AI1110 - Assignment 2

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Download all python codes from

https://github.com/TYCN129/AI1110–Assignments/tree/main/Assignment%202/Codes

and latex codes from

https://github.com/TYCN129/AI1110–Assignments/tree/main/Assignment%202

ICSE class 12 - 2019 paper

1 QUESTION - 18

Draw a sketch and find the area bounded by the curve $x^2 = y$ and $x + y = 2$

2 SOLUTION

We have the curves

$$x + y = 2 \tag{1}$$
$$x^2 = y \tag{2}$$

We can observe that the point $(1, 1)$ passes through the line. Let this point be called P. The line could be represented as,

$$\vec{X} = P + \lambda\vec{m} \tag{3}$$
$$\vec{X} = \begin{pmatrix}1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\m\end{pmatrix} \tag{4}$$

where m is slope of the line and is equal to -1

$$\vec{X} = \begin{pmatrix}1+\lambda\\1+\lambda m\end{pmatrix} \tag{5}$$
$$\vec{X} = \begin{pmatrix}1+\lambda\\1-\lambda\end{pmatrix} \tag{6}$$

Also the equation for parabola in vector form is,

$$\vec{X}^\top A\vec{X} + B^\top\vec{X} + C = 0 \tag{7}$$

For the parabola $x^2 - y = 0$, $A = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$, $B = \begin{pmatrix}0\\-1\end{pmatrix}$ and C is null matrix Thus,

$$\vec{X}^\top\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\vec{X} + \begin{pmatrix}0 & -1\end{pmatrix}\vec{X} = 0 \tag{8}$$

Substituting equation (6) in equation (8), we get

$$\begin{pmatrix}1+\lambda & 1-\lambda\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}1+\lambda\\1-\lambda\end{pmatrix} + \tag{9}$$
$$\begin{pmatrix}0 & -1\end{pmatrix}\begin{pmatrix}1+\lambda\\1-\lambda\end{pmatrix} = 0 \tag{10}$$
$$(1+\lambda)^2 - (1-\lambda) = 0 \tag{11}$$
$$\lambda^2 + 3\lambda = 0 \tag{12}$$

Equation (12) is quadratic and of the form $ax^2 + bx + c = 0$, where $a = 1, b = 3$ and $c = 0$
Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{13}$$
$$\lambda = \frac{-3 \pm \sqrt{3^2 - 4(1)(0))}}{2(1)} \tag{14}$$
$$\lambda = \frac{-3+3}{2}, \frac{-3-3}{2} \tag{15}$$
$$\lambda = 0, -3 \tag{16}$$

On substituting the values of λ in equation (6), we get the points of intersection,

$$\vec{X_1}^\top = \begin{pmatrix}1\\1\end{pmatrix} \tag{17}$$
$$\vec{X_2}^\top = \begin{pmatrix}-2\\4\end{pmatrix} \tag{18}$$

Thus, the two curves intersect at the points (-2 , 4) and (1 , 1)

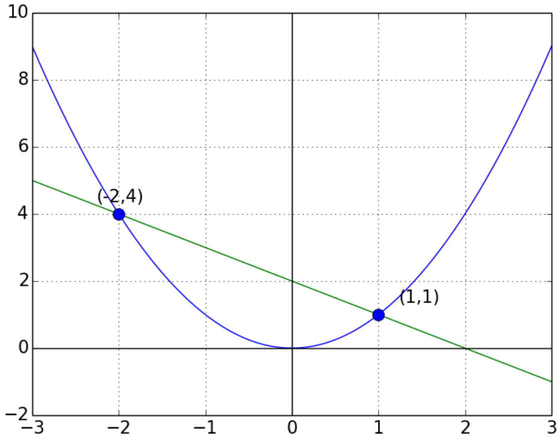


Fig 1: Graph Plot

2.1 *Area of trapezium ABCD* (A_1)

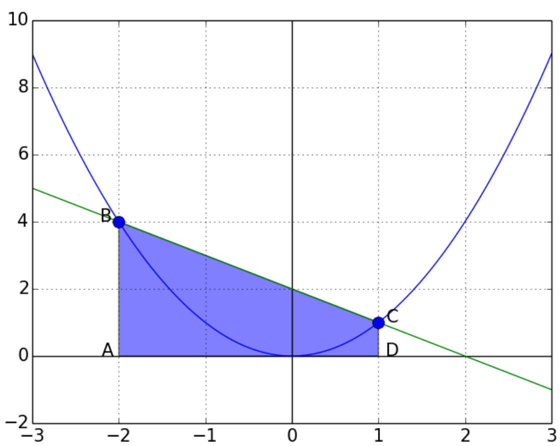


Fig 3: Trapezium ABCD

$$A_1 = \frac{1}{2} \times (AB + CD) \times AD \tag{19}$$

$$A_1 = \frac{1}{2} \times (4 + 1) \times 3 \tag{20}$$

$$\boxed{\mathrm{A}_1 = 7.5 sq.units}$$

2.2 *Area under the curve* $x^2 = y$ (A_2)

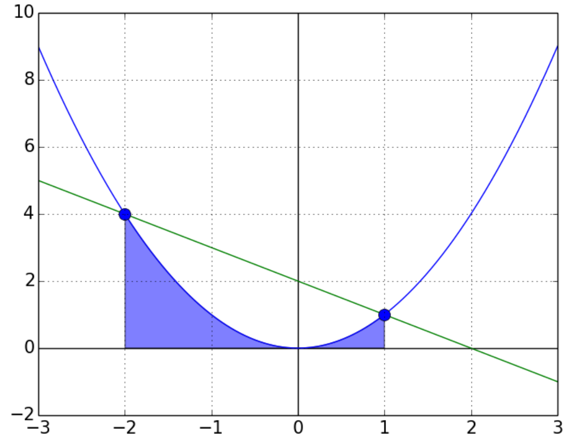


Fig 4: Region below curve $x^2 = y$

$$A_2 = \int_{-2}^{1} x^2 \, dx \tag{21}$$

$$A_2 = \left. \frac{x^3}{3} \right|_{-2}^{1} \tag{22}$$

$$A_2 = \frac{(1)^3}{3} - \frac{(-2)^3}{3} \tag{23}$$

$$\boxed{\mathrm{A}_2 = 3 sq.units}$$

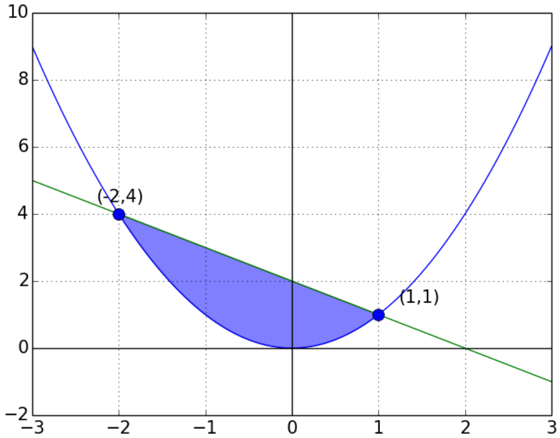
2.3 *Area of the shaded region*

$$A = A_1 - A_2 \tag{24}$$

$$A = 7.5 - 3 \tag{25}$$

$$\tag{26}$$

$$\boxed{\textbf{A = 4.5 sq.units}}$$



The area bound by the curves $x^2 = y$ and $x + y = 2$ is 4.5 sq.units