Design and Analysis of Algorithm

Lecture-26:
Graph Algorithms

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1. Topological Sorting
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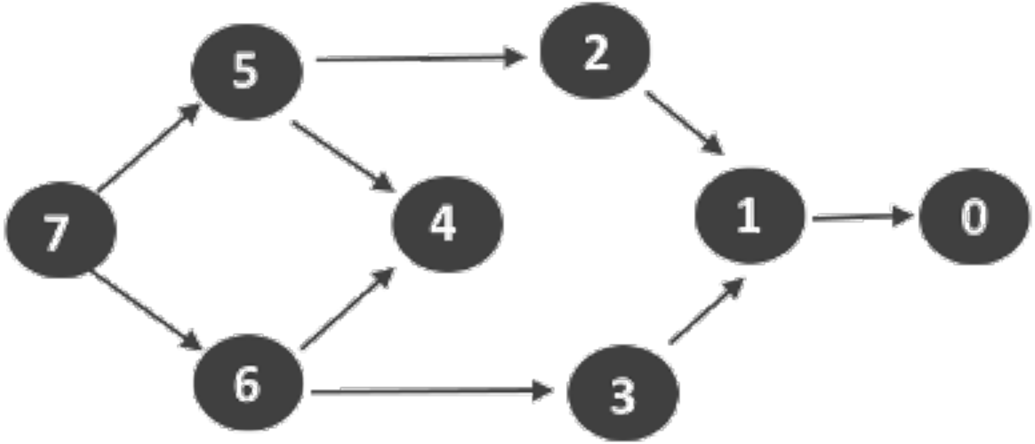
Introduction

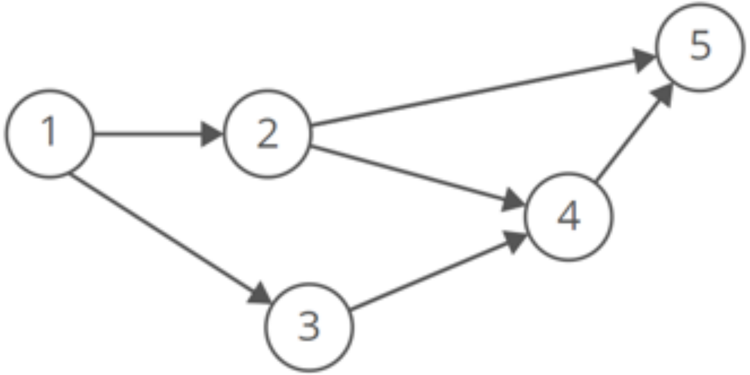
Topological sort algorithm takes a directed graph and returns an array of the nodes where each node appears *before* all the nodes it points to.

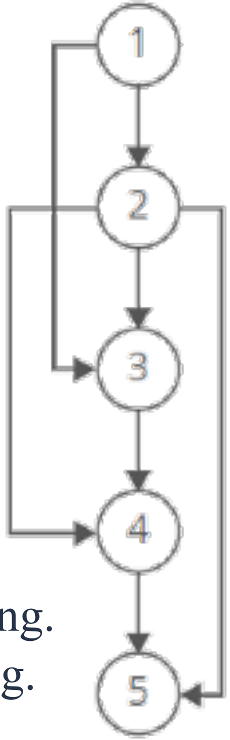
The ordering of the nodes in the array is called a *topological ordering*



Topological Sort : 7 6 5 4 3 2 1 0

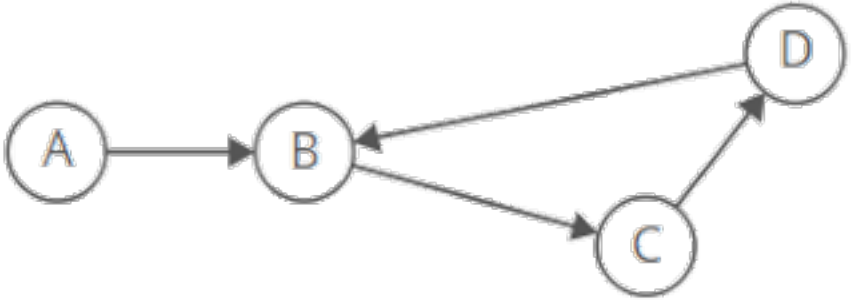
Example





Since node 1 points to nodes 2 and 3, node 1 appears before them in the ordering. And, since nodes 2 and 3 both point to node 4, they appear before it in the ordering.

