GATE ASSIGNMENT 3

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Download all python codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Gate_3/code.py

Download latex-tikz codes from

https://github.com/Dhatri−nanda/EE3900/blob/main/Gate_3/Gate_3.tex

QUESTION

The impulse response functions of four linear systems S_1, S_2, S_3, S_4 are given respectively by

$$h_1(t) = 1 \tag{0.0.1}$$
$$h_2(t) = U(t) \tag{0.0.2}$$
$$h_3(t) = \frac{U(t)}{t+1} \tag{0.0.3}$$
$$h_4(t) = e^{-3t}U(t) \tag{0.0.4}$$

where $U(t)$ is the unit step function, which of these systems is time invariant, casual and stable?

a) S_1 b) S_2 c) S_3 d) S_4

SOLUTION

Definitions:-

1) A continuous time signal x(t) is said to be **casual** if $x(t) = 0$ for every $t < 0$.
2) A time dependant system that is not a direct function of time is called **time-invariant** system.
3) A continuous time system is **stable** if and only if all the poles of it's transfer function occur in the left half of the complex plane. Whereas marginal stability correlates with zero real part.

The transfer function of an impulse response function by using laplace transform is

$$H(s) = \mathcal{L}\{h(t)\}(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt \tag{0.0.5}$$

U(t) is given as the unit step function,

$$U(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases} \tag{0.0.6}$$

Given, $h_1(t) = 1$
It is a non casual system as $h_1(t) \neq 0$ for any $t < 0$
And, since it is not a time dependant function, it is not time-invariant.
Next, the transfer function of $h_1(t)$ is

$$H_1(s) = \int_{-\infty}^{\infty} e^{-st}dt \tag{0.0.7}$$
$$= \left[\frac{e^{-st}}{s}\right]_{-\infty}^{\infty} = \infty \tag{0.0.8}$$

The transfer function is not defined.
$\therefore$ It is not stable.

Given, $h_2(t) = U(t)$
From (0.0.6)

$$h_2(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases} \tag{0.0.9}$$

It satisfies the condition for casuality.So, system S_2 is casual.
It is also time-invariant by the above given definition.
The transfer function of $h_2(t)$ is

$$H_2(t) = \int_{-\infty}^{\infty} U(t)e^{-st}dt \tag{0.0.10}$$
$$= \int_{0}^{\infty} e^{-st}dt \tag{0.0.11}$$
$$= \frac{1}{s} \tag{0.0.12}$$

Pole is $s = 0$ and ROC is $\mathcal{Re}\{s\} > 0$
$\therefore$ It is marginally stable.

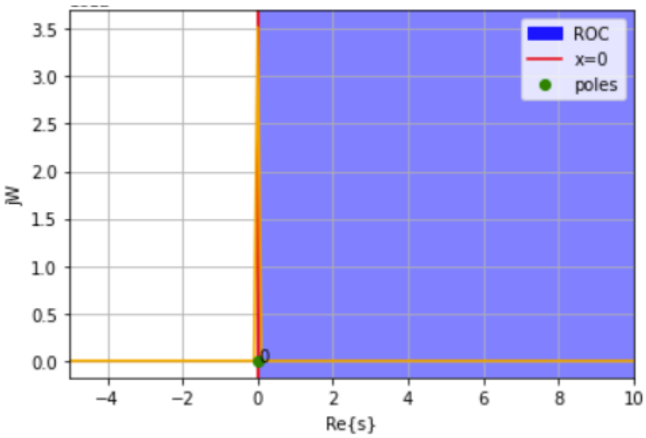

Fig. 1: plot for $H_2(t)$

Given, $h_3(t) = U(t)/(t+1)$
From, (0.0.6)

$$h_3(t) = \begin{cases} 0, t < 0, t \neq -1 \\ \text{not defined}, t = -1 \\ \frac{1}{t+1}, t > 0 \end{cases} \tag{0.0.13}$$

The system is not casual because

$$h_3(t) \neq 0, t = -1 \tag{0.0.14}$$

The denominator of $h_3(t)$ is a direct function of time, so it is not time-invariant.
The transfer function of $h_3(t)$ is

$$H_3(t) = \int_{-\infty}^{\infty} \frac{U(t)}{t+1} e^{-st} dt \tag{0.0.15}$$

$$= \int_{0}^{\infty} \frac{e^{-st}}{t+1} dt \tag{0.0.16}$$

Substituting x = st + s $\implies \frac{dx}{dt} = s \implies dt = \frac{1}{s} dx$

$$= \int_{s}^{\infty} \frac{e^{x}}{x} dx \tag{0.0.17}$$

The above integral is not a definite integral.
So,

$$H_3(t) = \infty \tag{0.0.18}$$

$\therefore$ The system S_3 is not stable.

Given, $h_4(t) = e^{-3t}U(t)$

From (0.0.6)

$$h_4(t) = \begin{cases} 0, t < 0 \\ e^{-3t}, t \geq 0 \end{cases} \tag{0.0.19}$$

It satisfies the condition for casuality. So the system is casual.
As e^{-3t} is a direct function of time, the system is not time-invariant.
The transfer function of $h_4(t)$ is

$$H_4(t) = \int_{-\infty}^{\infty} e^{-3t} U(t) e^{-st} dt \tag{0.0.20}$$

$$= \int_{0}^{\infty} e^{-(3+s)t} dt \tag{0.0.21}$$

$$= \frac{1}{3+s} \tag{0.0.22}$$

Pole is $s = -3$ and ROC is $\mathcal{Re}\{s\} > -3$
$\therefore$ It is a stable system.

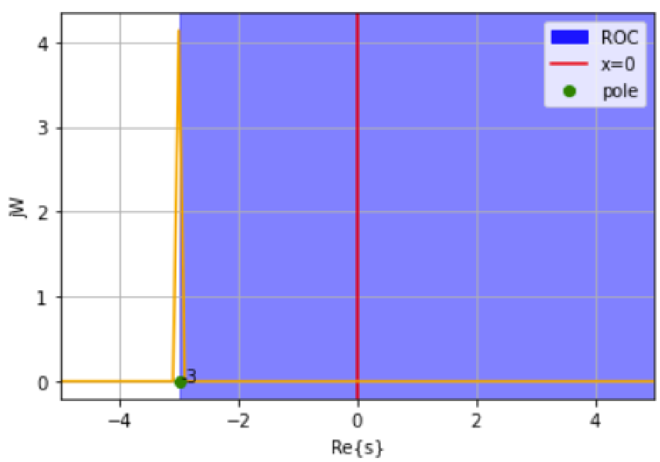

Fig. 2: plot for $H_4(t)$

Our Results:

System	casual	stable	time-invariant
1	no	yes	no
$U(t)$	yes	yes	yes
$U(t)/(t+1)$	no	no	no
$e^{-3t}U(t)$	yes	yes	no

$\therefore$ option B is correct.