GATE ASSIGNMENT 4

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Download latex-tikz codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Gate_4/Gate_4.tex

QUESTION

The Fourier Transform of the signal $f(t) = e^{-3t^2}$ is of the following form, where A and B are constants

A) $Ae^{-B|f|}$

B) Ae^{-Bf^2}

C) $A + B|f|^2$

D) Ae^{-Bf}

SOLUTION

The fourier transform of a given signal f(t) is given by,

$$X(f) = \int_{-\infty}^{\infty} f(t)e^{-2\pi jft}dt \tag{0.0.1}$$

Lemma 0.1. *If a signal $f(t)$ has a fourier transform $F(f)$, then*

$$f'(t) \overset{\mathcal{F}}{\rightleftharpoons} 2\pi jfF(f) \tag{0.0.2}$$

Proof. The fourier transform of $\frac{df(t)}{dt}$ is given by,

$$= \int_{-\infty}^{\infty} \frac{df(t)}{dt}e^{-2\pi jft}dt \tag{0.0.3}$$

$$= \left[e^{-2\pi jft}f(t)\right]_{-\infty}^{\infty} + 2\pi jf\int_{-\infty}^{\infty} f(t)e^{-2\pi jft}dt \tag{0.0.4}$$

$$= 2\pi jfF(f) \tag{0.0.5}$$

□

Lemma 0.2. *If a signal f(t) has fourier transform F(f) then,*

$$tf(t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{j}{2\pi}F'(f) \tag{0.0.6}$$

Proof. We prove it by finding the inverse fourier transform of $\frac{j}{2\pi}\frac{dF(f)}{df}$

The inverse fourier transform is given by

$$f_1(t) = \int_{-\infty}^{\infty} F_1(f)e^{j2\pi ft}df \tag{0.0.7}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} j\frac{dF(f)}{df}e^{j2\pi ft}df \tag{0.0.8}$$

$$= \frac{1}{2\pi}\left[je^{j2\pi ft}F(f)\right]_{-\infty}^{\infty} + \frac{1}{2\pi}\int_{-\infty}^{\infty} te^{j2\pi ft}F(f)df \tag{0.0.9}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} tF(f)e^{j2\pi ft}df \tag{0.0.10}$$

From (0.0.8),

$$f_1(t) = tf(t) \tag{0.0.11}$$

□

If we consider the signal

$$f(t) = e^{-\pi t^2} \tag{0.0.12}$$

On differentiating,

$$f'(t) = e^{-\pi t^2} \times (-2\pi t) \tag{0.0.13}$$

$$\implies f'(t) = f(t) \times (-2\pi t) \tag{0.0.14}$$

By applying fourier transformations using lemmas 0.1 and 0.2

$$2\pi jfF(f) = \frac{j}{2\pi}F'(f) \times (-2\pi) \tag{0.0.15}$$

$$\implies \frac{F'(f)}{F(f)} = -2\pi f \tag{0.0.16}$$

On integrating,

$$log(F(f)) = \frac{-2\pi f^2}{2} \tag{0.0.17}$$

So, the fourier transform of the signal $f(t) = e^{-\pi t^2}$is

$$F(f) = e^{-\pi f^2} \tag{0.0.18}$$

We know that, if

$$f(t) \stackrel{\mathcal{F}}{\rightleftharpoons} F(f) \tag{0.0.19}$$

$$\implies f(at) \stackrel{\mathcal{F}}{\rightleftharpoons} \frac{1}{a}F(f/a) \tag{0.0.20}$$

Therefore for the signal $f(t) = e^{-3t^2}$, the fourier transform is

$$F(f) = \frac{\sqrt{\pi}}{\sqrt{3}}e^{-\pi^2 f^2/3} \tag{0.0.21}$$

It is of the form

$$Ae^{-Bf^2} \tag{0.0.22}$$

So, correct option is B