Assignment 5

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AI20BTECH11002

Download all python codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Quiz_2/code.py

and latex-tikz codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Quiz_2/Quiz_2.tex

1 3.3(A)

Determine the z-transform of the following sequence. Include with your answer the region of convergence in the z-plane and a sketch of the pole-zero plot. Express all sums in closed form; α can be complex.

$$x_a[n] = \alpha^{|n|}, 0 < |\alpha| < 1 \tag{1.0.1}$$

2 Solution

The z- transform for any sequence x[n] is defined as

$$X(z) = \sum_{n=-\infty}^{n=\infty} x[n]z^{-n} \tag{2.0.1}$$

Now, z- transform of the sequence $x_a[n] = \alpha^{|n|}$ is

$$X_a(z) = \sum_{n=-\infty}^{-1} \alpha^{-n}z^{-n} + \sum_{n=0}^{\infty} \alpha^{n}z^{-n} \tag{2.0.2}$$

$$= \sum_{n=1}^{\infty} \alpha^{n}z^{n} + \sum_{n=0}^{\infty} \alpha^{n}z^{-n} \tag{2.0.3}$$

$$= \frac{\alpha z}{1-\alpha z} + \frac{1}{1-\alpha z^{-1}} \tag{2.0.4}$$

$$= \frac{z(1-\alpha^2)}{(1-\alpha z)(z-\alpha)} \tag{2.0.5}$$

where,
ROC in the z-plane is $|\alpha| < |z| < \frac{1}{|\alpha|}$

Here, the poles are

$$z = \alpha \tag{2.0.6}$$

$$z = \frac{1}{\alpha} \tag{2.0.7}$$

and the zero is

$$z = 0 \tag{2.0.8}$$

Now, considering $\alpha = 0.6$,

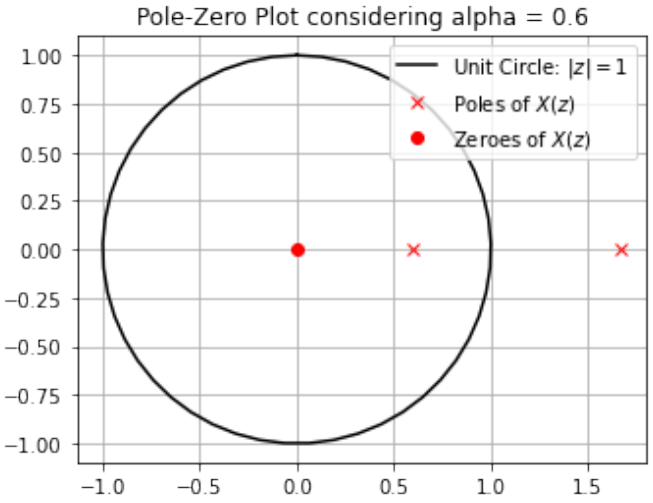

Fig. 0: plot