Assignment 2

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Download all python codes from

```
https://github.com/Dhatri−nanda/Miin/blob/main/
    Assignment2/code.py
```

and latex-tikz codes from

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https://github.com/Dhatri−nanda/Miin/blob/main/
    Assignment2/Assignment2.tex
```

1 Problem

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$\Pr(X = x) = \begin{cases} 0.1 & x = 0 \\ kx, & 1 \le x \le 2 \\ k(5-x), & 3 \le x \le 4 \\ 0, & otherwise \end{cases} \tag{1.0.1}$$

A) Find the value of k.
B) What is the probability that you study at least two hours? Exactly two hours? At most two hours?

2 Solution

If we expand the probabilities given further more by substituting the value of x and only considering 0 to 4 hours as the probability of studying in the remaining hours is zero, we get

x	0	1	2	3	4
$\Pr(X = x)$	0.1	k	2k	2k	k

TABLE 2: Given probabilities

we also know that,

$$\sum_{k=0}^{4} \Pr(X = k) = 1 \tag{2.0.1}$$

By substituting the probabilities in (2.0.1)

$$\implies 0.1 + k + 2k + 2k + k = 1 \tag{2.0.2}$$
$$\implies 6k = 0.9 \tag{2.0.3}$$

Therefore, from (2.0.3)

$$k = 0.15 \tag{2.0.4}$$

So from 2

x	0	1	2	3	4
$\Pr(X = x)$	0.1	0.15	0.3	0.3	0.15

TABLE 2: Probabilities after finding k

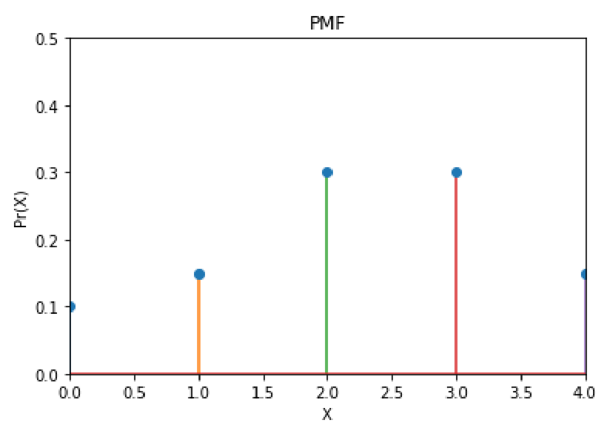

Fig. 2: Probability Mass Function (PMF)

We know that, Cumulative Distributive Function (CDF)

$$F(x) = \Pr(X \le x) \tag{2.0.5}$$

x	0	1	2	3	4
$F(X)$	0.1	0.25	0.55	0.85	1

TABLE 2: CDF

And also,

$$\Pr(x < X \le y) = F(y) - F(x) \tag{2.0.6}$$

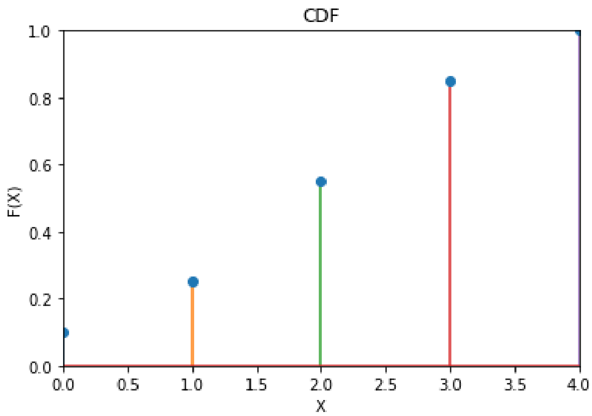

Fig. 2: Cumulative Distributive Function (CDF)

1) Probability of studying at least two hours

$$\implies \sum_{k=2}^{4} \Pr(X = k) = \Pr(X \geq 2) \tag{2.0.7}$$

$$\implies \Pr(1 < X \leq 4) \tag{2.0.8}$$

From (2.0.6) and (2)

$$= F(4) - F(1) \tag{2.0.9}$$

$$= 1 - 0.25 \tag{2.0.10}$$

$$= 0.75 \tag{2.0.11}$$

2) Probability of studying exactly two hours

$$= \Pr(X = 2) \tag{2.0.12}$$

$$= 0.3 \tag{2.0.13}$$

3) Probability of studying at most two hours

$$\implies \sum_{k=0}^{2} \Pr(X = k) = \Pr(X \leq 2) \tag{2.0.14}$$

From (2)

$$= F(2) \tag{2.0.15}$$

$$= 0.55 \tag{2.0.16}$$

$\Pr(X \geq 2)$	$\Pr(X = 2)$	$\Pr(X \leq 2)$
0.75	0.3	0.55
Case 1	Case 2	Case 3

TABLE 3: Final solution

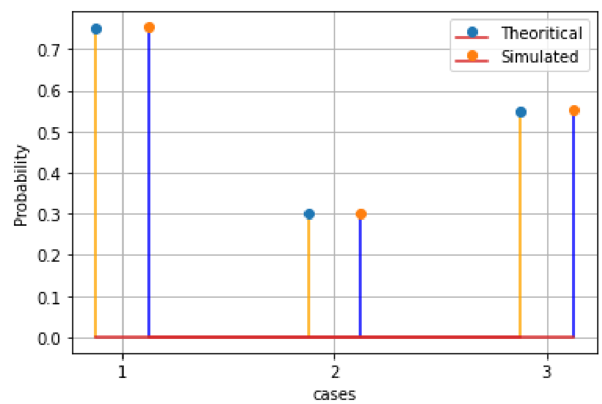

Fig. 3: Simulation and Theoretical Comparison