Assignment 2 Presentation

Akyam L Dhatri Nanda

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Question

Matrix Q.2.59

If A $= \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and I $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find k so that $A^2 = kA - 2I$

Theorem

The Cayley-Hamilton theorem states that any $N \times N$ matrix satisfies it's characteristic equation.

Characteristic equation of a matrix

For any square matrix M, the characteristic equation is $|M - \lambda I| = 0$

Solution

The characteristic equation of Matrix A is

$$|A - \lambda I| = 0 \tag{1}$$

$$\implies \left| \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0 \tag{2}$$

$$\implies \begin{vmatrix} 3-\lambda & -2-\lambda \\ 4 & -2-\lambda \end{vmatrix} = 0 \tag{3}$$

$$\implies (3-\lambda)(-2-\lambda) - 4(-2-\lambda) = 0 \tag{4}$$

$$\implies \lambda^2 - \lambda + 2 = 0 \tag{5}$$

From Cayley-Hamilton theorem,

$$A^2 - A + 2I = 0 \tag{6}$$

$$\implies k = 1 \tag{7}$$