - used in training and testing
- RBF Kernel Folder
  - rbf_kernel.py
  - Data Folder
    - data used to make the kernels
  - FinalKernels Folder
    - 1 image per kernel (for visualization)
    - rbf_weights (used in training)

Graphics

- most_confusing_test_samples.png
- test_train_error_perc.png
- confusion_matrix.png

Libraries

- pandas
- torch
- torchvision
- PIL
- torchsummary
- numpy
- matplotlib
- torcheval
- seaborn

How to Run First, *LeNet5_data.py* was used to normalize the data from online and store the data in *MNIST Data Problem 1* Folder. Then we used LeNet5_train.py to train and save the model to *LeNet5_Final.pth.* Then we used *LeNet5_test.py* to do the **Performance Evaluation** section bellow.

The input to the model must be **32x32 images** each pixel value being from $[-.1, 1.175]$. **X tensors must be shape float32 (samples, 1, 32, 32) and y tensors must be of shape int64 (samples). (NOT one-hot-encoded for training.)**

1.1 Architecture

The goal of Problem 1 was to implement LeNet5 as described in the original research paper. Keeping that in mind, here is the architecture:

C1 Layer: 1 in channel, 6 out channels, 5x5 kernel, stride = 1, padding = 0. TanH as an activation function.

S2 Layer: in channels=6, 2x2 kernel, stride=2. TanH as an activation function. This was a custom pytorch layer. It summed all the values in the 2x2 frame, multiplied by a weight, and then added a bias.

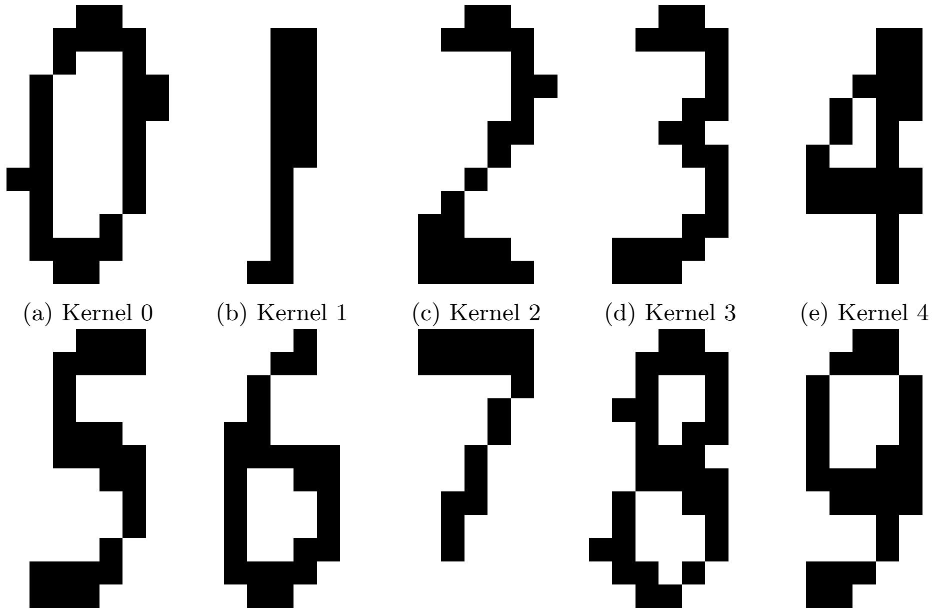
C3 Layer: in channels = 6, out channels = 16, 5x5 kernel, stride = 1, padding = 0. TanH as an activation function. This was a selective convolution layer. It was implemented as a custom pytorch layer. Basically, each of the six input layers was mapped to 10 of the 16 output layers. The mapping were specficed from the research paper.

S4 Layer: in channel = 16, 2x2 kernel, stride=2. TanH as an activation function. Similar to the S2 Layer.

C5 Layer: in channels = 16, out channels = 120, 5x5 kernel, stride = 1, padding = 0. TanH as an activation function. After, this channel is flattened into an tensor of size Batch x 120

F6 Layer: in features = 120, out features = 84. Just a normal Fully connected layer.

