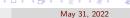
# Assignment 10: Papoulis Chapter 6

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## Outline

Question

Part a

Part b

## Question

#### Problem 6.19

The random variables x and y are independent with Rayleigh densities

$$f_{x}(x) = \frac{x}{\alpha^{2}} e^{-\frac{x^{2}}{2\alpha^{2}}} U(x) \text{ and } f_{y}(y) = \frac{y}{\beta^{2}} e^{-\frac{y^{2}}{2\beta^{2}}} U(y)$$

(a) Show that if  $z = \frac{x}{y}$ , then

$$f_{z}(z) = \frac{2\alpha^{2}}{\beta^{2}} \frac{z}{\left(z^{2} + \frac{\alpha^{2}}{\beta^{2}}\right)^{2}} U(z)$$
 (1)

(b) Using (1), show that for any k>0,  $P\left\{x\leq ky\right\}=\frac{k^2}{k^2+\frac{\alpha^2}{\beta^2}}$ 



### Solution

(a)

$$f_{z}(z) = \int_{-\infty}^{\infty} |y| f_{x}(yz) f_{y}(y) dy$$
 (2)

$$= \int_0^\infty y \times \frac{yz}{\alpha^2} e^{-\frac{yz^2}{2\alpha^2}} \times \frac{y}{\beta^2} e^{-\frac{y^2}{2\beta^2}} dy \tag{3}$$

$$=\frac{z}{\alpha^2\beta^2}\int_0^\infty y^3 e^{-cy^2} dy \tag{4}$$

$$= \frac{z}{2\alpha^2\beta^2 \left(\frac{z^2}{2\alpha^2} + \frac{1}{2\beta^2}\right)^2} \tag{5}$$

$$=\frac{2\alpha^2}{\beta^2}\frac{z}{\left(z^2+\frac{\alpha^2}{\beta^2}\right)^2}\tag{6}$$



### Solution

(b)

$$F_{z}(z) = \int_{0}^{z} \frac{2\alpha^{2}}{\beta^{2}} \frac{z}{\left(z^{2} + \frac{\alpha^{2}}{\beta^{2}}\right)^{2}} dz$$
 (7)

$$=\frac{\alpha^2}{\beta^2} \int_{\frac{\alpha^2}{\beta^2}}^{\frac{\alpha^2}{\beta^2} + z^2} \frac{dt}{t^2} \tag{8}$$

$$=\frac{z^2}{z^2 + \frac{\alpha^2}{\beta^2}}\tag{9}$$

$$= P\left\{x \le zy\right\} \tag{10}$$

Hence, proved.

