

Binary Search

Condition: All element should be in monotonic function

→ Means Sorted

Algorithm:-

- find mid element
- compare mid element with key
- $>$ go to right and repeat the same
- $<$ go to left and repeat same

Example:-

0	1	2	3	4	5	6	7
3	7	11	13	19	27	6	15

$n = 8$

key = 27

← means we want to find 27

now we go to the mid element of

array.

$$\text{mid element} = \left(\frac{\text{start} + \text{end index}}{2} \right)$$

in int

$$\text{Here mid} = \frac{7}{2} = 3.5$$

$$= 3^{\text{rd}} \text{ index}$$

0	1	2	3	4	5	6	7
3	7	11	13	19	27	6	15

$n = 8$

now compare it with key.

$$\text{Key} = 27 \quad \underline{\underline{27 > 13}}$$

Because 27 is greater than 13 we will move to right.

4	5	6	7
19	27	6	15

Now we find mid again.

$$\begin{aligned} \text{mid} &= \left(\frac{4 + 7}{2} \right) \\ &= \left(\frac{11}{2} \right) = \underline{\underline{5.5}} \\ &= 5 \end{aligned}$$

at index = 5

$$27 = 27$$

Binary Search										
Search 23	0	1	2	3	4	5	6	7	8	9
	2	5	8	12	16	23	38	56	72	91
23 > 16 take 2 nd half	L=0	1	2	3	M=4	5	6	7	8	H=9
	2	5	8	12	16	23	38	56	72	91
23 < 56 take 1 st half	0	1	2	3	4	L=5	6	M=7	8	H=9
	2	5	8	12	16	23	38	56	72	91
Found 23, Return 5	0	1	2	3	4	L=5, M=5	H=6	7	8	9
	2	5	8	12	16	23	38	56	72	91

```
#include <iostream>
using namespace std;

int binarySearch(int arr[], int n, int target, int hi, int lo)
{
    while(lo <= hi)
    {
        int mid = lo + (hi-lo)/2;
        if(arr[mid] == target)          return mid;
        else if(arr[mid] < target)
            lo = mid + 1;
        else if(arr[mid] > target)
            hi = mid - 1;
    }
    return 0;
}

int main()
{
    int arr[10] = {1,12,14,16,18,20,24,26,28,30};
    int n = 10;
    int target = 26;

    int lo = 0;
    int hi = n-1;
    cout<<"The index of the target is - "<<binarySearch(arr, n, target, hi,
lo);
    return 0;
}
```

Here

low = start = first index

high: end = final index

Here is different formula.

$$\text{mid} = \text{start} + \left(\frac{\text{end} - \text{start}}{2} \right)$$

Because range of int is

$$2^{31} - 1$$

what if my array is big enough that it's some is out of int range.

Then we need this formula.

Finding time complexity

Here there is some sequence is followed in binary search.

while first iteration $= \frac{N}{2}$

second iteration $= \frac{N}{4} = \frac{N}{2^2}$

third iteration $= \frac{N}{8} = \frac{N}{2^3}$

Here N = size of array.

final equation: $\frac{N}{2^k}$

Now according to decreasing infinite geometric series.

(S.P.) $= \frac{r}{1-r}$

Here $r = \frac{1}{2}$

$$\frac{1}{2^k} = \frac{1}{1 - \frac{1}{2}}$$

$$= 1$$

means

$$\frac{1}{2^k} = 1$$

$$\frac{N}{2^k} = 1$$

$$N = 2^k$$

$$k = \log n$$

upper bound $k = \log n$

Time complexity = $O(\log N)$