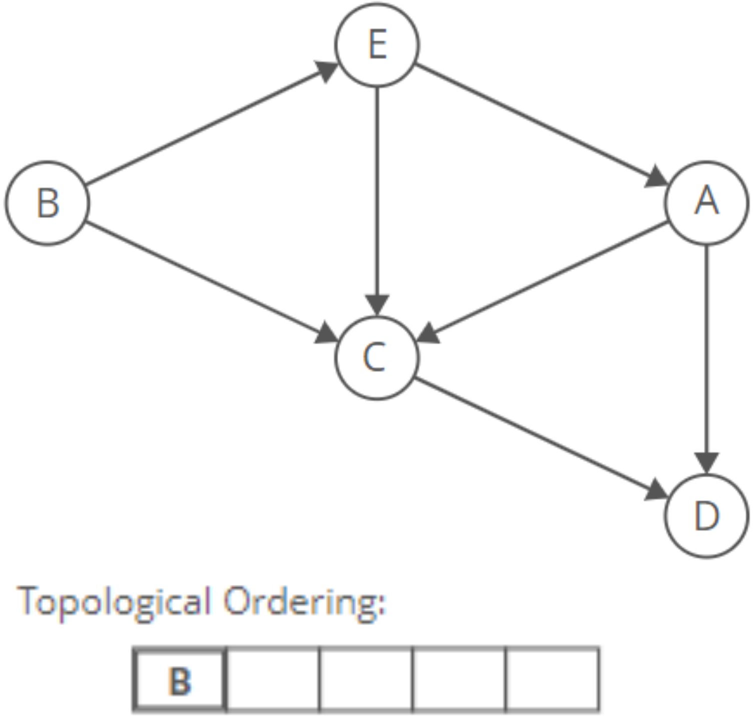
Cyclic Graphs



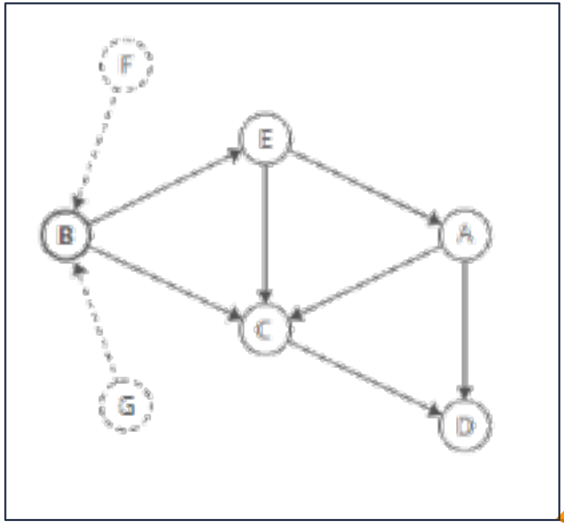
The cycle creates an impossible set of constraints—B has to be before and after D in the ordering.

As a rule, cyclic graphs don't have valid topological orderings.

Topological Sorting Algo

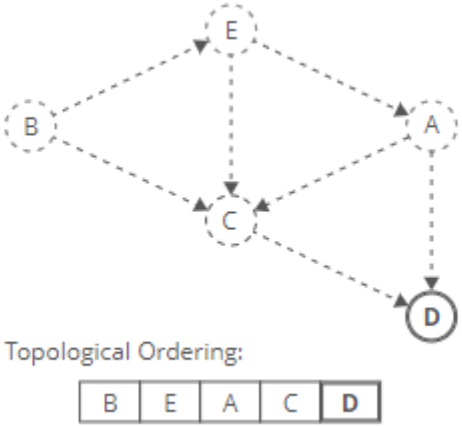


First node in the topological ordering is the node that doesn't have any incoming directed edges; it must have an indegree of zero.



Topological Sorting Algo

Once a node is added to the topological ordering, we can take the node, and its outgoing edges, out of the graph.



Topological Ordering:

B	E	A	C	**D**

1. Identify a node with no incoming edges.
2. Add that node to the ordering.
3. Remove it from the graph.
4. Repeat.

Questions

Q1. Topological sort of a Directed Acyclic graph is?

a) Always unique
b) Always Not unique
c) Sometimes unique and sometimes not unique
d) Always unique if graph has even number of vertices

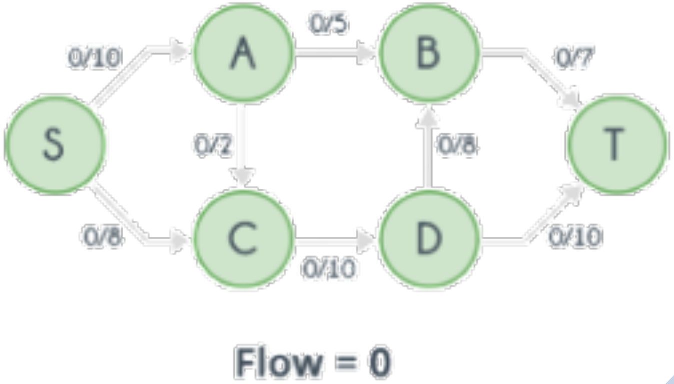
Q2. Topological sort can be implemented by?

