Assignment 1

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Download all python codes from

https://github.com/Dhatri-nanda/EE3900/blob/main/Assignment_1/code.py

and latex-tikz codes from

https://github.com/Dhatri-nanda/EE3900/blob/main/Assignment_1/Assignment_1.tex

1 Problem

Prove that the points $\begin{pmatrix}21\\-2\end{pmatrix}$, $\begin{pmatrix}15\\10\end{pmatrix}$, $\begin{pmatrix}-5\\0\end{pmatrix}$, $\begin{pmatrix}1\\-12\end{pmatrix}$ are the vertices of a rectangle, and find the coordinates of its centre.

2 Solution

Let's name the points as A,B,C and D respectively i.e.,

$$A = \begin{pmatrix}21\\-2\end{pmatrix} B = \begin{pmatrix}15\\10\end{pmatrix} C = \begin{pmatrix}-5\\0\end{pmatrix} D = \begin{pmatrix}1\\-12\end{pmatrix} \tag{2.0.1}$$

Let us find the length of the sides

$$AB^2 = (\mathbf{A}-\mathbf{B})^\top(\mathbf{A}-\mathbf{B}) \tag{2.0.2}$$
$$= \begin{pmatrix}6 & -12\end{pmatrix}\begin{pmatrix}6\\-12\end{pmatrix} \tag{2.0.3}$$
$$= (6)^2 + (12)^2 \tag{2.0.4}$$
$$= 36 + 144 \tag{2.0.5}$$
$$= 180 \tag{2.0.6}$$

$$BC^2 = (\mathbf{B}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C}) \tag{2.0.7}$$
$$= \begin{pmatrix}20 & 10\end{pmatrix}\begin{pmatrix}20\\10\end{pmatrix} \tag{2.0.8}$$
$$= (20)^2 + (10)^2 \tag{2.0.9}$$
$$= 400 + 100 \tag{2.0.10}$$
$$= 500 \tag{2.0.11}$$

$$CD^2 = (\mathbf{C}-\mathbf{D})^\top(\mathbf{C}-\mathbf{D}) \tag{2.0.12}$$
$$= \begin{pmatrix}-6 & 12\end{pmatrix}\begin{pmatrix}-6\\12\end{pmatrix} \tag{2.0.13}$$
$$= (-6)^2 + (12)^2 \tag{2.0.14}$$
$$= 36 + 144 \tag{2.0.15}$$
$$= 180 \tag{2.0.16}$$

$$DA^2 = (\mathbf{D}-\mathbf{A})^\top(\mathbf{D}-\mathbf{A}) \tag{2.0.17}$$
$$= \begin{pmatrix}-20 & -10\end{pmatrix}\begin{pmatrix}-20\\-10\end{pmatrix} \tag{2.0.18}$$
$$= (-20)^2 + (10)^2 \tag{2.0.19}$$
$$= 400 + 100 \tag{2.0.20}$$
$$= 500 \tag{2.0.21}$$

Here, opposite sides of the quadrilateral are equal Now,

$$\angle B = (\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix}-6 & 12\end{pmatrix}\begin{pmatrix}-20\\-10\end{pmatrix} = 0 \tag{2.0.22}$$

$$\angle C = (\mathbf{C}-\mathbf{B})^\top(\mathbf{D}-\mathbf{C}) = \begin{pmatrix}-20 & -10\end{pmatrix}\begin{pmatrix}6\\-12\end{pmatrix} = 0 \tag{2.0.23}$$

The two adjacent angles are right angles. And by congruency, remaining angles are also 90 degrees.
A quadrilateral with opposite sides equal and all the angles as right angles is a rectangle.
Therefore, the given points(ABCD) are vertices of a rectangle.
Center of rectangle is the midpoint of any diagonal

$$O = \frac{A+C}{2} \tag{2.0.24}$$
$$= \begin{pmatrix}8\\-1\end{pmatrix} \tag{2.0.25}$$

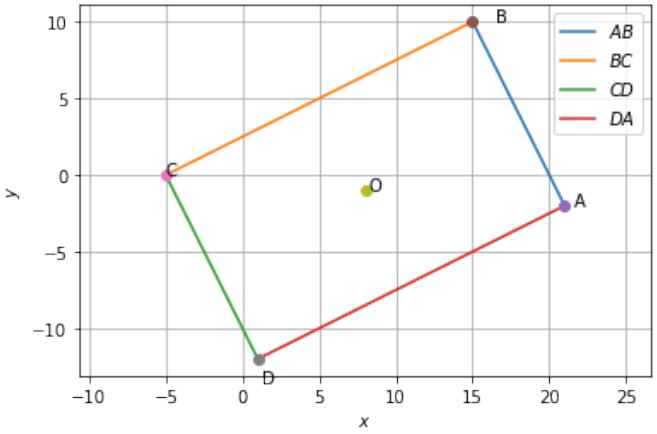

Fig. 0: plot