Design and Analysis of Algorithm

Lecture-22:
Backtracking

Contents

1 Bounding Function and Solution for 4-Queen Problem

2 Sum of Subsets

3 Graph Coloring Problem

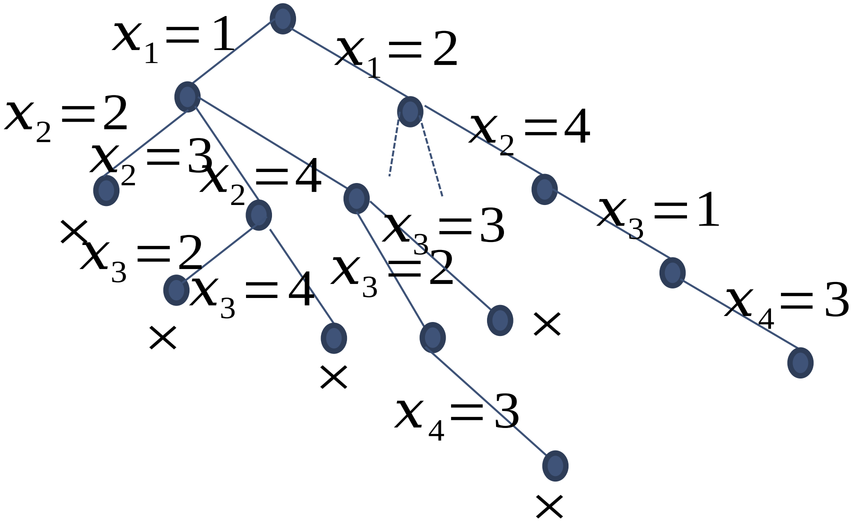
4-Queen Problem

Bounding condition: The condition which needs to be checked before placing the queen at a particular cell location is called bounding condition

Brute Force approach $\boxed{{}^{16}C_{4}}$

Condition: The Queen should not be under attack of any other queen

Finding all Solution



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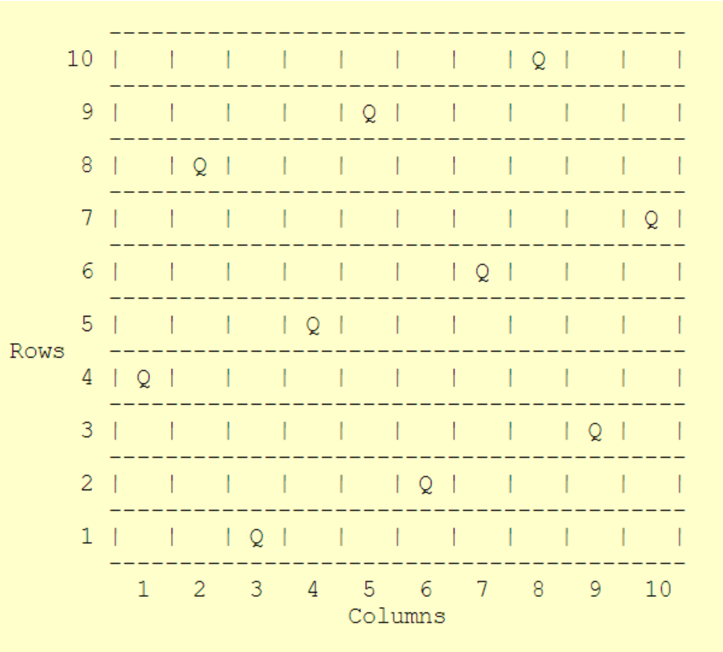
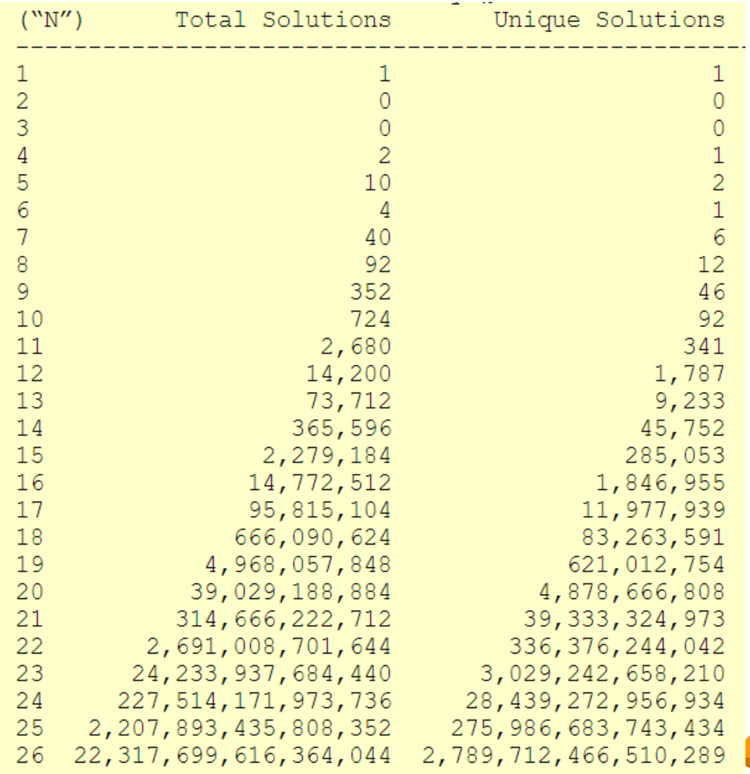
	●	●	●●
●	●●	●	●
		●●	

