GATE ASSIGNMENT 4

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Download latex-tikz codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Gate_4/Gate_4.tex

QUESTION

The Fourier Transform of the signal $f(t) = e^{-3t^2}$ is of the following form, where A and B are constants

A) $Ae^{-B|f|}$

B) Ae^{-Bf^2}

C) $A + B|f|^2$

D) Ae^{-Bf}

SOLUTION

Given,

$$f(t) = e^{-3t^2} \tag{0.0.1}$$

The fourier transform of a given signal f(t) is given by,

$$X(f) = \int_{-\infty}^{\infty} f(t)e^{-2\pi jft}dt \tag{0.0.2}$$

So let,

$$f(t) \overset{\mathcal{F}}{\rightleftharpoons} F(f) \tag{0.0.3}$$

The fourier transform of $\frac{df(t)}{dt}$ is given by,

$$= \int_{-\infty}^{\infty} \frac{df(t)}{dt}e^{-2\pi jft}dt \tag{0.0.4}$$

$$= \left[e^{-2\pi jft}f(t)\right]_{-\infty}^{\infty} + 2\pi jf\int_{-\infty}^{\infty} f(t)e^{-2\pi jft}dt \tag{0.0.5}$$

$$= 2\pi jfF(f) \tag{0.0.6}$$

Therefore,

$$\frac{df(t)}{dt} \overset{\mathcal{F}}{\rightleftharpoons} 2\pi jfF(f) \tag{0.0.7}$$

From (0.0.7) and (0.0.1)

$$te^{-at^2} \overset{\mathcal{F}}{\rightleftharpoons} \frac{-\pi fj}{3}F(f) \tag{0.0.8}$$

Lemma 0.1. $te^{-3t^2} \overset{\mathcal{F}}{\rightleftharpoons} \frac{j}{2\pi}\frac{dF(f)}{df}$

Proof. We prove it by finding the inverse fourier transform of $\frac{j}{2\pi}\frac{dF(f)}{df}$

The inverse fourier transform is given by

$$f^1(t) = \int_{-\infty}^{\infty} F^1(f)e^{j2\pi ft}df \tag{0.0.9}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} j\frac{dF(f)}{df}e^{j2\pi ft}df \tag{0.0.10}$$

$$= \frac{1}{2\pi}\left[je^{j2\pi ft}F(f)\right]_{-\infty}^{\infty} + \frac{1}{2\pi}\int_{-\infty}^{\infty} te^{j2\pi ft}F(f)df \tag{0.0.11}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} tF(f)e^{j2\pi ft}df \tag{0.0.12}$$

From (0.0.10),

$$f^1(t) = tf(t) \tag{0.0.13}$$

$$= te^{-3t^2} \tag{0.0.14}$$

□

From (0.0.7) and Lemma 0.1

$$\frac{j}{2\pi}\frac{dF(f)}{df} = \frac{-\pi fj}{3}F(f) \tag{0.0.15}$$

$$\implies \frac{dF(f)}{F(f)} = \frac{-2\pi^2}{3}fdf \tag{0.0.16}$$

On integrating,

$$log(F(f)) = \frac{-2\pi^2}{3}.\frac{f^2}{2} = -\frac{\pi^2 f^2}{3} \tag{0.0.17}$$

$$\implies F(f) = e^{-\frac{\pi^2 f^2}{3}} \tag{0.0.18}$$

∴ correct option is B