P1: Optimal pacing in swimming competitions

The goal of this project is to establish a simple model of a human swimmer and derive an optimal pacing strategy for a 100 m swimming race. For this, we first describe the simple mechanics of swimming used in this project: We assume there is only one distance s into a one-dimensional direction, where turns are omitted from the model. The swimmer can generate a forward propulsion force u acting against hydrodynamic drag, while being subject to drag forces propotional to the velocity. Following from a force balance we derive the following equation for the acceleration of the body

$$m\dot{v} = \gamma u - 0.5\rho c_w A v^2 \,, \tag{1}$$

where γ is the percentage of force directed to the forward propulsion, while A is the body area directed in drag direction and c_w the drag coefficient. This force u comes from the bodies energy system. This energy system provides a power P_{mech} that is required to contain the movement

$$P_{\text{mech}} = u\,v\,. \tag{2}$$

To achieve this mechanical power the body has to provide a power P_{body}, which is determined through the body efficency coefficient β

$$P_{\text{body}} = \frac{P_{\text{mech}}}{\beta}\,. \tag{3}$$

The body energy system contains three different forms of energies that can be provides. The first one is the alactacid energy systems, which is used first, as it is already available as an usable ATP energy source in the muscle cells. However, the amount is very limited, with a maximum energy amount $E_{\text{max,al}}$ and can provide a maximum power $P_{\text{max,al}}$. We assume that the power provided through this energy system P_{al} is propotional to the remaining energy E_{al}

$$\dot{E}_{\text{al}} = -P_{\text{al}} = -P_{\text{max,al}}\frac{E_{\text{al}}}{E_{\text{max,al}}}\,. \tag{4}$$

The more important energy systems are the aerobic (oxygen based) energy system and the anaerobic (lactate based) energy system. We assume that both are directly available if needed. In general, we assume that these three have to cover the required power

$$P_{\text{body}} = P_{\text{al}} + P_{\text{anaerobic}} + P_{\text{aerob}}\,. \tag{5}$$

The body can use a maximum amount of power $P_{\text{max,aerobic}}$ from the aerobic energy system, which is instantly provided and can be used the entire time as oxygen is provided throughout the swim. However, the anaerobic system can be used if more is power is required, which we assume will only be provided if $P_{\text{max, aerobic}}$ is reached. This power comes with the caveat that in the process $H+$-ions will be freed in the muscle cells (as a part of the lactic acid increase) that in turns increases the fatigue of the system. For simplicity, we assume that this fatigue F can be measured as lactic acid, where

$$\dot{F} = -\frac{P_{\text{anaerobic}}}{d}\,. \tag{6}$$

This fatigue in turn has an influence on the maximum power u_{max} that can be provided

$$u + \alpha F \leq u_{\text{max}} \tag{7}$$

The parameter α is an empirical scaling factor linking fatigue to a reduction in maximum producible force. The parameters of the system are provided in Table 1.

Parameter	Value	Parameter	Value
γ	0.5	$P_{\text{max,al}}$	$1000\,\text{kW}$
c_w	2	$P_{\text{max,aerobic}}$	$300\,\text{kW}$
A	$0.02\,\text{m}^2$	$E_{\text{max,al}}$	$1000\,\text{kJ}$
β	0.2	ρ	$997\,\text{kgm}^{-3}$
m	$70\,\text{kg}$	u_{max}	$200\,\text{kN}$
d	$5000\,\text{kJmmol}^{-1}\text{l}$	α	$10\,\text{kNmmol}^{-1}\text{l}$

Table 1: Model parameters.

Mandatory Tasks

The following tasks **have to be completed** in order to pass the project.

- State the system equations, inputs and state variables you used for the system. **Hint**: States are used to describe variables, that can be described through an ordinary differential equation. To derive the input vector, find the variables that the swimmer can influence during the swim. Implement the continuous-time system using orthogonal collocation for discretization.
- To test your system assume a swimmer pushes from the block with an initial velocity of $3\,\text{m/s}$, no fatigue and a full alactacid energy system without providing effort. Test how far the swimmer floats until the velocity is zero. Plot the velocity profile for this task.
- Design an MPC controller that computes an optimal pacing strategy to minimize the race completion time over 100 m. Think of two suitable cost functions and compare them against each other. Determine the better cost function for your project.
- To visualize your results in the report, plot all kinetic and energetic variables for this swim. Discuss what happens in which phase of the swim and which parameters limits the performance.
- Discuss the the performance for different prediction horizons and explain the behavior.

Additional tasks

Below are **suggested** additional tasks to obtain an excellent grads for the project. We want to emphasize that students are encouraged to come up with their own ideas for additional investigations and not all of the suggestions below must be included for an excellent grade.

- Experiment with further costs to do time optimal control under the fixed discretiztation used for your MPC. What are potential benefits in comparison to the cost functions already used.
- Simulate and analyse an optimal pacing strategy for a 400 m race. Discuss how the dominant limiting factors differ from the 100 m case.

Allowed packages

You are allowed to use the following packages in your project: **casadi**, **numpy**, **matplotlib**. For every other package you would like to use, please contact your supervisor in advance.

Deliverables

The following materials have to be submitted in order to pass the semester project:

- **Recorded final presentation** (video screencast). The presentation must be **5-7 minutes** (for the entire group) and the file should not exceed **200 mb**. Highlight on the slides which group member(s) are responsible.

- **Written report** to present and discuss the obtained results. You must use the **supplied template on Moodle** (Tex or Word) and write no more than **3-4 pages** (for the entire group). Highlight which group member worked on which section.
- **Source code** of your project. Please ensure that the code is executable and optionally add a short explanation of the structure (readme).

Please ensure that all formal conditions are (e.g. page limits, highlight responsible author) are satisfied, as we will deduct points for significant violations. Deadline for the submission is 05.02.2026. Please submit all deliverables via moodle.

Supervisor

Please address questions to:

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