Output Layer: To get to the output layer, which is of size Batch x 10, we used RBF Kernels. Their weights are frozen. We have 10 RBF kernels, one for each digit. We used the DIGIT dataset linked in the instructions pdf. For each digit, we made a mean image of size. Then we performed a center crop of 100 x 80 to get to closer to the final aspect ratio and also we noticed there were empty space in the edges so we were able to crop without getting rid of information. Then we resized it to a 7x12 image, and then clamped the values. So values less than .5 would be 0 and values greater than .5 would be 1. Then we flattened each digit to a 84 value 1D tensor and saved it in *./FinalKernels/rbf_weights.* We did 84 beceause the F6 layer had 84 out features.



(a) Kernel 0 (b) Kernel 1 (c) Kernel 2 (d) Kernel 3 (e) Kernel 4

(f) Kernel 5 (g) Kernel 6 (h) Kernel 7 (i) Kernel 8 (j) Kernel 9

Figure 1: 7x12 RBF Kernels per digit

Other notes: The parameters for each layer are initialized between $[-2.4/Fi, 2.4/Fi]$. Fi is the number of inputs in the layer. Each layer up to F6 has a tanh activation function $f(a) = Atanh(Sa)$. $A = 1.7159$. $S = 2/3$. Num of epochs = 20 and batch size = 1.

1.2 Training

1.2.1 Data

The LeNet5_data.py file handles are input data for us. First we convert each 28x28 image to a 32x32 image by adding 2 pixel padding on each side. Then we normalize them each pixel value to between $[-.1, 1.175]$. Then we export the digits into 4 files: XTrain, yTrain, XTest, yTest.

1.2.2 Optimizer

We used the built in pytorch SGD optimized with a learning rate of .0001.

1.2.3 Criterion (Loss Function)

$$E(W) = \frac{1}{P}\sum_{p=1}^{P}\left(y_{Dp}(Z^p, W) + \log\left(e^{-j} + \sum_{i} e^{-y_i(Z^p, W)}\right)\right)$$

This is the loss function that is used. $j = .1$. P are the number of samples in the batch. $y_{Dp}(Z^p, W)$ is the output value of the $Dpth$ RBF kernel, where Dp is the correct classification for sample p. In the second summation, we exponentiate the negative output value of all the other incorrect digit RBF kernels. So when using this log function, the **argmin** of the logits refers to the predicted class. To use the loss function in python, the Y tensor must be converted to a one-hot-encoded tensor.

1.3 Performance Evaluation

1.3.1 Error Rates

On epoch 20, the Train Error was 0.82%, and the Test Error was 1.37%.

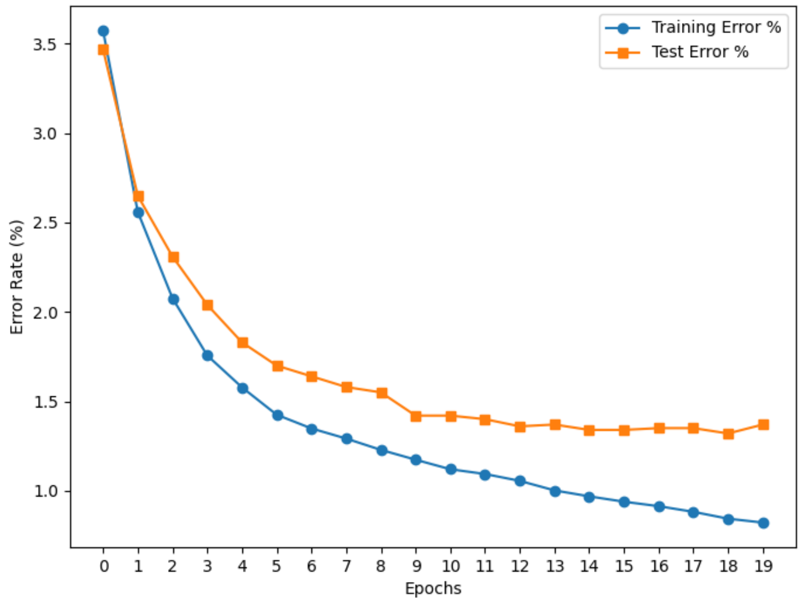


Figure 2: Error Rates. Shows that around epoch 10-11, the test error stabilized, but the train error continued decrease which indicated slight over fitting. It is not major over fitting because the test error is not getting worse.