No. of solution



```
        -----------------------------------------
    10  |   |   |   |   |   |   |   | Q |   |   |
        -----------------------------------------
     9  |   |   |   |   | Q |   |   |   |   |   |
        -----------------------------------------
     8  |   | Q |   |   |   |   |   |   |   |   |
        -----------------------------------------
     7  |   |   |   |   |   |   |   |   |   | Q |
        -----------------------------------------
     6  |   |   |   |   |   |   | Q |   |   |   |
        -----------------------------------------
     5  |   |   |   | Q |   |   |   |   |   |   |
Rows    -----------------------------------------
     4  | Q |   |   |   |   |   |   |   |   |   |
        -----------------------------------------
     3  |   |   |   |   |   |   |   |   | Q |   |
        -----------------------------------------
     2  |   |   |   |   |   | Q |   |   |   |   |
        -----------------------------------------
     1  |   |   | Q |   |   |   |   |   |   |   |
        -----------------------------------------
          1   2   3   4   5   6   7   8   9   10
                       Columns
```



("N")	Total Solutions	Unique Solutions
1	1	1
2	0	0
3	0	0
4	2	1
5	10	2
6	4	1
7	40	6
8	92	12
9	352	46
10	724	92
11	2,680	341
12	14,200	1,787
13	73,712	9,233
14	365,596	45,752
15	2,279,184	285,053
16	14,772,512	1,846,955
17	95,815,104	11,977,939
18	666,090,624	83,263,591
19	4,968,057,848	621,012,754
20	39,029,188,884	4,878,666,808
21	314,666,222,712	39,333,324,973
22	2,691,008,701,644	336,376,244,042
23	24,233,937,684,440	3,029,242,658,210
24	227,514,171,973,736	28,439,272,956,934
25	2,207,893,435,808,352	275,986,683,743,434
26	22,317,699,616,364,044	2,789,712,466,510,289

Objective Questions

Q1: Backtracking algorithm is implemented by constructing a tree of choices called as?

a) [✓ State-space tree]
b) State-chart tree
c) Node tree
d) Backtracking tree

Q1: In what manner is a state-space tree for a backtracking algorithm constructed?

a) [✓ Depth-first search]
b) Breadth-first search
c) Twice around the tree
d) Nearest neighbor first

Sum of Subsets

Given positive integers and a positive integer S. Find all subset of that sum to S.

Example

Solution:

Sum of subsets

We create a binary state space tree

$$Time\,Complexity = O(2^n)$$

The left child indicates the inclusion and right child indicates the exclusion of the element.

Node at each depth (or level) indicates inclusion or exclusion of the elements of the set

