GATE ASSIGNMENT 4

Dhatri Nanda - AI20BTECH11002

Download latex-tikz codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Gate_4/Gate_4.tex

QUESTION

The Fourier Transform of the signal $f(t) = e^{-3t^2}$ is of the following form, where A and B are constants

A) $Ae^{-B|f|}$

B) Ae^{-Bf^2}

C) $A + B|f|^2$

D) Ae^{-Bf}

SOLUTION

Given,

$$f(t) = e^{-at^2} \tag{0.0.1}$$

The fourier transform of a given signal f(t) is given by,

$$X(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \tag{0.0.2}$$

So let,

$$f(t) \overset{\mathcal{F}}{\rightleftharpoons} F(\omega) \tag{0.0.3}$$

The fourier transform of $\frac{df(t)}{dt}$ is given by,

$$= \int_{-\infty}^{\infty} \frac{df(t)}{dt}e^{-j\omega t}dt \tag{0.0.4}$$

$$= \left[e^{-j\omega t}f(t)\right]_{-\infty}^{\infty} + j\omega\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \tag{0.0.5}$$

$$= j\omega F(\omega) \tag{0.0.6}$$

Therefore,

$$\frac{df(t)}{dt} \overset{\mathcal{F}}{\rightleftharpoons} j\omega F(\omega) \tag{0.0.7}$$

From (0.0.7) and (0.0.1)

$$te^{-at^2} \overset{\mathcal{F}}{\rightleftharpoons} \frac{-j\omega}{2a}F(\omega) \tag{0.0.8}$$

Lemma 0.1. $te^{-at^2} \overset{\mathcal{F}}{\rightleftharpoons} j\frac{dF(\omega)}{d\omega}$

Proof. We prove it by finding the inverse fourier transform of $j\frac{dF(\omega)}{d\omega}$

The inverse fourier transform is given by

$$f^1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F^1(\omega)e^{-j\omega t}d\omega \tag{0.0.9}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} j\frac{dF(\omega)}{d\omega}e^{j\omega t}d\omega \tag{0.0.10}$$

$$= \frac{1}{2\pi}\left[je^{j\omega t}F(\omega)\right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} te^{j\omega t}F(\omega)d\omega \tag{0.0.11}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} tF(\omega)e^{j\omega t}d\omega \tag{0.0.12}$$

From (0.0.10),

$$f^1(t) = tf(t) \tag{0.0.13}$$

$$= te^{-at^2} \tag{0.0.14}$$

□

From (0.0.7) and Lemma 0.1

$$j\frac{dF(\omega)}{d\omega} = \frac{-j\omega}{2a}F(\omega) \tag{0.0.15}$$

$$\implies \frac{dF(\omega)}{F(\omega)} = \frac{-1}{2a}\omega d\omega \tag{0.0.16}$$

On integrating,

$$log(F(\omega)) = \frac{-1}{2a}.\frac{\omega^2}{2} = -\frac{\omega^2}{4a} \tag{0.0.17}$$

$$\implies F(\omega) = e^{-\frac{\omega^2}{4a}} \tag{0.0.18}$$

Putting $\omega = 2\pi f$ and $a = 3$

$$F(\omega) = e^{\frac{-\pi^2 f^2}{3}} \tag{0.0.19}$$

$\therefore$ correct option is B