1.3.2 Confusion Matrix

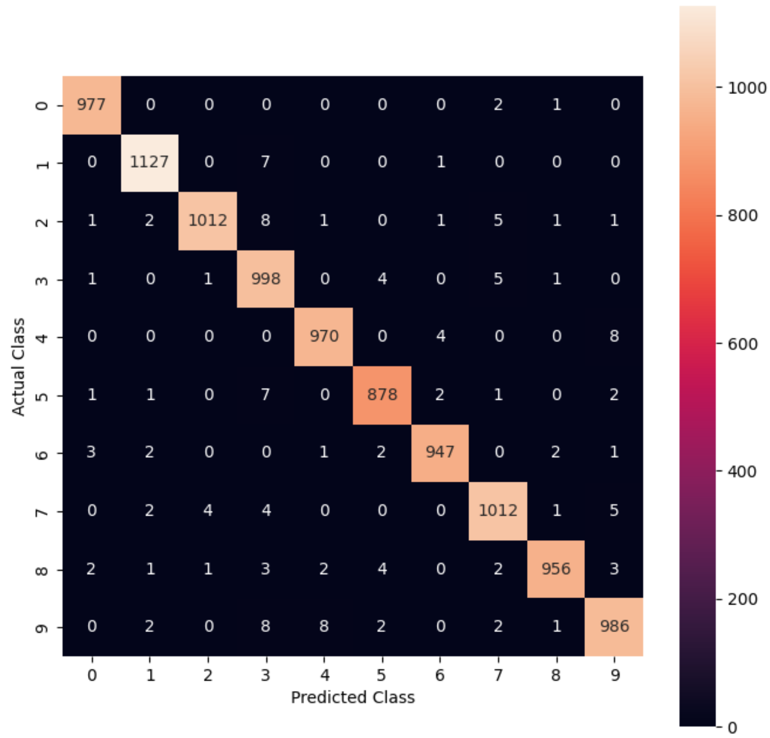


Figure 3: Confusion Matrix on test data of 10000 samples. The rows represent the actual digit labels, while the columns represent the predicted digit labels. Each cell indicates the number of instances for the corresponding actual and predicted digit combination.

1.3.3 Most Confusing

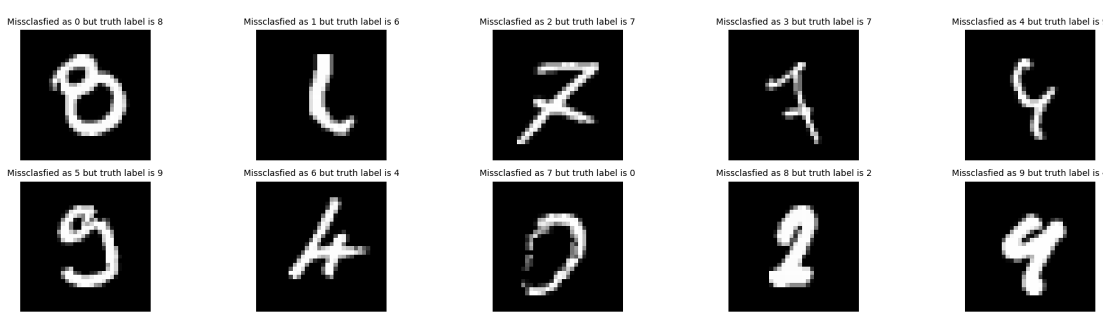


Figure 4: Most Confusing Sample. This figure answers the question: "What image was misclassified with the highest confidence as the digit "x." So just to explain further, the last image in the figure above was misclassified with the highest confidence as "9" but was in reality a 4.