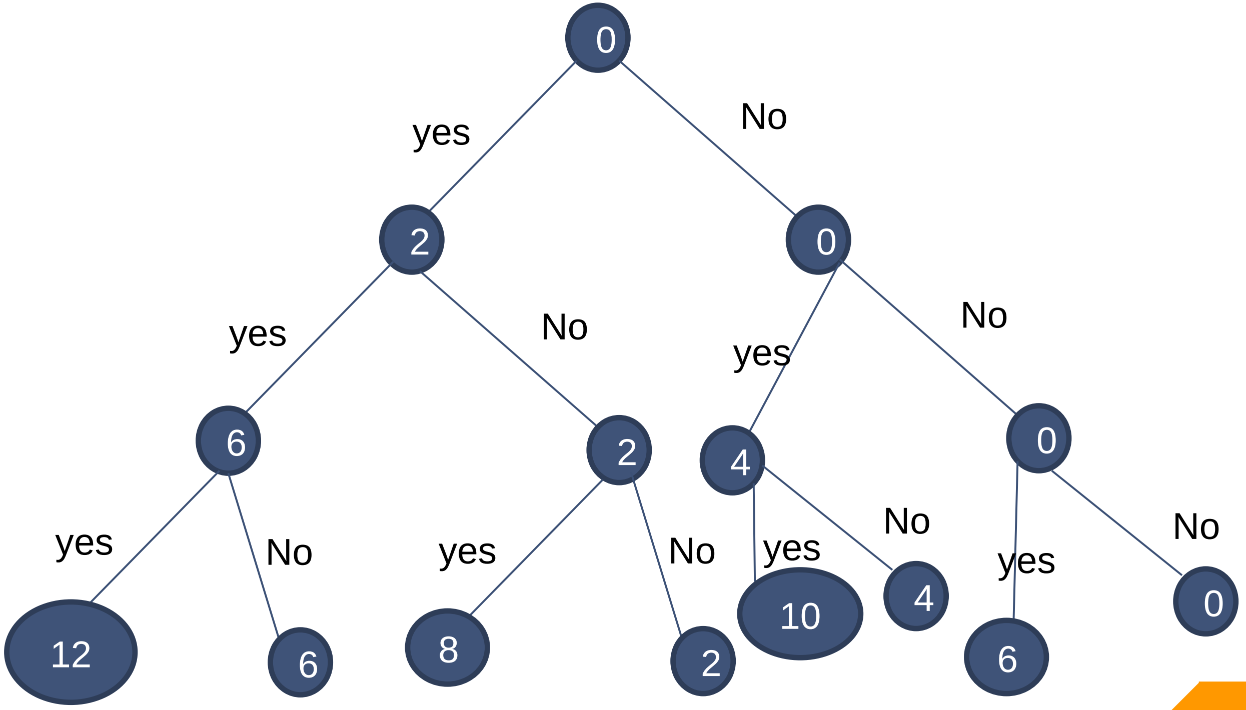
Nodes contain the sum of weights included so far

State Space tree for Three Items

$W_1 = 2$

$W_2 = 4$

$W_3 = 6$



Concept

The element of the solution is marked one or zero depending on whether the weight is included or not.

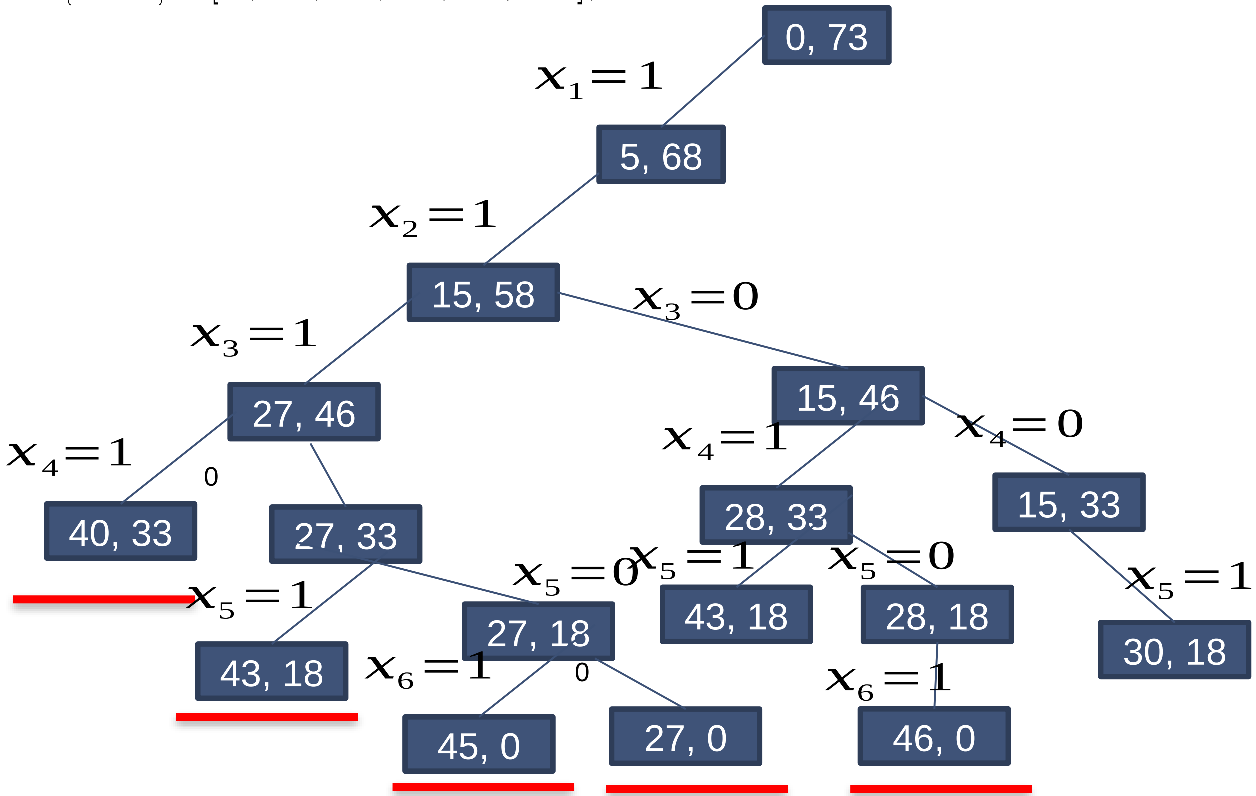
As discussed in the last example, For a node at level the left child corresponds to and right side corresponds to

