Assignment 12: Papoulis Chapter 9

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Outline

Question

Problem 9.53

(a) $E\{y^2(t)\}$ if $y(0) = y'(0) = 0$ and $y''(t) + 7y'(t) + 10y(t) = x(t)$
$R_x(\tau) = 5\delta(\tau)$

(b) $E\{y^2(n)\}$ if $y(-1) = y(-2) = 0$ and
$8y(n) - 6y(n-1) + y(n-2) = x(n)$ $R_x(m) = 5\delta(m)$

Solving a

If $y(0) = y'(0) = 0$ then y(t) is the output of a system with input
x(t)U(t) and impulse response h(t) such that
$h''(t) + 7h'(t) + 10h(t) = \delta(t)$
$h(0^-) = h(0^+) = 0$
$h(t) = \frac{1}{3}\left(e^{-2t} - e^{-5t}\right)U(t)$ and with $q(t) = 5U(t)$

$$E\left\{y^2(t)\right\} = \int_0^t \left(e^{-2t} - e^{-5t}\right)^2 d\tau \tag{1}$$

$$= \frac{e^{-4t}}{-4} + \frac{1}{4} + \frac{e^{-10t}}{-10} + \frac{1}{10} + 2\frac{e^{-7t}}{7} + \frac{2}{7} \tag{2}$$

$$= \frac{89}{140} - \frac{e^{-4t}}{4} - \frac{e^{-10t}}{10} + 2\frac{e^{-7t}}{7} \tag{3}$$

Solving b

If $y(-1) = y'(-2) = 0$ then y(n) is the output of a system with input
x(n)U(n) and delta response h(n) such that
$8h(n) - 6h(n-1) + h(n-2) = \delta(n)$
$h(-1) = h(-2) = 0$
$h(n) = \left(\dfrac{1}{2^{n+2}} - \dfrac{1}{2^{2n+3}} \right) U(n)$ and with $q(n) = 5U(n)$

$$E\left\{ y^2(n) \right\} = 5\Sigma_{k=0}^{n} \left(\frac{1}{2^{k+2}} - \frac{1}{2^{2k+3}} \right)^2 \tag{4}$$