2 Question 2 - Modified LeNet5

Files

- LeNet5_2.pth - The Model
  - BOX LINK: https://rutgers.box.com/s/2emopqzvsnwi58yr5didjfe1ageblu1x
- test2.py
  - **used for grading**
  - has the proper **pre-processing transforms**
  - has the proper **test code**
- ModifiedLeNet5_data.py
- ModifiedLeNet5_train.py
- ModifiedLeNet5_test.py
- affNIST Data Problem 2 Folder
  - outputed data from ModifiedLeNet5_data.py
  - used in training and validation
- Problem 2 Models Folder
  - used to store the models after each epoch

Graphics

- P2_test_validation_error_perc.png
- P2_transformed_data_sample.png

How to Run First, ModifiedLeNet5_data.py is used to collect and prepare the data and then is stored in *affNIST Data Problem 2 Folder*. Then we use *ModifiedLeNet5_train.py* to train the model and store each model after each epoch in the *Problem 2 Models Folder*. Then I examine the *P2_test_validation_error_perc.png* graph to see if there is any over fitting and the copy and move over the 'best' model from the *Problem 2 Models Folder* to *ModifiedLeNet5_Final.pth*.

The input to the model must be **40x40 images** with each pixel value being from $[0, 1]$. **X tensors must be shape float32 (samples, 1, 40, 40) and y tensors must be of shape int64 (samples). (NOT one-hot-encoded for training.)**

2.1 Overview

So the goal of Question 2 is to make your own model that handles transformed MNIST data. To tackle that, we have taking a 3-prong approach: **Data Augmentation**, **Modern Blocks**, and **Early Stopping**.

2.2 Data Augmentation

So for **training** we made our own augmented data from the original MNSIT data set. For each of 60,000 original training samples, we applied 10 transformations to reach a total of **600,000 training samples**. The code can be scene in ModifiedLeNet5_data.py. First we converted each MNIST image from a 28x28 image to a 40x40 image. This expansion gave us room to play around with our transformations without digits being cut of at the edges. Then we applied random affine transformations with the following values: degrees=[-20, 20], shear=[-.2, .2], translate=(0.3, 0.3), scale=(0.7, 1.3). The reasoning for doing this data augmentation was because we knew that we would be tested on unseen MNSIT data in a random style. So the easiest way to come up with new data was to apply random transformation to the original data and train on it. We could have used the provided the training data from the affNIST data set online, but we decided not to, so we can practice Data Augmentation.

Then for **validation**, we ended up using the affNIST data set from online, because we wanted a different source of unseen data that would have different transformation so we can thoroughly pick the best model.

All data for problem 2 is stored in the *affNIST Data Problem 2* Folder.