Bounding Function

Example

$$\sum_{i=1}^{k} w_i x_i + w_{k+1} \le m \quad \sum_{i=1}^{k} w_i x_i + \sum_{i=k+1}^{n} W_i \ge m$$

$$w\{1:6\} = [5,10,12,13,15,18], m = 30$$



1	1	0	0	1	0

Objective Questions

Q1: In general, backtracking can be used to solve?

a) Numerical problems
b) Exhaustive search
c) [✓ Combinatorial problems]
d) Graph coloring problems

Q1: In what manner is a state-space tree for a backtracking algorithm constructed?

a) [✓ Depth-first search]
b) Breadth-first search
c) Twice around the tree
d) Nearest neighbour first

Contents

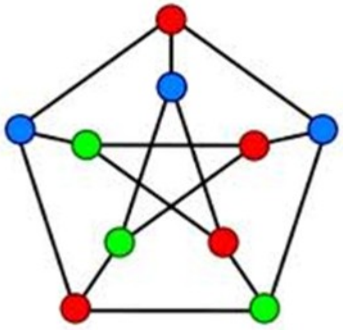
1 Graph Coloring Problem

Graph Coloring Problem

decision problem

Let G be a graph and m be a given positive integer. We want to discover whether the nodes of G can be colored in such a way that no two adjacent nodes have the same color yet only m colors are used

if is the degree of the given graph, then it can be colored with colors.

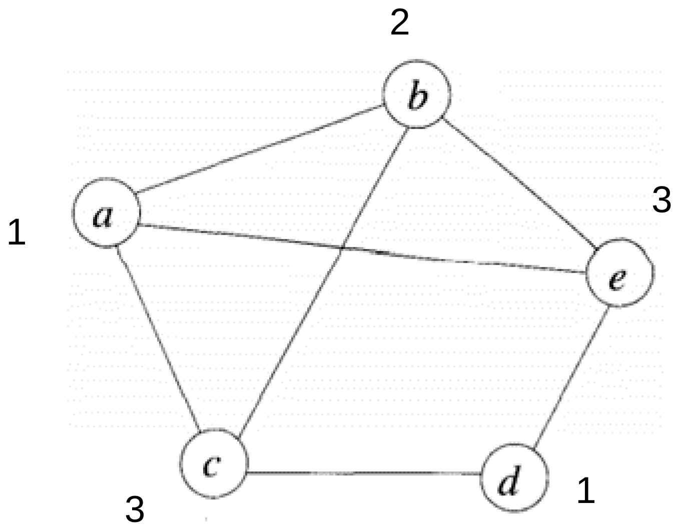


The optimization problem asks for the smallest integer for which the graph can be colored

Example

The graph can be colored with three colors1,2,and 3.

It can also be seen that three colors are needed to color this graph and hence this graph's chromatic number is 3.