a) Using Depth First Search
b) Using Breadth First Search
c) Using Depth and Breadth First Search
d) Using level ordered search

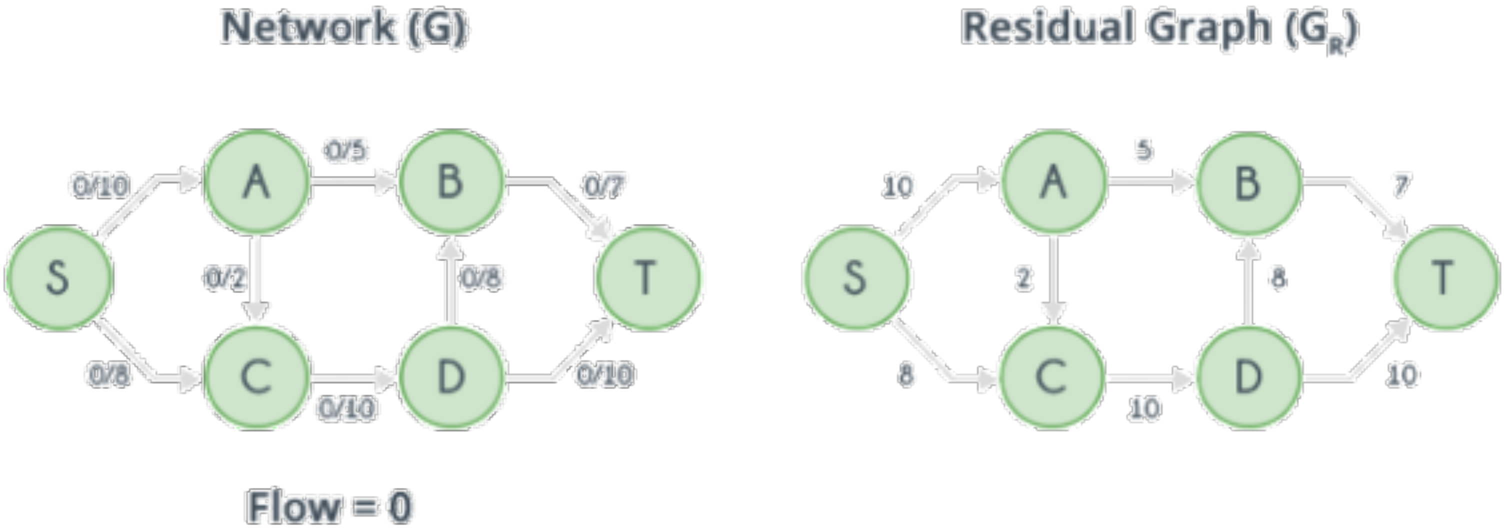
Ford-Fulkerson Algorithm

Ford-Fulkerson algorithm is a greedy approach for calculating the maximum possible flow in a network or a graph.

A term, flow network, is used to describe a network of vertices and edges with a source (S) and a sink (T). Each vertex, except S and T, can receive and send an equal amount of stuff through it. S can only send and T can only receive stuff.