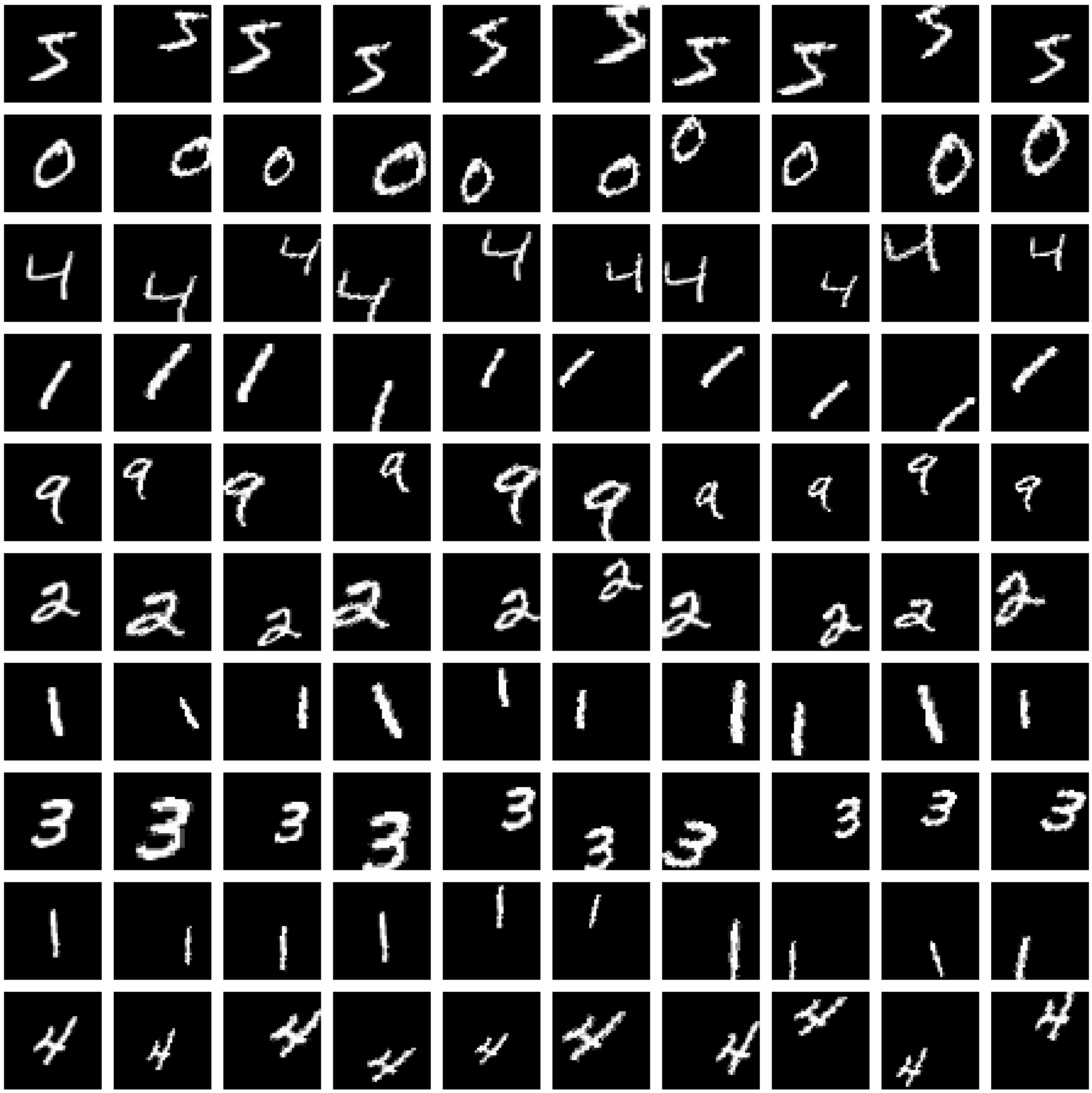


Figure 5: Sample Data from X_train

2.3 Modern Blocks

So the original LeNet5, as implemented in Problem 1, was made in 1998. During that time, modern blocks were not thought off. So we decided to make 4 changes to the architecture.

First we replaced the sub sampling layers with Max Pool. The original LeNet-5 employed subsampling, which averages the 2x2 pixels within a region to reduce dimensionality. This approach smooths the feature maps and provides some robustness to small variations, but it often loses critical high-frequency details like edges or corners, which are essential for distinguishing digits, especially under transformations like rotation or scaling. Max pooling, on the other hand, preserves these distinctive features by selecting the maximum activation in a given 2x2 reigon. This not only helps in retaining the most prominent patterns but also makes the network more invariant to slight positional shifts in the input data. As a result, max pooling enhances the model's ability to generalize to unseen transformed MNIST data by ensuring that vital features remain intact throughout the forward propagation.

Second we replace the selective C3 layer with a normal fully connected C3 Conv2d layer. In the original LeNet-5, the selective C3 layer used a predefined connectivity pattern, where only specific subsets of input feature maps contributed to certain output feature maps. This design reduced the computational complexity for 1998 but limited the network's capacity to learn diverse and complex feature interactions. By replacing the selective C3 layer with a fully connected Conv2D layer, the new architecture allows all feature maps from the previous layer to contribute to every output feature map. This increased flexibility enables the model to capture a more transformations like rotations, distortions, or noise in modified data.

Third we replace the tanh activation functions with LeakyReLU. The tanh activation function, used in the original LeNet-5, squashes values into a narrow range, which can result in vanishing gradients for extreme input values. This significantly slows down learning, especially in deeper networks, and makes it harder to adapt to variations in input data. In contrast, LeakyReLU introduces a small slope for negative input values, allowing gradients to flow even when inputs are negative. This eliminates the dead neuron problem faced by normal RELU and ensures that learning remains efficient throughout the network.

