GATE ASSIGNMENT 2

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QUESTION

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Consider the Z-transform $X(z) = 5z^2 + 4z^{-1} + 3$,

 $0 < |z| < \infty$. The inverse Z-transform x[n] is

A)
$$5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$$

B)
$$5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$$

C)
$$5u[n+2] + 3u[n] + 4u[n-1]$$

D)
$$5u[n-2] + 3u[n] + 4u[n+1]$$

SOLUTION

Given, Z-transform

$$X(z) = 5z^2 + 4z^{-1} + 3 (0.0.1)$$

 $ROC = 0 < |z| < \infty$

If X(z) is the Z-transform of x(n),

$$X(z) = \sum_{n = -\infty}^{\infty} x(n)z^{-n}$$
 (0.0.2)

And if δ is the impulse response, then

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$
 (0.0.3)

Therefore, from (0.0.2) and (0.0.3)

$$\delta(n+a) \stackrel{z}{\longleftrightarrow} z^a \tag{0.0.4}$$

The inverse transform

$$x(n) = Z^{-1}[X(z)] (0.0.5)$$

$$= 5\delta[n+2] + 3\delta[n] + 4\delta[n-1] \qquad (0.0.6)$$

Option (A) is correct.