ASSIGNMENT 3

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AI20BTECH11002

Download all python codes from

```
https://github.com/Dhatri−nanda/EE3900/blob/main
    /Assignment_3/code.py
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and latex-tikz codes from

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https://github.com/Dhatri−nanda/EE3900/blob/main
    /Assignment_3/Assignment_3.tex
```

1 Construction 2.9

Can you construct the quadrilateral PLAN if $PL = 6, LA = 9.5, \angle P = 75, \angle L = 150$ and $\angle A = 140$

2 Solution

Lemma 2.1. *Given*

$$PL = 6 \tag{2.0.1}$$
$$LA = 4.5 \tag{2.0.2}$$
$$\angle P = 75^\circ \tag{2.0.3}$$
$$\angle L = 150^\circ \tag{2.0.4}$$
$$\angle A = 140^\circ \tag{2.0.5}$$

Let

$$\angle P = \theta \tag{2.0.6}$$
$$\angle L = \alpha \tag{2.0.7}$$
$$\angle A = \delta \tag{2.0.8}$$
$$||\mathbf{L} - \mathbf{P}|| = a, \tag{2.0.9}$$
$$||\mathbf{A} - \mathbf{L}|| = b, \tag{2.0.10}$$
$$||\mathbf{A} - \mathbf{N}|| = c \tag{2.0.11}$$
$$||\mathbf{P} - \mathbf{N}|| = d \tag{2.0.12}$$
$$||\mathbf{P} - \mathbf{A}|| = e \tag{2.0.13}$$
$$\theta = \theta_1 + \theta_2 \tag{2.0.14}$$
$$\delta_1 = \angle NAP \tag{2.0.15}$$
$$\delta_2 = \angle LAP \tag{2.0.16}$$
$$\gamma = \angle N \tag{2.0.17}$$

If three angles and two sides of a quadrilateral are known, then the coordinates of the vertices can be expressed as

$$\mathbf{A} = \mathbf{L} + b \times \begin{pmatrix} \cos(180^\circ - \alpha) \\ \sin(180^\circ - \alpha) \end{pmatrix} \tag{2.0.18}$$
$$\mathbf{N} = d \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.19}$$

Where

$$d = e \times \left(\frac{\sin\left(\delta - \sin^{-1}\left[\sin\alpha \times \left(\frac{b}{e}\right)\right]\right)}{\sin(360^\circ - (\alpha + \theta + \delta))} \right) \tag{2.0.20}$$
$$e = \sqrt{a^2 + b^2 - 2 \times a \times b \cos\alpha} \tag{2.0.21}$$

Proof. Using angle sum rule of quadrilaterals

$$\gamma = 360^\circ - (\alpha + \theta + \delta) \tag{2.0.22}$$

Now, using cosine formula in $\triangle PLA$ we can find e:

$$e^2 = a^2 + b^2 - 2 \times a \times b \cos\alpha \tag{2.0.23}$$

Using sine rule,

$$\frac{\sin\alpha}{e} = \frac{\sin\delta_2}{b} \tag{2.0.24}$$
$$\delta_2 = \sin^{-1}\left[\sin\alpha \times \left(\frac{b}{e}\right)\right] \tag{2.0.25}$$
$$\tag{2.0.26}$$

Now in $\triangle MER$,

$$\delta_1 = \delta - \delta_2 \tag{2.0.27}$$

Using sine law of triangle,

$$\frac{\sin\gamma}{e} = \frac{\sin\delta_1}{d} \tag{2.0.28}$$
$$\implies d = e \times \left(\frac{\sin\delta_1}{\sin\gamma}\right) \tag{2.0.29}$$

From the above equations, we get

$$d = e \times \left(\frac{\sin\left(\delta - \sin^{-1}\left[\sin\alpha \times \left(\frac{b}{e}\right)\right]\right)}{\sin(360^\circ - (\alpha + \theta + \delta))} \right) \tag{2.0.30}$$

where

$$e = \sqrt{a^2 + b^2 - 2 \times a \times b \cos \alpha} \tag{2.0.31}$$

$\square$

Calculating e

$$e = \sqrt{6^2 + 9.5^2 - 2 \times 6.5 \times 9 \times \cos 150} \tag{2.0.32}$$

$$\approx 15 \tag{2.0.33}$$

Calculating d

$$d = 15 \times \left(\frac{\sin\left(140 - \sin^{-1}\left[\sin 150 \times \left(\frac{9.5}{15}\right)\right]\right)}{\sin\left(360^\circ - (75 + 150 + 140)\right)} \right) \tag{2.0.34}$$

Here, the denominator is $\sin(-5)$, which is a negative value

As d is a side of the quadrilateral, it cannot be negative, so we cannot construct the quadrilateral PLAN.