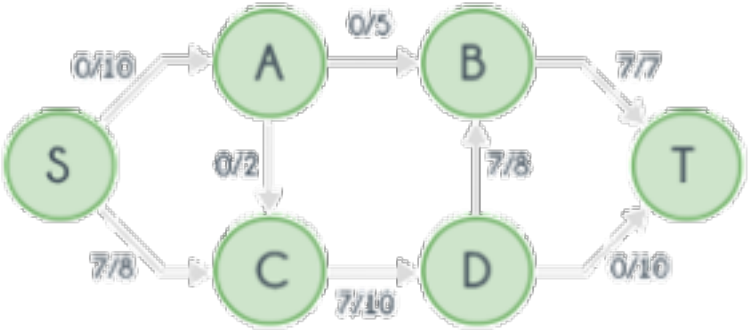


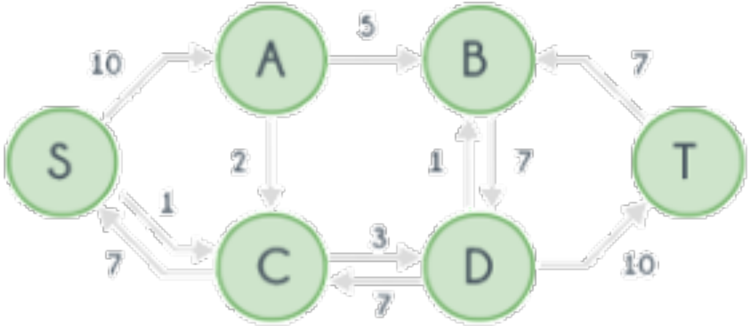
Ford-Fulkerson Algorithm



Ford-Fulkerson Algorithm

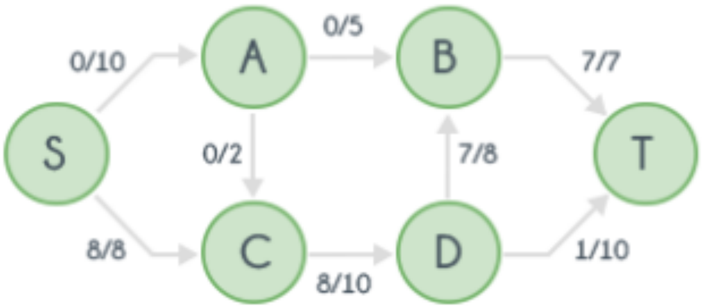
Path 1: S - C - D - B - T ⟶ Flow = Flow + 7

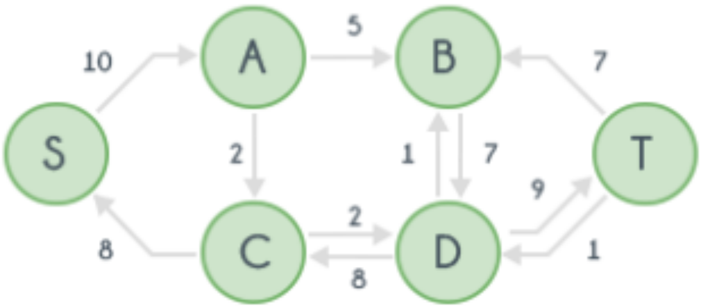




Ford-Fulkerson Algorithm

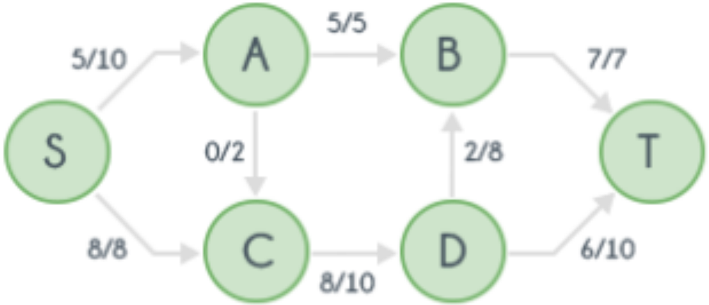
Path 2: S - C - D - T → Flow = Flow + 1

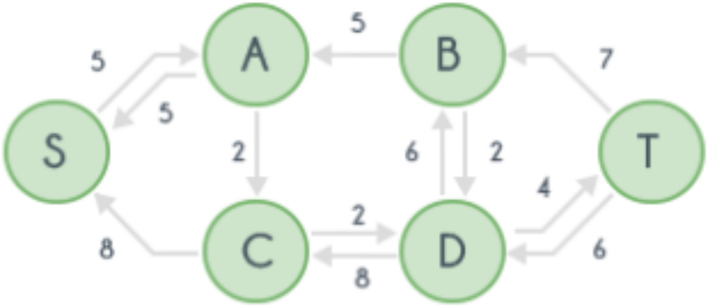




Ford-Fulkerson Algorithm

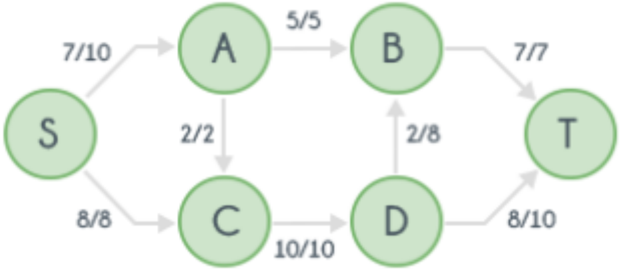
Path 3: S - A - B - T → Flow = Flow + 5

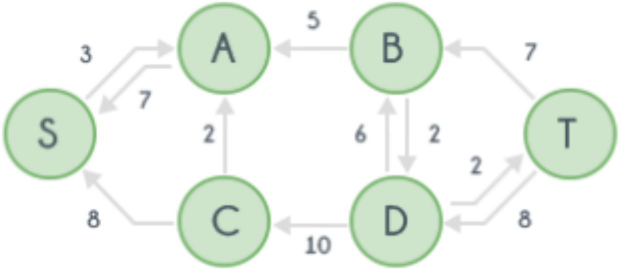




Ford-Fulkerson Algorithm

Path 4: S - A - C - D - T → Flow = Flow + 2





No More Paths Left

Max Flow = 15