Assignment 1

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Download all python codes from

```
https://github.com/Dhatri−nanda/EE3900/blob/main
    /Assignment_1/code.py
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and latex-tikz codes from

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https://github.com/Dhatri−nanda/EE3900/blob/main
    /Assignment_1/Assignment_1.tex
```

1 Problem

Prove that the points $\begin{pmatrix}21\\-2\end{pmatrix}$, $\begin{pmatrix}15\\10\end{pmatrix}$, $\begin{pmatrix}-5\\0\end{pmatrix}$, $\begin{pmatrix}1\\-12\end{pmatrix}$ are the vertices of a rectangle, and find the coordinates of its centre.

2 Solution

Lemma 2.1. *In a quadrilateral ABCD having center O, when the diagonals bisect each other, by "side-angle-side" congruency of the inner triangles, opposite sides of the quadrilateral are equal.*

Proof. For $\triangle AOD$ and $\triangle COB$

$$AO = OC \tag{2.0.1}$$

$$DO = OB \tag{2.0.2}$$

$$\angle AOD = \angle COB \tag{2.0.3}$$

So, by SAS congruency, $\triangle AOD \cong \triangle COB$
Similar goes for $\triangle AOB$ and $\triangle COD$
So, AD = BC and AB = CD □

Lemma 2.2. *In a quadrilateral ABCD with center O and having equal opposite sides, by "side-angle-side" congruency, the opposite angle is same as the known angle.*

Proof. For $\triangle ABC and \triangle CDA$

$$AB = CD \tag{2.0.4}$$

$$AC = AC \tag{2.0.5}$$

$$BC = DA \tag{2.0.6}$$

By SAS congruency, $\triangle ABC \cong \triangle CDA$
So, $\angle ABC = \angle CDA$
Similar goes for $\triangle BCD$ and $\triangle DAB$ □

A rectangle is a quadrilateral in which all angles are right angles and with equal opposite sides(and unequal adjacent sides).
Let's name the points as A,B,C and D respectively i.e.,

$$\mathbf{A} = \begin{pmatrix}21\\-2\end{pmatrix} \mathbf{B} = \begin{pmatrix}15\\10\end{pmatrix} \mathbf{C} = \begin{pmatrix}-5\\0\end{pmatrix} \mathbf{D} = \begin{pmatrix}1\\-12\end{pmatrix} \tag{2.0.7}$$

So, first we see if the diagonals bisect each other (but not with right angles). Next, we check if the angle between sides is right angle.
Finding midpoints of both the diagonals, to see if they bisect each other.
Midpoint of diagonal AC

$$= \frac{\mathbf{A} + \mathbf{C}}{2} \tag{2.0.8}$$

From (2.0.7)

$$= \begin{pmatrix}8\\-1\end{pmatrix} \tag{2.0.9}$$

Midpoint of diagonal AD

$$= \frac{\mathbf{B} + \mathbf{D}}{2} \tag{2.0.10}$$

From (2.0.7)

$$= \begin{pmatrix}8\\-1\end{pmatrix} \tag{2.0.11}$$

As the midpoints are equal, they bisect each other.
Let the midpoint be **O**.
Now we check the angle between diagonals

$$(\mathbf{A} - \mathbf{O})^\top(\mathbf{C} - \mathbf{O}) = \begin{pmatrix}13 & -1\end{pmatrix}\begin{pmatrix}-13\\1\end{pmatrix} \tag{2.0.12}$$

$$= -170 \tag{2.0.13}$$

$$\neq 0 \tag{2.0.14}$$

Therefore, the diagonals do not intersect at right angles and so not all sides are equal.

The angle between sides

$$\angle B = (\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B}) \tag{2.0.15}$$

$$= \begin{pmatrix} -6 & 12 \end{pmatrix} \begin{pmatrix} -20 \\ -10 \end{pmatrix} \tag{2.0.16}$$

$$= 0 \tag{2.0.17}$$

Therefore, one of the angle is right angle.
From lemma 2.1 and lemma 2.2, we have AD = BC and AB = CD Also, remaining angles are right angles.
The above stated conditions are sufficient to get to the definition of rectangle mentioned above.
So, given vertices form a rectangle.
The center of the rectangle from above is $\mathbf{O}$ i.e.,

$$\mathbf{O} = \begin{pmatrix} 8 \\ -1 \end{pmatrix} \tag{2.0.18}$$

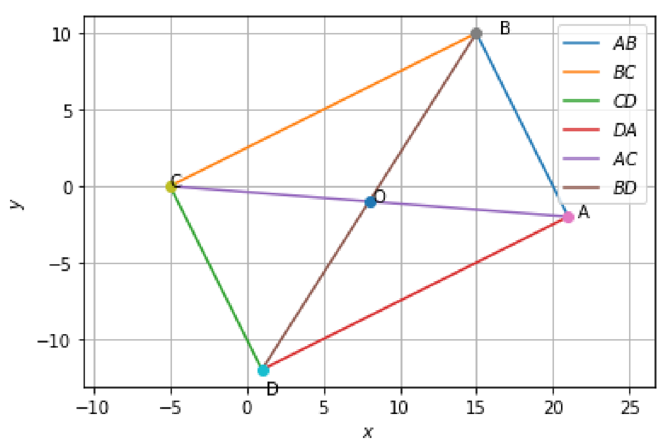

Fig. 0: plot