AI1110 - Assignment 2

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ICSE class 12 - 2019 paper

1 QUESTION - 18

Draw a sketch and find the area bounded by the curve $x^2 = y$ and $x + y = 2$

2 SOLUTION

Let us find out the points of intersection of the two curves. We have the curves

$$x^2 = y \tag{1}$$
$$x + y = 2 \tag{2}$$

Substituting $y = x^2$ (from equation 1) in equation 2.

$$x + x^2 = 2 \tag{3}$$
$$x^2 + x - 2 = 0 \tag{4}$$
$$(x + 2)(x - 1) = 0 \tag{5}$$
$$\boxed{x = \text{-2, 1}} \tag{6}$$

Substituting value of x in equation 2, we get.

$$\boxed{y = \text{4, 1}} \tag{7}$$

Let (x_1, y_1) and (x_2, y_2) be the two points of intersection.

$$(x_1, y_1) = (-2, 4) \tag{8}$$
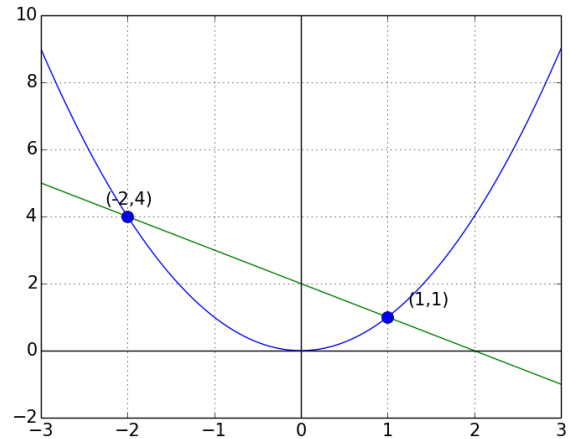$$(x_2, y_2) = (1, 1) \tag{9}$$

Figure 1: Graph Plot

We need to find out the area of the shaded region shown below
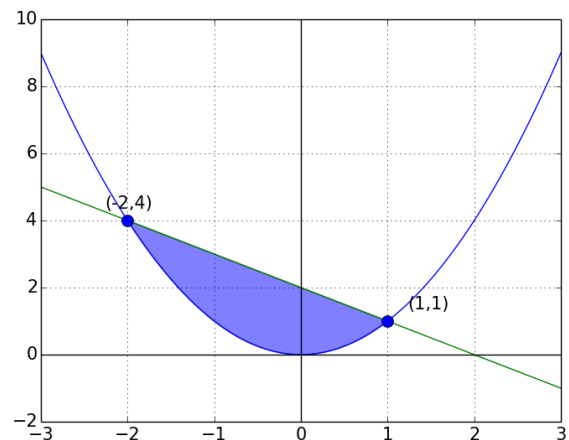

Figure 2: Region bound by the two curves

From the figure,

Area between the curves(A)= Area of trapezium ABCD(A_1) - Area under the curve $x^2 = y(A_2)$
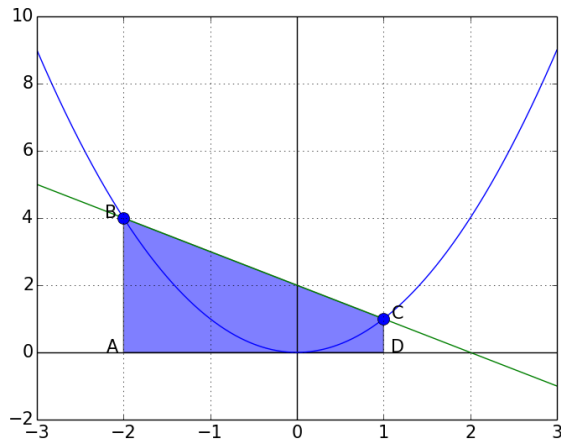
2.1 Area of trapezium ABCD (A_1)

Figure 3: Trapezium ABCD

$$A_1 = \frac{1}{2} \times (AB + CD) \times AD \tag{10}$$

$$A_1 = \frac{1}{2} \times (4 + 1) \times 3 \tag{11}$$

$$\boxed{A_1 = 7.5 sq.units}$$
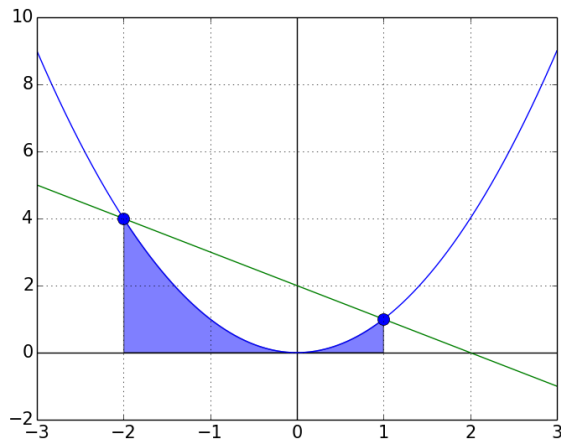
2.2 Area under the curve $x^2 = y$ (A_2)

Figure 4: Region below curve $x^2 = y$

$$A_2 = \int_{-2}^{1} x^2 \, dx \tag{12}$$

$$A_2 = \left. \frac{x^3}{3} \right|_{-2}^{1} \tag{13}$$

$$A_2 = \frac{(1)^3}{3} - \frac{(-2)^3}{3} \tag{14}$$

$$\boxed{A_2 = 3 sq.units}$$

Therefore,

$$A = A_1 - A_2 \tag{15}$$
$$A = 7.5 - 3 \tag{16}$$
$$\tag{17}$$

$$\boxed{\textbf{A = 4.5 sq.units}}$$
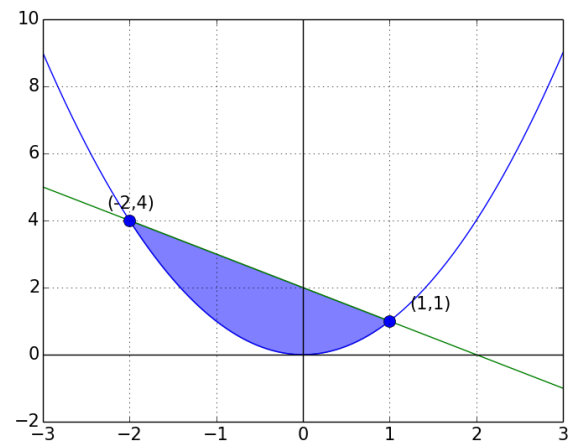
The area bound by the curves $x^2 = y$ and $x + y = 2$ is 4.5 sq.units