GATE ASSIGNMENT 3

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Download all python codes from

```
https://github.com/Dhatri−nanda/EE3900/blob/main
    /Gate_3/code.py
```

Download latex-tikz codes from

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https://github.com/Dhatri−nanda/EE3900/blob/main
    /Gate_3/Gate_3.tex
```

QUESTION

The impulse response functions of four linear systems S_1, S_2, S_3, S_4 are given respectively by

$$h_1(t) = 1 \tag{0.0.1}$$
$$h_2(t) = U(t) \tag{0.0.2}$$
$$h_3(t) = \frac{U(t)}{t+1} \tag{0.0.3}$$
$$h_4(t) = e^{-3t}U(t) \tag{0.0.4}$$

where $U(t)$ is the unit step function, which of these systems is time invariant, casual and stable?

a) S_1 b) S_2 c) S_3 d) S_4

SOLUTION

Definitions:-

1) A continuous time signal x(t) is said to be **casual** if $x(t) = 0$ for every $t < 0$.

2) A time dependant system that is not a direct function of time is called **time-invariant** system.

3) A continuous time system is **stable** if and only if all the poles of it's transfer function occur in the left half of the complex plane. Whereas marginal stability correlates with zero real part.

4) A continuous time system $h(t)$ is said to be BIBO stable if and only if it is absolutely integrable

$$\int_{-\infty}^{\infty} h(t)dt < \infty \tag{0.0.5}$$

The transfer function of an impulse response function by using laplace transform is

$$H(s) = \mathcal{L}\{h(t)\}(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt \tag{0.0.6}$$

U(t) is given as the unit step function,

$$U(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases} \tag{0.0.7}$$

Given, $h_1(t) = 1$
It is a non casual system as $h_1(t) \neq 0$ for any $t < 0$
And, since it is not a time dependant function, it is not time-invariant.
Next, the transfer function of $h_1(t)$ is

$$H_1(s) = \int_{-\infty}^{\infty} e^{-st}dt \tag{0.0.8}$$
$$= \left[\frac{e^{-st}}{s}\right]_{-\infty}^{\infty} = \infty \tag{0.0.9}$$

The transfer function is not defined, so we cannot decide the stability.
$\therefore$ We check the BIBO stability.

$$= \int_{-\infty}^{\infty} U(t)dt \tag{0.0.10}$$
$$= \int_{0}^{\infty} 1dt \tag{0.0.11}$$
$$= [t]_0^{\infty} = \infty \tag{0.0.12}$$

$\therefore$ The system S_1 is not stable.

Given, $h_2(t) = U(t)$
From (0.0.7)

