

Assignment 2

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Abstract—This document contains the solution for Assignment 2 (ICSE Class 12 Maths 2019 Q.18)

Q18 [ICSE 12 2019]: Draw a sketch and find the area bounded by the curve $x^2 = y$ and $x + y = 2$

Solution: The given parabola $x^2 - y = 0$ can be written in vector form as

$$\mathbf{x}^T \mathbf{a} \mathbf{x} + \mathbf{b}^T \mathbf{x} + \mathbf{c} = 0 \quad (1)$$

with the parameters,

$$\mathbf{a} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{c} = 0 \quad (2)$$

The line $x + y = 2$ can be written as

$$\mathbf{x} = \mathbf{p} + \lambda \mathbf{m} \quad (3)$$

where \mathbf{p} is a point and \mathbf{m} is the direction vector of the line

Choosing \mathbf{p} as $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, we get:

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (4)$$

The intersection of this line with the parabola is given by

$$\mathbf{x}_i = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_i \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (5)$$

where λ_i is given by,

$$\lambda_i = \frac{1}{\mathbf{m}^T \mathbf{a} \mathbf{m}} \left(-\mathbf{m}^T (\mathbf{a} \mathbf{p} + \mathbf{b}) \pm \sqrt{[\mathbf{m}^T (\mathbf{a} \mathbf{p} + \mathbf{b})]^2 - (\mathbf{p}^T \mathbf{a} \mathbf{p} + 2\mathbf{u}^T \mathbf{p} + \mathbf{c}) (\mathbf{m}^T \mathbf{a} \mathbf{m})} \right) \quad (6)$$

Substituting the values in above equation, we get

$$\lambda_i = 0, -3 \quad (7)$$

Using these values of λ , the intersection points are,

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (8)$$

From the figure below, the required area is equal to,

Area of trapezium ABCD - Area under the parabola (9)

$$= \frac{1}{2} \times (4 + 1) \times 3 - \int_{-2}^1 x^2 dx = 7.5 - \left[\frac{x^3}{3} \right]_{-2}^1 = 4.5 \text{ sq. units} \quad (10)$$

The blue shaded region in the figure below represents this area.

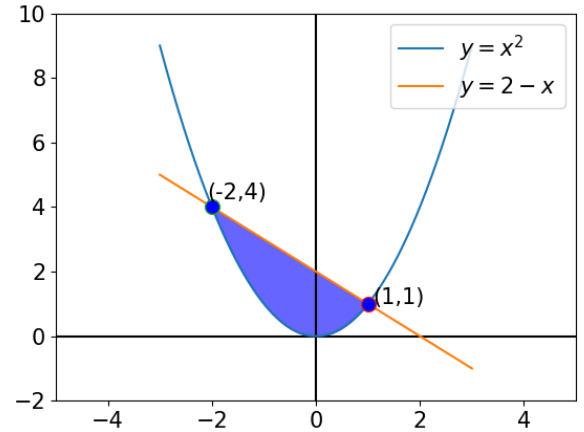


Fig. 1. Plot of line and parabola