Assignment 1

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Question: 8 (a)
The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference.

Solution:
Let the first three terms of the arithmetic expression be a_1, a_2, a_3.

So, $a_1 + a_3 = 2 \times a_2$

Given,
Sum of the first three terms is 42.

$$\begin{aligned}
&\implies a_1 + a_2 + a_3 = 42 \\
&\implies 2 \times a_2 + a_2 = 42 \\
&\implies 3 \times a_2 = 42 \\
&\implies a_2 = 14
\end{aligned}$$

Let the common difference of the Arithmetic Progression be d.

So, $a_2 - a_1 = d$ and $a_3 - a_2 = d$
Hence, we can write $a_1 = a_2 - d$ and $a_3 = a_2 + d$.

Product of the first and third term is 52.

$$\begin{aligned}
&\implies a_1 \times a_3 = 52 \\
&\implies (a_2 - d) \times (a_2 + d) = 52 \\
&\implies a_2^2 - d^2 = 52 \\
&\implies d^2 = a_2^2 - 52 \\
&\qquad = 14^2 - 52 \\
&\qquad = 144 \\
&\implies d = \pm\sqrt{144} \\
&\qquad = \pm 12
\end{aligned}$$

Case 1:
Common difference $(d) = 12$
First term $(a_1) = a_2 - d = 14 - 12 = 2$

Case 2:
Common difference $(d) = -12$
First term $(a_1) = a_2 - d = 14 + 12 = 26$