

Assignment 5

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Download all latex-tikz codes from

https://github.com/Dhatri-nanda/EE3900/blob/main/Quiz_1/Quiz_1.tex

1 2.19(A,B,C)

For each of the following impulse response of LTI systems, indicate whether or not the system is stable

$$h[n] = 4^n u[n] \quad (1.0.1)$$

$$h[n] = u[n] - u[n - 10] \quad (1.0.2)$$

$$h[n] = 3^n u[-n - 1] \quad (1.0.3)$$

2 SOLUTION

An impulse response $h[n]$ is stable if it is absolutely integrable i.e.,

$$\int_{-\infty}^{\infty} h[n] dn < \infty \quad (2.0.1)$$

$u(n)$ is the unit step function,

$$u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases} \quad (2.0.2)$$

$$1) h[n] = 4^n u[n]$$

$$\Rightarrow \int_{-\infty}^{\infty} 4^n u[n] dn \quad (2.0.3)$$

$$= \int_0^{\infty} 4^n dn \quad (2.0.4)$$

$$= \frac{1}{\log 4} [4^n]_0^{\infty} \quad (2.0.5)$$

$$= \infty \quad (2.0.6)$$

Therefore, it is not a stable system.

$$2) h[n] = u[n] - u[n - 10]$$

$$\Rightarrow \int_{-\infty}^{\infty} u[n] - u[n - 10] \quad (2.0.7)$$

$$= \int_{-\infty}^0 0 dn + \int_0^{10} 1 dn + \int_{10}^{\infty} 0 dn \quad (2.0.8)$$

$$= [n]_0^{10} \quad (2.0.9)$$

$$= 10 < \infty \quad (2.0.10)$$

Therefore, it is a stable system.

$$3) h[n] = 3^n u[-n - 1]$$

$$\Rightarrow \int_{-\infty}^{\infty} 3^n u[-n - 1] \quad (2.0.11)$$

$$= \int_{-\infty}^{-1} 3^n dn + \int_{-1}^{\infty} 0 dn \quad (2.0.12)$$

$$= \left[\frac{3^n}{\log 3} \right]_{-\infty}^{-1} \quad (2.0.13)$$

$$= \frac{1}{\log 3} \left[\frac{1}{3} - 0 \right] \quad (2.0.14)$$

$$= \frac{1}{3 \log 3} < \infty \quad (2.0.15)$$

Therefore, it is a stable system.