4-color Problem

A graph is said to be planar it can be drawn in a plane in such a way that no two edges cross each other

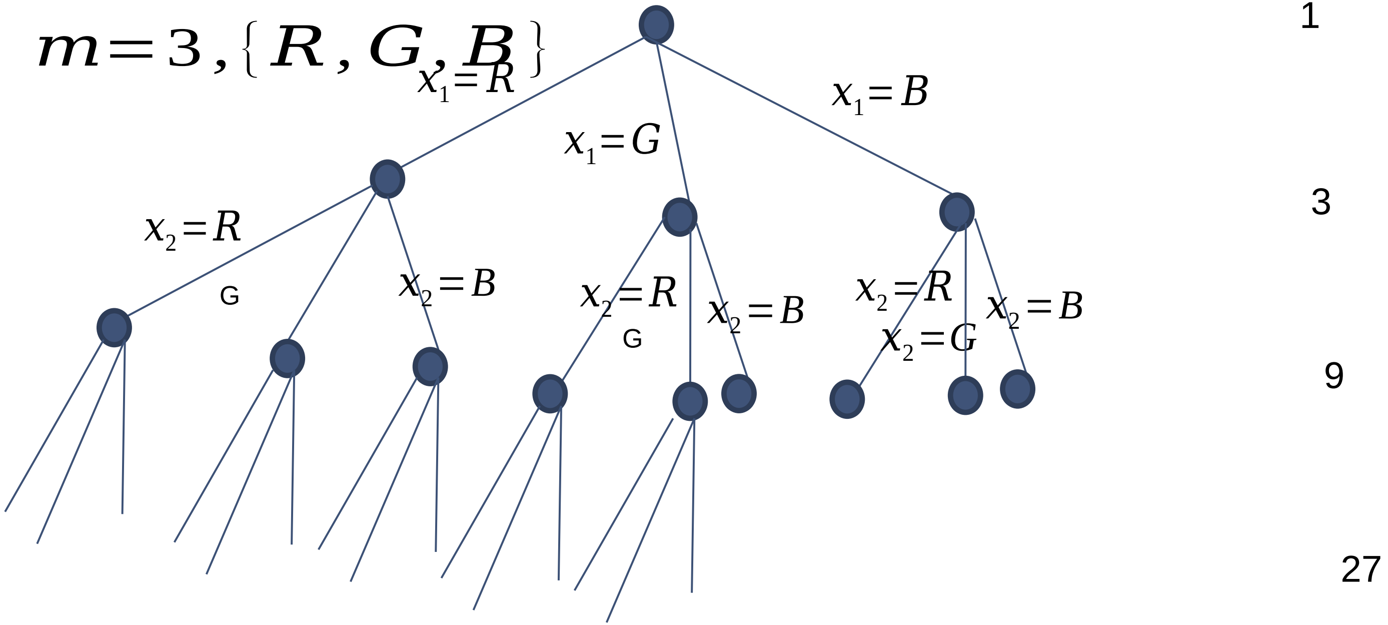
This problem asks the following question:

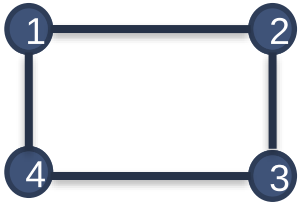
Given any map, can the regions be colored in such a way that no two adjacent regions have the same color yet only four colors are needed

We are interested in determining all the different ways in which a given graph can be colored using at most color

