Design and Analysis of Algorithm

Lecture-17:
Dynamic Programming

Contents

1 Reliability Design

Reliability

Definition

It is the probability that system/component will perform its intended task adequately when operated under stated operating condition for specified period of time.

Example of a simple system

Electric lamp made by a light bulb, socket, switch, wires, plug, and the lamp body.

Example of complex system

An aircraft, containing tens of thousands of mechanical, hydraulic, or electric elements.

A suitable arrangement can even increase the reliability of the system.

The resultant system reliability depends on the reliability of the individual elements and their number and mutual arrangement.

Possible arrangement of components

a) <u>Series system</u>: In a series configuration, a failure of any component results in the failure of the entire system.

<u>Example</u>

A personal computer may consist of four basic subsystems: the motherboard, the hard drive, the power supply and the processor. These are reliability-wise in series and a failure of any of these subsystems will cause a system failure. In other words, all of the units in a series system must succeed for the system to succeed.

System Reliability=$R_1 \times R_2 \times R_3$

In General

$$R_s = R_1 \times R_2 \times \cdots \ldots \ldots \ldots \times R_n$$

$$R_s = \prod_{i=1}^{n} R_i$$

Example: Series System

What will be the reliability of the system for a 100-hour mission, the system has three subsystems are reliability-wise in series, Subsystem 1 has a reliability of 99.5%, subsystem 2 has a reliability of 98.7% and subsystem 3 has a reliability of 97.3%

(a) 0.94
(b) 0.945
(c) 0.95
(d) 0.955

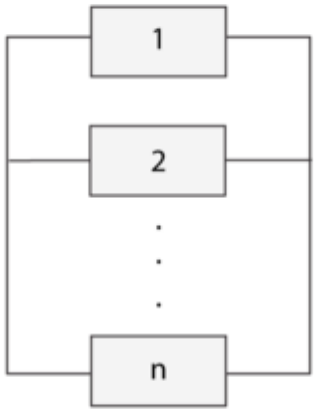
$$R_s = 0.995 \times 0.987 \times 0.973 = 0.9555$$

Possible arrangement of components

b) Parallel system: In a parallel configuration, at least one of the units must succeed for the system to succeed.

Example

Applications include the RAID computer hard drive systems, brake systems and support cables in bridges.



System Reliability$=1 - Q_s$

$$R_s = 1 - (Q_1 \times Q_2 \times Q_3 \ldots . Q_n)$$

$$R_s = 1 - (1 - R_1)(1 - R_2) \ldots \ldots (1 - R_n)$$

$$R_s = 1 - \prod_{i=1}^{n} (1 - R_i)$$

Example: Parallel System

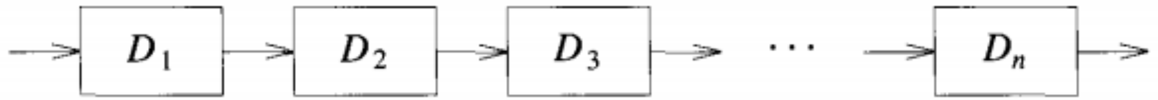
What will be the reliability of the system for a 100-hour mission, the system has three subsystems are reliability-wise in parallel, Subsystem 1 has a reliability of 99.5%, subsystem 2 has a reliability of 98.7% and subsystem 3 has a reliability of 97.3%

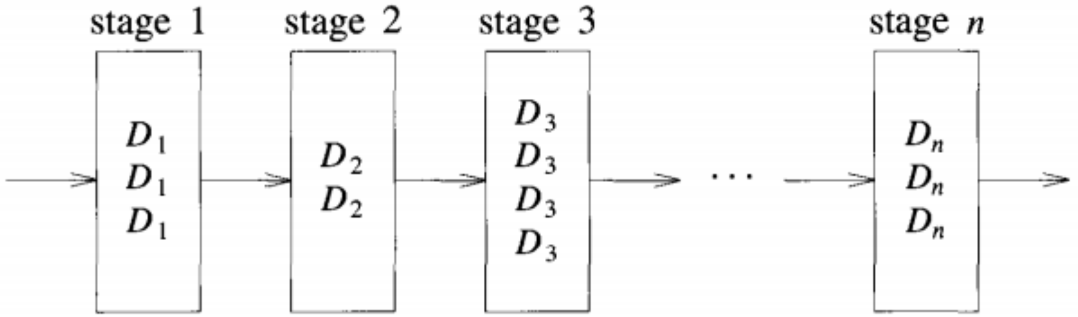
(a) 0.99
(b) 0.95
(c) 0.98
(d) 0.96

$$R_S = 1 - (1 - 0.995)\times(1 - 0.987)\times(1 - 0.973) = 1 - .005\times.013\times.027 = 0.99$$

Reliability Design Problem

Our problem is to use device duplication to maximize reliability





This maximization is to be carried out under a cost constraint

Assume that the reliability of each stage is given by $\phi_i(m_i)$

Then Reliability of entire system will be $\prod_{i=1}^{n} \phi_i(m_i)$

We wish to solve the following maximization problem: $\begin{cases} maximize \prod_{i=1}^{n} \phi_i(m_i) \\ subject\ to\ \sum_{i=1}^{n} c_i m_i < c \end{cases}$

Example

Given the reliability and cost of 3 devices as, (0.9,30), (0.8,15) and (0.5,20), design the system by device duplication so as to have maximum reliability of the system within the cost of 105 units.

D_i	C_i	r_i	U_i
D_1	30	0.9	1+1=2
D_2	15	0.8	1+2=3
D_3	20	0.5	1+2=3

With the given information compute the upper limit for each device

$$remaining\ cost = 105 - (30 + 15 + 20) = 40$$

Extra Copies of D_1 with 40=$\lfloor 40/30 \rfloor$ =1

Extra Copies of D_2 with 40=$\lfloor 40/15 \rfloor = 2$

Extra Copies of D_3 with 40=$\lfloor 40/20 \rfloor = 2$

Example

$S_1^1 = \{(30{,}0.9)\}$ $S_1^2 = \{\ (60, (1-(1-0.9)^2) = 0.99)\}$

$S_1 = \{(30{,}0.9), (60, 0.99)\}$

$S_2^1 = \{(45{,}0.72), (75{,}0.792)\}$ $S_2^2 = \{\left(60, 0.9\times(1-(1-0.8)^2)\right), (90\ ,--\ -)\}$

$S_2^2 = \{\underline{(60, 0.864)}\}$

$S_2^3 = \{\left(75, 0.9\times(1-(1-0.8)^3)\right), (105,--\ -)\}$

$S_2 = \{(45{,}0.72), \underline{(60, 0.864)}, (75{,}0.792), (75, 0.8928)\}$

$S_3^1 = \{(65{,}0.36), (80{,}0.432), (95, 0.446)\}$ $S_3^2 = \{(85{,}0.54), \underline{(100{,}0.648)}\}$ $S_3^3 = \{(105{,}0.63)\}$

$S_3 = \{(65{,}0.36), (80{,}0.432), (85{,}0.54), (95, 0.446), \underline{(100{,}0.648)}, (105{,}0.63)\}$

D_i	C_i	r_i	U_i
D_1	30	0.9	1+1=2
D_2	15	0.8	1+2=3
D_3	20	0.5	1+2=3

Copies of $D_3 = 2$ Copies of $D_2 = 2$ Copies of $D_1 = 1$

RBD