Design and Analysis of Algorithm

Lecture-29:
P and NP problem

Contents

Polynomial and Exponential Algorithm

- Polynomial Algorithm

Linear Search	n
Binary Search	$\log n$
Insertion Sort	n^2
Merge Sort	$n \log n$
Matrix Multiplication	n^3

- Exponential Algorithm

0/1 Knapsack	2^n
TSP	2^n
Sum of Subsets	2^n
Graph Coloring	2^n
Hamiltonian Cycle	2^n

Introduction

Almost all the algorithms we have studied thus far have been polynomial-time algorithms: on inputs of size n, their worst-case running time is $O(n^k)$ for some constant k.

Can all problems be solved in polynomial time?

NO

✓ Turing's famous "Halting Problem," cannot be solved by any computer, no matter how much time we allow.

Generally, we think of problems that are solvable by polynomial-time algorithms as being tractable, or easy, and problems that require superpolynomial time as being intractable, or hard.

The Class P problem

P: the class of problems that have polynomial-time deterministic algorithms.

– That is, they are solvable in $O(p(n))$, where $p(n)$ is a polynomial on n

– A deterministic algorithm is (essentially) one that always computes the correct answer

Example

- Fractional Knapsack
- MST
- Sorting

The class NP

NP: the class of decision problems that are solvable in polynomial time on a nondeterministic machine (or with a nondeterministic algorithm)

– A nondeterministic computer is one that can "guess" the right answer or solution

• Note that NP stands for "Nondeterministic Polynomial-time"

Example

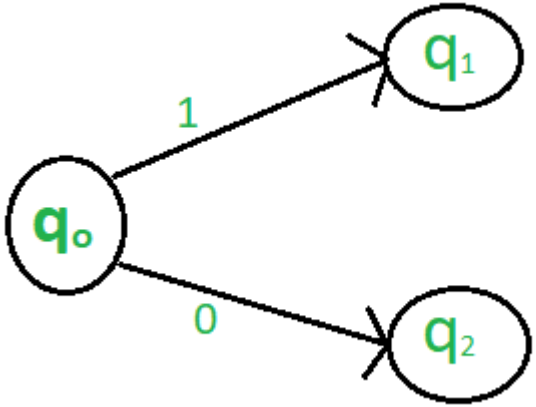
– *Traveling Salesman*
– *Graph Coloring*

NP Problems

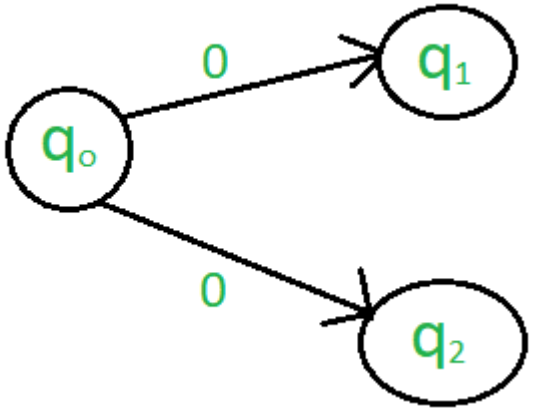
Why we need to understand NP Problems?

Deterministic and Non-deterministic Algorithm

The algorithms in which the result of every algorithm is uniquely defined are known as the Deterministic Algorithm.



The algorithms in which the result of every step is not uniquely defined and result could be random are known as the Non-Deterministic Algorithm.



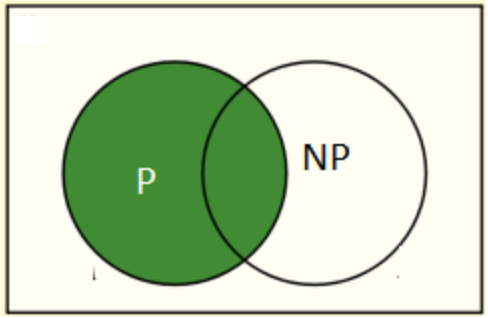
Question

Which of the following graph problems are known to be in NP?

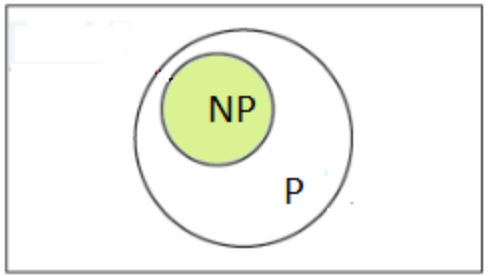
A. Is the length of the longest simple path $\leq k$?
B. Is the length of the longest simple path $\geq k$?
C. Is the length of the longest simple path $= k$?
D. Find the length of the longest simple path.
E. All of the above.

Question

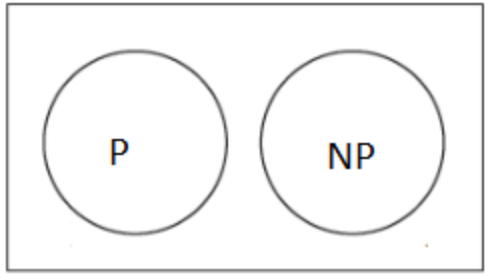
Which of the following correctly represents the relation between P and NP?



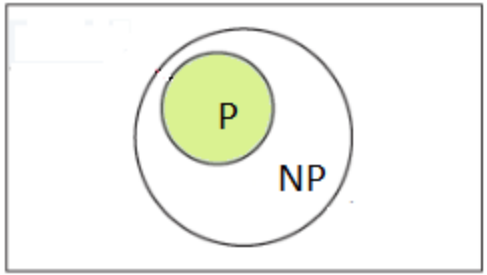
A



B



C



D

Tractable and Non-Tractable problem

Tractable Problem: A problem that is solvable by a polynomial-time algorithm.

- Searching an unordered list
- Searching an ordered list
- Sorting a list

Intractable Problem: a problem that cannot be solved by a polynomial-time algorithm. The lower bound is exponential.

- 0/1 Knapsack Problem