GATE ASSIGNMENT 2

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Download latex-tikz codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Gate_2/Gate_2.tex

QUESTION

GATE 2010 Q.14

Consider the Z-transform $X(z) = 5z^2 + 4z^{-1} + 3$, $0 < |z| < \infty$. The inverse Z-transform x[n] is

A) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$
B) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$
C) $5u[n+2] + 3u[n] + 4u[n-1]$
D) $5u[n-2] + 3u[n] + 4u[n+1]$

SOLUTION

Given, Z-transform

$$X(z) = 5z^2 + 4z^{-1} + 3 \tag{0.0.1}$$

ROC $= 0 < |z| < \infty$

The inverse Z-transform of X(z) is defined as

$$x[n] = \frac{1}{2\pi j}\oint_c X(z)z^{n-1}dz \tag{0.0.2}$$

where c is a counter clockwise contour in the ROC of X(z) encircling the origin.

$\implies$ $x[n] = \sum$[residues of $X(z)z^{n-1}$ at the poles inside c]

The residue at $z = d_0$ is defined as

$$\frac{1}{(s-1)!}\left[\frac{d^{s-1}\psi(z)}{dz^{s-1}}\right]_{z=d_0} \tag{0.0.3}$$

where

$$X(z)Z^{n-1} = \frac{\psi(z)}{(z-d_0)^s} \tag{0.0.4}$$

Now,

$$X(z)Z^{n-1} = \frac{5z^{n+2} + 3z^n + 4z^{n-1}}{z} \tag{0.0.5}$$

$$\implies \psi(z) = 5z^{n+2} + 3z^n + 4z^{n-1}, \tag{0.0.6}$$

$$d_0 = 0, s = 1 \tag{0.0.7}$$

The residue at z=0,

$$x[n] = \frac{1}{0!}\left[5z^{n+2} + 3z^n + 4z^{n-1}\right]_{z=0} \tag{0.0.8}$$

$$n = -2 \implies x[-2] = 5 \tag{0.0.9}$$

$$n = 0 \implies x[0] = 3 \tag{0.0.10}$$

$$n = 1 \implies x[1] = 4 \tag{0.0.11}$$

And, $x[n]$ is 0 for all remaining n

We know that x[n] and the impulse sequence $\delta[n]$ are related by,

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{0.0.12}$$

Therefore,

$$x[n] = 5\delta[n+2] + 3\delta[n] + 4\delta[n-1] \tag{0.0.13}$$

$\therefore$ Option (A) is correct