$$h_2(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases} \tag{0.0.13}$$

It satisfies the condition for casuality.So, system S_2 is casual.
It is also time-invariant by the above given definition.
The transfer function of $h_2(t)$ is

$$H_2(t) = \int_{-\infty}^{\infty} U(t)e^{-st}dt \tag{0.0.14}$$
$$= \int_{0}^{\infty} e^{-st}dt \tag{0.0.15}$$
$$= \frac{1}{s} \tag{0.0.16}$$

Pole is $s = 0$ and ROC is $\mathcal{Re}\{s\} > 0$
$\therefore$ The system S_2 is marginally stable.

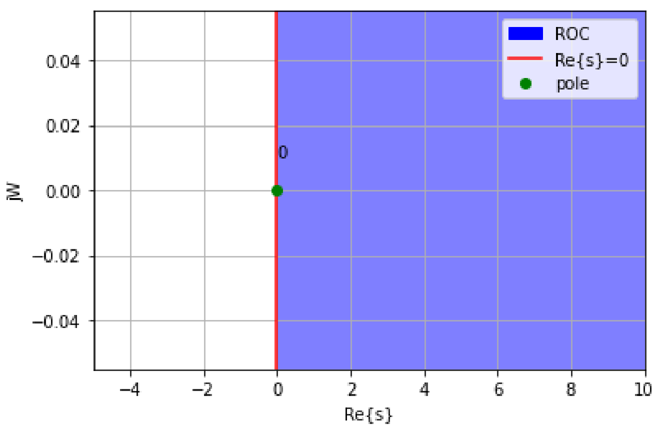

Fig. 1: plot for $H_2(t)$

Given, $h_3(t) = U(t)/(t+1)$
From, (0.0.7)

$$h_3(t) = \begin{cases} 0, t < 0, t \neq -1 \\ \text{not defined}, t = -1 \\ \frac{1}{t+1}, t > 0 \end{cases} \tag{0.0.17}$$

The system is not casual because

$$h_3(t) \neq 0, t = -1 \tag{0.0.18}$$

The denominator of $h_3(t)$ is a direct function of time, so it is not time-invariant.
The transfer function of $h_3(t)$ is

$$H_3(t) = \int_{-\infty}^{\infty} \frac{U(t)}{t+1} e^{-st}dt \tag{0.0.19}$$
$$= \int_{0}^{\infty} \frac{e^{-st}}{t+1}dt \tag{0.0.20}$$

Substituting x = st + s $\implies \frac{dx}{dt} = s \implies dt = \frac{1}{s}dx$

$$= \int_{s}^{\infty} \frac{e^x}{x}dx \tag{0.0.21}$$

The above integral is not a definite integral.
So,

$$H_3(t) = \infty \tag{0.0.22}$$

As the transfer function is not defined, we check the BIBO stability.

$$= \int_{-\infty}^{\infty} \frac{U(t)}{t+1}dt \tag{0.0.23}$$
$$= \int_{0}^{\infty} \frac{1}{t+1}dt \tag{0.0.24}$$
$$= [log(t+1)]_0^{\infty} = \infty \tag{0.0.25}$$

$\therefore$ The system S_3 is not stable.

Given, $h_4(t) = e^{-3t}U(t)$
From (0.0.7)

$$h_4(t) = \begin{cases} 0, t < 0 \\ e^{-3t}, t \geq 0 \end{cases} \tag{0.0.26}$$

It satisfies the condition for casuality. So the system is casual.
As e^{-3t} is a direct function of time, the system is not time-invariant.
The transfer function of $h_4(t)$ is

$$H_4(t) = \int_{-\infty}^{\infty} e^{-3t}U(t)e^{-st}dt \tag{0.0.27}$$
$$= \int_{0}^{\infty} e^{-(3+s)t}dt \tag{0.0.28}$$
$$= \frac{1}{3+s} \tag{0.0.29}$$

Pole is $s = -3$ and ROC is $\mathcal{Re}\{s\} > -3$
$\therefore$ S_4 is a stable system.

Our Results:

System	casual	stable	time-invariant
1	no	no	no
$U(t)$	yes	yes	yes
$U(t)/(t+1)$	no	no	no
$e^{-3t}U(t)$	yes	yes	no

$\therefore$ option B is correct.

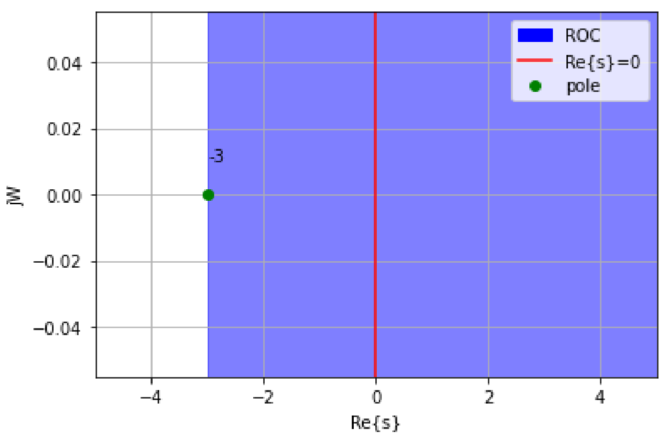

Fig. 2: plot for $H_4(t)$