

Assignment 1

Velma Dhatri Reddy
AI21BTECH11030

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Question 8(a) The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference.

Solution:

The parameters involved in the questions are listed in Table (I):

Parameter	Symbol/Formula	Value
Sum of the first three terms	S	42
Product of first and third term	P	52
Common difference	$d = \pm \sqrt{\left(\frac{S}{3}\right)^2 - P}$?
First term	$a_1 = \frac{S}{3} - \pm \sqrt{\left(\frac{S}{3}\right)^2 - P}$?

TABLE I

Let the first three terms of the arithmetic expression be a_1, a_2, a_3 . So, $a_1 + a_3 = 2 \times a_2$.

Given, sum of the first three terms(S) is 42.

$$\begin{aligned} \Rightarrow a_1 + a_2 + a_3 &= S & (1) \\ \Rightarrow 2 \times a_2 + a_2 &= S & (2) \\ \Rightarrow 3 \times a_2 &= S & (3) \\ \Rightarrow a_2 &= \frac{S}{3} & (4) \end{aligned}$$

Let the common difference of the Arithmetic Progression be d . So, $a_2 - a_1 = d$ and $a_3 - a_2 = d$. Hence, we can write $a_1 = a_2 - d$ and $a_3 = a_2 + d$.

Given, product of the first and third term(P) is 52.

$$\Rightarrow a_1 \times a_3 = P \quad (5)$$

$$\Rightarrow (a_2 - d) \times (a_2 + d) = P \quad (6)$$

$$\Rightarrow a_2^2 - d^2 = P \quad (7)$$

$$\Rightarrow d^2 = a_2^2 - P \quad (8)$$

$$= \left(\frac{S}{3}\right)^2 - P \quad (9)$$

$$\Rightarrow d = \pm \sqrt{\left(\frac{S}{3}\right)^2 - P} \quad (10)$$

$$(11)$$

On substituting the values, we get:

$$d = \pm \sqrt{\left(\frac{42}{3}\right)^2 - 52} \quad (12)$$

$$= \pm \sqrt{14^2 - 52} \quad (13)$$

$$= \pm \sqrt{144} \quad (14)$$

$$= \pm 12 \quad (15)$$

Case 1:

$$\begin{aligned} \text{(i) Common difference } (d) &= 12 \\ \text{(ii) First term } (a_1) &= a_2 - d = \frac{S}{3} - d = \frac{42}{3} - 12 = 14 - 12 = 2 \end{aligned}$$

Case 2:

$$\begin{aligned} \text{(i) Common difference } (d) &= -12 \\ \text{(ii) First term } (a_1) &= a_2 - d = \frac{S}{3} - d = \frac{42}{3} + 12 = 14 + 12 = 26 \end{aligned}$$