Example

$$m=3,\{R,G,B\}$$



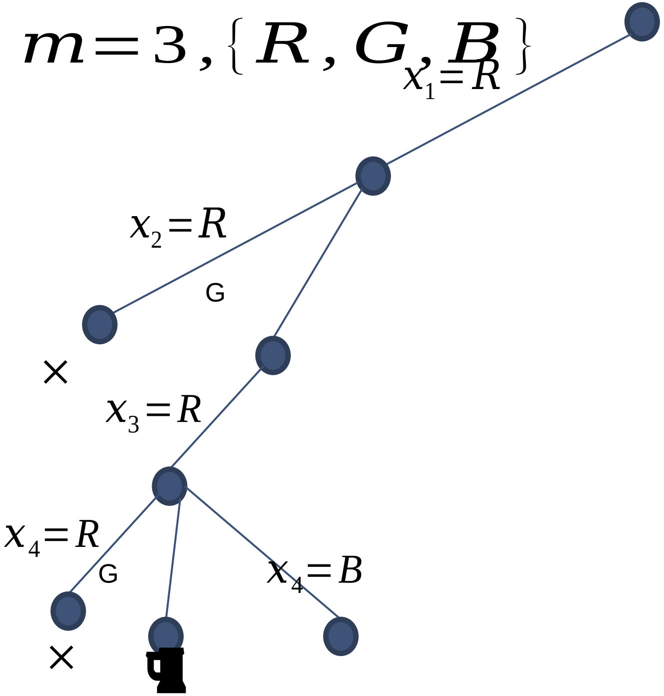


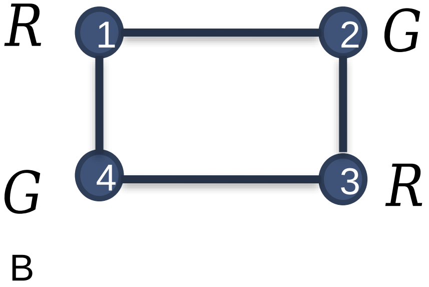
$$1+3+9+27\ldots.3^n=\frac{3^{n+1}-1}{3-1}=O(3^n)$$

In general for colors and nodes=

Example

$$m=3,\{R,G,B\}$$



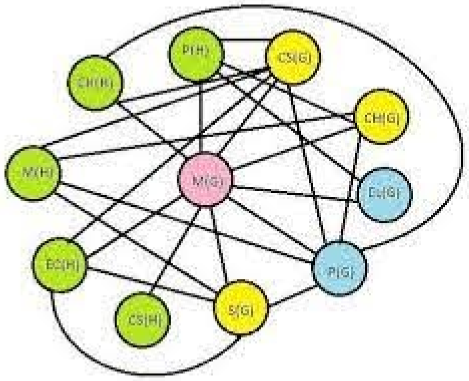


Application

As there are very busy cities in this world they may face a problem with train tracks. In some cities we have very few tracks and many trains in which their will be a moving trains one after the other from both sides. So , their shouldn’t be any colliding they must have some perception before itself. This may help in avoiding train trafficking and train accidents.

Solution

This problem can be solved by using graph coloring concept by coloring the vertices by using chromatic number concept to color vertices with minimum number of colours. Colors got by edges are the tracks for which we are allocating tracks for trains.



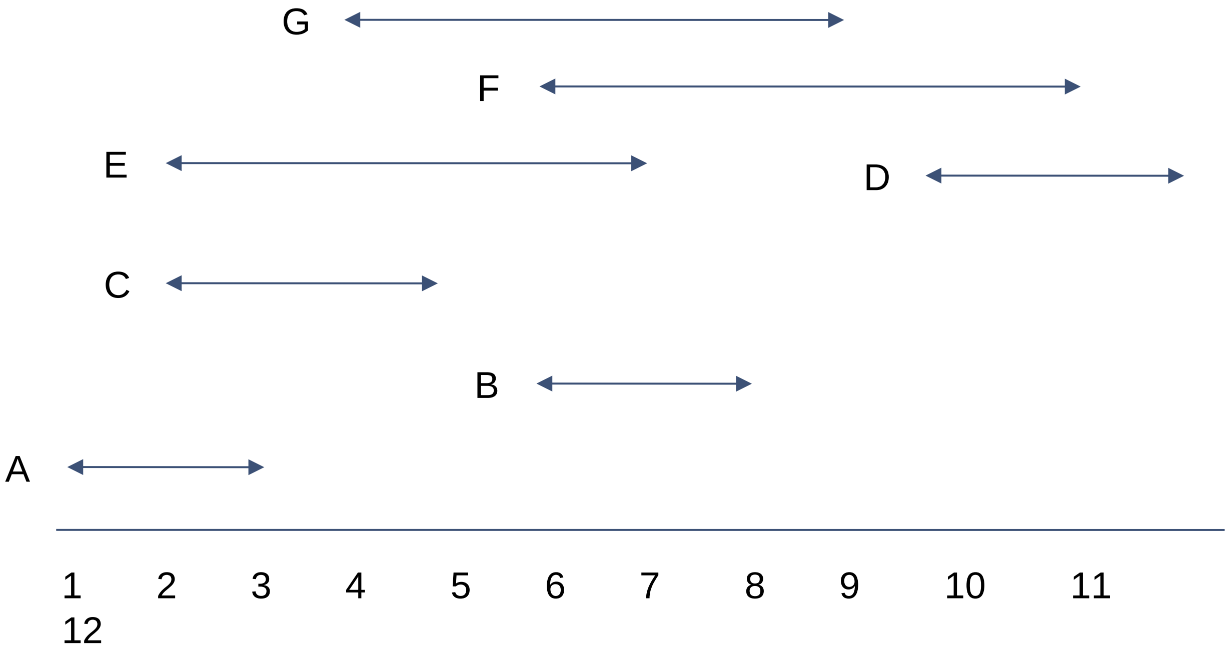
Example

- Let’s consider seven trains are to be scheduled to a station at their respective time. How we can group the train so that the trains can be scheduled at the station without waiting.

