Assignment 13: Papoulis Chapter 11

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Outline

Question

Problem 11.4

The process x(t) is WSS and $y''(t) + 3y'(t) + 2y(t) = x(t)$ Show that

(a) $R''_{yx}(\tau) + 3R'_{yx}(\tau) + 2R_{yx}(\tau) = R_{xx}(\tau)$
$R''_{yy}(\tau) + 3R'_{yy}(\tau) + 2R_{yy}(\tau) = R_{xy}(\tau)$ all τ

(b) If $R_{yx}(\tau) = q\delta(\tau)$, then $R_{yx}(\tau) = 0$ for $\tau < 0$ and for $\tau > 0$:
$R''_{yx}(\tau) + 3R'_{yx}(\tau) + 2R_{yx}(\tau) = 0$ $R_{yx}(0) = 0$ $R'_{yx}(0^+) = q$
$R''_{yy}(\tau) + 3R'_{yy}(\tau) + 2R_{yy}(\tau) = 0$ $R_{yy}(0) = \frac{q}{12}$ $R'_{yy}(0) = 0$

Solving a

By multiplying both sides of the equation given in question by x$(t-\tau)$ and y$(t+\tau)$ we can conclude that
$R''_{yx}(\tau)+3R'_{yx}(\tau)+2R_{yx}(\tau)=R_{xx}(\tau)$
$R''_{yy}(\tau)+3R'_{yy}(\tau)+2R_{yy}(\tau)=R_{xy}(\tau)$

Solving b

From (a) it follows that $R''_{yx}(\tau) + 3R'_{yx}(\tau) + 2R_{yx}(\tau) = q\delta(\tau)$
Since $R_{xx}(\tau) = 0$ for $\tau < 0$, the above shows that $R_{yx}(\tau) = 0$ for $\tau < 0^-$
$R'_{yx}(0^-) = 0$

$$S_{yx}(s) = \frac{q}{s^2 + 3s + 2} \tag{1}$$

$$R_{yx}\left(0^+\right) = \lim_{s\to\infty} sS_{yx}(s) = q \tag{2}$$

$$R'_{yx}\left(0^+\right) = \lim_{s\to\infty} s^2 S_{yx}(s) = 0 \tag{3}$$

$R''_{yy}(\tau) + 3R'_{yy}(\tau) + 2R_{yy}(\tau) = R_{xy}(\tau) = R_{xy}(-\tau) = 0$ for $\tau > 0$

Solving b

$$S_{yy}(s) = \frac{q}{(s^2+3s+2)(s^2-3s+2)} \tag{4}$$

$$= \frac{\frac{qs}{12}+\frac{q}{4}}{s^2+3s+2} + \frac{-\frac{qs}{12}+\frac{q}{4}}{s^2-3s+2} \tag{5}$$

$$S_{yy}^{+}(s) = \frac{\frac{qs}{12}+\frac{q}{4}}{s^2+3s+2} \tag{6}$$

$$R_{yy}^{+}\left(0^{+}\right) = R_{yy}(0) = \lim_{s\to\infty} s^2 S_{yy}^{+}(s) = \frac{q}{12} \tag{7}$$

$$R_{yy}'(0) = \lim_{s\to\infty} s\left(sS_{yy}^{+}(s) - \frac{q}{12}\right) = 0 \tag{8}$$