Forth we replace the last RBF Layer with a simple Fully Connected Linear Layer. The RBF kernel layer in the original LeNet-5 acted as a distance-based classifier, where outputs were determined by the radial distance from fixed weights, ie the average shape of each digit. While this approach works well for datasets with limited variability, it struggles with unseen transformations, as the weights may not adequately represent the transformed data. Replacing the RBF kernel with a linear fully connected layer introduces greater flexibility by enabling the model to learn a set of adaptable weights for classification. For unseen transformed MNIST data, a fully connected layer provides the adaptability to new patterns and variations, significantly improving the model's robustness and accuracy.

2.4 Early Stopping

The last major change we did was to combat over fitting. Since we have so much training data, we were afraid that the model might over fit. So what implemented our own version of "early stopping." We saved the model after each epoch and kept track of the model's training and validation errors. Then after 20 epochs, we looked at the error graph bellow, and picked the best model. We decided to pick the model after epoch 16 because right before we saw the validation error jump. We think that at this epoch the model could generalize well to unseen data. One thing to note is that after each epoch, we did not calculate the error rates on the FULL train and validation sets, instead we used a random subset of the training and validation sets to plot the error rates, because if we used the FULL sets, it would take too long.

2.5 Results

Final Validation Error on the full validation set is 4.185% .

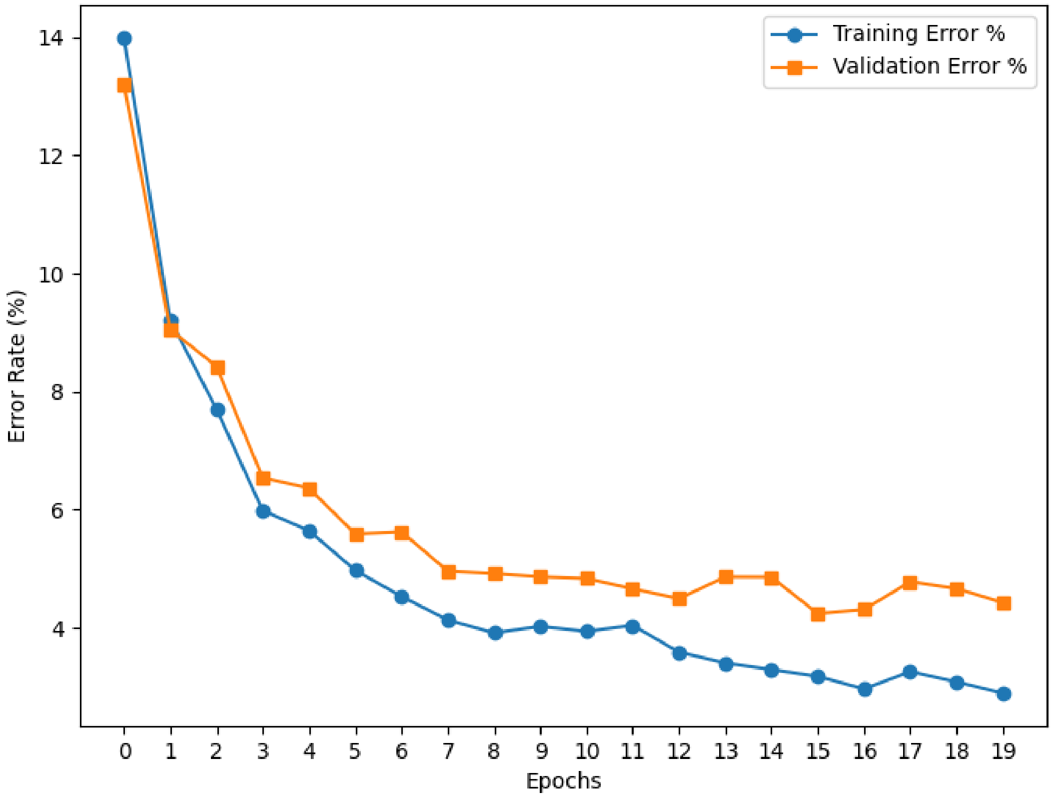


Figure 6: This is the error rates graph for Problem 2. Read the Early Stopping Section to see what model we picked.