Assignment 1

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1 PROBLEM

Prove that the points $\begin{pmatrix} 21 \\ -2 \end{pmatrix}, \begin{pmatrix} 15 \\ 10 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -12 \end{pmatrix}$ are the vertices of a rectangle, and find the coordinates of its centre.

2 SOLUTION

Let's name the points as A,B,C and D respectively i.e.,

$$A = \begin{pmatrix} 21 \\ -2 \end{pmatrix} B = \begin{pmatrix} 15 \\ 10 \end{pmatrix} C = \begin{pmatrix} -5 \\ 0 \end{pmatrix} D = \begin{pmatrix} 1 \\ -12 \end{pmatrix} \quad (2.0.1)$$

A quadrilateral having equal opposite sides and four right angles is called a rectangle.
First, let us find the length of the sides

$$AB^2 = (\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B}) \quad (2.0.2)$$

$$= \begin{pmatrix} 6 & -12 \end{pmatrix} \begin{pmatrix} 6 \\ -12 \end{pmatrix} \quad (2.0.3)$$

$$= (6)^2 + (12)^2 \quad (2.0.4)$$

$$= 36 + 144 \quad (2.0.5)$$

$$= 180 \quad (2.0.6)$$

$$BC^2 = (\mathbf{B} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) \quad (2.0.7)$$

$$= \begin{pmatrix} 20 & 10 \end{pmatrix} \begin{pmatrix} 20 \\ 10 \end{pmatrix} \quad (2.0.8)$$

$$= (20)^2 + (10)^2 \quad (2.0.9)$$

$$= 400 + 100 \quad (2.0.10)$$

$$= 500 \quad (2.0.11)$$

$$CD^2 = (\mathbf{C} - \mathbf{D})^\top (\mathbf{C} - \mathbf{D}) \quad (2.0.12)$$

$$= \begin{pmatrix} -6 & 12 \end{pmatrix} \begin{pmatrix} -6 \\ 12 \end{pmatrix} \quad (2.0.13)$$

$$= (-6)^2 + (12)^2 \quad (2.0.14)$$

$$= 36 + 144 \quad (2.0.15)$$

$$= 180 \quad (2.0.16)$$

$$DA^2 = (\mathbf{D} - \mathbf{A})^\top (\mathbf{D} - \mathbf{A}) \quad (2.0.17)$$

$$= \begin{pmatrix} -20 & -10 \end{pmatrix} \begin{pmatrix} -20 \\ -10 \end{pmatrix} \quad (2.0.18)$$

$$= (-20)^2 + (10)^2 \quad (2.0.19)$$

$$= 400 + 100 \quad (2.0.20)$$

$$= 500 \quad (2.0.21)$$

Here, opposite sides of the quadrilateral are equal Now,

$$\angle B = (\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} -6 & 12 \end{pmatrix} \begin{pmatrix} -20 \\ -10 \end{pmatrix} = 0 \quad (2.0.22)$$

$$\angle C = (\mathbf{C} - \mathbf{B})^\top (\mathbf{D} - \mathbf{C}) = \begin{pmatrix} -20 & -10 \end{pmatrix} \begin{pmatrix} 6 \\ -12 \end{pmatrix} = 0 \quad (2.0.23)$$

The two adjacent angles are right angles. And by congruency, remaining angles are also 90 degrees. As it satisfies the properties of a ractangle, the given points(ABCD) are vertices of a rectangle.
Center of rectangle is the midpoint of any diagonal

$$O = \frac{A + C}{2} \quad (2.0.24)$$
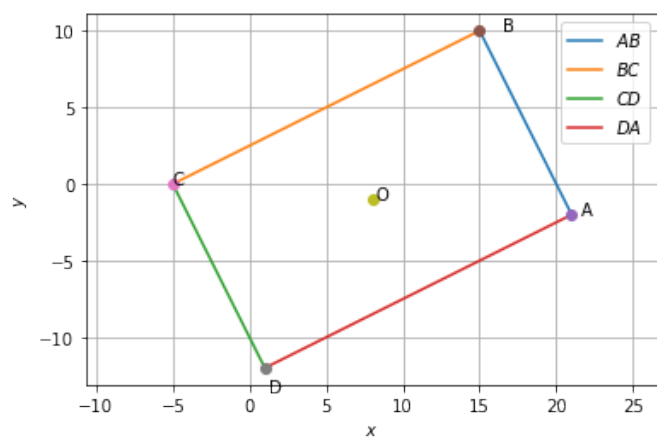
$$= \begin{pmatrix} 8 \\ -1 \end{pmatrix} \quad (2.0.25)$$

Fig. 0: plot