Assignment 2

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Download all python codes from

https://github.com/Dhatri-nanda/EE3900/blob/main/Assignment_2/code.py

and latex-tikz codes from

https://github.com/Dhatri-nanda/EE3900/blob/main/Assignment_2/Assignment_2.tex

1 Problem

If A = $\begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find k so that $A^2 = kA - 2I$

2 Solution

The Cayley-Hamilton theorem states that any $N \times N$ matrix satisfies it's characteristic equation.
Finding the characteristic equation of the given matrix.
The characteristic equation of a matrix A is

$$|A - \lambda I| = 0 \tag{2.0.1}$$

$$\implies \left| \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0 \tag{2.0.2}$$

$$\implies \begin{vmatrix} 3-\lambda & -2-\lambda \\ 4 & -2-\lambda \end{vmatrix} = 0 \tag{2.0.3}$$

$$\implies (3-\lambda)(-2-\lambda) - 4(-2-\lambda) = 0 \tag{2.0.4}$$

$$\implies \lambda^2 - \lambda + 2 = 0 \tag{2.0.5}$$

$$\tag{2.0.6}$$

From (2.0.5) and (2.0.1)

$$A^2 - A + 2I = 0 \tag{2.0.7}$$

As we compare the given equation with the equation acquired on solving, we get

$$k = 1 \tag{2.0.8}$$