Train	Time duration
A	1-3
B	6-8
C	2-5
D	10-12
E	2-7
F	6-11
G	4-9

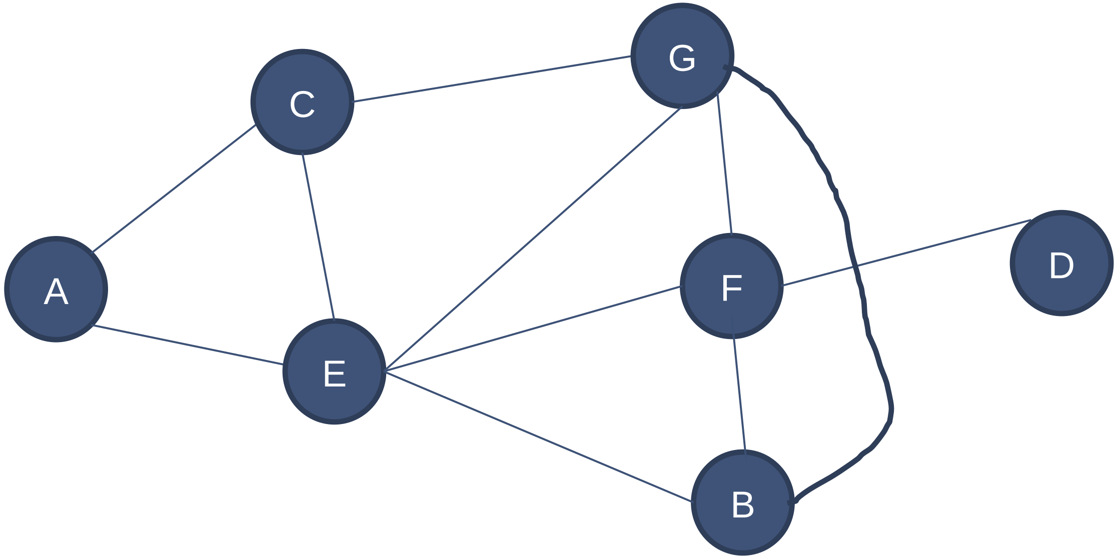
Solution

Arrange the schedule of trains in interval graph.

