# GATE 2 Presentation

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# Question

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Consider the Z-transform $X(z) = 5z^2 + 4z^{-1} + 3$, $0 < |z| < \infty$. The inverse Z-transform x[n] is

- $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$
- $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$
- $5u[n+2] + 3u[n] + 4u[n-1]$
- $5u[n-2] + 3u[n] + 4u[n+1]$

## Theorem

The inverse Z-transform of X(z) is defined as

$$x[n] = \frac{1}{2\pi j} \oint_c X(z) z^{n-1} dz \tag{1}$$

where c is a counter clockwise contour in the ROC of X(z) encircling the origin.

$\implies x[n] = \sum$[residues of $X(z)z^{n-1}$ at the poles inside c]

The residue at $z = d_0$ is defined as

$$\frac{1}{(s-1)!} \left[ \frac{d^{s-1}\psi(z)}{dz^{s-1}} \right]_{z=d_0} \tag{2}$$

where

$$X(z)Z^{n-1} = \frac{\psi(z)}{(z-d_0)^s} \tag{3}$$

# Solution

Given, Z-transform

$$X(z) = 5z^2 + 4z^{-1} + 3 \tag{4}$$

ROC $= 0 < |z| < \infty$
Now,

$$X(z)z^{n-1} = \frac{5z^{n+2} + 3z^n + 4z^{n-1}}{z} \tag{5}$$

$$\implies \psi(z) = 5z^{n+2} + 3z^n + 4z^{n-1}, \tag{6}$$

$$d_0 = 0, s = 1 \tag{7}$$

From (1)

$$x[n] = \frac{1}{0!}\left[5z^{n+2} + 3z^{n} + 4z^{n-1}\right]_{z=0} \tag{8}$$

$$n = -2 \implies x[-2] = 5 \tag{9}$$

$$n = 0 \implies x[0] = 3 \tag{10}$$

$$n = 1 \implies x[1] = 4 \tag{11}$$

Therefore,

$$x[n] = \begin{cases} 5, & n = -2 \\ 3, & n = 0 \\ 4, & n = 1 \\ 0, & \text{Otherwise} \end{cases} \tag{12}$$

We know that x[n] and the impulse sequence $\delta[n]$ are related by,

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{13}$$

Therefore,

$$x[n] = 5\delta[n+2] + 3\delta[n] + 4\delta[n-1] \tag{14}$$