Assignment 5

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Download all python codes from

https://github.com/Dhatri−nanda/AS5/blob/main/Assignment5/code.py

and latex-tikz codes from

https://github.com/Dhatri−nanda/AS5/blob/main/Assignment5/Assignment5.tex

1 Problem

Let $X_1, X_2, X_3, ...$ be a sequence of i.i.d random variables with mean 1. If N is a geometric random variable with the probability mass function $P(N = k) = \frac{1}{2^k}$; k = 1, 2, 3,... and it is independent of the X_i's, then $E\left(X_1 + X_2 + X_3 + ... + X_n\right)$ is equal to

2 Solution

The expectation operator,

$$E\left(X_1 + ... + X_n\right) = E\left(X_1\right) + ... + E\left(X_n\right) \quad (2.0.1)$$

We know that,

$$E\left(X\right) = \sum_{i=1}^{\infty} x_i \Pr\left(X = x_i\right) \quad (2.0.2)$$

$$= \sum_{i=1}^{\infty} k_i \Pr\left(X = k_i\right) \quad (2.0.3)$$

So now,

$$E\left(X_1\right) = k_1 \Pr\left(X = k_1\right) = 1\left(\frac{1}{2}\right) \quad (2.0.4)$$

Similarly,

$$E\left(X_2\right) = k_2 \Pr\left(X = k_2\right) = 2\left(\frac{1}{2}\right)^2 \quad (2.0.5)$$

and the pattern follows.
Let

$$E\left(X_1 + ... + X_n\right) = S \quad (2.0.6)$$

By substituting (2.0.4) and (2.0.5) in (2.0.1)

$$S = 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + ... \quad (2.0.7)$$

Dividing by 2 on both sides

$$\frac{S}{2} = 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{2}\right)^1 + 2\left(\frac{1}{2}\right)^2 + ... \quad (2.0.8)$$

Subtracting (2.0.8) from (2.0.7)

$$\frac{S}{2} = \frac{1}{2} + \frac{1}{2}^2 + \frac{1}{2}^3 + ... \quad (2.0.9)$$

$$= \frac{1/2}{1 - 1/2} \quad (2.0.10)$$

$$= 1 \quad (2.0.11)$$

Therefore, from (2.0.6)

$$E\left(X_1 + X_2 + ... + X_n\right) = 2 \quad (2.0.12)$$

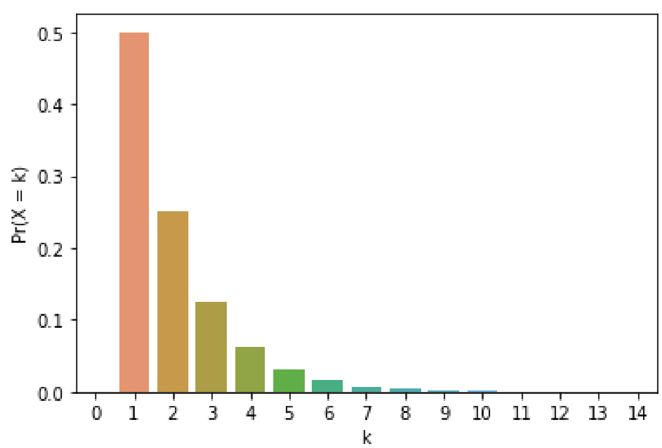

Fig. 0: PMF of X