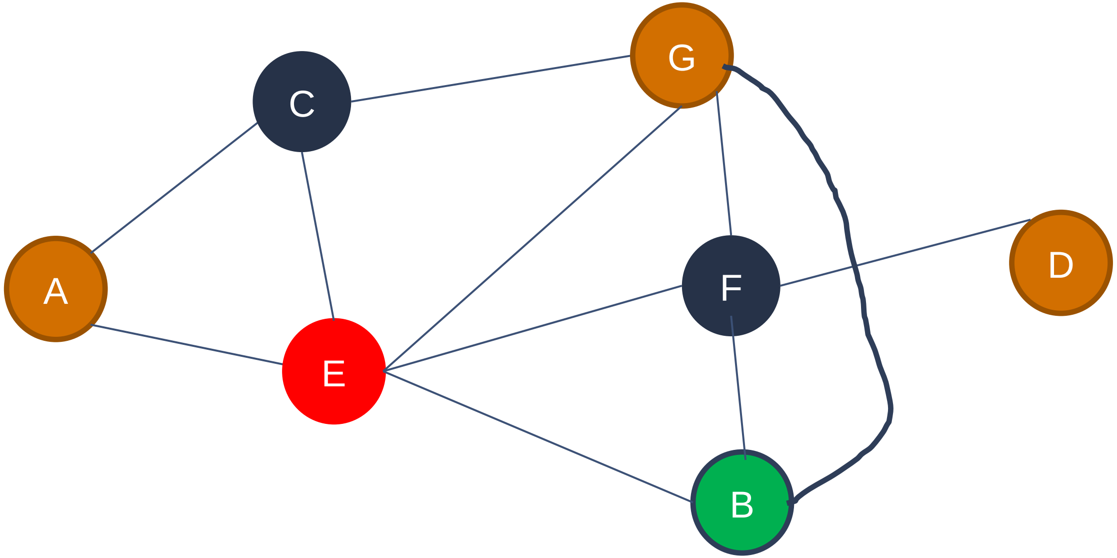


A	: 1-3
B	: 6-8
C	: 2-5
D	: 10-12
E	: 2-7
F	: 6-11
G	: 4-9

Solution

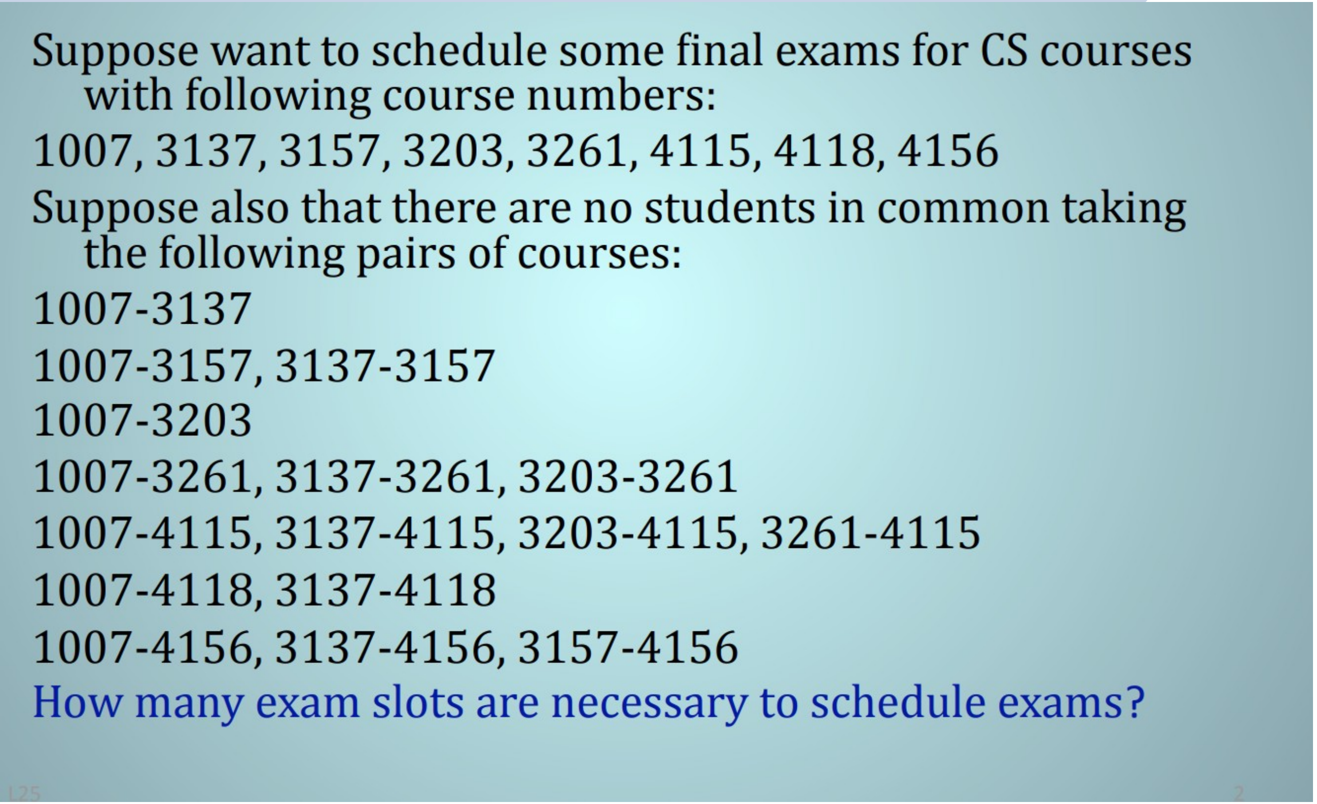


Solution



1	2	3	4
Purple	Yellow	Blue	Green
A,G,D	C,F	E	B

Assignment



Suppose want to schedule some final exams for CS courses with following course numbers:

1007, 3137, 3157, 3203, 3261, 4115, 4118, 4156

Suppose also that there are no students in common taking the following pairs of courses:

1007-3137

1007-3157, 3137-3157

1007-3203

1007-3261, 3137-3261, 3203-3261

1007-4115, 3137-4115, 3203-4115, 3261-4115

1007-4118, 3137-4118

1007-4156, 3137-4156, 3157-4156

How many exam slots are necessary to schedule exams?